Exam Solution

Course: WM0324LR Ethics and Engineering for Aerospace Engineering
Exam Source:Goldmine
Exam Date: 2015-07-02

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21-01-2020

1 Solar System Dynamics, Meteorites & Cratering

a)

Orbital period and semi-major axis are related by:

i)

$$T = 2\pi\sqrt{\frac{a^3}{GM}} \tag{1}$$

As GM is a property of the sun (heaviest body) and all planets in question orbit the sun, we do not have to compute it. We then get:

$$\begin{aligned}
T_{Mars} &= \sqrt{a_{Mars}^3}\sqrt{(4\pi^2 GM)^{-1}} = \sqrt{1.5^3}\sqrt{(4\pi^2 GM)^{-1}} = 1.837\sqrt{(4\pi^2 GM)^{-1}} \text{ Earth years} \\
T_{Jupiter} &= \sqrt{a_{Jupiter}^3}\sqrt{(4\pi^2 GM)^{-1}} = \sqrt{5^3}\sqrt{(4\pi^2 GM)^{-1}} = 11.18\sqrt{(4\pi^2 GM)^{-1}} \text{ Earth years} \\
T_{Saturn} &= \sqrt{a_{Saturn}^3}\sqrt{(4\pi^2 GM)^{-1}} = \sqrt{9.54^3}\sqrt{(4\pi^2 GM)^{-1}} = 29.466\sqrt{(4\pi^2 GM)^{-1}} \text{ Earth years}
\end{aligned} \tag{2}$$

This leads to:

$$\begin{aligned}
T_{Mars} &= \frac{1.837}{11.18} \cdot T_{Jupiter} = 0.16 \text{ Jovian years} \\
T_{Saturn} &= \frac{29.466}{11.18} \cdot T_{Jupiter} = 2.64 \text{ Jovian years}
\end{aligned} \tag{3}$$

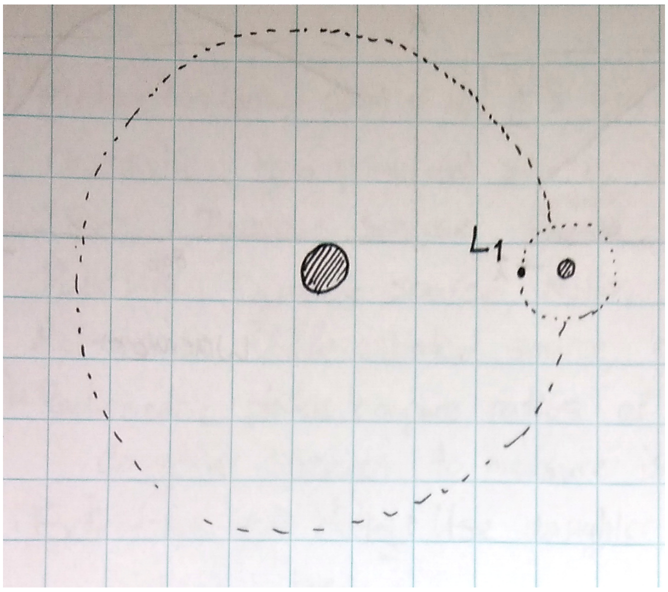
ii)

Lagrange point L1 is the point in between two bodies where the gravity potentials of the two bodies are equal. Formally, to find this point from the Hill equations, one needs to compute where:

$$\frac{\partial U}{\partial x} = \frac{\partial U}{\partial y} = \frac{\partial U}{\partial z} = 0 \tag{4}$$

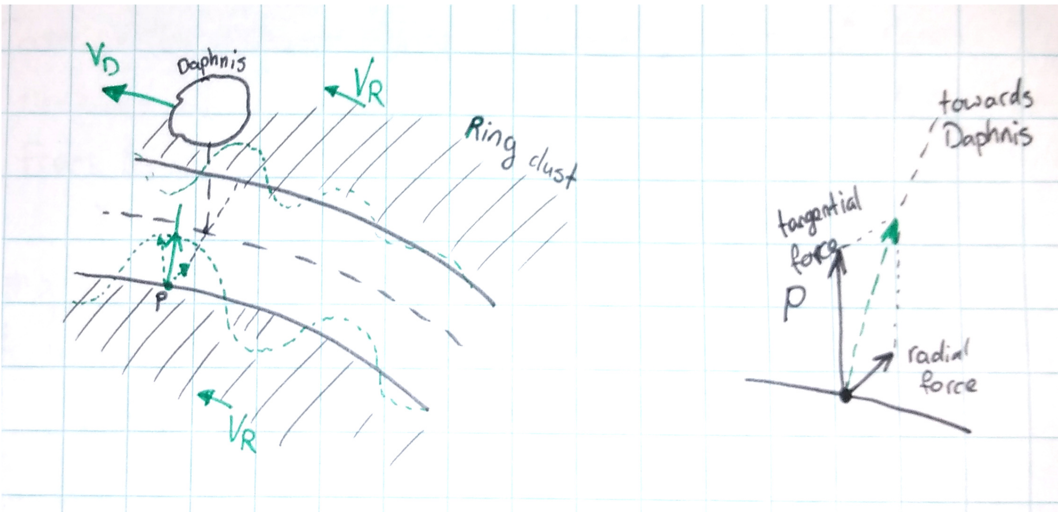
However, since we're looking for the point in between the bodies where the gravity potentials are equal, we know that this point lies on the edge of the Hill sphere, as indicated in the figure. The radius of this sphere is:

$$R_h = a \cdot \sqrt{\frac{m_q}{3(m_p + m_q)}} \tag{5}$$



iii)

The waves are caused by the gravitational attraction of Daphnis, which is moving faster than the material of the rings. There is a perpendicular component to this wave, as Daphnis' motion is inclined with respect to the ring-plane. This can also be seen in the figure, where a certain particle P experiences the gravitational attraction from Daphnis.



One can determine the density of Daphnis if one knows the volume and the mass. If one assumes a certain spheroid shape for Daphnis, and estimates the radius from imagery, one can find an approximation for the volume. From the wave properties in the image, one can find $\mu_{Daphnis}$, from which the mass of Daphnis can be obtained. The density is then the mass divided by the volume.

b)

i)

Laplace equation:

$$\nabla^2 U = 0$$
$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0 \tag{6}$$

For bodies of non-uniform density, there are disturbances and the potential starts embodying oscillatory behaviour, which are typically modelled using Legendre polynomials.

ii)

The Legendre polynomials must be known to model this, to gain insight in the nature of the disturbance.

c)

i)

The gravitational attraction of the Moon creates tidal bulges in the surface body of water on Earth. These bulges lag behind substantially compared to the motion of the Moon. This causes a net positive torque on the Moon, slightly enhancing its orbit over time.

ii)

The effects of tidal energy dissipation are most pronounced when the orbiting moon is near its planetary host. For an eccentric orbit, this periodically occurs in the perigee, and over time the orbit will become more circular as a result.

iii)

[PS & SOD Reader p.223]

1. Raising of lunar orbit dissipates $0.121 \cdot 10^{12}$ W.
2. The change of Earth's axial rotation dissipates $2.441 \cdot 10^{12}$ W.
3. The change of the Moon's axial rotation dissipates $2.977 \cdot 10^{6}$ W.

Editor's note: I believe that the exact values are not expected here, and that indicating the relative differences (i.e. #2 is 20 times as large as #1, and #3 is about a millionth of #2) is sufficient.

d)

i)

[Meteorites lecture slides - slide 16], [Lissauer - p. 286]

1. Chondritics: These are of primordial origin, typically condensed from nebulae.
2. Irons: Igneous source, origin is molten cores of asteroids.
3. Pallasites: Igneous source, origin is molten mantles or cores of asteroids.
4. Achondritics: Typical origin is differentiated bodies, near the surface (i.e. crust) of asteroids or planets.

ii)

1. Radiometric dating: Compare ratios of parent species and daughter species with a reference to estimate the age of an asteroid sample.
2. Extinct-nuclide dating: Use daughter species products to estimate age instead. Can be used instead of radiometric dating if daughter species has fully decayed, or to obtain greater accuracy.
3. Cosmic ray exposure dating: Determine presence of rare nuclides typical to cosmic rays in meteorite, such as ^{21}Ne.

iii)

When a meteorite is entering the atmosphere of a planet, it is pulled down by its weight force, and pulled up by the atmospheric drag force. From Newton's 2nd law we know that:

$$F = ma$$
$$mg - \frac{1}{2}C_D \rho A V^2 = ma$$

However, at terminal velocity, we know that the velocity is constant, and therefore the acceleration equals zero:

$$\frac{1}{2}C_D \rho A V_t^2 = mg$$
$$V_t^2 = \frac{2mg}{C_D \rho A}$$
$$V_t = \sqrt{\frac{2mg}{C_D \rho A}}$$

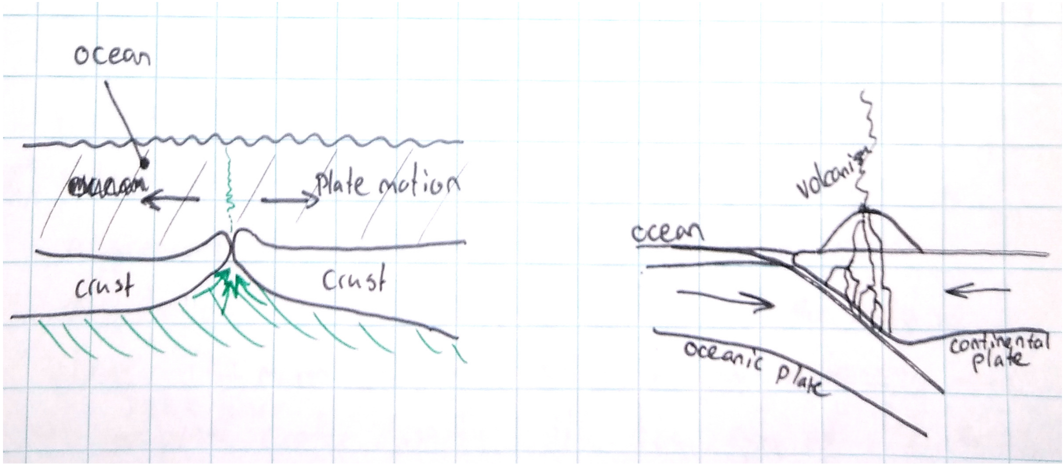
iv)

A dense crater distribution implies that the body is old, given that the body features no active volcanism, tectonic activity or aggressive atmospheric erosion. Since the moon has none of these, it makes for an interesting case study, since more than half of its surface can be relatively easily mapped. The crater distribution can then be compared to the ages of the lunar rock samples brought back from the Apollo missions.

2 Filler lecture:Volcanism and Planetary Interiors

a)

A mid-ocean ridge is the border between two oceanic plates that are moving away from each other. From the local thinning of the plates, and eventually the gap between the plates, matter from below the crust can seep upwards, until it solidifies on the ocean floor, forming long, elevated ridges, as seen in the left figure. In the right figure, there is an active continental margin. It is a continental margin, the boundary between a continental plate and an oceanic plate, where active volcanism occurs. Matter from the ocean floor is transported down along with the oceanic plate, and as it travels down, heats up and melts. This causes the molten material to rise upward and manifest in a volcanically active mountain ridge, also known as subduction volcanism.



b)

The three types of volcanism that occur on Earth are:

1. Mid-ocean ridge volcanism - Between two ocean plates that move apart.
2. Subduction zone volcanism - Between an oceanic plate diving underneath a continental plate
3. Hotspot volcanism - Most notable examples on Earth occur in ocean, caused by a local hotspot (plume) in the mantle rising upward and pushing hot material through the crust. The plates may move slowly over-top the hotspots, creating ridges on the ocean floor, or even islands (e.g. Hawaii).

c)

Cryovolcanism is a type of volcanism where instead of molten rock and metal, fluids such as water or methane, are expelled. This type of volcanism occurs under significantly lower temperatures too, hence the name. Cryovolcanism is most often encountered on icy moons, such as Europa, Enceladus, and Triton.

d)

1. Measure the gravity field: From this the moment of inertia of the planet can be assessed. If the planet is differentiated, more mass will be centered around the core.
2. Measure the magnetic field: Planetary magnetic fields typically require a molten metallic core, and properties of this core can be extracted from magnetic field readings.
3. Atmospheric properties (e.g. black-body radiation measurements, emission spectroscopy, etc.): If the composition and general temperature of the atmosphere is measured, one can conclude some things about the interior convection profile, which contains information about the internal structure.
4. Probing the planet surface with a lander: This can be used to detect certain geological features (evidence of tectonic activity, volcanism, etc.) and perform seismic measurements.

3 Filler lecture:Magnetism

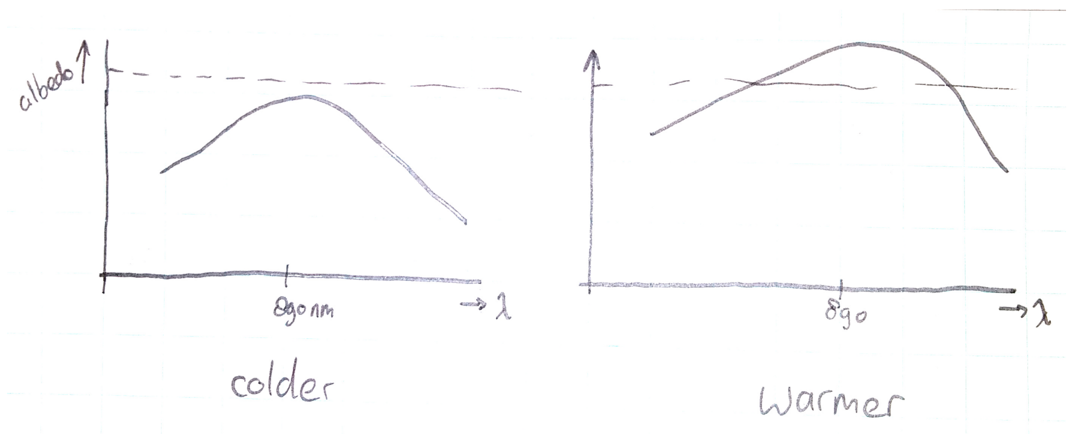
- Omitted as magnetism is no exam topic for upcoming exam -

4 Filler lecture:Atmospheres

a)

First, Uranus and Neptune are located much further away from the sun, making their equilibrium temperature lower. This causes more condensation of methane, which changes the emissions spectrum. Especially higher-frequency molecule vibrations will be less common when the temperature is lower, reflected in the the much lower albedo values for the lower frequencies. Secondly, Uranus and Neptune have different mass and atmospheric compositions to Jupiter and Saturn, which can impact things like atmosphere transparency, which in turn impacts the measured albedo.

b)



The main take-away is that the albedo increases as the temperature increases, and the temperature is directly affected by the pressure.

c)

First, we make two assumptions:

1. We assume that the two planets have the same bond albedo A_B.
2. We assume that the two planets have the same emissivity ϵ, since their atmospheric composition is similar.

The equilibrium temperature is then purely dependant on the planet's distance to the sun. Since we already know the temperature of Uranus, we substitute the planet-specific data in the equation given in the question, and then divide the two:

$$\begin{aligned} \frac{T^4_{Neptune}}{T^4_{Uranus}} &= \frac{\frac{(1-A_B)}{4\epsilon\sigma}\frac{F_O}{r^2_{Neptune}}}{\frac{(1-A_B)}{4\epsilon\sigma}\frac{F_O}{r^2_{Uranus}}} \\ \frac{T^4_{Neptune}}{T^4_{Uranus}} &= \frac{r^2_{Uranus}}{r^2_{Neptune}} \\ T^4_{Neptune} &= T_{Uranus}\sqrt[4]{\frac{r^2_{Uranus}}{r^2_{Neptune}}} \end{aligned} \tag{7}$$

And so:

$$T^4_{Neptune} = 60 \cdot \sqrt[4]{\frac{20^2}{40^2}} = 42.43\ K \tag{8}$$

d)

- Atmospheric composition can contribute to raising equilibrium temperature, especially if large amounts of greenhouse gases are present, such as CO_2 or CH_4. An extreme example of this is Venus, which is blanketed in a dense, heat-retaining atmosphere.
- Internal heating, for example from lingering formation heat or radioactive decay of isotopes such as ^{40}K or ^{232}Th.

e)

There are numerous effects, but an important one is the notion that Uranus' orbit is eccentric, causing it to move closer to and further from the sun. The formula presented in the question assumes a circular orbit of constant radius, and therefore the nuance of this effect is completely neglected.

f)

5 Filler lecture:Exoplanets

a)

From observation, the short periodic function has a period of 0.05 year (planet 1), whilst the long periodic function has a period of 0.7 year (planet 2). We can then calculate the radius of each orbit as follows:

$$\begin{aligned} T &= 2\pi\sqrt{\frac{r^3}{GM}} \\ r_{orbit} &= \sqrt[3]{GM\left(\frac{T}{2\pi}\right)^2} \end{aligned} \tag{9}$$

And so we get:

$$\begin{aligned} r_1 &= \sqrt[3]{6.674E-11\cdot 2E30\cdot\left(\frac{0.05\cdot 365.25\cdot 24\cdot 60^2}{2\pi}\right)^2} = 2.03E10\ m = 0.135\ AU \\ r_2 &= \sqrt[3]{6.674E-11\cdot 2E30\cdot\left(\frac{0.7\cdot 365.25\cdot 24\cdot 60^2}{2\pi}\right)^2} = 1.18E11\ m = 0.786\ AU \end{aligned}$$

We now assume that since the observer is far away, the star and the planet are effectively flat discs, and the reduction in stellar flux is directly proportional to the reduction in area from the planet moving in front of the star. Further, we will indicate the reduced flux, as measured in the data, by ϕ_r. This means that:

$$\begin{aligned} \frac{A_{star}-A_{planet}}{A_{star}} &= \phi_r \\ A_{planet} = 2\pi r_{planet}^2 &= A_{star}(1-\phi_r) \\ r_{planet} &= \sqrt{\frac{A_{star}(1-\phi_r)}{2\pi}} \end{aligned} \tag{10}$$

Which leads to:

$$\begin{aligned} r_1 &= \sqrt{\frac{2\pi\cdot(7E8)^2(1-0.990)}{2\pi}} = 7.00E7\ m \\ r_2 &= \sqrt{\frac{2\pi\cdot(7E8)^2(1-0.994)}{2\pi}} = 5.42E7\ m \end{aligned} \tag{11}$$

b1)

1. Radial Velocity Method
2. Gravitational Micro-lensing
3. Timing method

b2)

1. Mass, period
2. Mass, period

b3)

1. High masses
2. High masses

c)

d)

The question is rather non-specific as to the nature of this tail, so we will make some assumptions first:

1. The tail is completely opaque
2. The tail remains the same size and remains opaque for at least the transfer time.

This will cause a band to form across the star with left-behind planetary matter to blot out the star partially. This is illustrated in the figure too. We approximate the area of this band by a rectangle, ignoring the curvature of the sun at the edges. Once the planet has completed the full transfer, and the band crosses the whole sun, the area reduction is:

$$\frac{A_star - 2r_{star} \cdot 2r_{planet}}{A_star} = \frac{2\pi \cdot (7E8)^2 - 2 \cdot (7E7) \cdot 2(7E8)}{2\pi \cdot (7E8)^2} = 0.936. \tag{12}$$

This results in the curve found in the figure. First, the planet enters the scene, reducing the stellar flux by 1%, but gradually the flux reduces further, as the band passes over the star. At the end of the transfer at 250 minutes, only 93.6% of the light is left, but then it stops reducing.