Part I

Worksheet 2

1 Natural Languages

1. Write a paragraph on the following open-ended point to discuss with your supervisor

   - Is frequency information or structural information more important when considering processing difficulty?

   I consider structural information to be more important when considering processing difficulty. While the probabilities of words are relevant, there are simply structures which are challenging to parse. I give an example from Shakespeare's Macbeth:

   > Doubtful it stood;
   > As two spent swimmers, that do cling together
   > And choke their art. The merciless Macdonwald–
   > Worthy to be a rebel, for to that
   > The multiplying villanies of nature
   > Do swarm upon him–from the western isles
   > Of kerns and gallowglasses is supplied;
   > And fortune, on his damned quarrel smiling,
   > Show'd like a rebel's whore: but all's too weak:
   > For brave Macbeth–well he deserves that name–
   > Disdaining fortune, with his brandish'd steel,
   > Which smoked with bloody execution,
   > Like valour's minion carved out his passage
   > Till he faced the slave;
   > Which ne'er shook hands, nor bade farewell to him,
   > Till he unseam'd him from the nave to the chaps,
   > And fix'd his head upon our battlements.

   This quote describes a brutal battle with "brave Macbeth" slaying the rebel Macdonwald. However, it's almost impossible to read. All the sentences are individually comprehensible; however once they're combined into one long sentence, they're impossible to parse. The words which Shakespeare uses are used on modern English; all with moderately high probability. However, the sentence structures which he use are outdated and not found in modern English. This makes any extended sentences impossible to parse. If these sentences were restructured to use modern English structures, we would be able to parse them far easier. I believe there is *no assignment of words* such that a sentence with the same structure as this quote from Macbeth would be easily understandable to a normal English speaker. While there are many sentence structures such that the same words could be used to convey Shakespeare's original meaning:

   > It stood doubtful; like two spent swimmers clinging together and choking each other.

2. Give examples and counter-examples of sentences in English (or any other language) that would support theories of constant information rate (try to think of examples that are different from those provided in the slides!)

   Consider the following passage:

> The quality of programmers is a decreasing function of the density of **go to** statements in their programs. I recently discovered the disastrous effects the **go to** statement has, and became convinced the **go to** statement should be abolished from all "higher level" languages. I did not attach importance to this; I submit considerations for publication because when the subject turned up I was urged to do so.

Now consider the original passage from Edskar Dijkstra's "Go To Statement Considerd Harmful":

> For number of years I have been familiar with the observation that the quality of programmers is a decreasing function of the density of **go to** statements in the programs they produce. More recently I discovered why the use of the **go to** statement has such disastrous effects, and I became convinced that the **go to** statement should be abolished from all "higher level" programming languages (i.e. everything except, perhaps, plain machine code). At that time I did not attach too much importance to this discovery; I now submit my considerations for publication because in very recent discussions in which the subject turned up, I have been urged to do so.

Throughout this quote, Edgskar is discussing a topic which (at the time) was not commonly discussed. Furthermore, this is an introduction and so people have little contextual information about which to decrease the information rate. Therefore, the entropy of every sentence is high. As such, in the original quote, connectives are introduced at every opportunity to decrease the entropy rate as far as possible. The modified version removed all connectives to increase the entropy rate and as such is harder to parse.

2 Formal Languages and Learnability

1. Consider a rigid classic categorial grammar $C_{cg} = (\Sigma, P_r, S, \mathcal{R})$ where $P_r = \{S, X\}$, $\Sigma = \{a, b, c, d, e\}$ and $S = S$. If a, b have type X and you know that $abc \in \mathcal{L}(C_c)$, $abdc \in \mathcal{L}(C_{cg})$, $ebc \in \mathcal{L}(C_{cg})$, give possible types for each of c, d and e?

$$\{(c, X \setminus X \setminus S), (d, X \setminus X), (e, X)\}$$

2. Explain why a finite class of finite languages is learnable within Gold's paradigm.

   Under Golds paradigm, if the language which we are recognising is finite then the learner will be fed exhaustive data – every sample in the data. It's therefore trivial to build a learning function $\mathcal{F}$ which output the language containing the set S of strings s which $\mathcal{F}$ has been fed.

3. Describe a learning paradigm where a learner could learn from two sources simultaneously (bilingual)

   Define a grammatical system $(\mathcal{H}, \Omega, \mathcal{L})$ as follows:

   - $\mathcal{H}$ is the hypothesis space – the set of all pairs of languages (L_1, L_2) of the form $((\mathcal{N}_1, \Sigma_1, \mathcal{P}_1, S_1), (\mathcal{N}_2, \Sigma_2, \mathcal{P}_2, S_2))$ where $\mathcal{P}$ is restricted to allow only grammars below a certain complexity.

- Ω is the set of strings which are visible. For example, the set (Σ_1^*, Σ_2^*).
- $\mathcal{L}$ is a function from $\mathcal{H} \to \Omega$ which specifies the set of languages which each of the grammars can generate.

The learning paradigm is then specified as follows:

- A pair $(L_1, L_2) \in \mathcal{L}$ is selected as the target language
- All samples (s_1, s_2) are taken from the hypothesis space Ω
- The learner receives samples (s_1, s_2) from the learner in an infinite sequence.
- After receiving each sample, the learner produces a hypothesis $(G_{i_1}, G_{i_2}) \in \mathcal{G}$.
- Learning is successful when (G_1, G_2) have both been identified in the limit.

3 Information Theory

1. A binary symmetric channel is one where the input x_i and the output y_i are both in $\{0, 1\}$. The channel is characterised by p the probability that an input bit is transmitted as the opposite bit. If q is the probability that the source sends $x = 0$, and $1 - q$ the probability of $x = 1$, show tht the mutual information is maximised when zeros and ones are transmitted with equal probability (i.e when $q = 0.5$).

   Using the definition for mutual information and distributivity of partial differentiation over addition:

$$I(X_i; Y_i) = H(Y_i) - H(Y_i|X_i)$$
$$\frac{\partial I(X_i; Y_i)}{\partial q} = \frac{\partial H(Y_i)}{\partial q} - \frac{\partial H(Y_i|X_i)}{\partial q}$$

   I will now consider each case separately:

$$\begin{aligned}
H(Y_i) &= -\sum_{y_i \in \{0,1\}} p(y_i) \lg p(y_i) \\
&= -(pq + (1-p)(1-q)) \lg(pq + (1-p)(1-q)) - (p(1-q) + (1-p)q) \lg(p(1-q) + (1-p)q) \\
&= -(1 - p - q + 2pq) \lg(1 - p - q + 2pq) - (p + q - 2pq) \lg(p + q - 2pq) \\
\frac{\partial H(Y_i)}{\partial q} &= -(2p-1) \lg(1 - p - q + 2pq) - (2p - 1) + (2p - 1) \lg(p + q - 2pq) + (2p - 1) \\
&= (2p - 1) \lg \frac{p + q - 2pq}{1 - p - q + 2pq}
\end{aligned}$$

$$\begin{aligned}
H(Y_i|X_i) &= -\sum_{x_i \in \{0,1\}} \sum_{y_i \in \{0,1\}} p(x_i, y_i) \lg p(y_i|x_i) \\
&= -p(1-q) \lg p - pq \lg p - (1-p)q \lg(1-p) - (1-p)(1-q) \lg(1-p) \\
\frac{\partial H(Y_i|X_i)}{\partial q} &= p \lg p - p \lg p - (1-p) \lg(1-p) + (1-p) \lg(1-p) \\
&= 0
\end{aligned}$$

Substituting into the original formula gives:

$$\begin{aligned}
\frac{\partial I(X_i|Y_i)}{\partial q} &= (2p-1)\lg\frac{p+q-2pq}{1-p-q+2pq} \\
0 &= (2p-1)\lg\frac{p+q-2pq}{1-p-q+2pq} \\
0 &= \lg\frac{p+q-2pq}{1-p-q+2pq} \\
1 &= \frac{p+q-2pq}{1-p-q+2pq} \\
1-p-q+2pq &= p+q-2pq \\
0 &= 4pq-2p-2q+1 \\
0 &= (2p-1)(2q-1) \\
0 &= 2q-1 \\
q &= \frac{1}{2}
\end{aligned}$$

2. Using the processed Alice in Wonderland file write some simple code to generate some good candidates for nonsense words by:

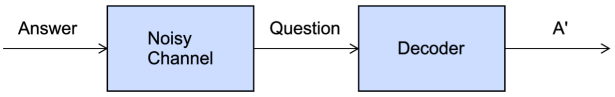
   - finding the probability distribution defined by a bigram langauge model
   - generating some words using the probability distribution
   - selecting the 10 words whose information rate is lowest

word	information rate
a	0.023684605006745612
i	0.01536501274171789
ani	0.007926312071608285
thchi	0.007209095652393102
thi	0.004004583532064527
ini	0.0031108113059844437
ali	0.002623860223613271
wii	0.002328867707379849
ati	0.0018429980959495836
bei	0.0017422835462293014

3. Describe how you could frame the following tasks as noisy channel problems:

   - automatically answering questions

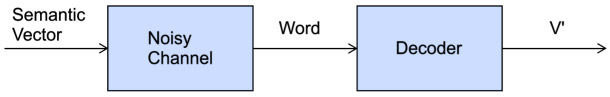
     We could consider the following framework:



     In this framework, we phrase the problem of answering a question as "the answer which is most likely to have caused the user to answer the question". This allows us to use Bayes' Rule – since $P(Q|A)$ is likely easier to compute than $P(A|Q)$. For example, this problem would become trivial if we restricted the format of answers to yes/no and modelled questions as a bi-gram model while the other direction would remain almost intractable.

- disambiguating multiple senses of a word

  Consider the following framework:



  We represent this problem as "trying to establish the semantic meaning of a word from the word presented". Phrasing this as "establishing the vector which was most likely to have generated that word" is far easier than directly approaching the reverse direction.

4 Distributional Models

1. Describe how you might use word distributions to compare the similarity of two characters in a text. What might any *similarity* be telling us about the characters.

   I would build a vector representation of each of the characters – the simplest approach would use a count model, although a predict model would yield better results. I would then consider the unit vector in the direction of the representation of the characters.

   An even better approach to building a vector representation would use an approach similar to that taken by a GNN. Consider each word to be a node and let there be an edge $u \to v$ if word u occurs close to word v with the weight of the edge as the number of co-occurrences of v near u divided by the total number words which occur near u. Then let the vector representation of a node be an aggregation of the count model representation of all adjacent nodes (possibly after some dimensionality reduction i.e from the encoder of an encoder-decoder architecture).

5 2022 Paper 7 Question 5

https://www.cl.cam.ac.uk/teaching/exams/pastpapers/y2022p7q5.pdf

Most of this was done under exam conditions and in exam time. Part of (b) is in italics – this was done when typesetting the question. Apologies for the strange typesetting of probabilities. That was done in exam time.

(a) You have the following sentences translated from English into Triposi.

English	Triposi
she drinks water	mwamni sileg
the rain soaks the teacher	sileng mworob sesesrakan
the teacher drinks here	sesesrakan mwamni mwabma
the teacher keeps drinking	sesesrakan mwatbo mwamni

(i) Translate the following English sentences into Triposi:
the rain keeps soaking the teacher
she keeps drinking water here

sileng mworob mwabma sesesrakan

mwatbo mwamni mwamba

(ii) Describe how we can calculate the likelihood of a translation using the noisy channel framework: you will need to give and explain the equation for decoding

from one language into another, and explain how you can obtain the information needed to carry out the calculations.

The noisy channel framework is as follows: Input X sent across a noisy channel as form Y (with some probability of corruption). The receiver then receives Y and has to estimate X' – the X with maximal probability of having generated Y.

We can use this to translate from one language to another using the following framework:

English -> Triposi -> English'

So we try to find the English sentence which was most likely to have generated the Triposi sentence.

Firstly, we design a model and then fit it using a corpus of training data (or prior knowledge about the triposi grammar).

We then take a data-science approach by using Bayes rule to find a maximum apriori estimator for English' given the Triposi.

$$English' = \text{argmax}_{English} P(English|Triposi) = \text{argmax}_{English} \frac{P(Triposi|English)P(English)}{P(Triposi)}$$

We can find this either by numerical optimisation (gradient search or a known formula) or mathematically by differentiation.

(iii) What problem do the following underlined English words present, given the training data we have so far, and how can you still translate them into Triposi with a machine? *the <u>teachers</u> drink <u>Perrier</u>*

The underlined English words are not present in the training corpus. This means distributional models will be unable to even represent their existence – let alone translate them! And any model taking a learning approach will not have a means of translation.

We could use a distributional semantics to build a predict language model, which represents each word in a vector space. Then, we could replace the absent words with their closest English word for which we have a translation. This would largely preserve meaning – although not entirely. The likely translation would be "the teacher drinks water" which captures some but not all of the essence of the sentence.

(b) You know that your optical character recognition model has made errors on every predicted instance of 'e'. Explain what information you need in order to automatically correct these errors with a noisy channel approach.

We need to know (or have an estimate of) the *probability* of different types of errors occurring – the probability of "e" being mistranslated to any other letter. The noisy channel approach attempts to discover $X'\text{argmax}_X P(X|Y)$ – its not possible to do this without knowing what the error rate of the channel is.

Furthermore we must also know the prior probability of the letter 'e' being sent across the channel. A good model would take this probability given the context (i.e using a 2nd order model), however any model would be sufficient. Bayes rule needs priors to work.

$$English' = \text{argmax}_{English} P(English|Triposi) = \text{argmax}_{English} \frac{P(Triposi|English)P(English)}{P(Triposi)}$$

(c) A signaller sends you Morse code messages, but you know that they send a dot when they should send a dash two times in five, and a dash instead of a dot three times in

ten. You also know that they use the character M (−−) 3 times in 100, N (−·) and I (··) 7/100 and A (·−) 8/100.

You receive the message "·· −· ·−". What is the likelihood that it represents MIN, MAN, NAN or AIM?

Note that I use likelihoods – the actual probability depends on the probability of the message being sent at all – for which we have insufficient information.

The probability of a dash transitioning to a dot is 0.4

The probability of a dot transitioning to a dash os 0.3

The probability of M is 0.03

The probability of N is 0.07

The probability of I is 0.07

The probability of A is 0.08

We can use Bayes rule to work out the probability.

MIN is (– .. -.) The probability of MIN|.. -. .- is

$$\frac{P(Message|Min)P(Min)}{P(M)} = \kappa P(Message|Min)P(Min) = \kappa(0.4{*}0.4{*}0.3{*}0.7{*}0.4{*}0.3){*}(0.03{*}0.07{*}0.07)$$

MAN is (– .. -.) The probability of MAN|.. -. .- is

$$\frac{P(Message|Man)P(Man)}{P(M)} = \kappa P(Message|Man)P(Man) = \kappa(0.4{*}0.4{*}0.3{*}0.4{*}0.4{*}0.3){*}(0.03{*}0.08{*}0.07)$$

NAN is (– .. -.) The probability of NAN|.. -. .- is

$$\frac{P(Message|Nan)P(Nan)}{P(M)} = \kappa P(Message|Nan)P(Nan) = \kappa(0.4{*}0.7{*}0.3{*}0.4{*}0.4{*}0.3){*}(0.07{*}0.08{*}0.07)$$

AIM is (– .. -.) The probability of AIM|.. -. .- is

$$\frac{P(Message|Aim)P(Aim)}{P(M)} = \kappa P(Message|Aim)P(Aim) = \kappa(0.7{*}0.4{*}0.3{*}0.7{*}0.4{*}0.6){*}(0.08{*}0.07{*}0.03)$$

(d) Discuss the similarities and differences between the noisy channel and human processing of spoken English.

The noisy channel model (which we have discussed) is a 1st order model. Human processing of spoken English is certainly a 2nd order model – one which takes the full context into account (this greatly decreases ambiguity).

The noisy channel model only takes sentence units into account – spoken English contains extra information such as prosidy (intonation, tone) or disfluency / false starts (which indicate a topic is more challenging).

Furthermore, there is evidence that humans slow their speaking to make the information rate uniform.

The noisy channel model also assumes perfect knowledge – humans do not have perfect knowledge. When we encounter a person for the first time (or a person initiates a conversation) we often struggle to understand them. I believe this is because our priors for what this person may be saying are not set. We therefore expect them to say something totally different. The noisy channel framework would be unable to deal with a situation such as this (or otherwise have an overly simplistic model).