Operational Space Control

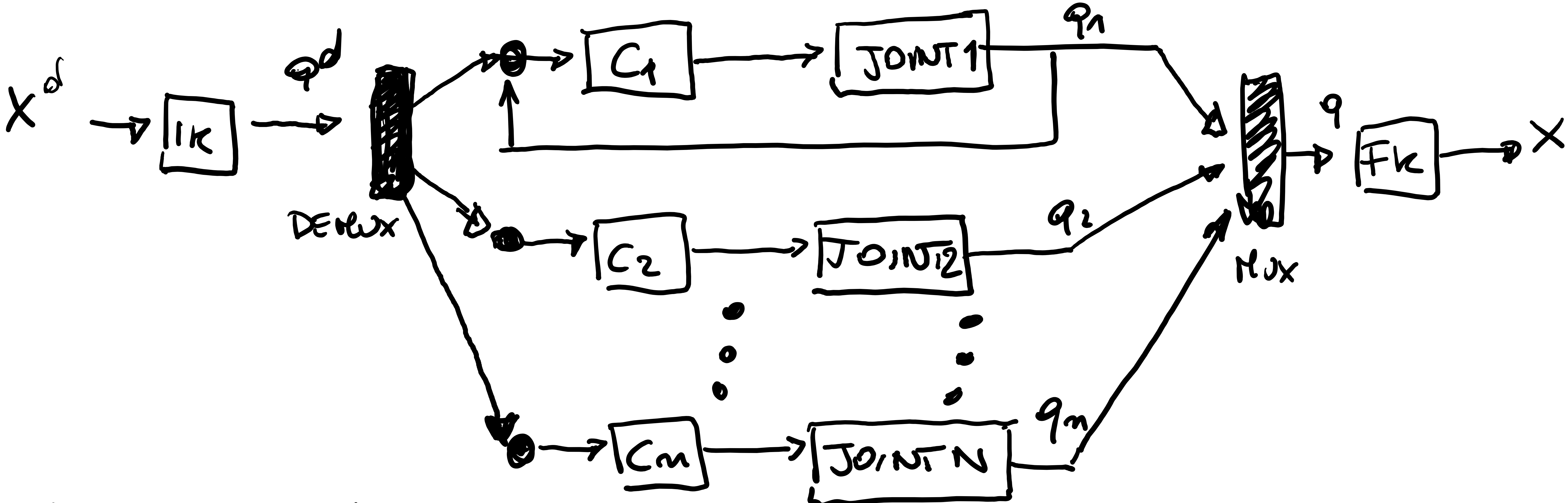
Inverse Kinematics and Joint Motion Control

Resolved Motion Rate Control

Direct Operational Space Control

INVERSE KINEMATICS FOR END-EFFECTOR CONTROL

we can control end-effector by controlling joint space motion



if $q \to q^d$ then $x \to x^d$

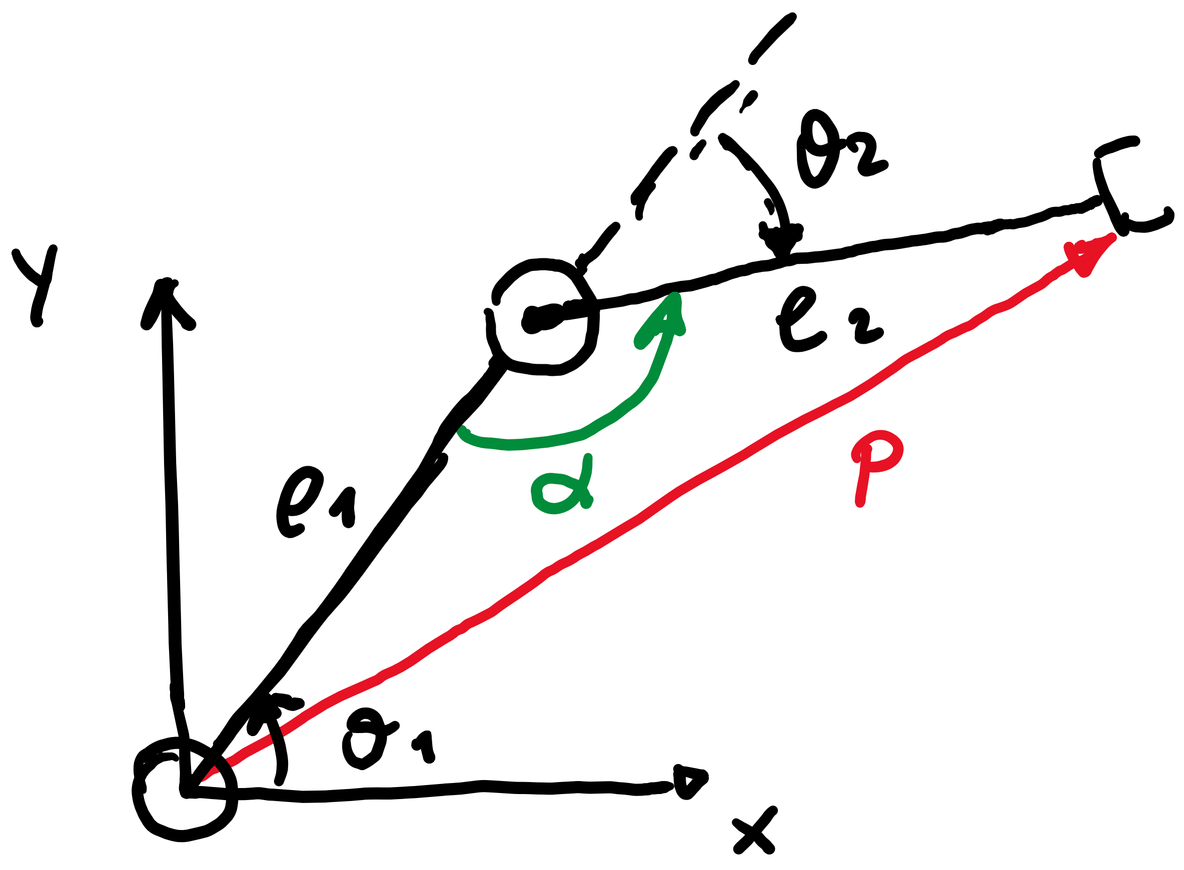
⊕ simple

⊖ existence of closed form solution to IK is not guaranteed

⊖ numerical implementation is slow

INVERSE KINEMATICS COMPUTATION

closed form solution:

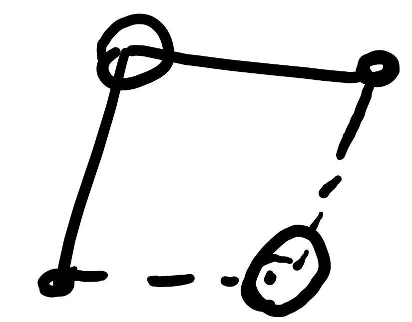


cosine theorem

$$p_x^2 + p_y^2 = l_1^2 + l_2^2 + 2 l_1 l_2 \, c_\alpha$$

$$c_\alpha = \frac{p_x^2 + p_y^2 - l_1^2 - l_2^2}{2 l_1 l_2} = c_{\theta_2}$$

$$s_{\theta_2} = \pm \sqrt{1 - c_{\theta_2}^2}$$



Elbow up/down

$$\theta_2 = \mathrm{atan2}(s_{\theta_2}, c_{\theta_2})$$

θ_2 + FWKIN $\longrightarrow$ θ_1

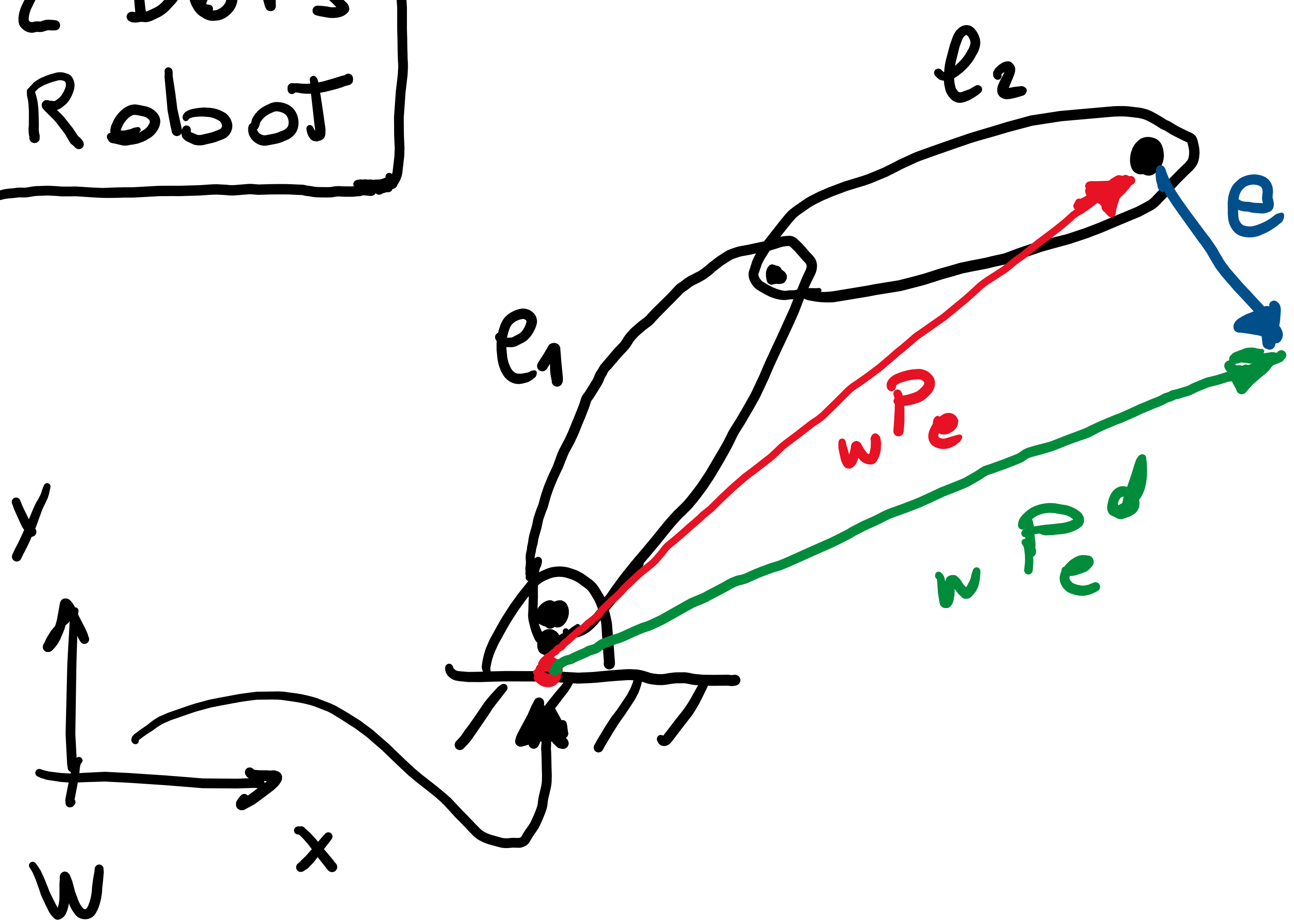
⊖ requires algebraic intuition

⊕ super fast

otherwise... numerical solution techniques

NUMERICAL INVERSE KINEMATICS

2 DoFs Robot



$$_wP_e = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \end{bmatrix}$$

$$_wJ_e = \left[\begin{array}{c|c} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ \hline l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{array}\right]$$

NUMERICAL IK STEPS (IMPUT P_e^d, OUTUPUT q)

① sequence of q^i

② compute error $e_i = P_e^d - P_{e_i}(q^i)$

③ make $\|e_i\| \to 0$ as $i \to \infty$

in practice, stop when $\|e_i\| <$ Tolerance

- To minimize e we need its derivative
- Jacobian provides a linearization of FWD KIN.

$$de = \cancel{dp_e^d}^{=0} - dp_e = -J_e\, dq \qquad \text{infinitesimal}$$

$$\boxed{\Delta e = -J_e \Delta q} \qquad \text{non infinitesimal}$$

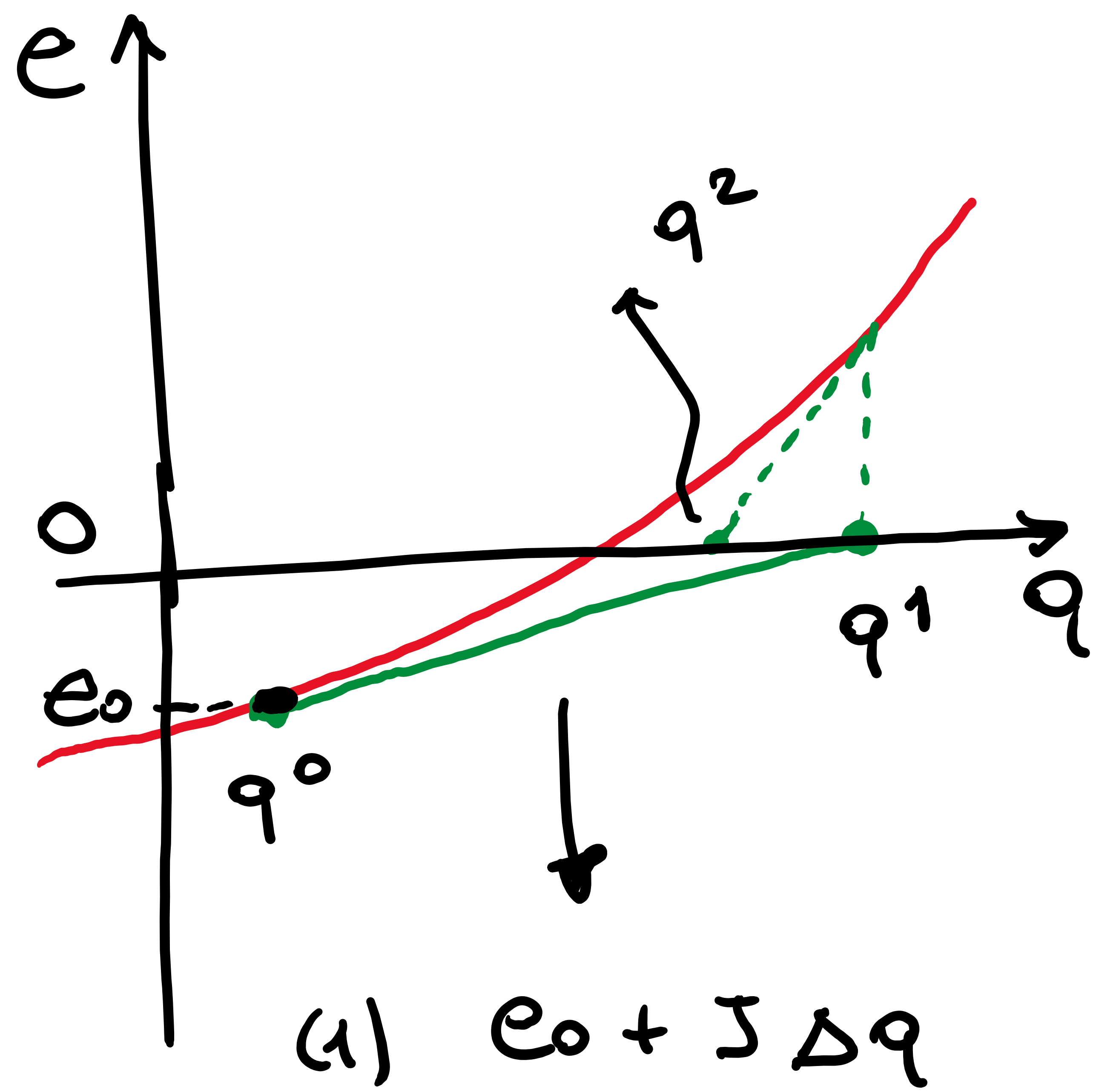
NEWTON STEP:

1. evaluate J at q^i
2. find a value of Δq that makes (1) intersect $\emptyset$

$$-J(q^i)\Delta q + e_i = 0$$

$$\Downarrow$$

$$\boxed{\Delta q = J_e^{-1}(q^i)\, e_i}$$

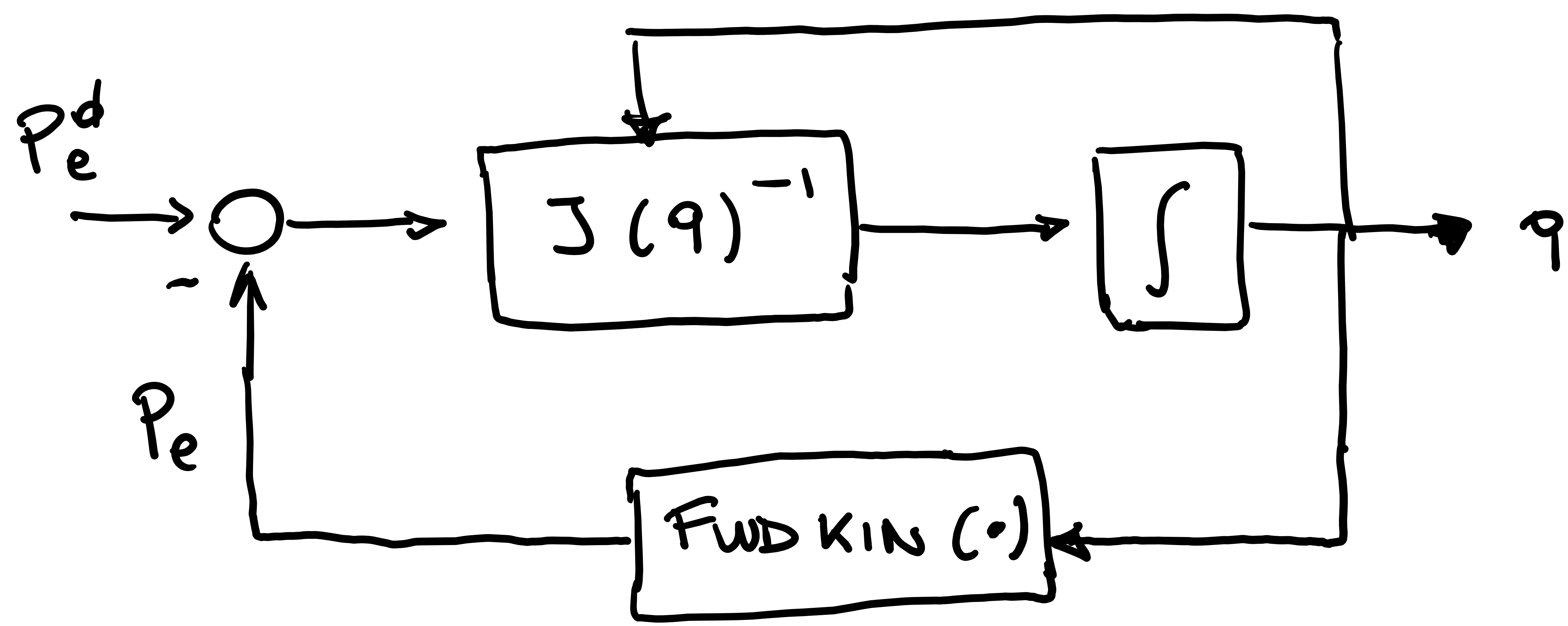


3 add Δq To $q^{(i)}$

$$q^{i+1} = q^{i} + \Delta q = q^{i} + J_e^{-1}(q^{i})\, e_i$$

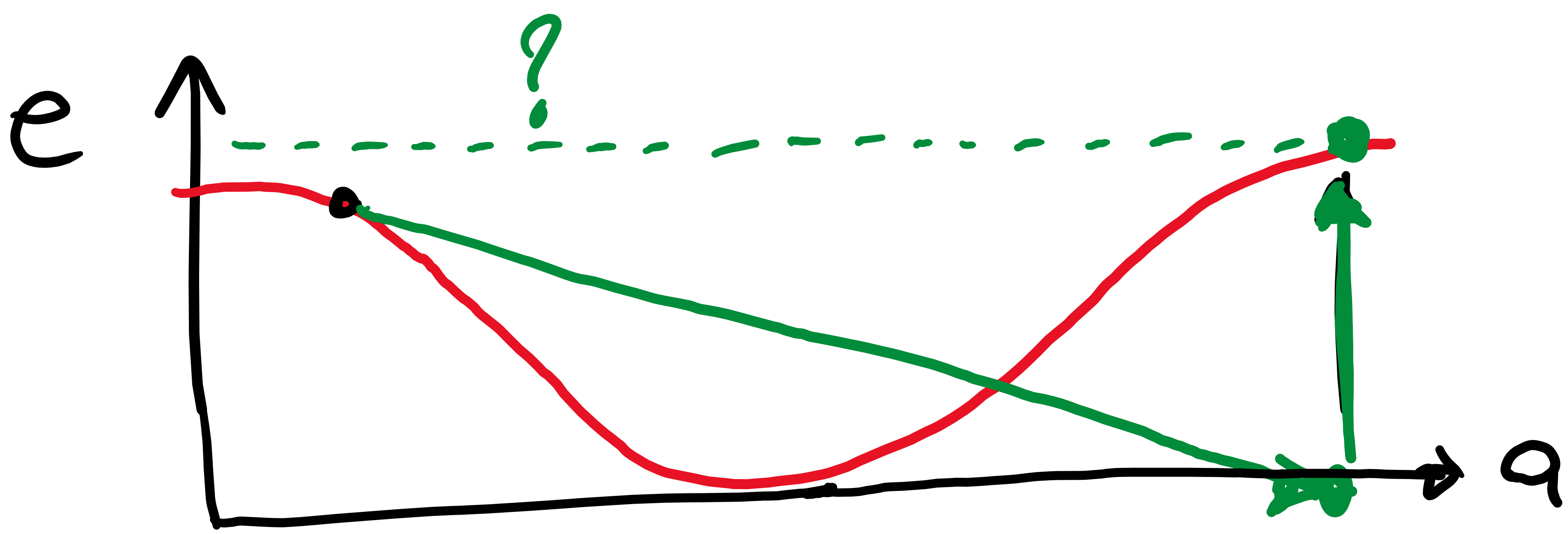
4 go To 1, stop when $\| e_i \| < Tol$

BLOCK DIAGRAM



PROBLEMS

Ⓐ you can get stuck in points with $\emptyset$ slope



J becomes singular

$\Downarrow$

$\nexists\ \Delta q$

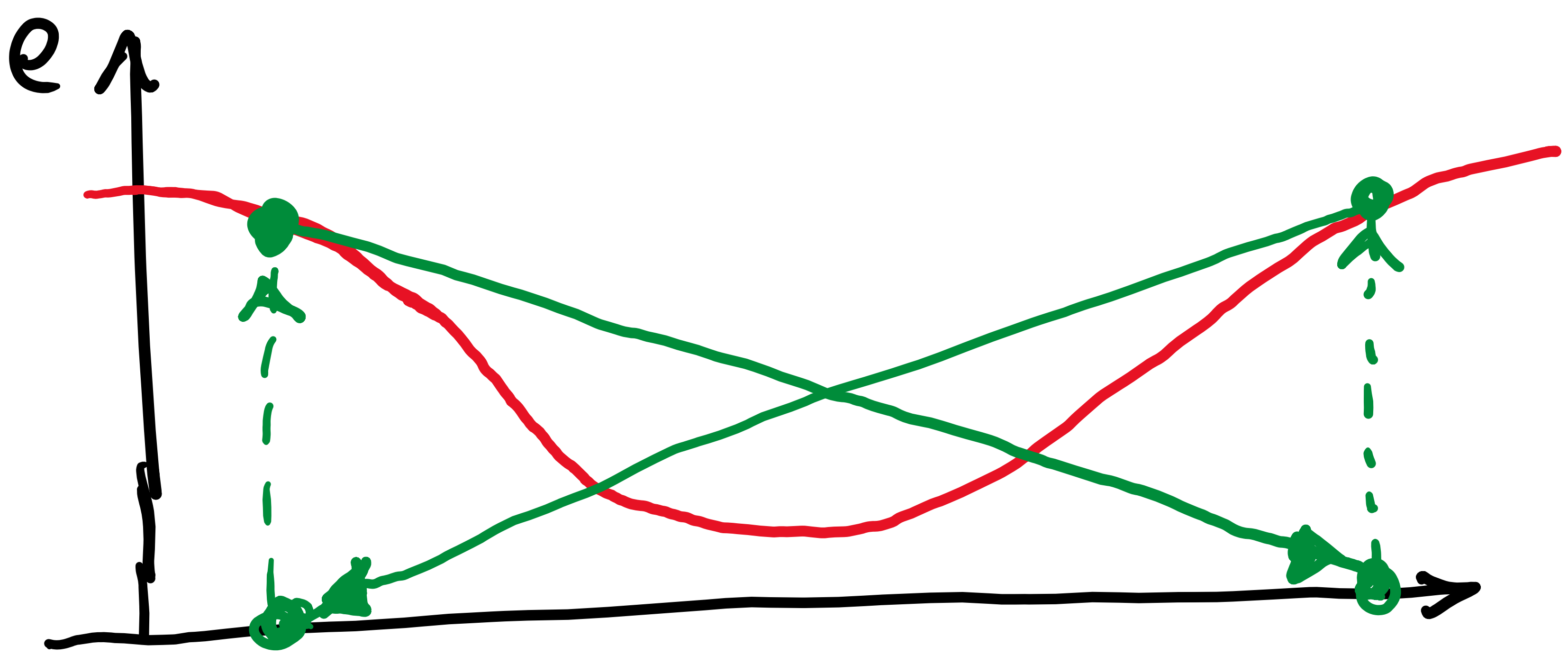
SOLUTION Ⓐ: damped pseudo-inverse to compute Δq

$$J^{-1} \approx J^T (JJ^T + \lambda I)^{-1} \qquad \lambda \text{ small}$$

$$\boxed{\Delta q = J^T (JJ^T + \lambda I)^{-1} e_i}$$

$\rightarrow$ works also with redundancy !

Ⓑ Get stuck in cycles



Solution Ⓑ:

- we can avoid cycles by ensuring that e_i decreases at each iteration

- move along the Newton direction but take smaller steps.

$$q^{i+1} = q^{i} + \underbrace{B \, J(q^{i})^{-1} \Big[\overbrace{P_e^{d} - P_e(q^{i})}^{e_i} \Big]}_{\text{Newton step}}$$

STEP SIZE SELECTION

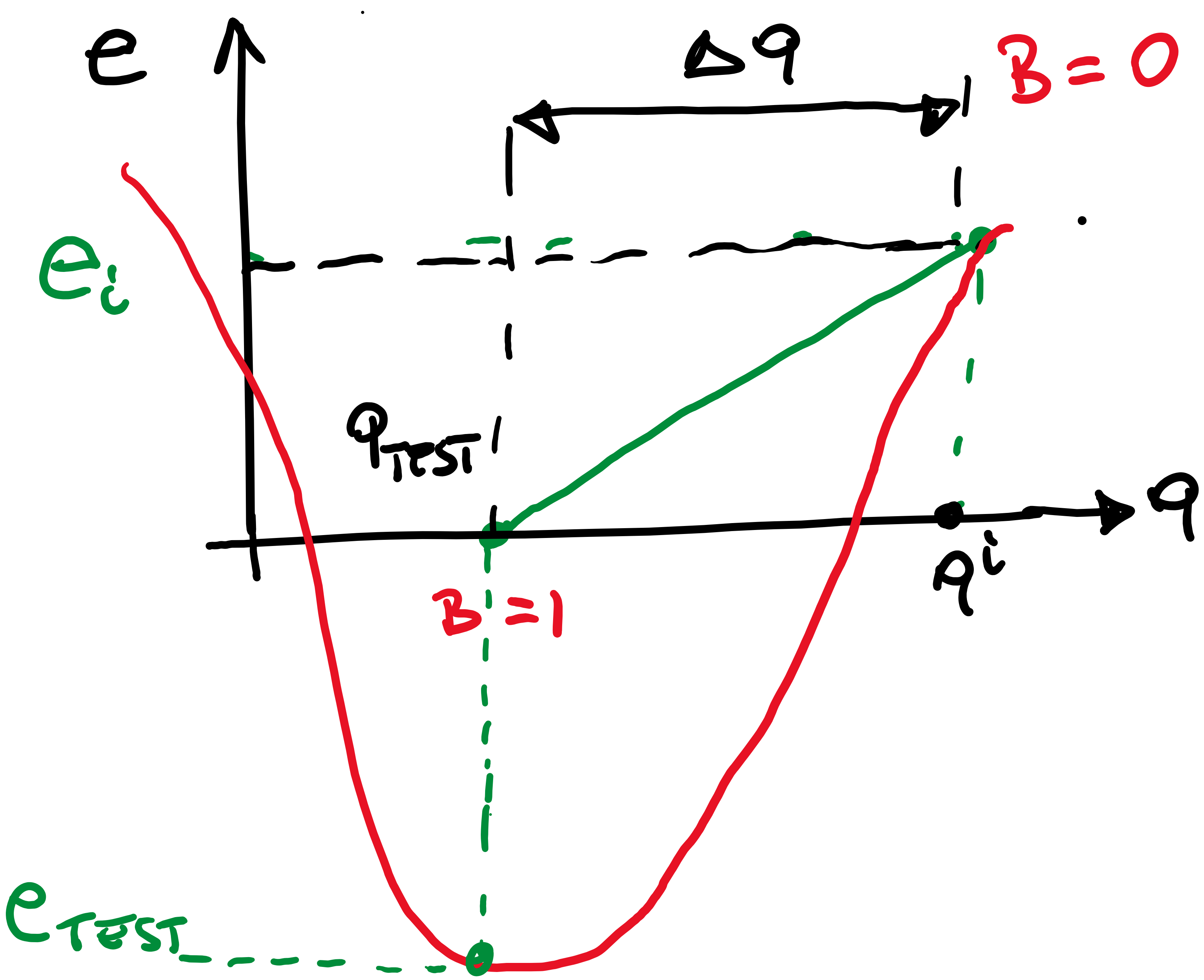
how to choose B ? $0 \leq B \leq 1$

① $e_i = P_e^d - P_e(q^i)$

② $\Delta q = J^{-1}(q^i)\, e_i$

③ $q_{TEST} = q^i + B\,\Delta q$

check the reduction of e with $B = 1$



$q_{TEST} = q^i + 1 \cdot \Delta q$

$\| e_i \| < \| e_{TEST} \|$ NOT OK

$\Rightarrow$ Take smaller step

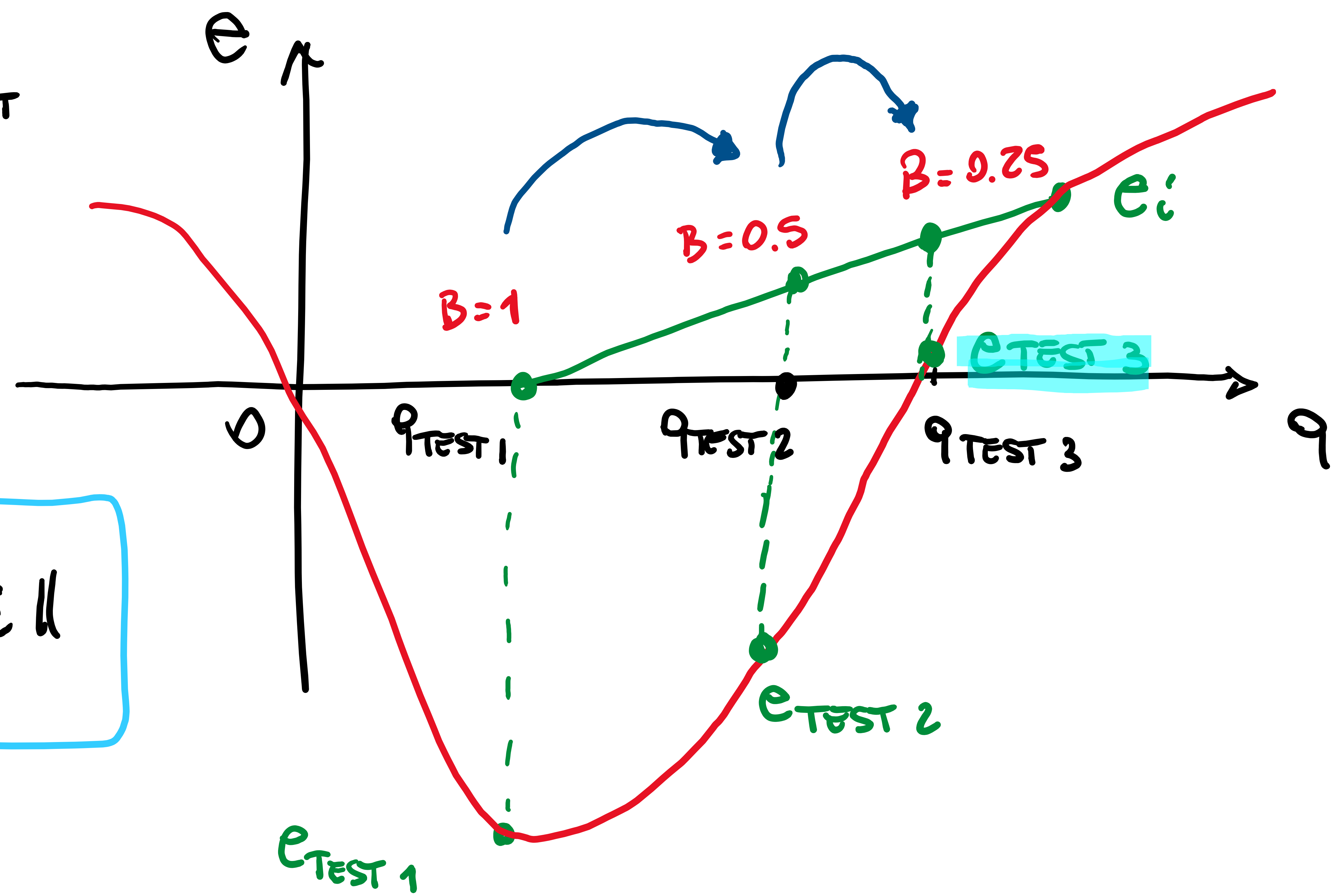
while $\|e_i\| \leq \|e_{TEST}\|$

$$B = B/2$$

$$q_{TEST} = q_i + B\,\Delta q$$

end

$$q^{i+1} = q_{TEST}$$



GOAL

$$\|e_{TEST}\| \leq \|e_i\|$$

B = 0,25

$$q^{i+1} = q_{TEST\,3}$$

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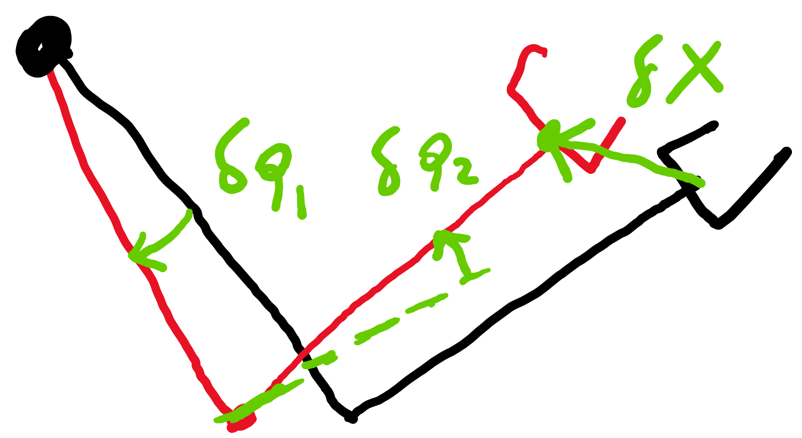
RESOLVED MOTION RATE CONTROL

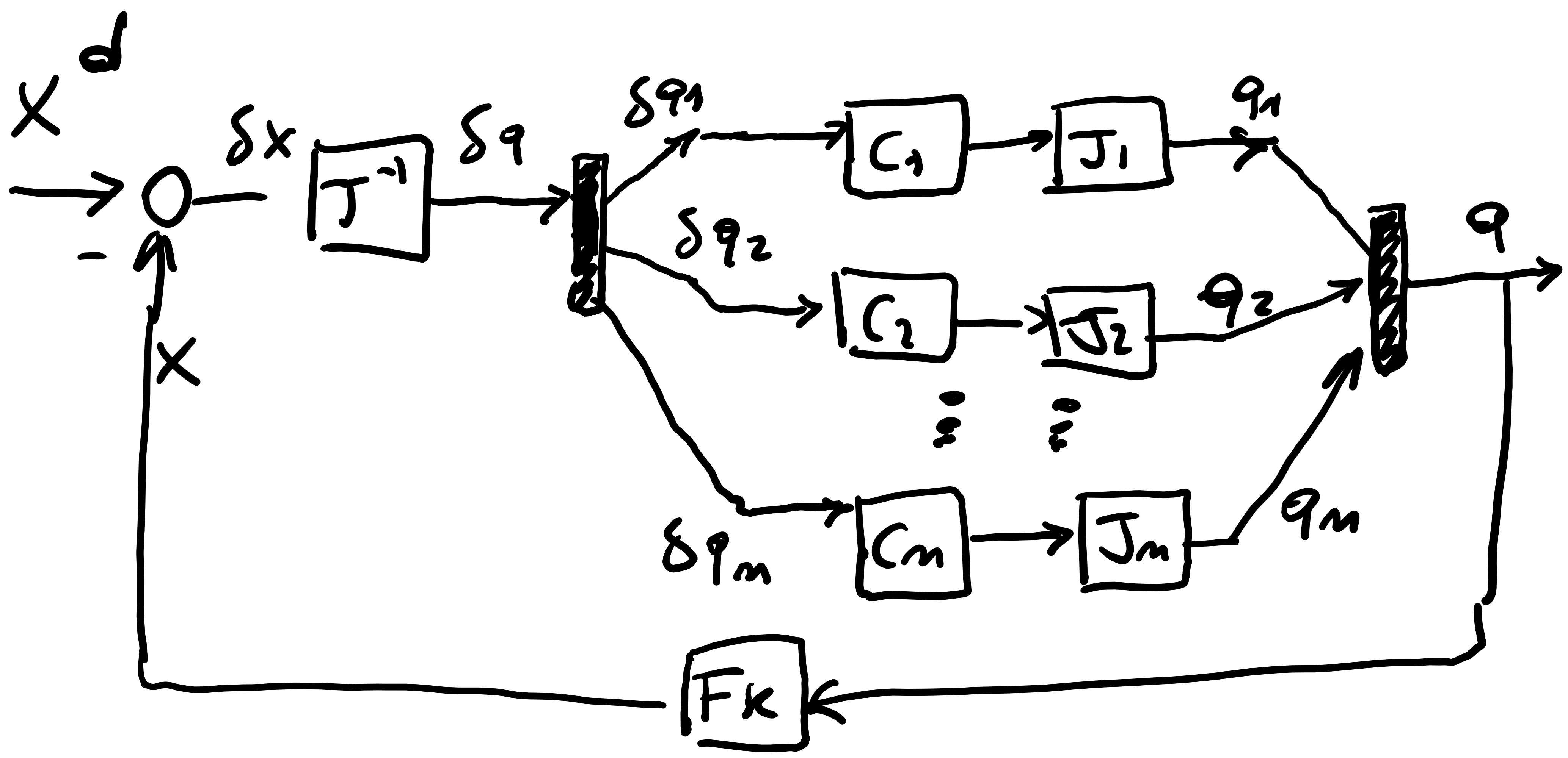
Directly find δq that corresponds to δx



$$\delta x = J(q)\,\delta q$$ differential kinematics

$$\Downarrow$$

$$\boxed{\delta q = J^{-1}(q)\,\delta x}$$ → always possible for non redundant manipulators



[whitney 1969]

- differently from IK we close the loop at the end-effector

⊕ simple

⊖ J can be ill conditioned (closed to singularity)

⊖ redundancy → $\exists\ \infty\ \delta q$ that achieve the same δx

⊖ discrepency in the metric of JACOBIAN (rad/s and m/s are not homogeneous)

⊖ we cannot set (as for IK case) a desired tracking behavior for end-effector (only at joint level)

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DIRECT OPERATIONAL SPACE CONTROL

[Kathib 1987]

IDEA: instead of defining a motion for joints directy control forces that act on end-effector

- we need to consider the dynamic behavior of manipulator at end-effector (OPERATIONAL POINT)

MAP JOINT SPACE DYNAMICS TO EE

(1) $M\ddot{q} + h = \tau$

(2) $\ddot{x} = J\ddot{q} + \dot{J}\dot{q} \Rightarrow \ddot{q} = J^{-1}(\ddot{x} - \dot{J}\dot{q}) \rightarrow (1)$

$$M J^{-1}(\ddot{x} - \dot{J}\dot{q}) + h = \tau$$

• premultiplying both sides for $(J^T)^{-1} = J^{-T}$

$$\underbrace{\left(J^{-T} M J^{-1}\right)}_{\Lambda(x)} \ddot{x} \underbrace{- \underbrace{J^{-T} M J^{-1}}_{\Lambda} \dot{J} \dot{q} + J^{-T} R}_{\mu(x,\dot{x})} = \underbrace{J^{-T} \tau}_{f}$$

↳ can be singular!

$$\boxed{\Lambda(x)\ddot{x} + \mu(x,\dot{x}) = f} \quad (3)$$

manipulator dynamics reflected at end-effector

• $\Lambda(x) = J^{-T} M J^{-1}$ operational space inertia

• $\mu(x,\dot{x}) = -\Lambda \dot{J} \dot{q} + J^{-T} R$ bias forces at end-effector

• f = Cartesian force at end-effector

Remark:

(3) is equivalent to project (1) with J^{-T}

$$J^{-T}\left[M\ddot{q} + R\right] = f$$

Note on Λ

if The J is not square (e.g. our task is To control only end-effector position, Then J cannot be inverted.

In this case is better To compute:

$$\boxed{\Lambda = (J M^{-1} J^T)^{-1}}$$ that always exists

OPERATIONAL SPACE INVERSE DYNAMICS

linearize (3):

$$(4) \quad \boxed{f = \hat{\Lambda} f^d + \hat{\mu}}$$

$\hat{\Lambda}$, $\hat{\mu}$ are estimates

better set $\hat{\mu} = 0$ than use a poor estimate (affects stability)

CLOSED LOOP DYNAMICS:

plug (4) into (3)

$$I_{m \times m} \ddot{x} = f^d \quad \text{(unit mass!)}$$

- define a controller at end-effector

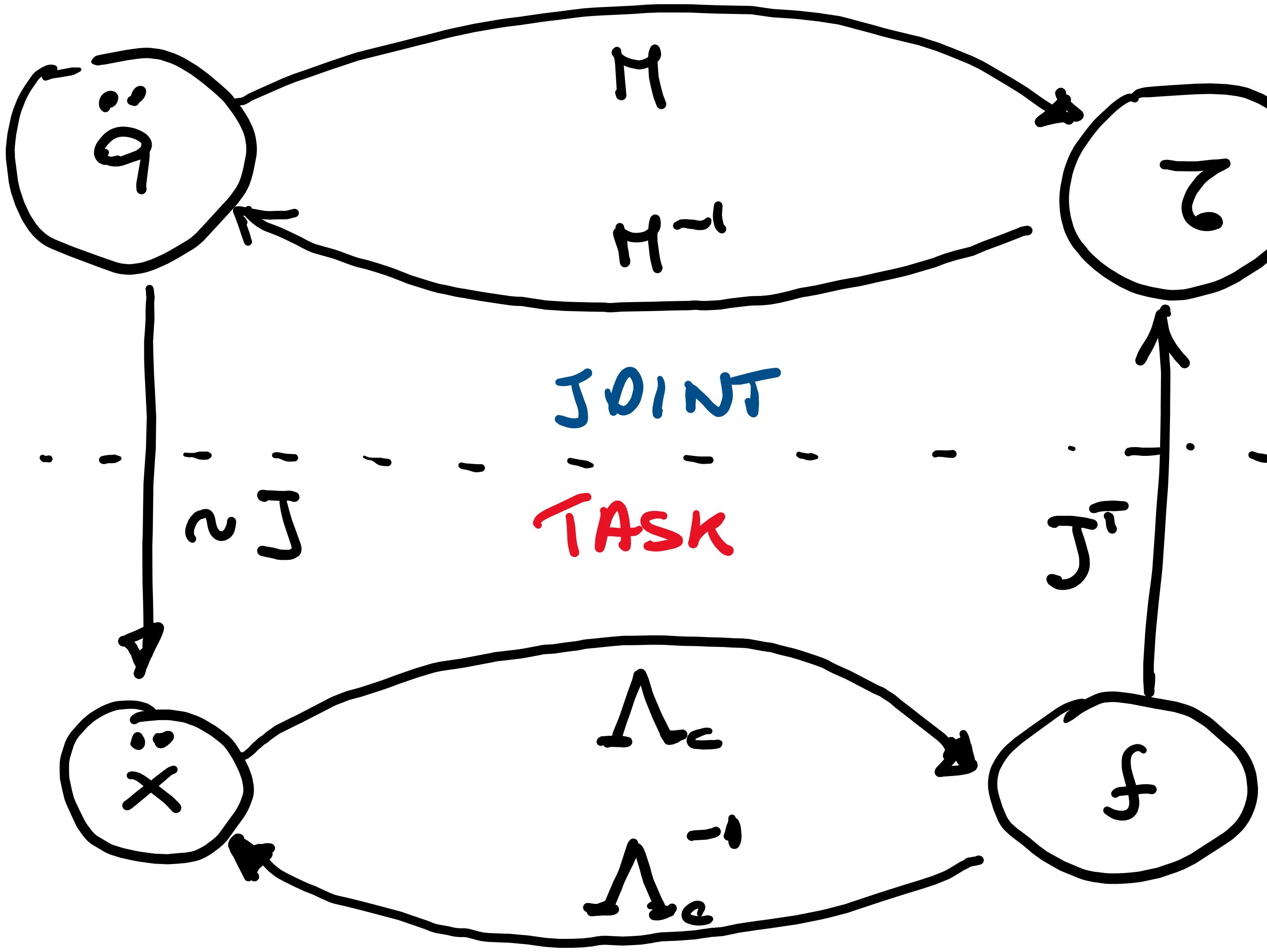
$$f^d = \ddot{x}^d + K_x (x^d - x) + D_x (\dot{x}^d - \dot{x}), \quad \ddot{x}^d(t)$$

- The error dynamics becomes $(e_x = x^d - x)$

$$I_{m \times m} \ddot{e}_x + D_x \dot{e}_x + K_x e_x = 0 \qquad e_x \to 0$$

- The Torque command is $\boxed{\tau = J^T (\hat{\Lambda} f^d + \hat{\mu})}$

RECAP MAPPINGS



M: "Joint accel" $\rightarrow$ "Joint Torques"

M^{-1}: "Joint Torques" $\rightarrow$ "Joint Accel"

J: "Joint vels" $\rightarrow$ "Task vels"

"Joint accels" $\rightarrow$ "Task accels"

J^T: "Task Forces" $\rightarrow$ "Joint Torques"

Newton law: $F = m\,a$

$\Lambda^{-1} = (J M^{-1} J^T)$: "Task forces" $\rightarrow$ "Task accels"

$\Lambda = (J^{-T} M J^{-1})$; "Task accels" $\rightarrow$ "Task forces"

References:

- D. E. Whitney, Resolved *motion rate* control of manipulators and human, 1969.

- O. Kathib, A Unified Approach for Motion and Force Control of Robot Manipulators: The Operational Space Formulation, 1987.

- P. Wensig, AME 50551 – Introduction To Robotics (L36): http://sites.nd.edu/pwensing/ame-50551-introduction-to-robotics/