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Section B

1. a)

$$n = \left(\begin{pmatrix}2\\-1\\2\end{pmatrix} - \begin{pmatrix}4\\1\\0\end{pmatrix}\right) \times \left(\begin{pmatrix}1\\2\\3\end{pmatrix} - \begin{pmatrix}4\\1\\0\end{pmatrix}\right)$$

$$n = \begin{pmatrix}-2\\-2\\2\end{pmatrix} \times \begin{pmatrix}-3\\1\\3\end{pmatrix}$$

$$n = \begin{pmatrix}-6-2\\-6+6\\-2-6\end{pmatrix} = \begin{pmatrix}-8\\0\\-8\end{pmatrix} \cancel{= \begin{pmatrix}1\\0\\1\end{pmatrix}}$$

n is in the direction of $\begin{pmatrix}1\\0\\1\end{pmatrix}$

$$r \cdot n = \begin{pmatrix}2\\-1\\2\end{pmatrix} \cdot n$$

$$x + z = 4$$

b) $a \cdot b \times c$

$$= \begin{pmatrix}-1\\1\\0\end{pmatrix} \cdot \left(\begin{pmatrix}0\\2\\1\end{pmatrix} \times \begin{pmatrix}1\\0\\-1\end{pmatrix}\right)$$

$$= \begin{pmatrix}-1\\1\\0\end{pmatrix} \cdot \begin{pmatrix}-2\\1\\-2\end{pmatrix} = 3 \neq 0$$

So a, b, c are not coplanar.

Let A, B, C be the reciprocal basis to a, b, c

$$A = \frac{b \times c}{a \cdot b \times c}$$

$$= \frac{1}{3}\begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix}$$

$$B = \frac{c \times a}{a \cdot b \times c}$$

$$= \frac{1}{3}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$C = \frac{a \times b}{a \cdot b \times c}$$

$$= \frac{1}{3}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

$$d = (d \cdot A)a + (d \cdot B)b + (d \cdot C)c$$

$$d = \frac{1}{3}\left((-4+1+2)a + (2+1-1)b + (2+1+2)c\right)$$

$$d = -\frac{1}{3}a + \frac{2}{3}b + \frac{5}{3}c$$

c) $\cosh z = -1$

$$z = \operatorname{arccosh} -1$$

$$z = \ln\left(x \pm \sqrt{x^2-1}\right)$$

$$z = \ln\left(-1 \pm \sqrt{\cancel{+}1-1}\right)$$

$$z = \ln -1$$

$$z = \ln e^{i\pi+2k\pi i}$$

$$z = (i\pi + 2k\pi i)\ln e$$

$$z = (1+2k)\pi i \text{ for any } k \in \mathbb{Z}$$

d) i) $$\frac{1}{\sin^2 x} - \frac{1}{x^2}$$

$$= \frac{x^2 - \sin^2 x}{x^2 \sin^2 x}$$

$$\approx \frac{x^2 - x^2\left(1 - \frac{x^2}{6} + O(x^4)\right)^2}{x^4 + O(x^6)}$$

$$\approx \frac{x^2 - x^2 + \frac{x^4}{3} + O(x^6)}{x^4}$$

$$\approx \frac{1}{3}\,\frac{x^4}{x^4}$$

$$\approx \frac{1}{3}$$

ii) $\dfrac{\sin x - \arctan x}{x^2 \ln(1+x)}$

$$\sin x \approx x - \frac{x^3}{6} + O(x^5)$$

$$\arctan x \approx x - \frac{x^3}{2} + O(x^5)$$

$$\ln(1+x) \approx x - \frac{x^2}{2} + O(x^3)$$

$$\frac{\sin x - \arctan x}{x^2 \ln(1+x)}$$

$$= \frac{x - \frac{x^3}{6} - x + \frac{x^3}{2} + O(x^5)}{x^2(x + O(x^2))}$$

$$= \frac{\frac{1}{3}x^3 + O(x^5)}{x^3 + O(x^4)}$$

$$= \frac{1}{3} + O(x^2)$$