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(a) Let A be the transition matrix. a_{ij} is the probability that the next state will be state j given that the current state is state i.

A is a 5×5 matrix (since there are three hidden states and a start and end state).

$$A = \begin{pmatrix} a_{SS} & a_{SF} & a_{SL_1} & a_{SL_2} & a_{SE} \\ a_{FS} & a_{FF} & a_{FL_1} & a_{FL_2} & a_{FE} \\ a_{L_1S} & a_{L_1F} & a_{L_1L_1} & a_{L_1L_2} & a_{L_1E} \\ a_{L_2S} & a_{L_2F} & a_{L_2L_1} & a_{L_1L_2} & a_{L_2E} \\ a_{ES} & a_{EF} & a_{EL_1} & a_{EL_2} & a_{EE} \end{pmatrix}$$

We can estimate a_{ij} by the formula:

$$a_{ij} = \frac{Count(i \rightarrow j)}{Count(i)}$$

The sum of each row in the transiiton matrix is 1 since every state must transition to some state.

By definition, since S is the start state and only occurs at the start of a sequence, the probability of a state transitioning to $S = 0$ for any state. Note also that the probability of E transitioning into any other state is also zero (the HMM would terminate when the state became E but for the purposes of this question I will mark the probability of E transitioning to E as 1 and any other state as 0).

Let B be the emission matrix. b_{ij} is the probability that hidden state i emits symbol j.

B is a 5×8 matrix – since there are 5 hidden states (including the two special states for start and end) and 8 symbols in the emission alphabet (one for each dice roll and two special ones [k_s and k_e] for start and end).

$$B = \begin{pmatrix} b_{Sk_s} & b_{S1} & b_{S2} & b_{S3} & b_{S4} & b_{S5} & b_{S6} & b_{Sk_e} \\ b_{Fk_s} & b_{F1} & b_{F2} & b_{F3} & b_{F4} & b_{F5} & b_{F6} & b_{Fk_e} \\ b_{L_1k_s} & b_{L_11} & b_{L_12} & b_{L_1L_3} & b_{L_14} & b_{L_15} & b_{L_16} & b_{L_1k_e} \\ b_{L_2k_s} & b_{L_21} & b_{L_22} & b_{L_13} & b_{L_24} & b_{L_25} & b_{L_26} * b_{L_2k_e} & \\ b_{Ek_s} & b_{E1} & b_{E2} & b_{E3} & b_{E4} & b_{E5} & b_{E6} & b_{Ek_e} \end{pmatrix}$$

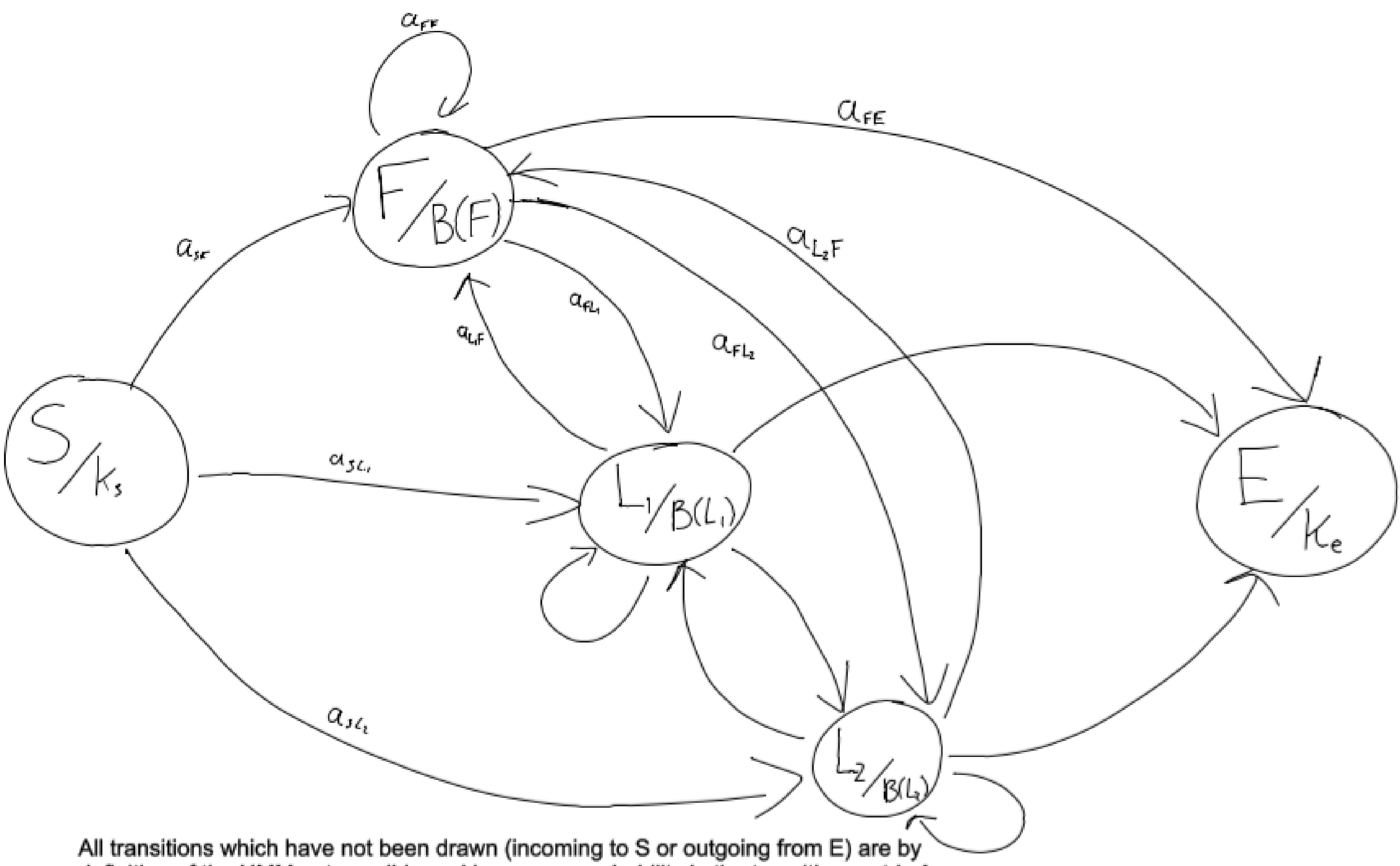
We can estimate b_{ij} by the formula:

$$b_{ij} = \frac{Count(i, j)}{Count(i)}$$

Note that since by definition, k_s is the state emitted by the start state; b_{Si} for $i \neq k_s$ will be 0 and $b_{Sk_s} = 1$. The same statement holds for the end state E and the end symbol k_e.

The sum of each row in the emission matrix sums to one – since every state is guaranteed to emit some symbol.

(b)



All transitions which have not been drawn (incoming to S or outgoing from E) are by definition of the HMM not possible and have zero probability in the transition matrix A

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(c) (a) The probability a_{FL_2} is equal to zero.

Setting this probability to zero means that it is impossible for the HMM to model a switch directly from F to L_2. The HMM would now model this.

(b) For all hidden states s, $s \neq F \iff a_{sE} = 0$.

IE the probability of transitioning from any state which is not F to the end state E is zero.

This model will ensure that every sequence ends on F – as the croupier does.

(c) $a_{L_2L_2} = 0$

IE the probability of transitioning from L_2 to L_2 is zero. So the HMM cannot model it happening and the HMM now models the new behavior appropriately.

We would have to maintain the invariant that the outgoing probabilities of L_2 sum to 1.

(d) This can't be modelled by the HMM †. By definition of the HMM, the next state is only dependent on the previous state – and not on the emissions. So the emission of the previous state can have no impact on the next state. So the HMM cannot model this behavior.

†with the exception of the case where one dice d always rolls a 6 – in this case $a_{dd} = 0$.