Equivariant Filter Results

Hiya Gada

July 8, 2021

1 Introduction

I will be referring to the notations and conventions used in [1].
The results shown in this paper are for the same set of landmarks (using the same random number seed).
We will consider three types of origins as given below:

- $\hat{X}(0) = id$ and $\xi^\circ = \hat{\xi}(0)$
- $\xi^\circ = (I_4, 0)$ and the corresponding $\hat{X}(0)$ such that $\phi(\hat{X}(0), \xi^\circ) = \hat{\xi}(0)$
- Make sure that the C° matrix remains precise, leading to the precision of Σ so that it doesn't overshoot and become singular or negative definite. This can be done by making sure the y_i° vectors are distributed around the origin (floating point number will be more precise).

2 Results

We run a few test cases for each type of origin.
The landmarks being in a box of $(0, 1) \times (0, 1)$, we take cases of the origin (for the landmarks) being inside the box, on the border of this box, and two cases of outside the box. We also include cases where a lot of noise is received from the sensors, and check the rate at which the error converges to zero.

2.1 Case 1

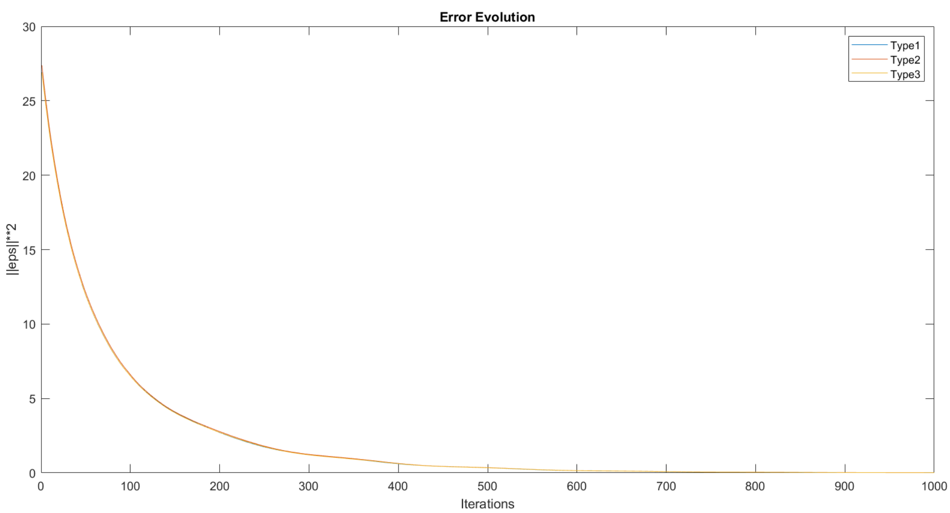

Figure 1: Case 1

The above figure but zoomed:

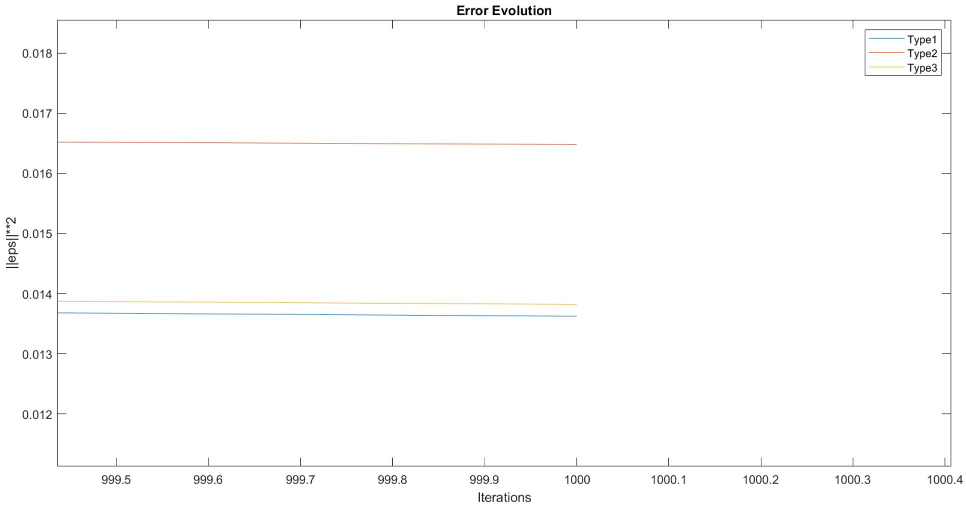

Figure 2: Case 1 zoomed

We consider the initial estimate which is closest to the actual value of the landmarks, that is, [0.5; 0.5; 0] for each landmark.
Error comparison: Type 1 < Type 3 « Type 2

2.2 Case 2

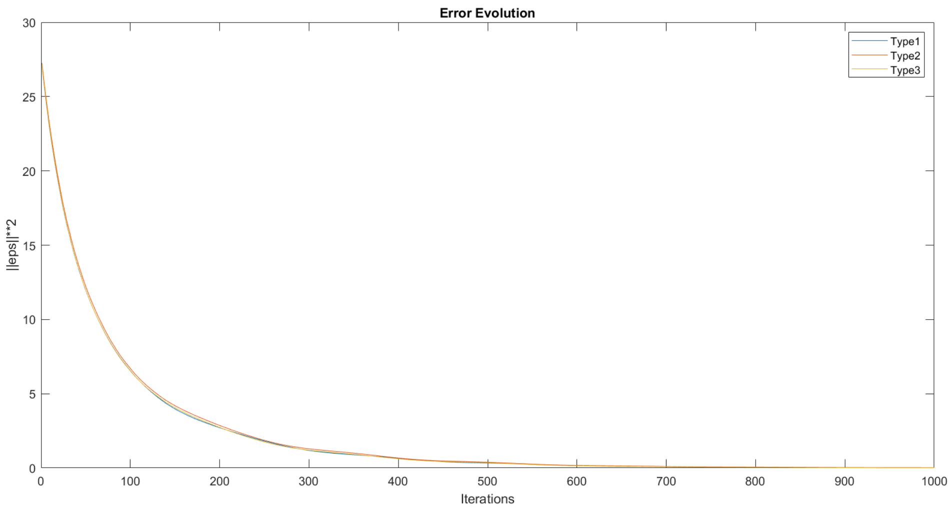

Figure 3: Case 2

The above picture zoomed:

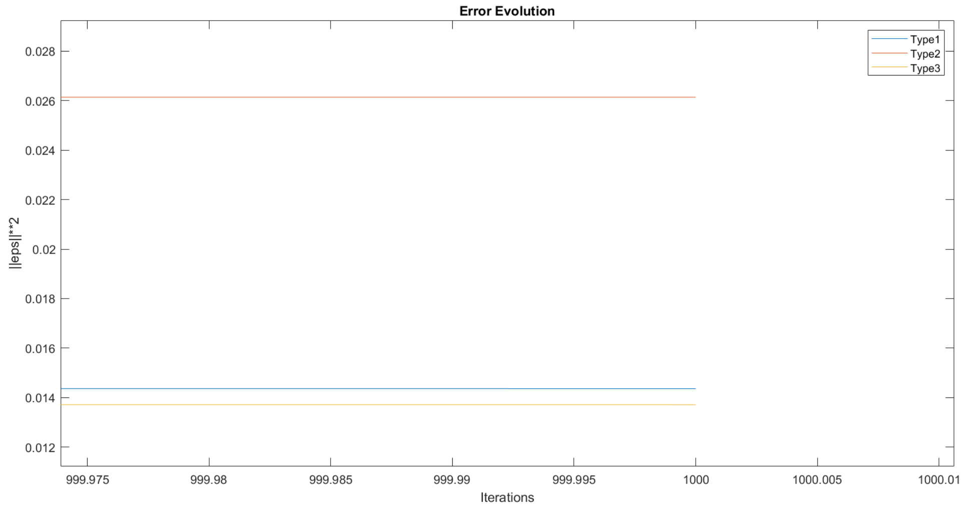

Figure 4: Case 2 zoomed

We consider the initial estimate, $\hat{\xi}(0)$ which is farther than the actual value of the landmarks (unlike Case 1), that is, [1; 1; 0] for each landmarks.
Error comparison: Type 3 $<$ Type 1 « Type 2

2.3 Case 3

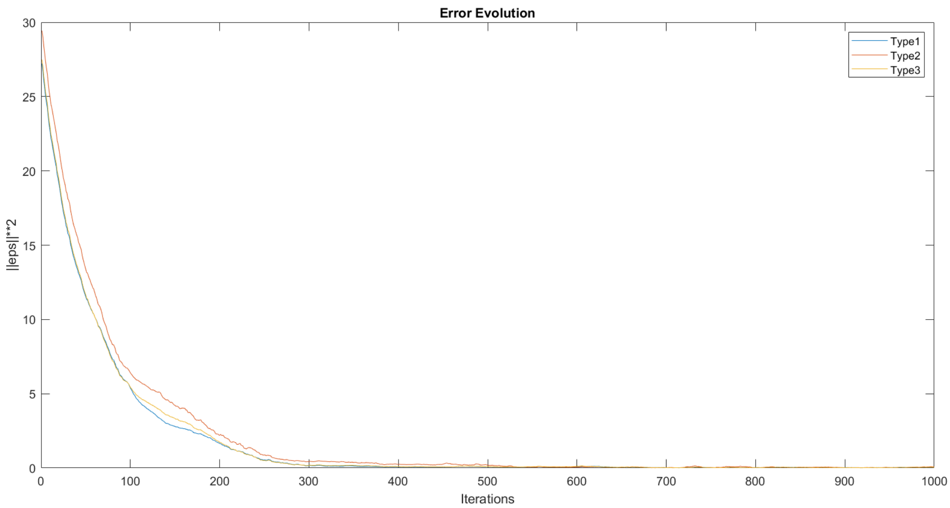

Figure 5: Case 3

The above picture zoomed:

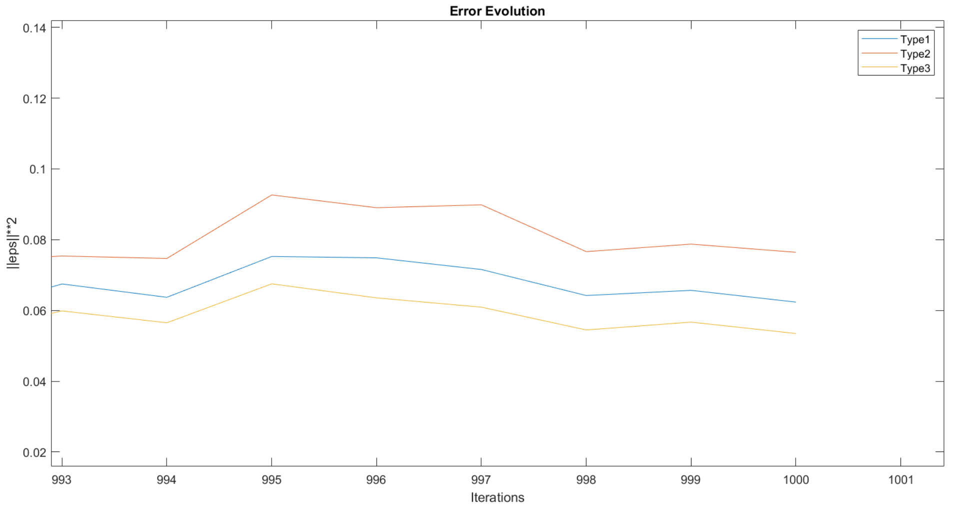

Figure 6: Case 3 zoomed

We consider **high noise** in this case and the same initial estimate as Case 2, i.e. [1; 1; 0] for each of the landmarks.
Error comparison: Type 3 < Type 1 < Type 2

2.4 Case 4

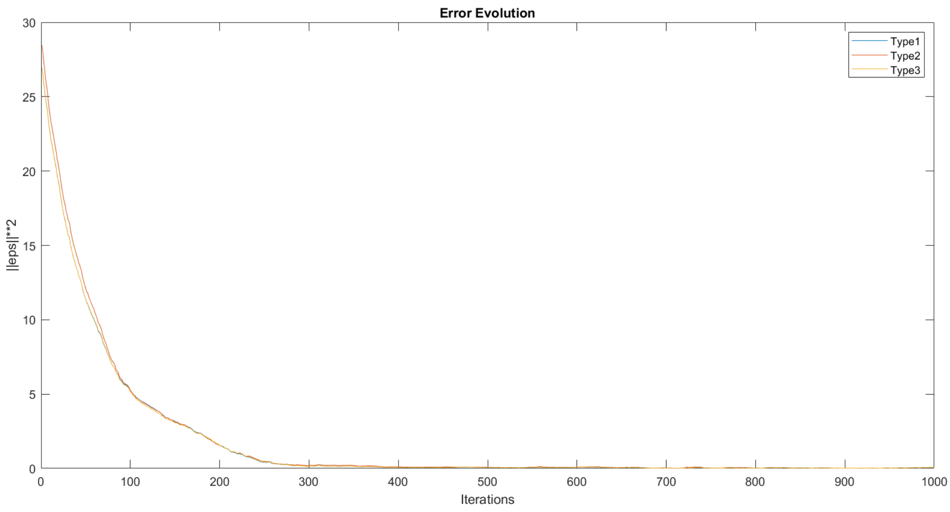

Figure 7: Case 4

The above picture zoomed:

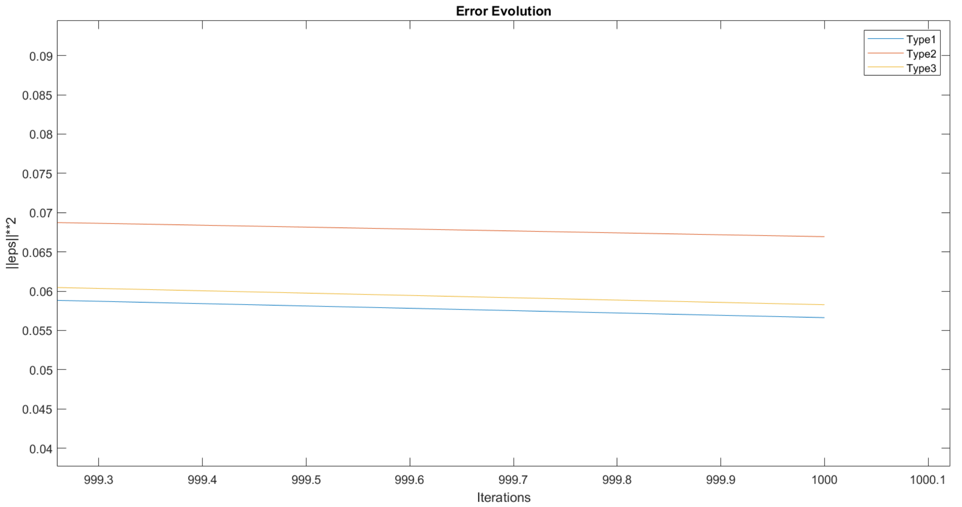

Figure 8: Case 4 zoomed

We consider **high noise** in this case and the same initial estimate as Case 1, i.e. [0.5; 0.5; 0] for each of the landmarks.
Error comparison: Type 1 $<$ Type 3 « Type 2

2.5 Case 5

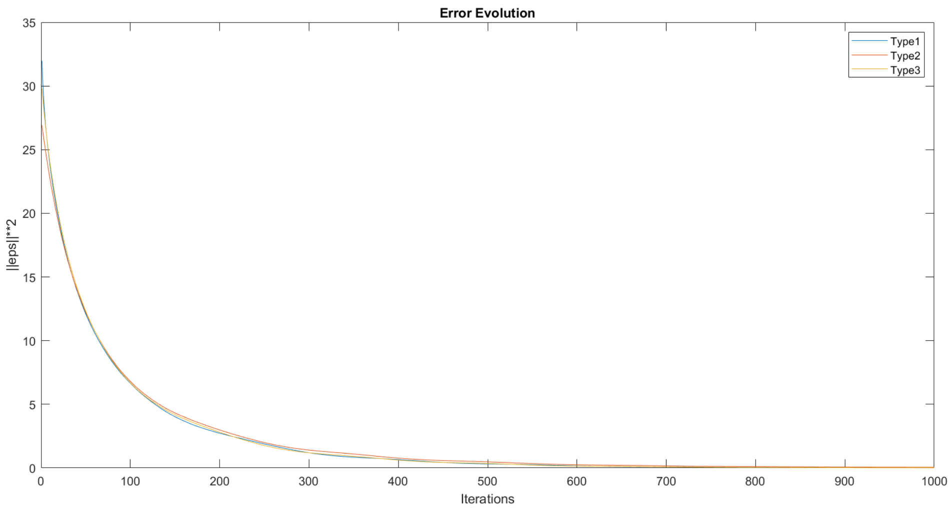

Figure 9: Case 5

The above picture zoomed:

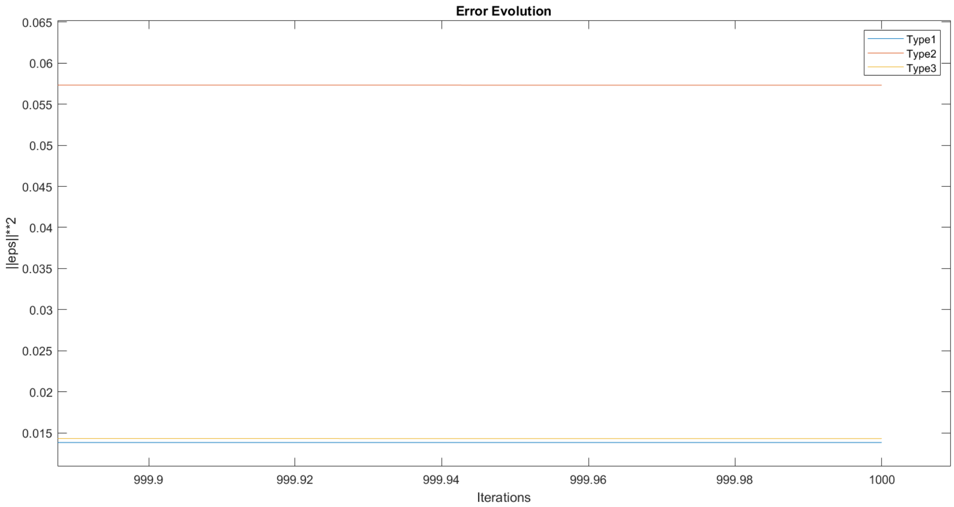

Figure 10: Case 5 zoomed

We consider the initial estimate as [2; 2; 0] for each of the landmarks (farther than Case 2).
Error comparison: Type 1 < Type 3 «< Type 2

2.6 Special case

In this case, we take the initial estimate farthest from the actual values, i.e. [5; 5; 0] for the landmarks.
Type 1 origin shows error when the code is run. Type 2 and Type 3 origins show no error when the code is run. This can be resolved by reducing the dt time interval used in our simulation (which may cause Σ to overshoot).

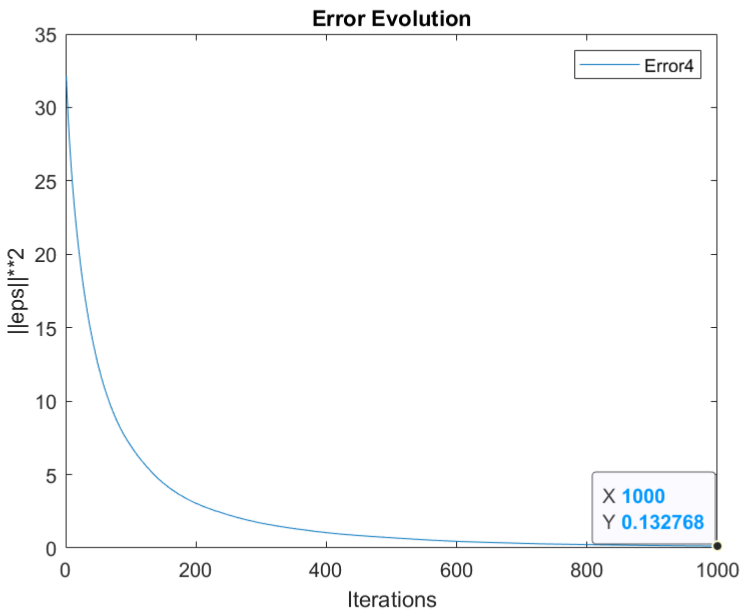

Figure 11: Type 2 special case

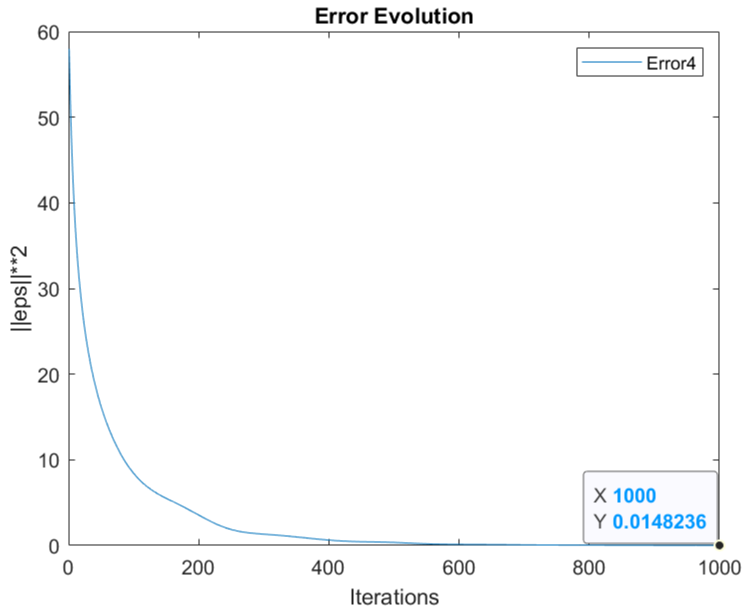

Figure 12: Type 3 special case

Error comparison: Type 3 «< Type 2
Type 3 and Type 2 origin ensures that the floating point number is distributed about 0, hence giving more precision and allowing to take origins which are far away from the landmarks. This was not possible in other cases.

3 Conclusion

- Type 2 showed the worst results in all cases.
- Type 3 should be preferred over Type 2 if the origin is far away.
- Type 1 gave good results only when the origin was closer.
- Type 3 is the best if our input has a lot of noise and when the origin is far away, (the worst possible case).

References

[1] Robert Mahony, Pieter van Goor, and Tarek Hamel. *Equivariant Filter (EqF).*

[2] Robert Mahony, Jochen Trumpf, and Tarek Hamel. *Equivariant Systems Theory and Observer Design.*