CONSTRAINED INVERSE DYNAMICS

GOAL: compute τ satisfying contact constraints $J\ddot{q} + \dot{J}\dot{q} = 0$ that realize a desired motion $\ddot{q}^d$ as a function of the current state $(q, \dot{q})$ of the robot

but... for floating base robots the problem is ill posed because contact forces are function of τ

$\Rightarrow$ we work in a reduced dimensional

$$N(M\ddot{q}^d + h) = N S^T \tau$$

$$\underbrace{N S^T}_{A} \underbrace{\tau}_{x} = \underbrace{N (M \ddot{q}^d + h)}_{b}$$

- differently from simulation where we want an accurate solution, in control we want to do "the best we can" to realize the motion $\ddot{q}^d$
- we see this as a LS problem (e.g. minimize $\|Ax - b\|$) rather than the solution of a sistem of equation, Therefore it is licit to use (damped) pseudo-inverse.

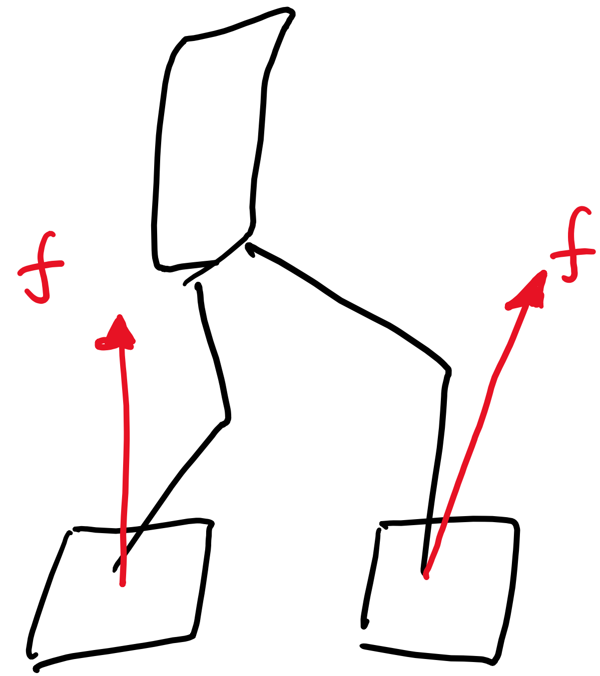
$$\boxed{\tau = (N S^T)^{\#} N (M \ddot{q}^d + h)}$$

CONSTRAINED FLOATING BASE INVERSE DYNAMICS

$$\begin{array}{cc} & n+6 \\ J = K & \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} \end{array}$$

$$N = I - J^T J^{T\#} = \begin{array}{cc} & n+6 \\ n+6 & \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & K \\ n+6 & \begin{bmatrix} \quad \\ \quad \end{bmatrix} \end{array} \begin{bmatrix} \quad \end{bmatrix}$$



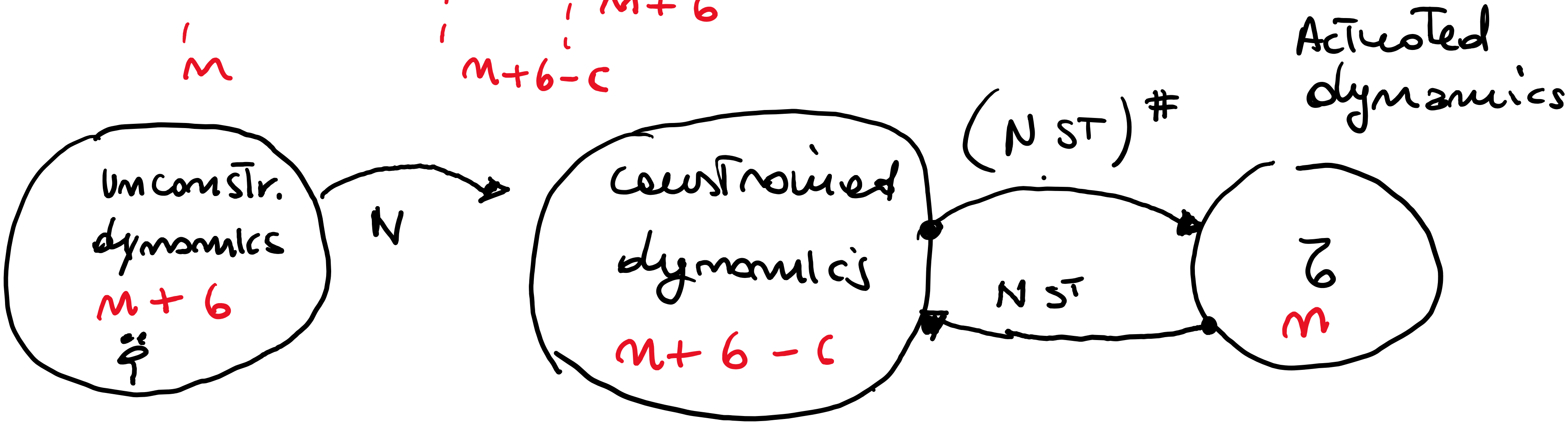
$$\begin{array}{cc} & n \\ S^T = n+6 & \begin{bmatrix} 0 \\ I_n \end{bmatrix} \end{array}$$

$$\Rightarrow NS^T = \begin{array}{cc} & n \\ n+6 & \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \end{array}$$

max rank $= n$

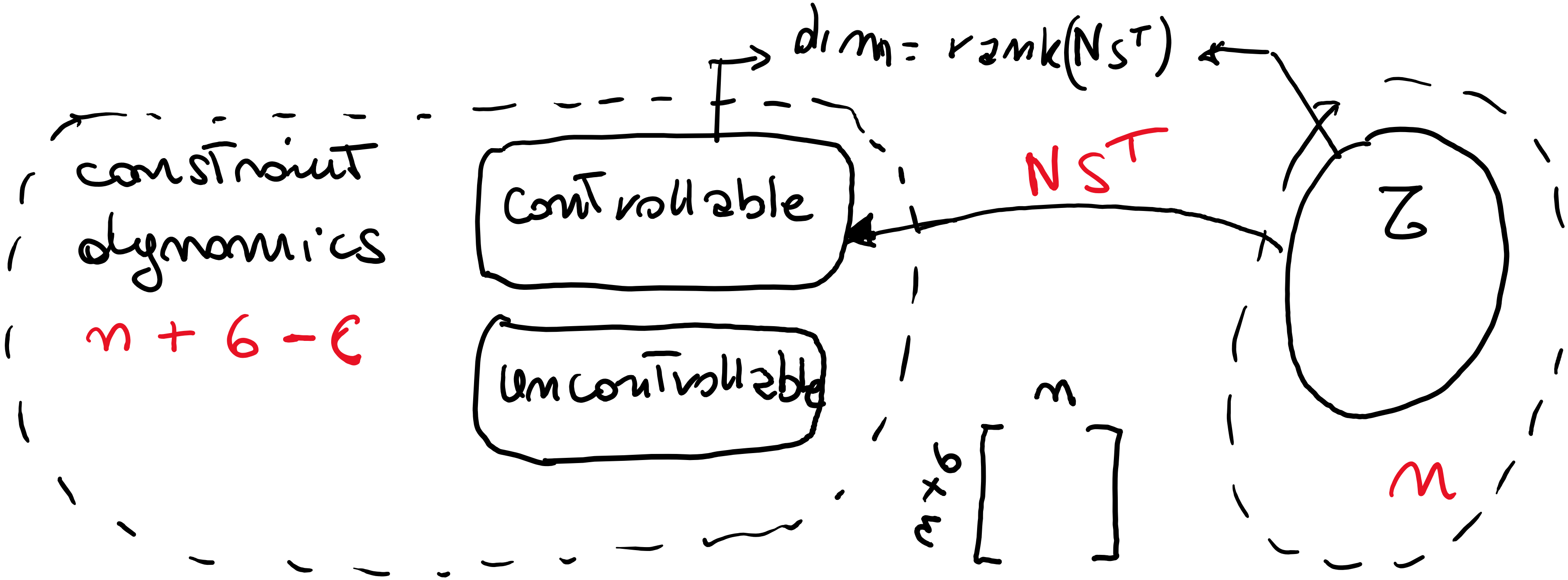
$$\tau = (N S^T)^{\#} \; N \; (M \ddot{q}^d + h) \; + \tau_{NS}$$

m $\qquad$ $m+6-c$ $\qquad$ $m+6$



$c = \#$ of <u>independent</u> constraints $\neq k$

ANALYSIS OF NS^T



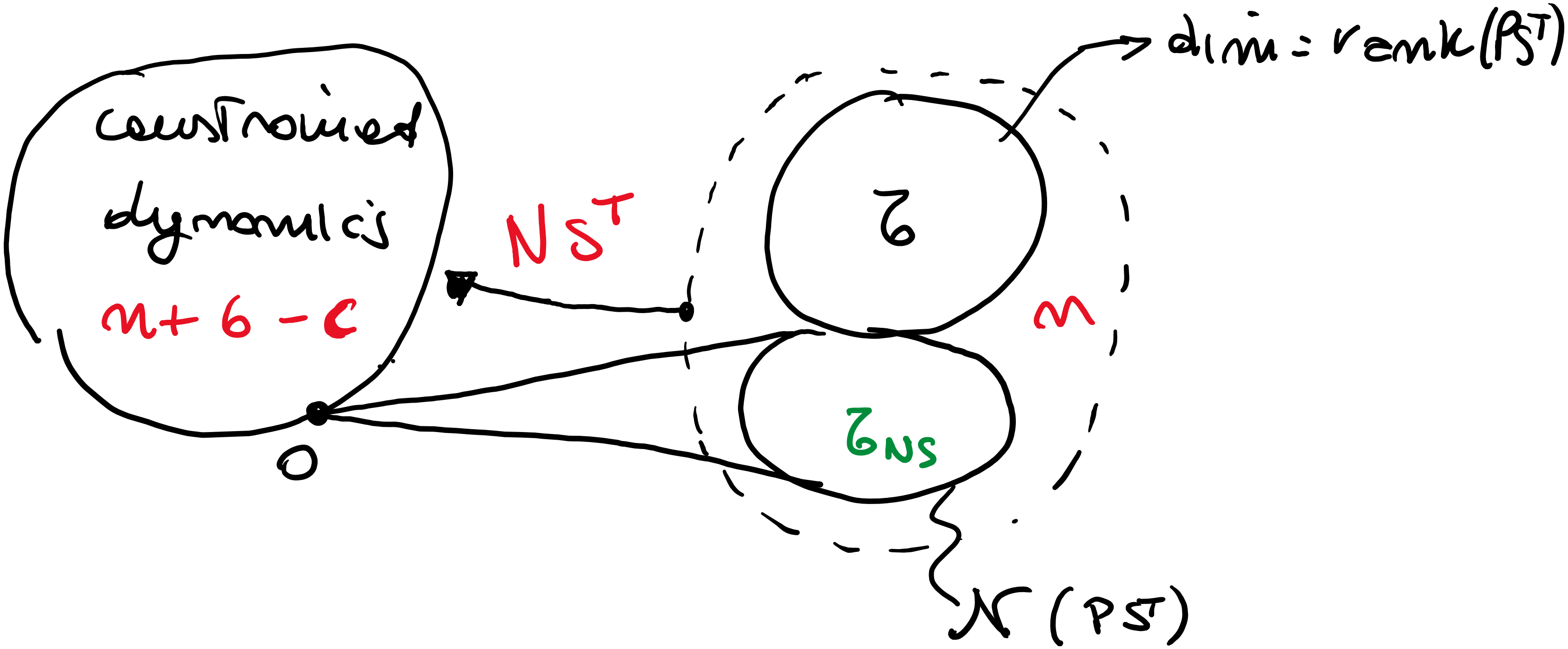
Ⓐ UNDERACTUATION / CONTROLLABILITY

To achive $\ddot{q}^d$ I need $\text{rank}(NS^T) = n+6-c$

$$\boxed{n_u = n + 6 - c - \text{rank}(NS^T)}$$

UNDERACTUATED DOFS

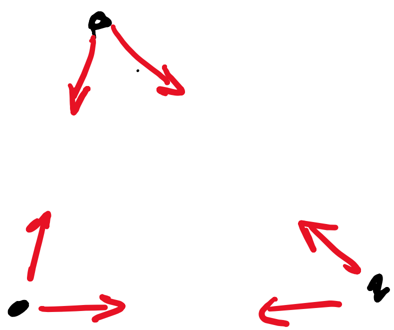
(B) OVER CONSTRAINED



- if $\text{rank}(NS^T) < m \Rightarrow \exists\ \tau_{NS} \in \mathcal{N}(NS^T)$

The system is OVER CONSTRAINED and I have force redundancy $\Rightarrow \exists$ internal forces (correspondent to τ_{NS}) that DO NOT AFFECT $\ddot{q}$

- These components correspond to contact forces that DO NOT AFFECT the motion of the floating base → because they nullify each other



TOP VIEW: 3 LEGS STANCE

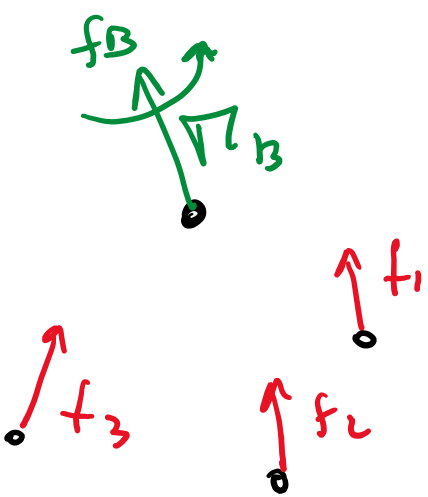
- The dimension of the space of internal Torques is the dimension of $\mathcal{N}(N S^T)$

$$\boxed{n_z = \dim \mathcal{N}(N S^T) = n - \operatorname{rank}(N S^T)}$$

- These Torques can be used for

LOAD DISTRIBUTION

- To compute the degree of underactuation or of force redundancy
- it is easier to reason in terms of base wrench and contact forces
- it can be proof is equivalent [HUTTER 2013]



$$W_B = \begin{bmatrix} f_B \\ \tau_B \end{bmatrix} \underset{6}{=} \underbrace{\overset{K}{\begin{bmatrix} J_{b1}^T & J_{b2}^T & J_{b3}^T \end{bmatrix}}}_{J_b^T} f$$

J_b^T relates contact forces f with base wrench

Ⓐ UNDERACTUATION $\triangleq$ $\text{rank}(J_b^T) < 6$

$$\boxed{n_u = 6 - \text{rank}(J_b^T)}$$

underactuated DOFs that cannot be controlled through $\underline{f}$

$$r\left\{\begin{bmatrix} & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\right. \rightarrow \exists \text{ Null space in CODOMAIN}$$

Ⓑ OVERCONSTRAINED $\triangleq$ $\text{rank}(J_b^T) < K$

$$\boxed{n_f = K - \text{rank}(J_b^T)}$$

dimension of null space of internal forces

$$\begin{array}{c} \overbrace{\qquad\qquad}^{r} \\ \begin{bmatrix} & & 0 & 0 & 0 \\ & & 0 & 0 & \\ & & 0 & 0 & \end{bmatrix} \end{array}$$

$\exists$ NULL SPACE in DOMAIN

$\Rightarrow$ we can have multiple solutions to $f = (J_b^T)^{\#} W_B$

EXAMPLE: 1 LEG ON THE GROUND

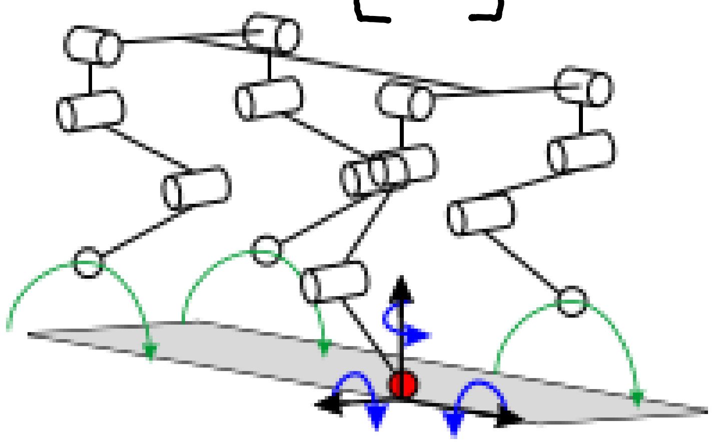
$$J_b^T = \begin{array}{cc} & 3 \\ 6 & \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \end{array}$$

$k = 3$

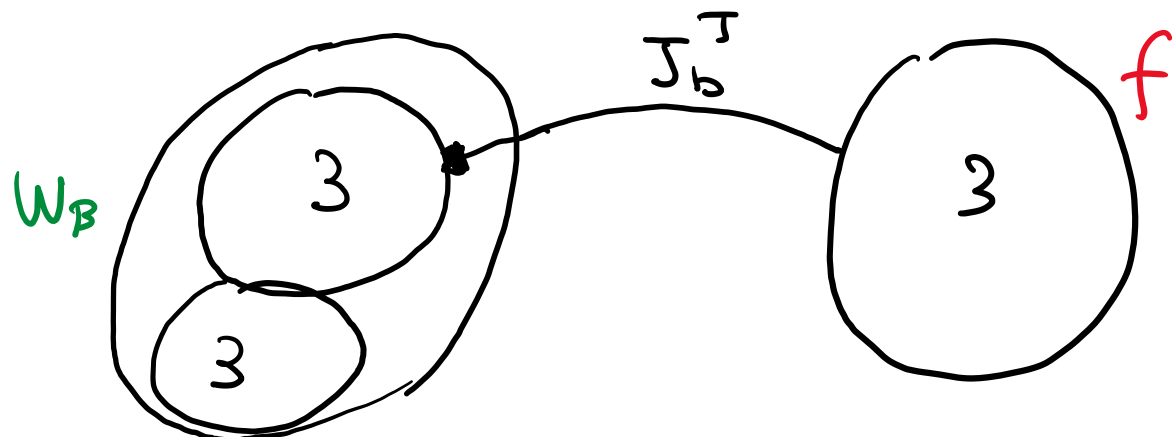
$\mathrm{rank}(J_b^T) = 3$

$n_u = 6 - 3 = 3$ uncontrollable DOFs

$n_f = k - 3 = 0$ no internal forces



⇒ cannot apply moments about contact



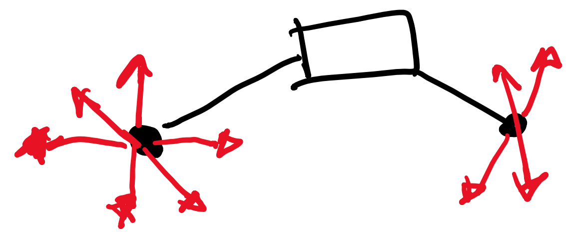
$\dim(N) = 3$ $\dim(N) = \emptyset$

EXAMPLE : 2 LEGS ON GROUND

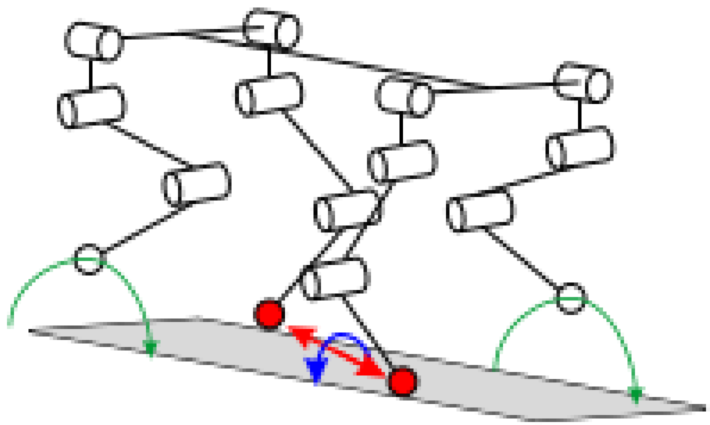
$$J_b^T = \begin{array}{cc} & 6 \\ 6 & \underbrace{\begin{bmatrix} \quad & \vdots & \quad \end{bmatrix}}_{r} \end{array}$$

$k = 6$

$\text{rank}(J_b^T) = 5$ why?



3 DOFs removed   2 DOF removed $= 5$

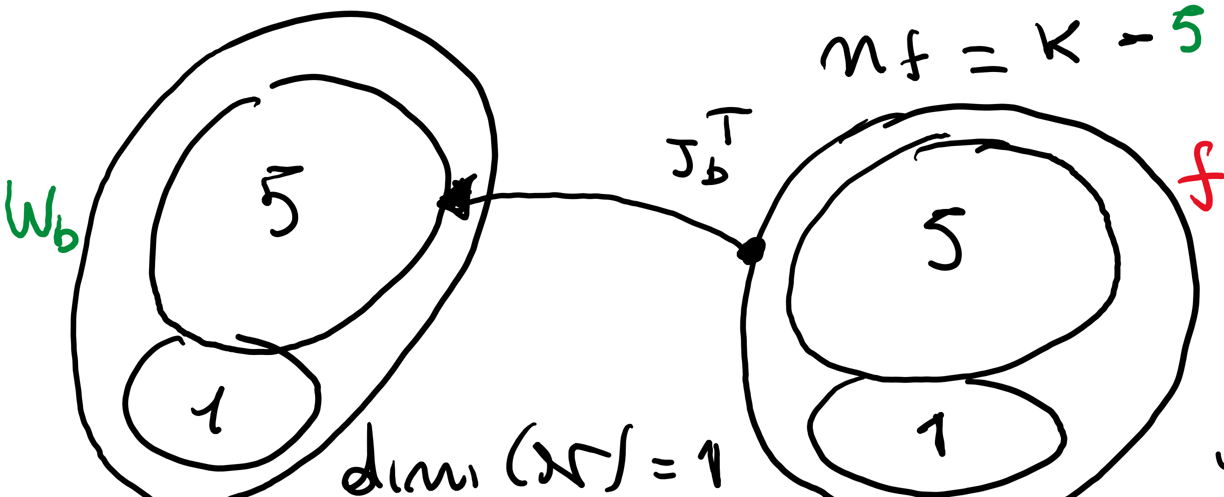


$n_u = 6 - 5 = 1$ uncontrollable DOF

$n_f = k - 5 = 1$ internal force

$\Downarrow$

underactuated AND over constrained



$\dim(N) = 1$

EXAMPLE: HUMANOID 1 FOOT ON GROUND

$$J_b^T = \begin{array}{cc} & 6 \\ 6 & \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \end{array}$$

$k = 6$

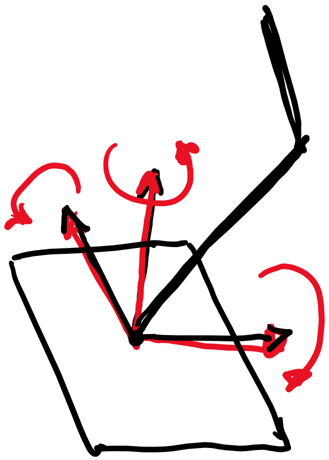
$\mathrm{rank}(J_b^T) = 6$

$n_u = 6 - 6 = \emptyset$

$n_\tau = k - 6 = \emptyset$

sufficiently constrained

no internal forces



$\Rightarrow$ both linear and rotational motion are constrained (6D contact)

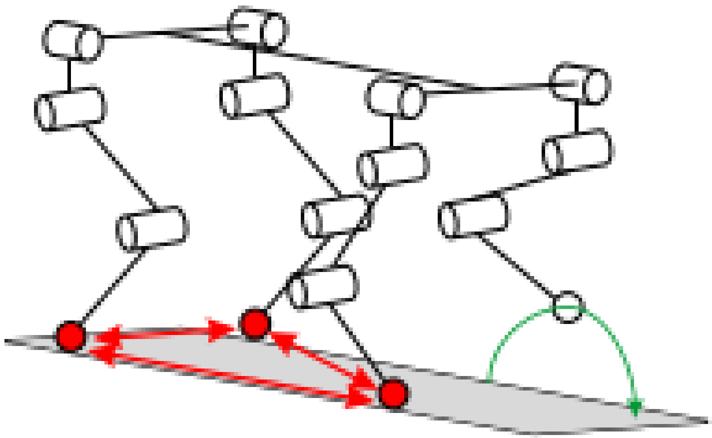
$\Rightarrow$ one unique solution

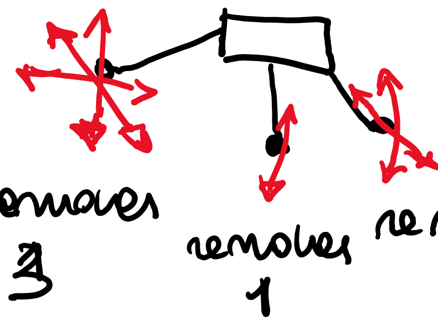
EXAMPLE : 3 LEGS ON THE GROUND

$$J_b^T = \begin{array}{c c} & 9 \\ 6 & \left[\begin{array}{c:c} \longleftrightarrow & \\ r = 6 & \end{array}\right] \end{array}$$

$k = 9$

$\text{rank}(J_b^T) = 6$





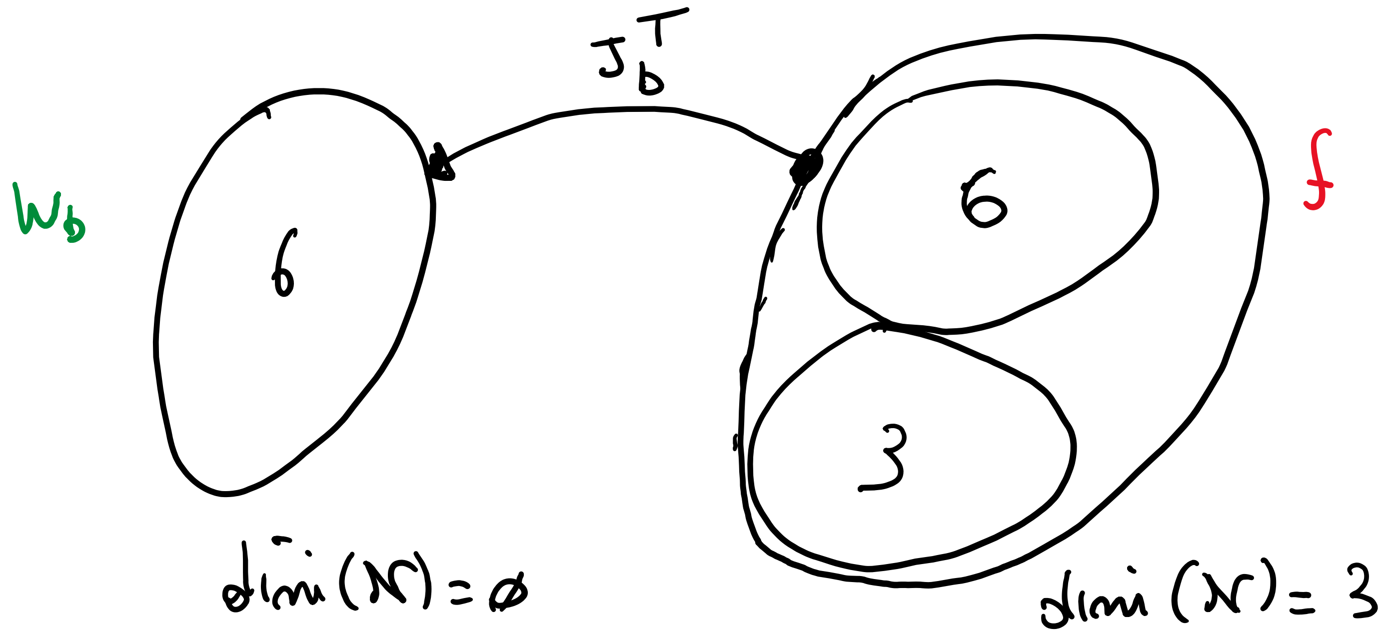
$= 6$

$n_u = 6 - 6 = \emptyset$ # uncontrollable DOFs

SUFFICIENTLY CONSTRAINED:

The system behaves as if it is FULLY ACTUATED and any motion can be achieved

$n_f = k - 6 = 3$ # internal forces $\Rightarrow$ over constrained


J_b^T
W_b
6
6
3
f
dim(N) = ∅
dim(N) = 3

EXAMPLE : 4 LEGS ON THE GROUND

$$J_b^T = \begin{array}{cc} & 12 \\ 6 & \left[\; \overleftrightarrow{\;r=6\;} \;\vdots\; \quad \right] \end{array}$$

$k = 12$

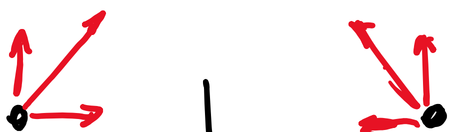
$\text{rank}(J_b^T) = 6$

$n_u = 6 - 6 = \emptyset$ # uncontrollable DOFs

$n_z = k - 6 = 6$ internal forces

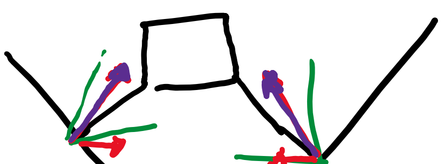
$\Downarrow$

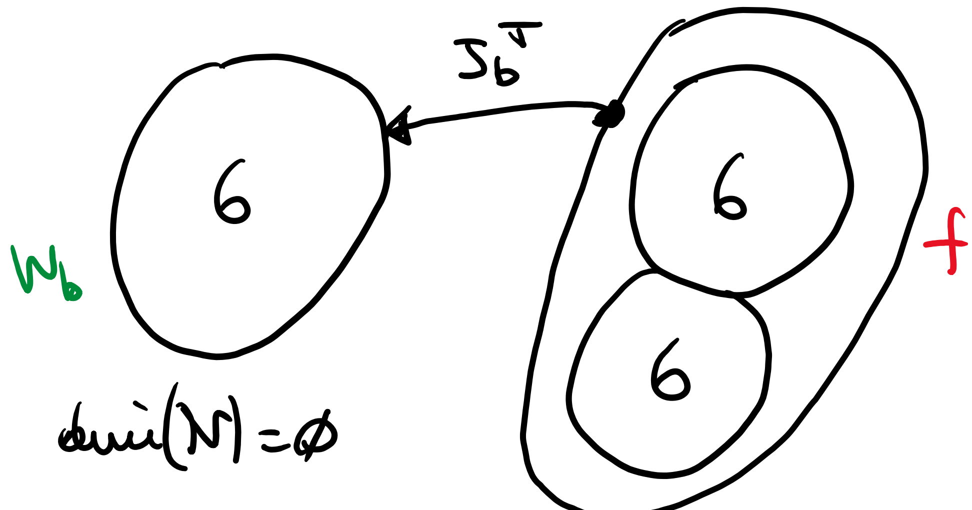
overconstrained



$\Downarrow$

Exploit for optimization





$\dim(N) = \emptyset$

References:

- Design and control of legged robots with compliant actuation – PhD Thesis, M.Hutter, 2013.
- Control of Contact Forces using Whole-Body Force and Tactile Sensors – PhD Thesis, A. Del Prete, 2013.