Equivariant Filter Derivation

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May 27, 2021

I will be referring to the notations and conventions used in [1]. These notes will skip many steps and will focus on the final result.

1 Functions

1.1 Introducing ϕ

We define a right action of a Lie Group $SLAM_n(3)$ on a manifold $\mathcal{M}$ which is a smooth map $\phi\colon SLAM_n(3) \times \mathcal{M} \to \mathcal{M}$, such that,

$$\phi(X, \xi) = (PA, p_i + R_P a_i) \tag{1}$$
$$\phi(id, \xi) = \xi \tag{2}$$

for any $X \in SLAM_n(3)$ and any $\xi \in \mathcal{M}$.

1.2 Introducing f

There is a function $f\colon \mathbb{V} \to \mathfrak{X}(\mathcal{M})$, where $\mathfrak{X}(\mathcal{M})$ denotes the space of vector fields over $\mathcal{M}$ and $\mathbb{V}$ denotes the velocity space, in our case $\mathfrak{se}(3)$. It is given by,

$$\dot{\xi} = f_U(\xi) = (PV, R_P v_i) \tag{3}$$

1.3 Defining $d_*\phi$

We define a function $d_*\phi\colon SLAM_n(3) \times \mathfrak{X}(\mathcal{M}) \to \mathfrak{X}(\mathcal{M})$,

$$d_*\phi(X, f_U) := d\phi_X \cdot f \circ \phi_X^{-1} \tag{4}$$
$$d_*\phi(X, f_U)(P) = (PA^{-1}VA, R_{PA^{-1}} v_i + R_{PA^{-1}V} a_i) \tag{5}$$

1.4 Deriving ψ

There exists a function $\psi\colon SLAM_n(3) \times \mathbb{V} \to \mathbb{V}$ such that,

$$\psi(X, U) = (A^{-1}VA, R_A^\top v_i + R_A^\top \Omega^\times a_i) \tag{6}$$

1.5 Deriving ρ

There exists an action $\rho\colon SLAM_n(3) \times \mathcal{N} \to \mathcal{N}$ such that,

$$\rho(X, q) = R_A^\top q + R_A^\top (a_i - x_A) \tag{7}$$

1.6 Defining the lift Λ

A lift is a map $\Lambda\colon \mathcal{M} \times \mathbb{V} \to \mathfrak{slam}_\mathfrak{n}(3)$ satisfying,

$$\Lambda(\xi, U) = U = (V, v_i) \tag{8}$$
$$X\Lambda(\xi, U) = (AV, R_A v_i) \tag{9}$$

2 Linearisation functions

2.1 Defining ε

We will fix a local coordinate chart $\varepsilon\colon \mathcal{U} \to \mathbb{R}^M$, where $M = dim(\mathcal{M})$ and $\mathcal{U} \subset \mathcal{M}$ is a neighbourhood of the fixed origin ξ°.

$$\varepsilon = \varepsilon(e), \quad \epsilon(\xi^\circ) = 0 \tag{10}$$
$$\varepsilon(P, p_i) = (\log(P^{0^{-1}}P)^\vee, p_i - p_i^0) \tag{11}$$
$$\varepsilon^{-1}(a, b_i) = (P^0 expm(a^\wedge), b_i + p_i^0) \tag{12}$$

2.2 Defining δ

We linearise the output error, using a local coordinate chart $\delta\colon \mathcal{U}_{y^\circ} \subset \mathcal{N} \to \mathbb{R}^N$,

$$\delta = \delta(h(e)) \tag{13}$$
$$\delta(y) = (y_i - y_i^\circ) \tag{14}$$

3 Matrices

3.1 Deriving A_t°

$$A_t^\circ = d\epsilon \cdot D_{E|id}\phi(E, \xi^\circ) \cdot D_{e|\xi^\circ}\Lambda(e, v^\circ) \cdot d\epsilon^{-1}[u] \tag{15}$$

Consider the third term,

$$D_{e|\xi^\circ}\Lambda(e, v^\circ)[\gamma] = \left.\frac{d}{ds}\right|_{s=0} \Lambda(e + \gamma s, v^\circ) \tag{16}$$
$$= \left.\frac{d}{ds}\right|_{s=0} v^\circ = 0 \tag{17}$$
$$\implies A_t^\circ = 0 \tag{18}$$

3.2 Deriving C_t

$$C^\circ := D_{y|y^\circ}\delta(y) \cdot D_{e|\xi^\circ}h(e) \cdot D_{\epsilon|0}\epsilon^{-1}(\epsilon) \tag{19}$$

Consider the third term,

$$d\varepsilon^{-1}[u_P, u_{p_i}] = (P^\circ u_P^\wedge, u_{p_i}) \tag{20}$$

Consider the second term,

$$D_{e|\xi^\circ} h(e)[P^\circ u_P^\wedge, u_{p_i}] = -R_{u_P^\wedge P^{\circ -1}} p_i^\circ - x_{u_P^\wedge P^{\circ -1}} + R_{P^\circ}^\top u_{p_i} \tag{21}$$

$$D_{e|\xi^\circ} h(e)[P^\circ u_P^\wedge, u_{p_i}] = -u_P^\wedge P^{\circ -1} \bar{p}_i{}^\circ + R_{P^\circ}^\top \bar{u}_{p_i} = \eta \tag{22}$$

$$\tag{23}$$

Consider the first term,

$$d\delta(y)[\eta] = \frac{d}{ds}\Big|_{s=0} \delta(y^\circ + s\eta) \tag{24}$$

$$= \frac{d}{ds}\Big|_{s=0} (s\eta) = \eta \tag{25}$$

$$= -u_P^\wedge y_i^\circ + R_{P^\circ}^\top \bar{u}_{p_i} \tag{26}$$

Thus we get,

$$C^\circ(u_P, u_{p_i}) = (-u_P^\wedge y_i^\circ + R_{P^\circ}^\top \bar{u}_{p_i}) \tag{27}$$

We know that,

$$-u_P^\wedge y_i^\circ = \begin{pmatrix} y_i^{\circ\times} & -I_3 \end{pmatrix} \begin{pmatrix} \Omega_{u_P} \\ V_{u_P} \end{pmatrix} \tag{28}$$

$$= \begin{pmatrix} y_i^{\circ\times} & -I_3 \end{pmatrix} u_P \tag{29}$$

Hence we get,

$$C^\circ \begin{pmatrix} u_P \\ u_{p_1} \\ \vdots \\ u_{p_n} \end{pmatrix} = \begin{pmatrix} y_1^{\circ\times} & -I_3 & R_{P^\circ}^\top & 0 & \cdots & 0 \\ y_2^{\circ\times} & -I_3 & 0 & R_{P^\circ}^\top & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ y_n^{\circ\times} & -I_3 & 0 & 0 & \cdots & R_{P^\circ}^\top \end{pmatrix} \begin{pmatrix} u_P \\ u_{p_1} \\ \vdots \\ u_{p_n} \end{pmatrix} \tag{30}$$

References

[1] Robert Mahony, Pieter van Goor, and Tarek Hamel. *Equivariant Filter (EqF).*

[2] Robert Mahony, Jochen Trumpf, and Tarek Hamel. *Equivariant Systems Theory and Observer Design.*

[3] Robert Mahony and Tarek Hamel. *A Geometric Nonlinear Observer for Simultaneous Localisation and Mapping*