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And(M_1, M_2)

Remember the and construction for the *And* of two DFA's M_1, M_2.

The formula is in the question sheet:

Given two DFA's:

$$M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$$
$$M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$$

The DFA defined as:

$$M = (Q_1 \times Q_2, \Sigma, \delta, (s_1, s_2), F_1 \times F_2)$$
$$\delta((q_1, q_2), a) = (\delta_1(q_1, a), \delta_2(q_2, a))$$

Is the DFA which is in an accepting state when *BOTH* M_1, M_2 are in accepting states.

Pumping Lemma

Note that L is a regular language implies that L enjoys the Pumping Lemma property.

A language L enjoys the Pumping Lemma property does not imply that it is regular:

Regular Language $\implies$ Pumping Lemma

Pumping Lemma $\not\Longleftarrow$ Regular

The Pumping Lemma states that for all regular languages, if they accept a string s of length $\geq \ell$ for some ℓ then there exists some $u_1 v u_2 = s$ where $|v| \geq 1$ ($v \neq \varepsilon$) and $|u_1 v| \leq \ell$ such that $\forall n \in \mathbb{N}. u_1 v^n u_2 \in L$.

To prove a language is not regular, you must find some w_ℓ for arbitrary ℓ which cannot be pumped.

So for $L = a^n b^n$, you find $w_\ell = a^\ell b^\ell$ and say that "since the first ℓ letters are a, v must be some sequence of a's of length s, u_1 is length r and u_2 is $a^{\ell-r-s} b^\ell$. Then consider v^2 then the string is now $a^{\ell+r} b^\ell$. $r \neq 0$ so $\ell + r \neq \ell$. So the string is not of the form $a^n b^n$. However it is still in the language L. So L cannot be pumped and hence L is not regular.