Modeling legged robots

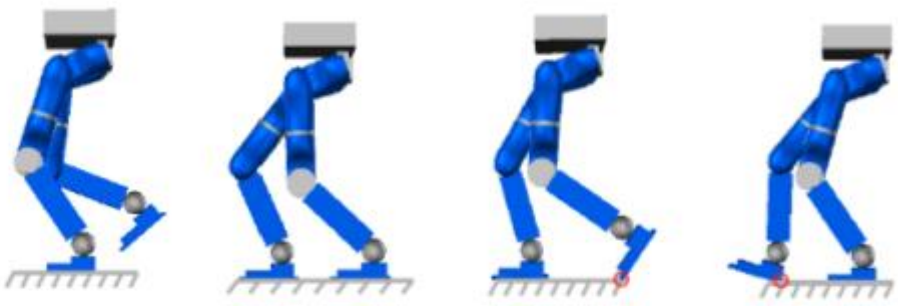
Minimal coordinates

Excess of Coordinates

Structure of the Floating Base Dynamics

Modeling legged robots

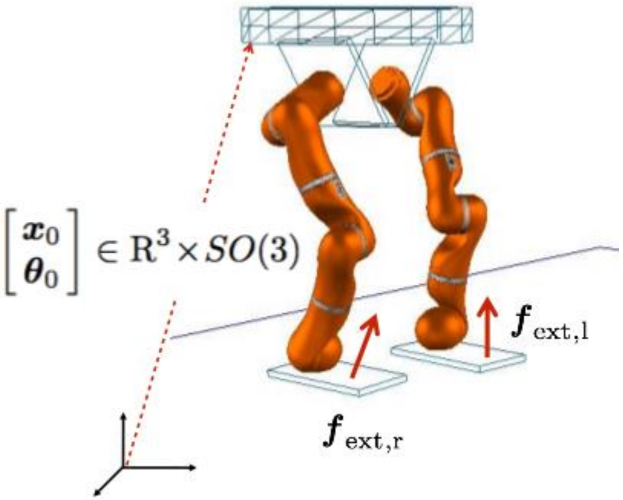
Minimal coordinates



look at these contact configurations as different fixed-base robots, each with a specific kinematic and dynamic models

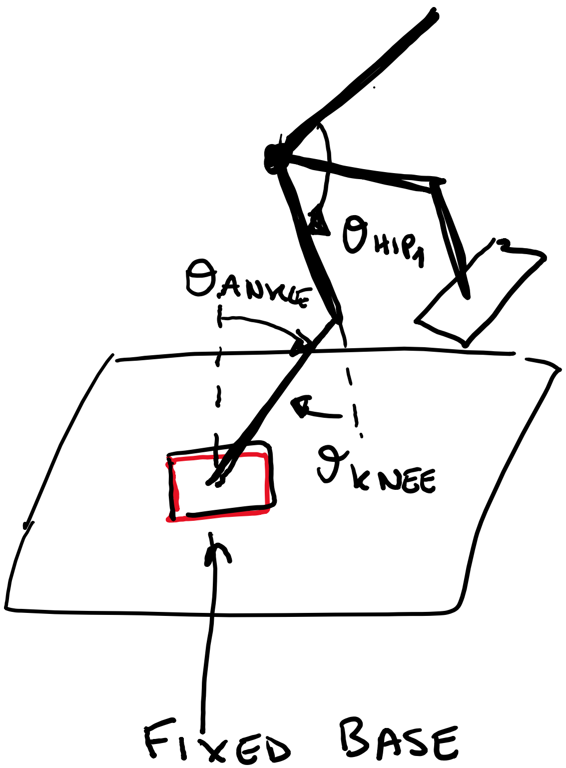
Excess of coordinates

single floating-base system with limbs that may establish contacts



Minimal coordinates

- get the minimum amount of coordinates for each mode:



STANCE MODE

FLIGHT MODE

$$q_{st} = [\theta_{ANKLE}, \theta_{KNEE}, \theta_{HIP}]$$

$$q_{fl} = [x_{HIP}, z_{HIP}, \theta_{TORSO}, \theta_{HIP}, \theta_{KNEE}]$$

⊕ simple dynamics:

$$M(q_J)\ddot{q}_J + R(q_J, \dot{q}_J) = \tau_J$$

$$q_J \in \mathbb{R}^n$$

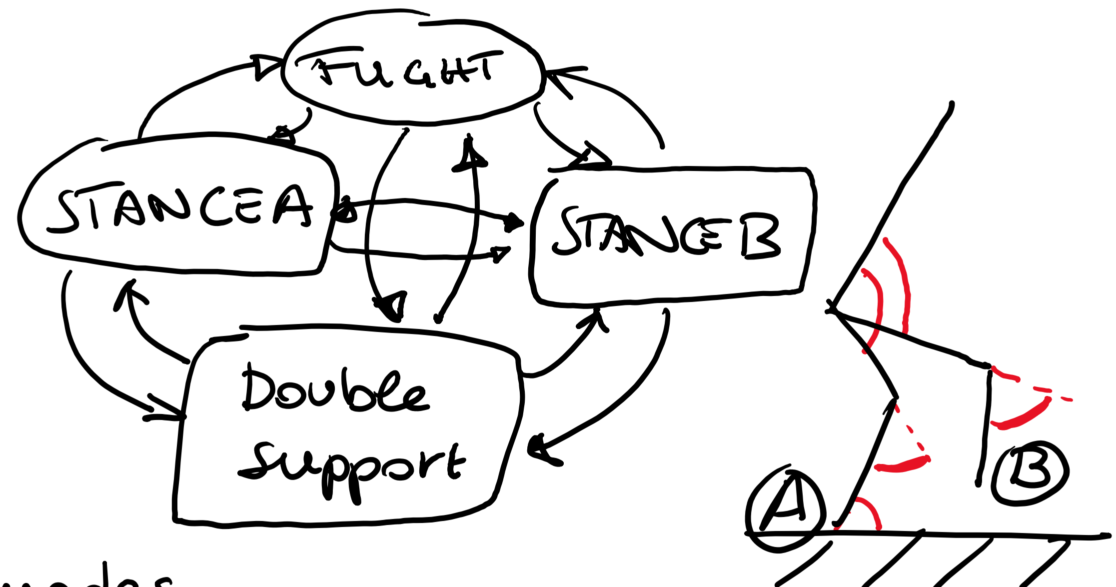
⊖ If put a foot on the ground IT loses 6 DOFs and we would need a different set of generalized coordinates

<u>BIPED</u>

- 4 modes
- 12 Transitions

$\Downarrow$

COMPLEX



<u>QUADRUPEDS</u> : 16 modes...

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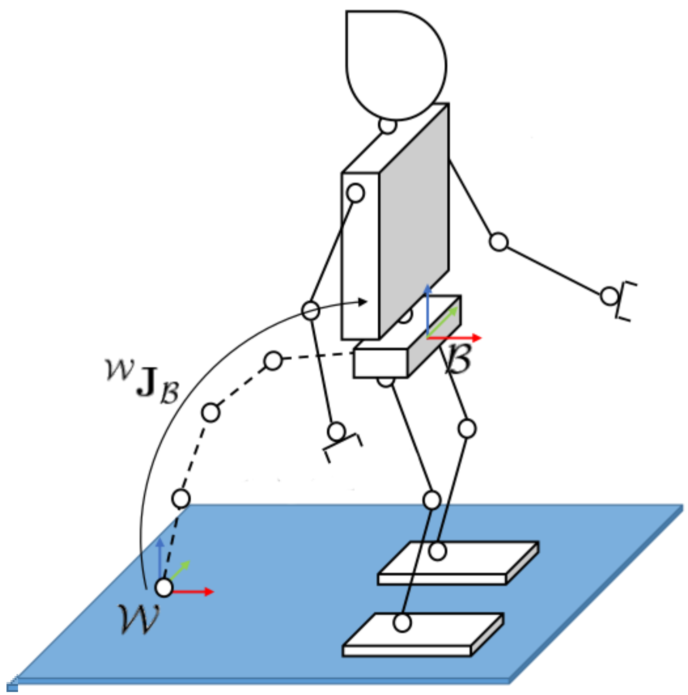


Joint angles are not enough to describe the robot configuration

Legged robots can be modeled as fixed-base robots
Also adding add a 6DoF **virtual kinematic chain** between the World W and the base B

$$q = \begin{bmatrix} q_b \\ q_J \end{bmatrix}$$

Where q_u denotes the configuration of B with respect to W

⇒ each link pose can be retrieved from the pose of the Base and the joint configurations

⇒ each point of the robot can be retrieved from the position / orientation (pose) of the links

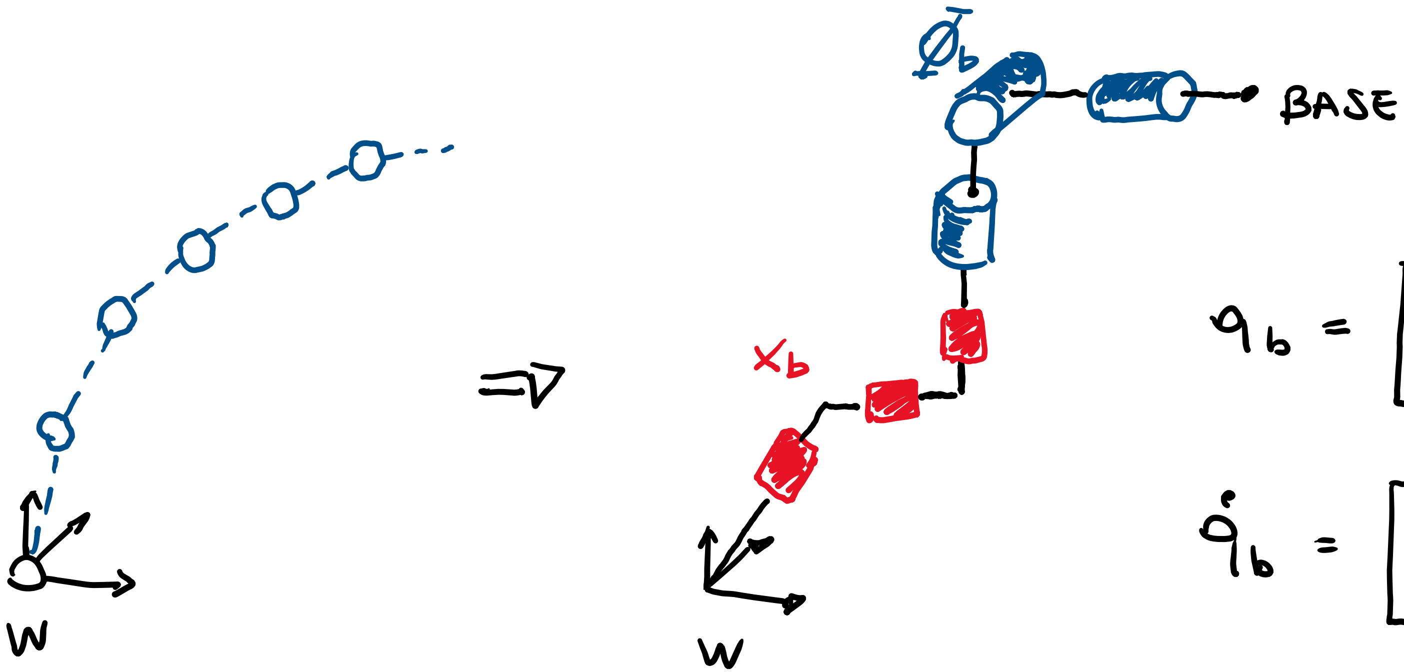
Different representations

Depending on how we parametrize the pose of B we can have different generalized velocities:

(A) VIRTUAL JOINTS

3 PRISMATIC → TRANSLATION $\in \mathbb{R}^3$

3 ROTATIONAL → ORIENTATION $\in \mathbb{R}^3$



EULER ANGLES

$$q_b = \begin{bmatrix} x_b \\ \phi_b \end{bmatrix}$$

$$\dot{q}_b = \begin{bmatrix} \dot{x}_b \\ \dot{\phi}_b \end{bmatrix}$$

EULER RATES

BASE JACOBIAN WITH VIRTUAL JOINTS

The base jacobian maps to the spatial velocity of the base

$$ {}_w V_B = \begin{bmatrix} \dot{x}_b \\ \omega_b \end{bmatrix} = {}_w J_b \, \dot{q}_b \quad , \quad \dot{q}_b = \begin{bmatrix} \dot{x}_b \\ \dot{\phi} \end{bmatrix} $$

$$ \boxed{ {}_w J_b = \begin{bmatrix} I & 0 \\ 0 & J_\omega \end{bmatrix} } \rightarrow \text{maps euler rates } \dot{\phi} \text{ into } \omega $$

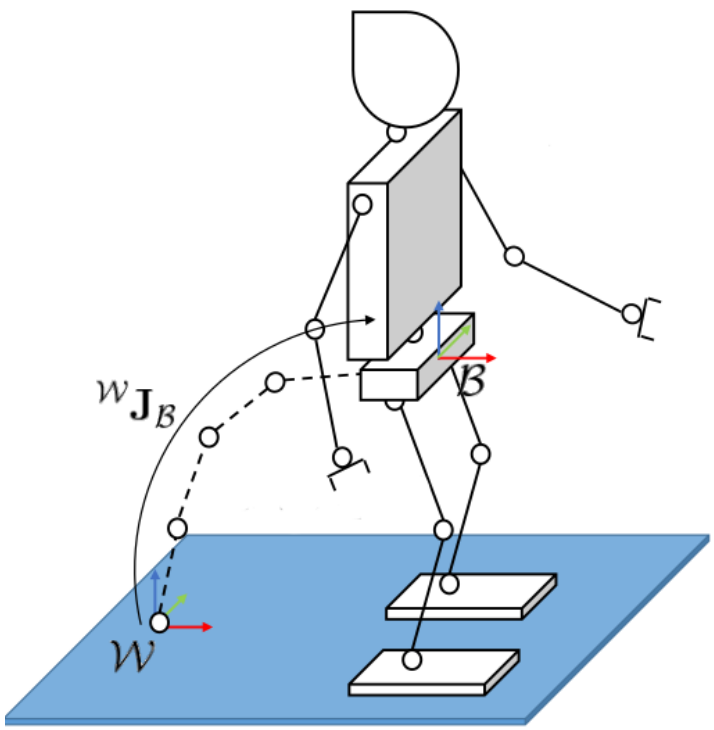
⊕ direct relation between virtual chain kinematics and generalized velocities

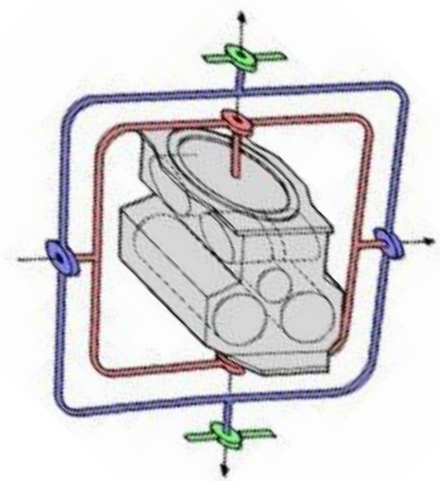
$$ q_b = \begin{bmatrix} x_b \\ \phi \end{bmatrix} , \quad \dot{q}_b = \frac{d}{dt}(q_b) $$

⊕ easy to numerically integrate

$$ q_{b,k+1} = q_{b,k} + dT \, \dot{q}_{b,k+1} $$

⊖ singularity associated to the Euler Angles





GIMBAL LOCK : 2 AXES ALIGNED

(B) FREE FLOATING BODY

$$\dot{q}_b = \begin{bmatrix} \dot{x}_b \\ \omega_b \end{bmatrix} \qquad q_b = \begin{bmatrix} x_b \\ ? \end{bmatrix} \rightarrow \text{depends on the parametrization}$$

(+) Jacobian is the identity

$$\dot{q}_b = {}_W V_B \Rightarrow \boxed{{}^W J_B = I_{6\times 6}}$$

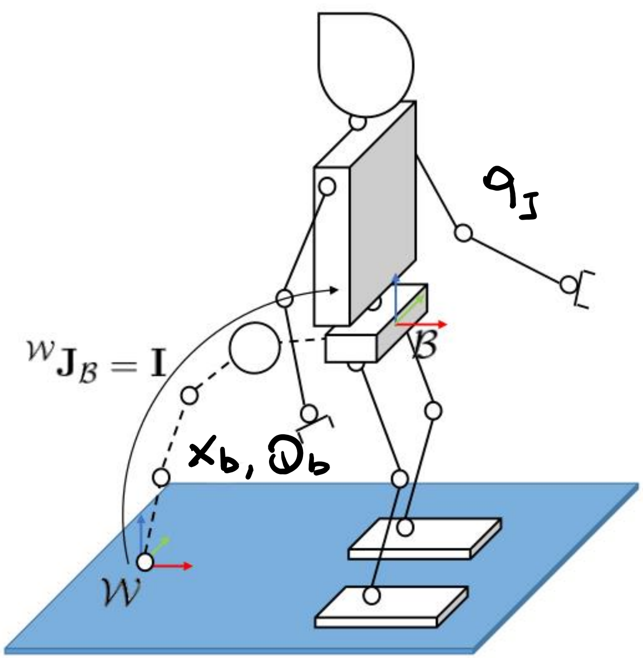
SPATIAL VELOCITY

- if $? =$ Quaternion Q_b

(+) Non minimal representation

→ No singularities

(−) Pose and velocity have not the same size



$$q_b = \begin{bmatrix} x_b \\ Q_b \end{bmatrix} \in \mathbb{R}^7 \qquad \dot{q}_b = \begin{bmatrix} \dot{x}_b \\ \omega \end{bmatrix} \in \mathbb{R}^6$$

$$\dot{q}_b \neq \frac{d}{dt} q_b \; !$$

⊖ Numerical integration is more complicated

$$q_{b,k+1} \quad q_{b,k} + dT \begin{bmatrix} \dot{x}_{b\,k+1} \\ \frac{1}{2} M_{k+1} \cdot Q_k \end{bmatrix}$$

Quaternion product

$M = [0, \omega]$

Therefore, using quaternions to represent orientation:

ROBOT CONFIGURATION: $q = (q_b, q_J) \in \mathbb{R}^{n+7}$

$\hookrightarrow q_b = (x_b, Q_b) \in \mathbb{R}^7$

slight liberty of notation dim $q \neq$ dim $\dot{q}$

ROBOT VELOCITY: $\dot{q} = (\dot{q}_b, \dot{q}_J) \in \mathbb{R}^{n+6}$

$\hookrightarrow \dot{q}_b = (\dot{x}_b, \omega_b) \in \mathbb{R}^6$

ROBOT ACCELERATION: $\ddot{q} = (\ddot{q}_b, \ddot{q}_J) \in \mathbb{R}^{n+6}$

$\hookrightarrow \ddot{q}_b = (\ddot{x}_b, \dot{\omega}_b) \in \mathbb{R}^6$

The full floating base dynamics:

$$M(q)\ddot{q} + h(q,\dot{q}) = S^T \tau + J_c(q)^T f$$

$\ddot{q} \in \mathbb{R}^{n+6}$

$\dot{q} \in \mathbb{R}^{n+6}$

$\tau \in \mathbb{R}^{n}$ $\longrightarrow$ $S^T = \begin{bmatrix} 0_{6\times n} \\ I_{n\times n} \end{bmatrix}$ $\rightarrow$ BASE IS UNDERACTUATED

$+$ CONTACT CONSTRAINTS

$$J_c \dot{q} = 0 \implies J_c \ddot{q} + \dot{J}_c \dot{q} = 0$$

In matrix form:

$$\begin{bmatrix} M & -J_c^T \\ J_c & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ f \end{bmatrix} = \begin{bmatrix} S^T \tau - h \\ -\dot{J}_c \dot{q} \end{bmatrix}$$

Summary minimal /excess of coordinates

MINIMAL COORDINATES

- different # DOFs in each contact mode

$$q_{FL} = [x_{HIP}, z_{HIP}, \theta_{HIP}, \theta_{KNEE}, \theta_{TORSO}]$$

$$q_{st} = [\theta_{ANKLE}, \theta_{KNEE}, \theta_{HIP}]$$

- Treatment of Transitions between modes
- contact constraints are embedded in the choice of coordinates
- suitable for systems with simple contact scenarios

EXCESS COORDINATES

- same # DOFs (use only one coordinate convention

$\Downarrow$

simplified code

- different # of constraints (J_c changes)

$$\begin{bmatrix} M & -J_c^T \\ J_c & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ f \end{bmatrix} = \begin{bmatrix} S^T\tau - h \\ -\dot{J}_c \dot{q} \end{bmatrix}$$

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STRUCTURE OF FB DYNAMICS

A floating base robot is a robot that has an underactuated base that is “floating” in the space, therefore we can partition the dynamics into actuated and underactuated parts, without “trusters” the Lagrangian dynamics has this structure:

$$\underbrace{\begin{bmatrix} M_b(q) & M_{bJ}(q) \\ M_{Jb}(q) & M_J(q) \end{bmatrix}}_{M} \begin{bmatrix} \ddot{q}_b \\ \ddot{q}_J \end{bmatrix} + \underbrace{\begin{bmatrix} h_b(q,\dot{q}) \\ h_J(q,\dot{q}) \end{bmatrix}}_{h} = \begin{bmatrix} 0 \\ \tau_J \end{bmatrix} + \begin{bmatrix} J_{cb}^T(q)\, f \\ J_{cJ}^T(q)\, f \end{bmatrix}$$

f: expressed in frame C with origin in contact and aligned with W

J_c: maps robot velocity into a twist at the contact

linear velocity of contact frame C $\leftarrow$

$$\begin{bmatrix} \dot{x}_f \\ \omega_f \end{bmatrix} = J_c \dot{q}$$

angular velocity of contact frame C $\leftarrow$

- J_c has the following structure:

$$J_c = \begin{bmatrix} J_{cb} & J_{cJ} \end{bmatrix} \in \mathbb{R}^{6 \times n+6}$$

$$\hookrightarrow \quad J_{cb} = \begin{bmatrix} I_{3\times3} & -[x_f - x_b]_\times \\ 0 & I_{3\times3} \end{bmatrix} \in \mathbb{R}^{6\times6}$$

$$J_{cJ} = \frac{\partial (x_f - x_b)}{\partial q}$$

- REMARK: all quantities should be expressed in the same frame (eg inertial)
- Mass matrix is partitioned as follows:

$$M = \begin{array}{c c} & \begin{array}{cc} 6 & n \end{array} \\ \begin{array}{c} 6 \\ n \end{array} & \begin{bmatrix} M_b & M_{bJ} \\ M_{bJ}^T & M_J \end{bmatrix} \end{array}$$

$\in \mathbb{R}^{(n+6)\times(n+6)}$

MOMENTUM MATRIX (JACOBIAN OF THE ARTICULATED BODY MOMENTUM): $A \in \mathbb{R}^{6\times(n+6)}$

$$h = A\dot{q} \in \mathbb{R}^6$$

JOINT MASS MATRIX $\in \mathbb{R}^{n\times n}$

COMPOSITE RIGID BODY INERTIA $\left(\sum_{L \in LINKS} {}_W X_L^* \, I_L \, {}_L X_W\right) \in \mathbb{R}^{6\times 6}$

AKA LOCKED INERTIA OF THE MULTI-BODY SYSTEM

PROPERTIES OF $\mathcal{M}$:

- symmetric $M^T = M$
- positive definite

- $R = C(q, \dot{q})\dot{q} + G$

  ↳ C is chosen such that $\dot{M} - 2C$ is skew symmetric (i.e. $A + A^T = 0$)

- $G = -M(q) \begin{bmatrix} {}^w g \\ 0_{3\times 1} \\ 0_{n\times 1} \end{bmatrix}$ → gravitational acceleration vector expressed in inertial frame

- KINETIC ENERGY

$$K = \frac{1}{2} \sum_{L \in LINKS} V_L^T I_L V_L = \frac{1}{2} \dot{q}^T M \dot{q}$$

Important facts about the floating base dynamics

$$\begin{bmatrix} M_b(q) & M_{bJ}(q) \\ M_{Jb}(q) & M_J(q) \end{bmatrix} \begin{bmatrix} \ddot{q}_b \\ \ddot{q}_J \end{bmatrix} + \begin{bmatrix} h_b(q,\dot{q}) \\ h_J(q,\dot{q}) \end{bmatrix} = \begin{bmatrix} 0 \\ \tau_J \end{bmatrix} + \begin{bmatrix} J_{cb}^T(q)\, f \\ J_{cJ}^T(q)\, f \end{bmatrix}$$

Fact 1: joint torques τ only affect joint coordinates! They can’t have a direct influence on the part on the floating base dynamics, until the system comes in contact with its environment.

Fact 2: If we neglect inertial/coriolis couplings, the floating base can be controlled only through the action of contact forces f

Fact 4: the contact forces f are limited by physical laws

Fact 5: also joint torque can be limited

Create limitations in robot ability to control its displacements

Fact 6: the underactuated part (first 6 rows) coincides with the **newton-euler equations** of motions. They link the variation of the linear and angular momentum of the whole system to the contact forces. They can also be obtained by balancing forces and moments acting on the robot as a whole.

Fact 7: when supplying a certain amount of joint torque $\boldsymbol{\tau}$, the environment reacts by producing the contact forces $\mathbf{f}$. Those very same forces act on the under-actuated dynamics to enable the robot to move inside the environment.

The locomotion problem can then be split into two consecutive stages:

1) find the force trajectories which drive the **under-actuated** dynamics and resulting centroidal trajectory

2) compute the required joint torque trajectory from the **actuated** dynamics, under the hypothesis of non sliding contacts

References:

- P. Wensig, AME 60621 – Optimization-Based Robotics (L06-L07): https://sites.nd.edu/pwensing/ame-60621/
- G. Oriolo, Autonomous and Mobile Robotics – Humanoid Robots 2: http://www.diag.uniroma1.it/oriolo/amr/slides/HumanoidRobots2_Slides.pdf