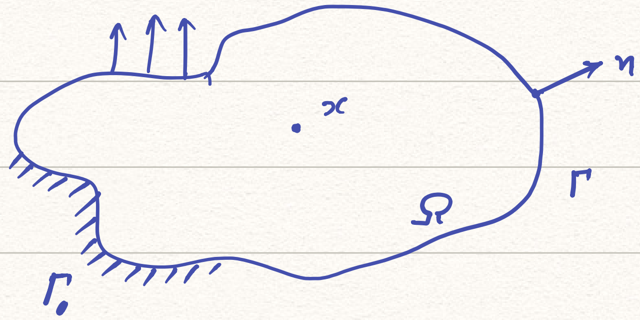

Displacement field $u : \Omega \to \mathbb{R}^3$

Strain tensor $\varepsilon(x) = \frac{1}{2}\left(\nabla u(x) + \nabla u(x)^T\right)$

Stress tensor $\sigma(x) = \lambda(\operatorname{tr}\varepsilon)I + 2\mu\varepsilon$

Governing equations :

$$\nabla \cdot C \nabla U(x) + \rho \omega^2 U(x) = 0 \qquad x \in \Omega$$

$$U(x) = 0 \qquad x \in \Gamma_0$$

$$\sigma(x) \cdot n(x) = T(x) \qquad x \in \Gamma$$

$$u(\cdot, 0) = u_0 = U(x) \qquad ?$$

$$u(x,t) = U(x) e^{i\omega t}$$

Variational formulation:

$p : \Omega \to \mathbb{R}^3$

$p(x) = 0$ for $x \in \Gamma_0$

$$\int_\Omega (\nabla \cdot C \nabla U) \cdot p \, dx + \int_\Omega \rho \omega^2 U \cdot p \, dx = 0$$

$$\int_\Gamma T \cdot p \, ds - \int_\Omega C \nabla U : \nabla p \, dx + \int_\Omega \rho \omega^2 U \cdot p \, dx = 0$$

code : $\lambda = \rho \omega^2$ $\boxed{\lambda = -\rho\omega^2}$

$$\lambda_h(x) = e\lambda + (1-e)\lambda\,\zeta(h(x))$$
$$\mu_h(x) = e\mu + (1-e)\mu\,\zeta(h(x))$$

$$\int_\Gamma T\cdot p\,ds + \int_\Omega \rho\omega^2 u\cdot p\,dx - \int_\Omega \sigma_h : \nabla p\,dx$$

$$\sigma_h = \lambda_h(\operatorname{tr}\varepsilon)I + 2\mu_h\varepsilon$$

$$J(h) = \omega$$

$$L(h,u,p) := \omega + \int_\Gamma T\cdot p\,ds + \int_\Omega \rho\omega^2 U\cdot p\,dx - \int_\Omega \sigma_h : \nabla p\,dx$$

Primal equation :

$$\delta L(h, U, p; \delta p) = 0$$

$$\int_\Gamma T\cdot \delta p\,ds + \int_\Omega \rho\omega^2 U\cdot\delta p\,dx - \int_\Omega \sigma_h : \nabla\delta p\,dx = 0$$

$$\left.\begin{aligned} \nabla\cdot\sigma_h + \rho\omega^2 U &= 0, && \Omega \\ U &= 0, && \Gamma_0 \\ \sigma_h\cdot n &= T, && \Gamma \end{aligned}\right\} \quad \text{(P)}$$

Adjoint Equation :

$\delta L (h, U, p ; \delta U) = 0$

$\int_\Omega \rho \omega^2 U \cdot p \, d$

$- \int_\Omega \sigma_h : \nabla p \, dx$

$$\frac{d}{dt}\Big|_{t=0} \int_\Omega \rho \omega^2 (U + t\delta U) \cdot p \, dx - \frac{d}{dt}\Big|_{t=0} \int \sigma_h (U + t\delta U) : \nabla p \, dx$$

$$\ldots \qquad - \int_\Omega \lambda_h (\nabla \cdot \delta U)(\nabla \cdot p) \, dx$$

$$- \int_\Omega \mu_h \nabla \delta U : \nabla p \, dx$$

$$- \int_\Omega \mu_h \nabla \delta U^T : \nabla p \, dx$$

$$\Rightarrow \quad - \int_\Gamma \delta U \cdot \sigma_h(p) \cdot n \, ds + \int_\Omega (\nabla \cdot \sigma_h(p)) \cdot \delta U \, dx$$

$$+ \int_\Omega \rho \omega^2 \delta U \cdot p \, dx$$

$$\left.\begin{aligned} \nabla \cdot \sigma_h(p) &= -\rho \omega^2 p \quad &, \ \Omega \\ p &= 0 \quad &, \ \Gamma_0 \\ \sigma_h(p) \cdot n &= 0 \quad &, \ \Gamma \end{aligned}\right\} \quad A$$

Gradient of Obj function:

$$\left\langle \frac{\partial L}{\partial h}, \delta h \right\rangle = -\frac{d}{dt}\Big|_{t=0} \int_{\Omega} \sigma_{h+t\delta h} : \nabla p \, dx$$

$$= -(1-e)\,\xi'(h)\,\sigma : \nabla p \, \delta h$$

$$= -(1-e)\,\xi'(h)\,\sigma(U) : \nabla p$$

U not u