Converting Between Binary and Decimal

What Is Binary?

The binary system is just a different way to represent numbers. It happens to be the system used by computers, because most electronic components use two states: “on” or “off”, which can be represented by a 1 or a 0 respectively.

So, it’s important for computer scientists to understand the relationship between binary and decimal. But before we dive into converting between the two systems, let’s review the decimal number system which we use every day.

How We Count in Decimal

The decimal system is based on ten digits (0-9) and powers of 10. Each position (or “place”) in a decimal number represents a power of 10, depending on its place value. This is how we humans normally count.

When we count, we increment (add one to) the rightmost digit by 1. But when that digit reaches the largest available digit (which is 9 in decimal), we run into a problem: how do we represent a number larger than 9 with only 10 digits?

The solution is to add another decimal place. We can use a combination of multiple digits to represent one number. For example, the number ten is created by combining the digits 1 and 0.

Another way to think about this: imagine there are an infinite number of zeroes to the left of any number. For example, the number 5 can be written as ...00005. We don’t write these leading zeroes because they don’t change the value of the number, but they’re there. When we need to create another decimal place, we’re just bumping an invisible 0 up to a 1, like an odometer on a car.

So if we needed to write out a formula for counting in decimal, it would look something like this:

1. Increment: increase the rightmost digit by 1 until you hit 10.
2. Carry over: if a digit reaches 9 and we want to add 1, we reset it to 0 and then increment the next digit to the left.

If you’re reading this I’m sure you’re very familiar with counting, so why are we going over this? Because the method for counting in binary is exactly the same as counting in decimal! The only difference is that we have two available digits (0-1) instead of ten (0-9).

How to Count in Binary

The binary number system is similar to decimal, but instead of ten digits, it only uses two digits: 0 and 1. The largest digit in binary is 1, just like the largest digit in decimal is 9.

In decimal, each decimal place represents a power of 10: the “ones” place, the “tens” place, “hundreds”, “thousands”, and so on. But in decimal, each position represents a power of 2. So we have the “ones” place, the “twos” place, “fours”, “eights”, “sixteens”, and so on.

This can be hard to wrap your head around at first, but it gets easier when you remember how we count in decimal and apply the same rules.

1. Increment: increase the rightmost digit by 1 until you hit 2.
2. Carry over: if a digit reaches 1 and we want to add 1, we reset it to 0 and then increment the next digit to the left.

So, for example, counting from one to ten in binary looks like this:

`1, 2, 3, 4, 5, 6, 7, 8, 9, 10`

`1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010`

Binary to Decimal Conversion

To convert a binary number to a decimal number:

1. Write down the binary number.
2. Label each bit position from right to left, starting at 1.
3. Multiply each bit based on its position.
4. Add all the results to get the decimal equivalent.

Example: Convert 11001 to Decimal

Step 1. Write the binary number: 11001

Step 2. Label the bit positions:

Step 3. Multiply each bit based on its position.

1 x 16 = **16;** 1 x 8 = **8;** 0 x 4 = **0;** 0 x 2 = **0;** 1 x 1 = **1**

Step 4. Sum of the results:

16 + 8 + 0 + 0 + 1 = **25**

Therefore, 11001 (in binary) equals **25** (in decimal).

Decimal to Binary Conversion

To convert a decimal number to binary:

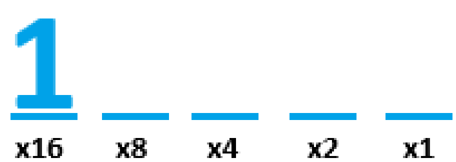
1. Find the highest power of 2 less than or equal to the decimal number and write a 1 in that binary position.
2. Subtract that value from the decimal number.
3. Move to the next lower power of 2.
    a. If the power fits into the remainder, write a 1 and subtract it.
    b. If not, write a 0.
4. Repeat until there's nothing left to subtract.

Example: Convert 19 to Binary

Step 1. Find the highest power of 2 below 19. We can easily find it by counting powers of 2 until we go past 19, then use the number before that.

1, 2, 4, 8, 16, **32**

32 is bigger than 19, which means we're starting with the 16s place. Let's write out all the positions we'll be using, starting at the 16s and working our way down to the 1s. We write a **1** in the 16s position, because 19 is bigger than 16.



Step 2. Subtract and record.

19 - 16 = **3**

Now we just repeat steps 1 and 2 until we hit zero.

8 is bigger than 3 so we write **0**.

4 is bigger than 3 so we write **0**.

2 is smaller than 3, so we write **1** and subtract.

3 - 2 = **1**

1 is equal to 1, so we write **1** and subtract.

`1 - 1 =` **0**

And we're done! 19 (in decimal) = **10011** (in binary).