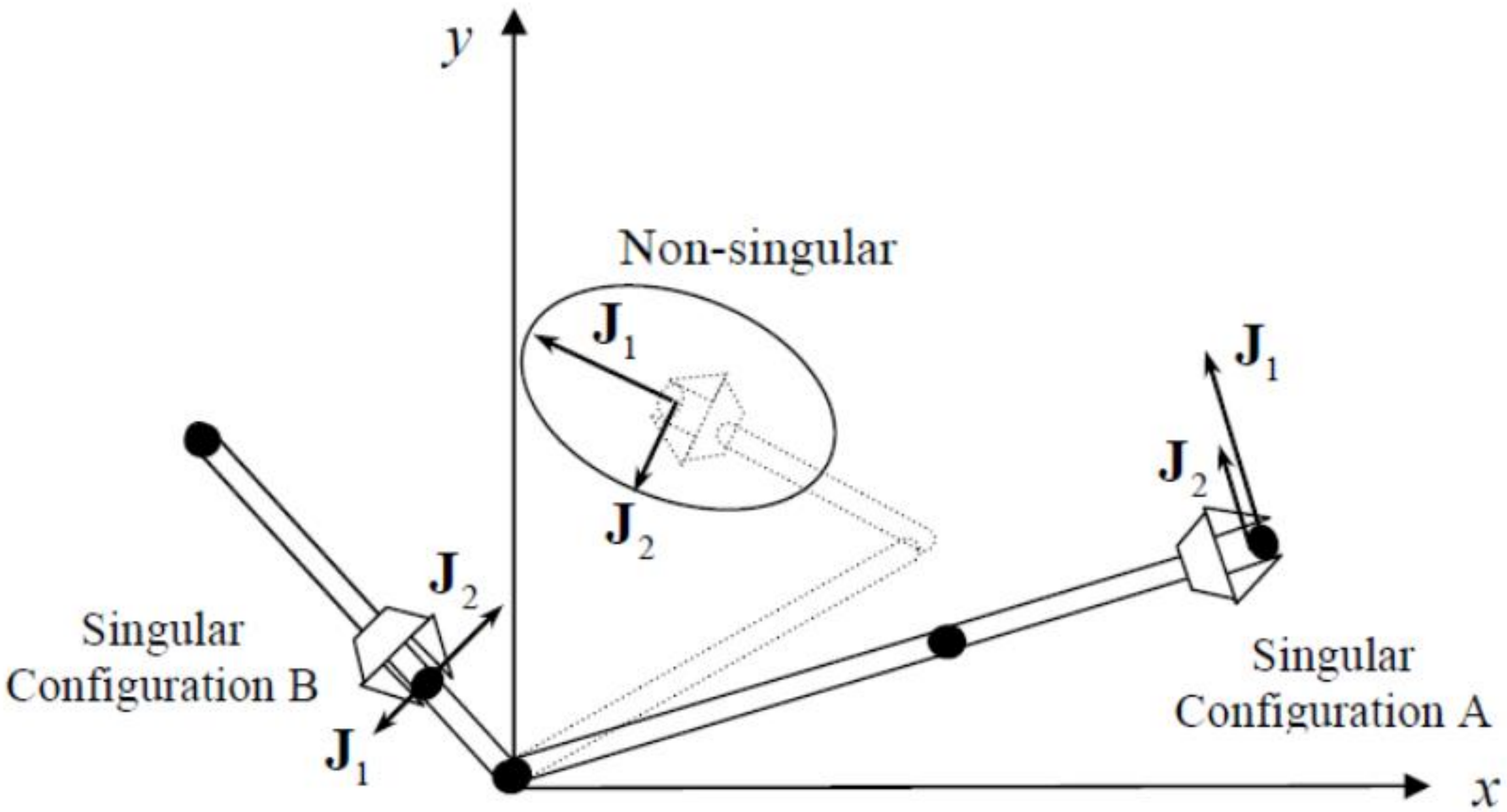
Extra / Reduced Mobility

Redundant Manipulators

Singularities

REDUNDANCY VS SINGULARITY

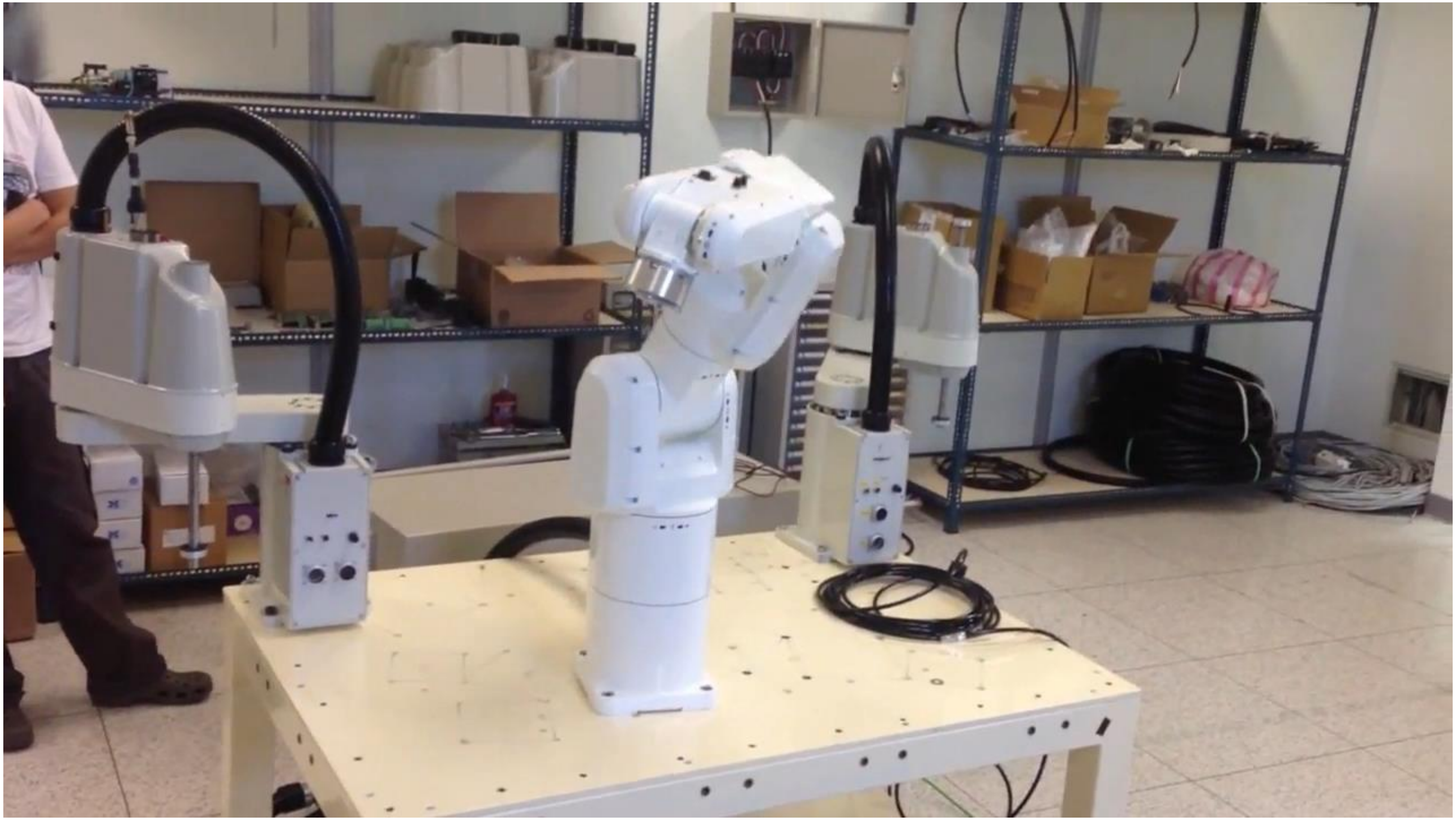


Redundancy

you have more mobility than you need

Singularity

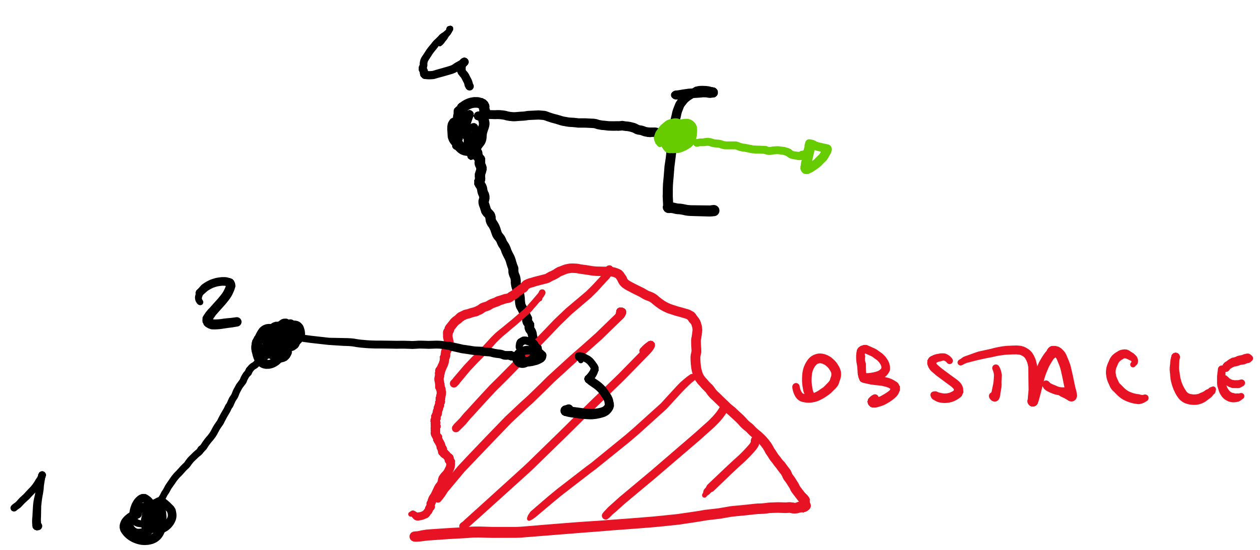
you have less mobility than you need

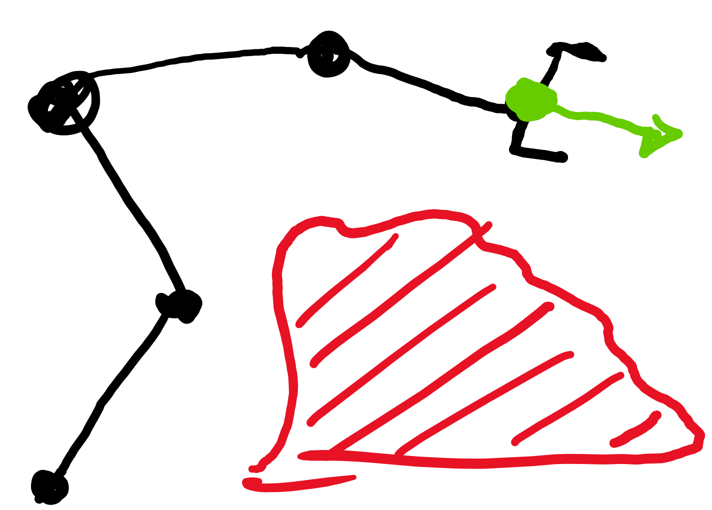


REDUNDANT MANIPULATOR

- The robot has more DoFs than the dimension of the Task (e.g. control position / orientation of end-effector)
- The redundant DoF can be used to increase DEXTERITY

EXAMPLE: 2D PLANE → end effector (2) position
(1) orientation

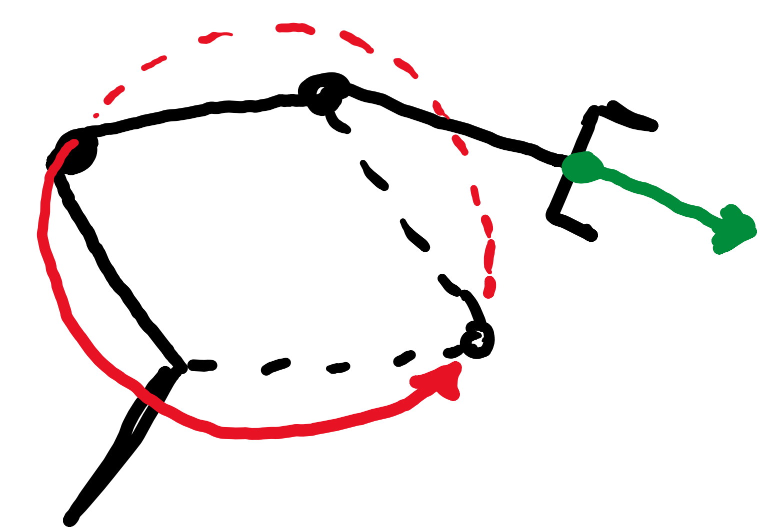




Task (m) : 2D
Robot (n) : 4 DoFs

$n > m$

⇒ There are ∞^{4-3} correspond To same position / orientation of EE



INVERSE KIN. FOR REDUNDANT MANIPULATORS

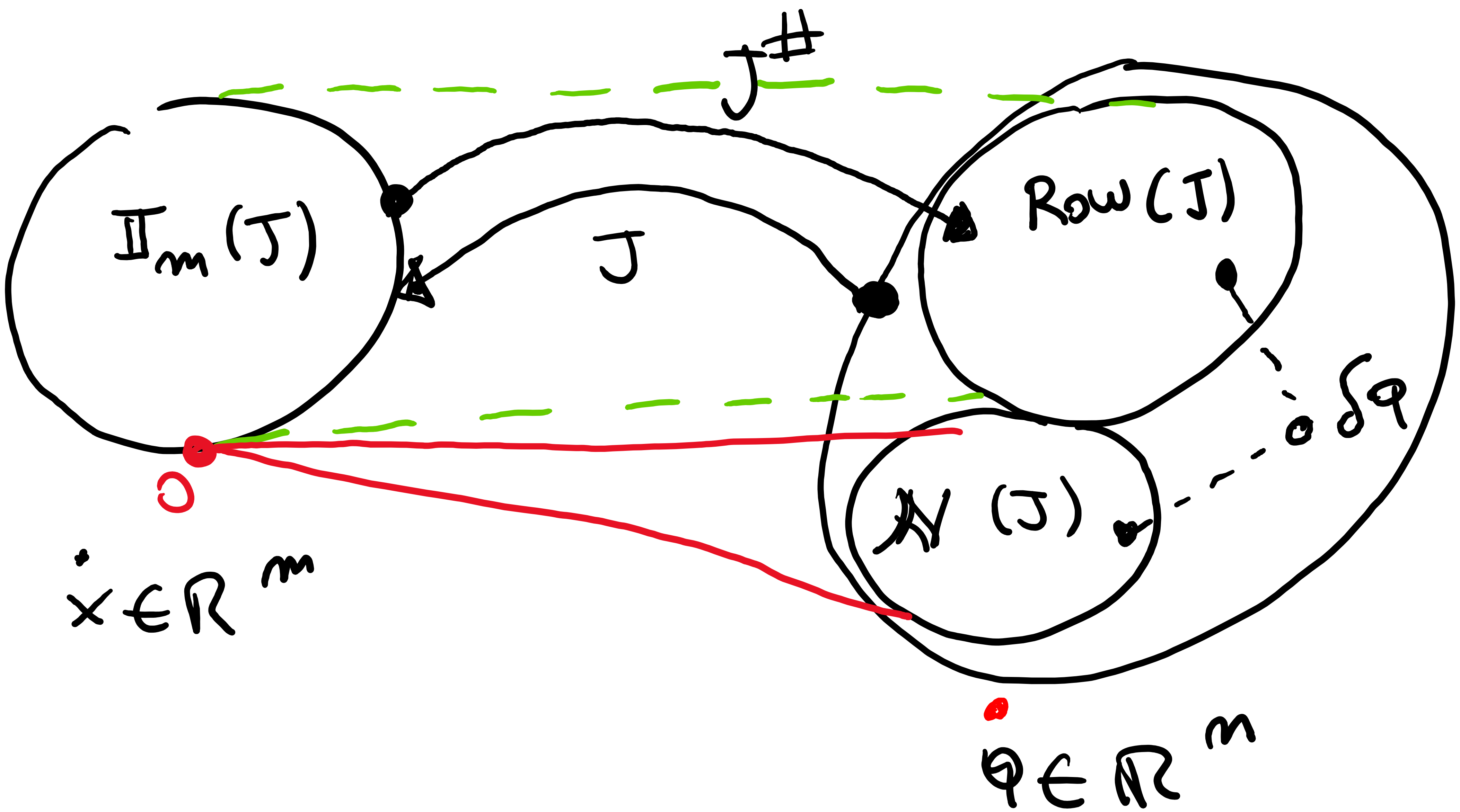
$\delta x = J(q)\,\delta q$

$$J = \begin{array}{c c} & n \\ m & \begin{bmatrix} \# & \# & \# & \# \\ \# & \# & \# & \# \\ \# & \# & \# & \# \end{bmatrix} \end{array}$$

Jacobian is rectangular (FAT) $m < n$

$$\delta q = \underset{\uparrow}{J^{\#}(q)}\,\delta x + \underbrace{\left[I - J^{\#} J \right]}_{\text{Null-space projector}} \delta q_0$$

generalized pseudo-inverse



$\exists\; N(J)$

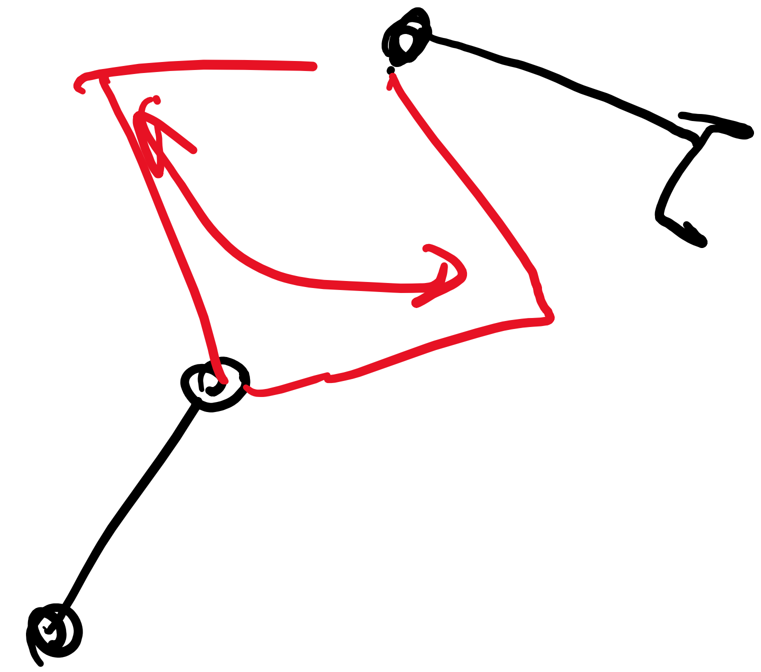
- $\dim \mathrm{Row}(J) + \dim N(J) = n$
- $R(J) \perp N(J)$

NULL SPACE JOINT VELOCITIES

$$0 = J\,[\,I - J^{\#}J\,]\,\delta q_0$$

$\dot{x}$

↳ motion of joints that result in no motion at the end-effector



$\oplus$ We can use δq_0 to do other tasks (e.g. increase dexterity / avoid obstacles)

MAPPING END-EFFECTOR FORCES

① Non-redundant manipulator $(m = n)$

$$\tau = J^T f$$

② Redundant manipulator $(m < n)$

$$J^T = \begin{array}{cc} & m \\ n & \begin{bmatrix} \# & \# & \# \\ \# & \# & \# \\ \# & \# & \# \\ \# & \# & \# \end{bmatrix} \end{array}$$

Jacobian^T is rectangular (SKINNY)

$$\tau = \underset{\uparrow}{J^T f} + \underset{\uparrow}{\left[I - (J^{T\#})^{\#} J^{T\#} \right] \tau_0}$$

Torques that produce force at end-effector

Torques that do not produce force at the end-effector (τ_{NS})

verify That:

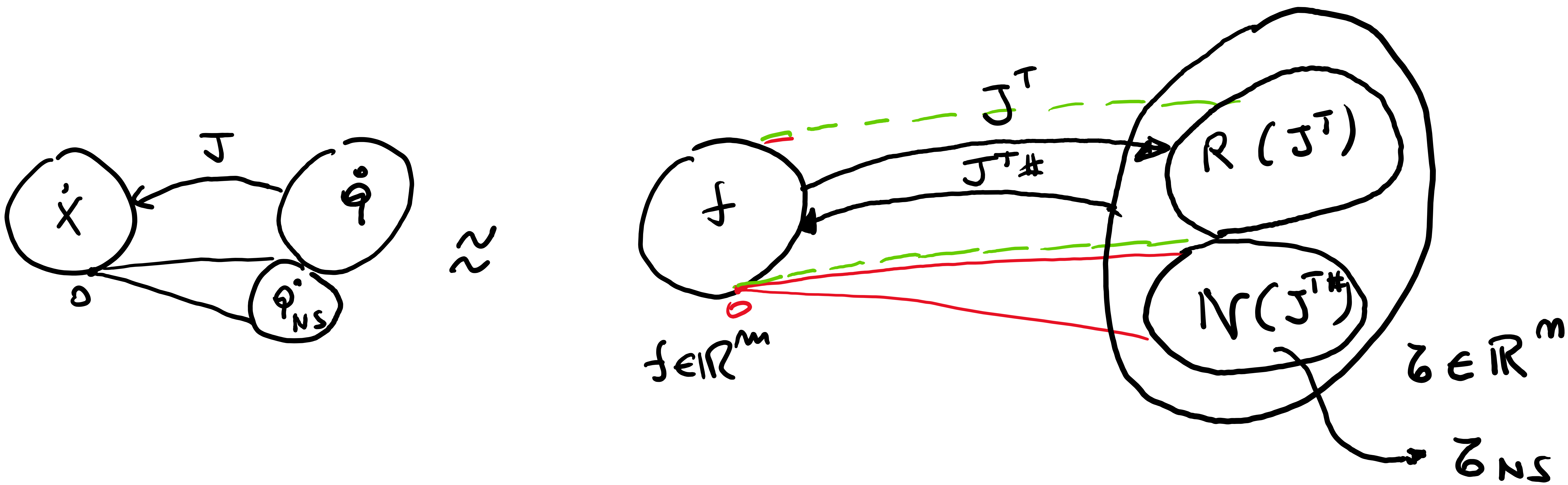
$$0 = J^{T\#} \cdot \left[I - J^{T} J^{T\#} \right] \tau_0$$

f

KINETO - STATIC DUALITY

0

- Range of J^T is the space of joint torques that can balance end-effector forces



$J^{T\#}$: generalized pseudo-inverse of J^T

PSEUDO-INVERSE COMPUTATION

Ⓐ FAT CASE

$$A = {}_m\begin{bmatrix} & \overset{n}{} & \end{bmatrix}$$

$$A^{\#} = ?$$

$$A^{\#} = A^T (A A^T)^{-1}$$

(AA^T) = [ ] invertible

RIGHT PSEUDO-INVERSE

$$AA^{\#} = [\;][\;] = [\;] = I_{m \times m}$$

Ⓑ SKINNY CASE

$$A = {}_m\begin{bmatrix} \overset{n}{} \end{bmatrix}$$

$$A^{\#} = [\;]$$

$$A^{\#} = \underbrace{(A^T A)}_{[\;]\ \text{invertible}}{}^{-1} A^T$$

LEFT PSEUDO-INVERSE

$$A^{\#} A = [\;][\;] = [\;] = I_{n \times n}$$

why not $A^{\#}A$ for fat matrix?

$$A^T(AA^T)^{-1}A \neq I_{m\times m}$$

$\hookrightarrow = A^{-T}A^{-1}$ only for A square but A is rectangular!

∞ PSEUDO-INVERSES EXIST

setting a weight W we can have different mappings:

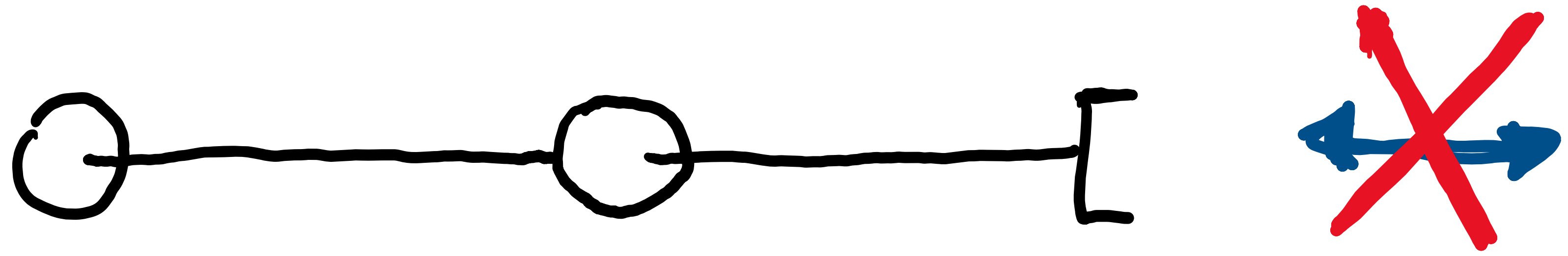
$$A^{\#}_W = WA^T(AWA^T)^{-1} \Rightarrow AA^{\#}_W = AWA^T(AWA^T)^{-1} = I_{m\times m}$$

ok

- if $W = I$ $\Rightarrow$ Moore-Penrose pseudo-inverse

$$A^+ = A^T(AA^T)^{-1}$$

SINGULAR CONFIGURATONS



In a singular configuration $\bar{q}_s$:

1. The end-effector mobility decreases (cannot move in some Cartesian directions)

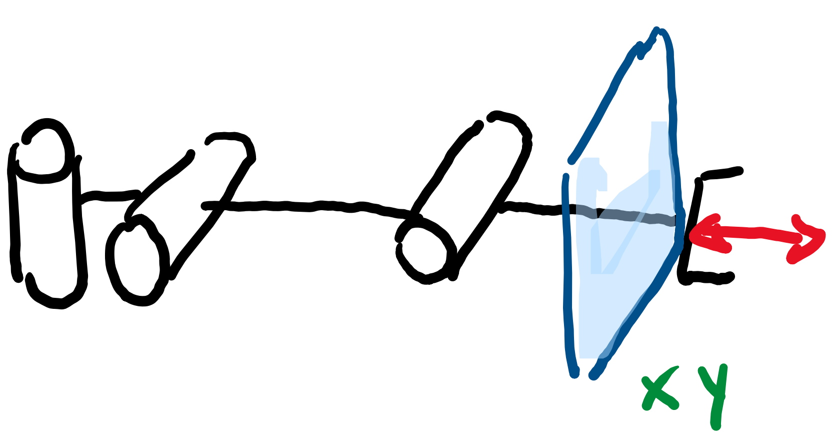
2. The Jacobian loses rank

$$J = \begin{array}{c} \\ \delta x \\ \delta y \\ \delta z \end{array} \begin{array}{c} \delta q \\ \left[\begin{array}{ccc} & & \\ & & \\ 0 & 0 & 0 \end{array}\right] \end{array}$$

no joint motion

can move along z

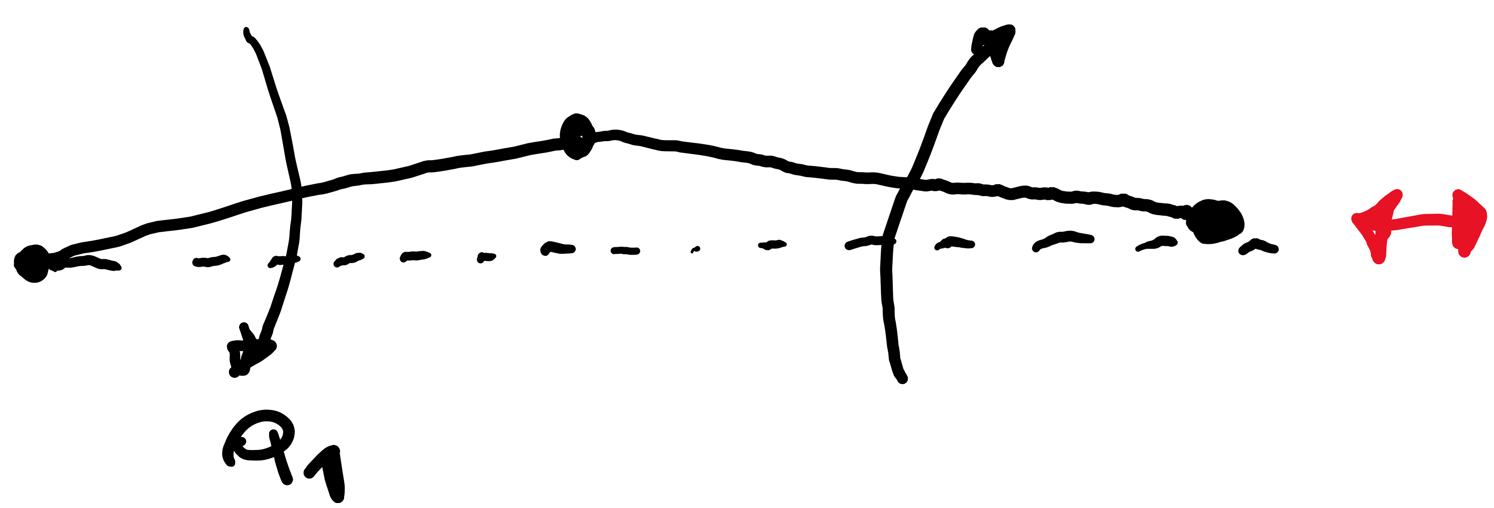
it becomes a redundant manipulator in subspace orthogonal to singular direction



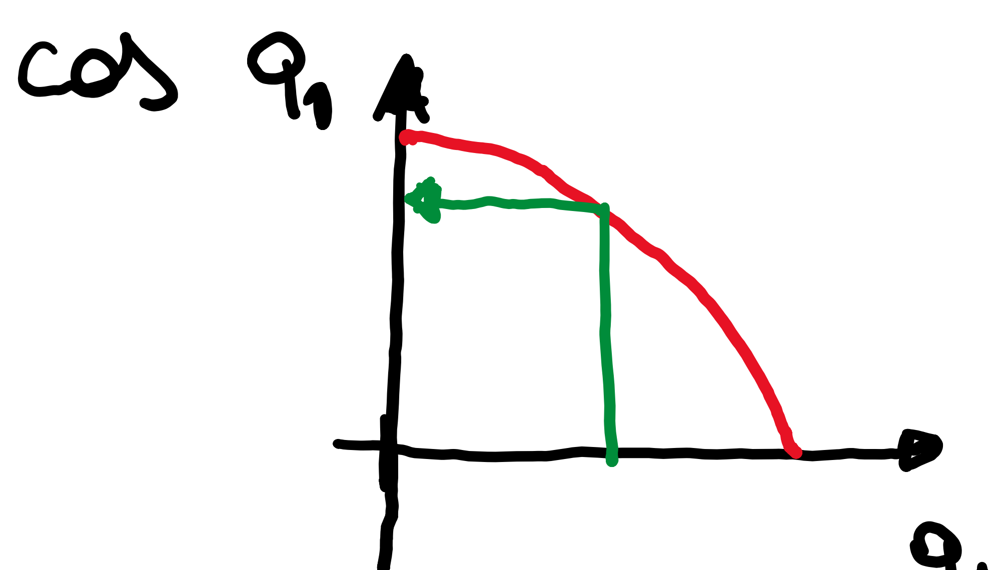
③ $\det(J(\bar{q}_s)) = 0$

④ $\exists\ \infty$ solutions to the IK problem

⑤ close to singularity you have big Δq for small Δx



$$\delta q = \underset{\approx 0}{J}^{-1}\,\delta x$$

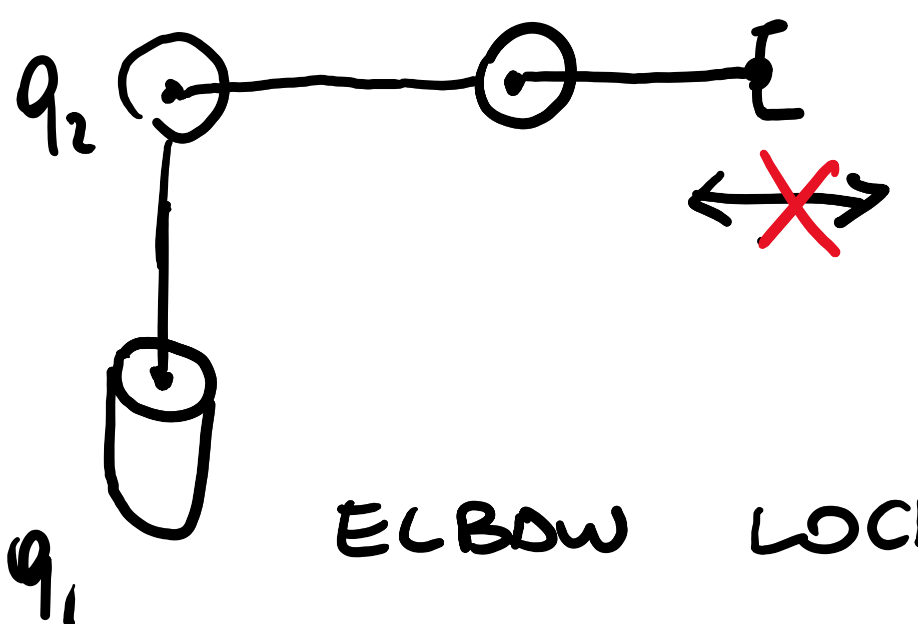


TYPES OF SINGULARITIES

TASK: control position in 3D space

BOUNDARY SINGULARITY

- stretched / retracted
- extreme of WS
- can be avoided



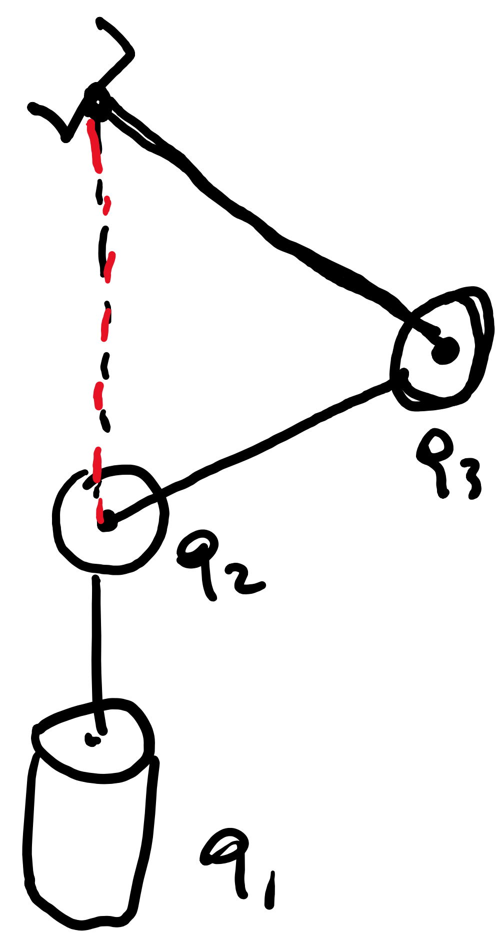
ELBOW LOCK

$\exists\, q_1, q_2, q_3$ to move

INTERNAL SINGULARITY

- inside the WS
- alignment of 2 or more axes

∞ q_1 for some position



SHOULDER LOCK

∞ q_1, q_3

WRIST LOCK

STABILIZE MANIPULATOR AT SINGULARITIES

They are bad but we still want to have control across them

① PSEUDO-INVERSE METHOD

$$\delta q = J^+ \delta x$$

- $J \delta q = \delta x$ is an undetermined system (more unknown than equations because J has a line of zeros)
- pseudoinverse gives the best possible solution to undet systems

$$\delta q^* = \arg\min \| J \delta q - \delta x \|^2$$

- we need to deal with the loss of rank of J

- damped pseudo-inverse

$$\delta q = J^T (J J^T + \lambda^2 I)^{-1} \delta x$$

↳ becomes invertible

$$\delta q^* = \arg\min \| J \delta q - \delta x \|^2 + \lambda^2 \| \delta q \|^2$$

$$\left\| \begin{bmatrix} J \\ \lambda I \end{bmatrix} \delta q - \begin{bmatrix} \delta x \\ 0 \end{bmatrix} \right\|^2$$

Trade-off BW LS and minimum norm condition

- singular value decomposition (SVD)

$$J = U D V^T$$

D → diagonal matrix

U, V → orthogonal matrix

$$D = \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_m \end{bmatrix}$$

singular values of J (one of them is zero!)

$$J^+ = V D^+ U^T \quad \text{with } D^+ = \mathrm{diag}(\sigma_i^+) \ \forall i$$

where $\sigma_i^+ = \begin{cases} 1/\sigma_i & \text{if } \sigma_i \neq 0 \\ 0 & \text{if } \sigma_i = 0 \end{cases}$

(B) JACOBIAN TRANSPOSE METHOD

use J^T instead than J^{-1}

$\delta q = \alpha J^T \delta x$ for some appropriate scalar α

References:

- Samuel R. Buss, Introduction to Inverse Kinematics with Jacobian Transpose, Pseudoinverse and Damped Least Squares methods, 2009.
- Robotics Modelling, Planning and Control – B. Siciliano, L. Sciavicco, L. Villani, G. Oriolo.
- Handbook of Robotics - Kinematically Redundant Manipulators, Stefano Chiaverini, Giuseppe Oriolo, Ian D. Walker.