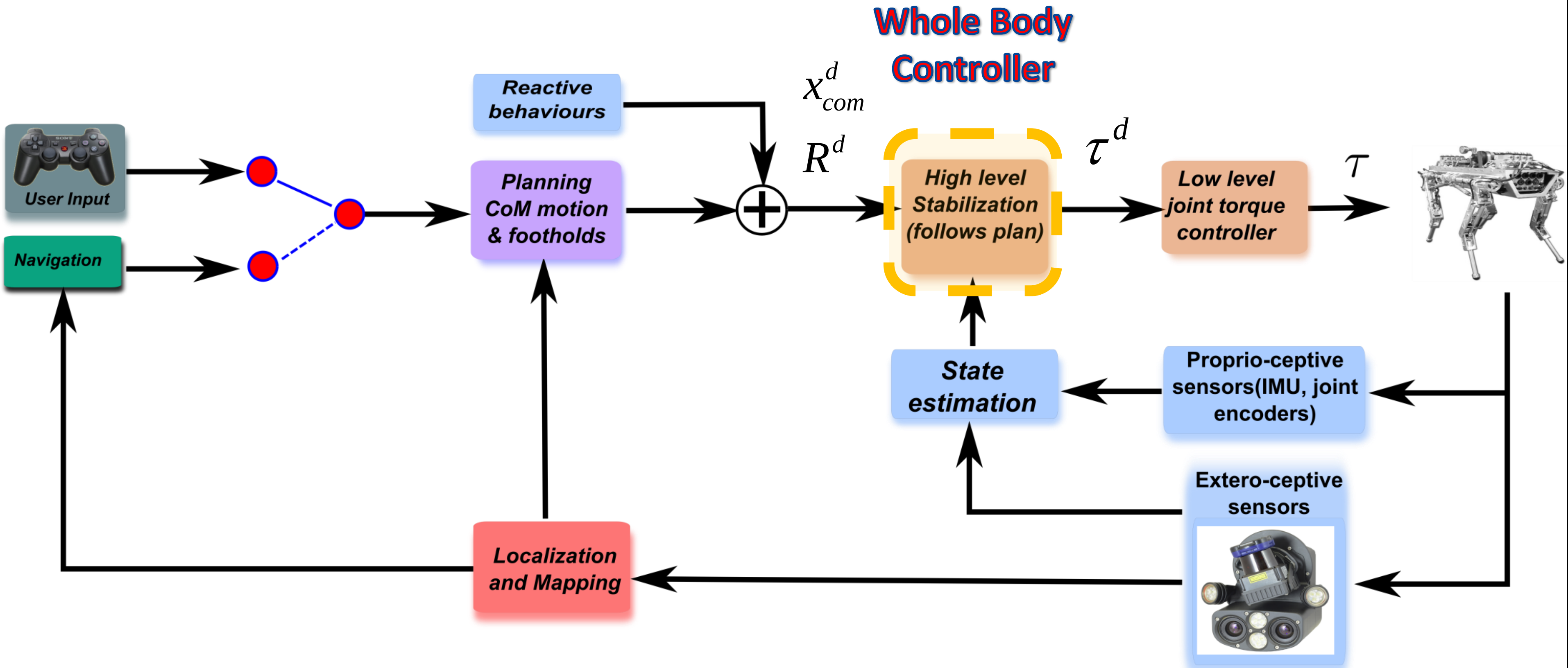
Control of Locomotion

Stability

Locomotion Framework

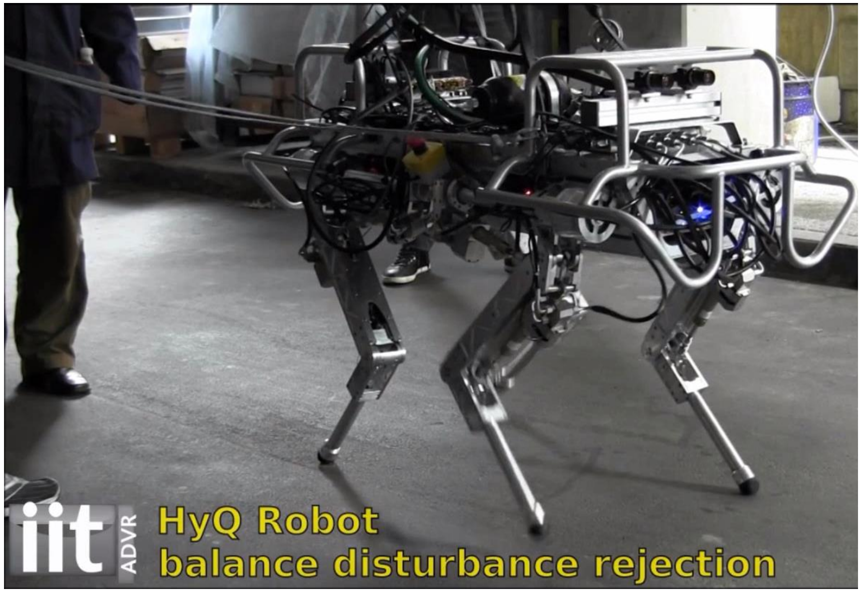


Whole-Body Controller

It computes joint torques:

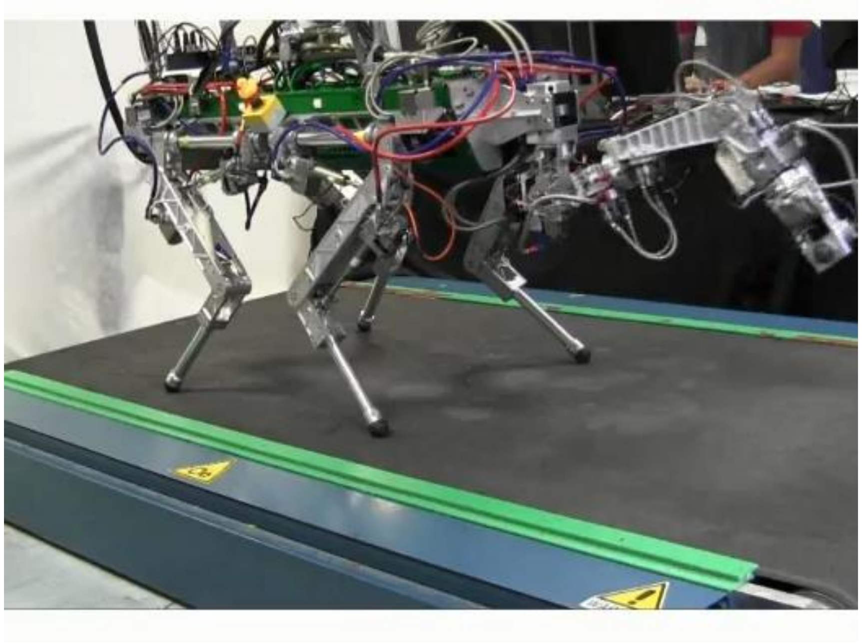
1) to stabilize trunk orientation

[Barasuol et al. *ICRA, 2013]*



2) to stabilize CoM position

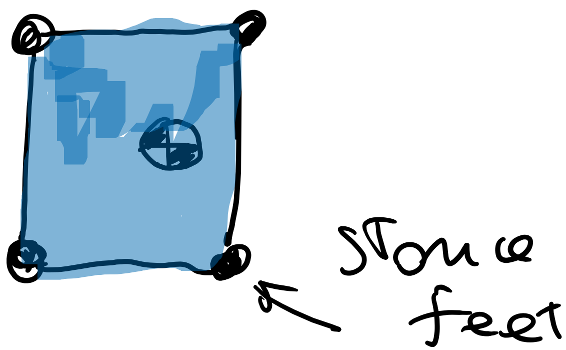
[Rehman et al. ICRA, *2017]*



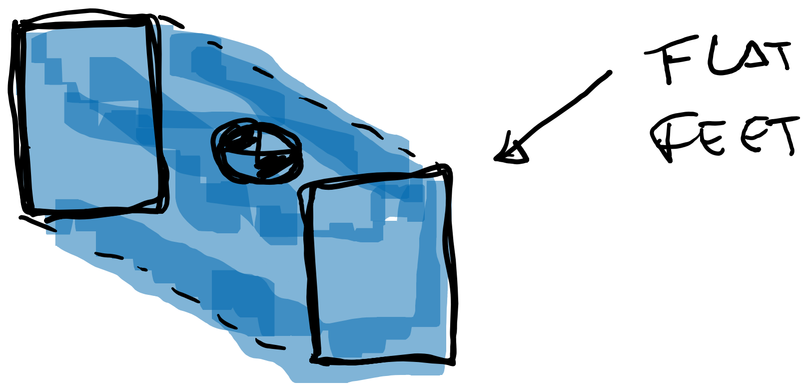
LOCOMOTION STABILITY

$\triangleq$ avoid falling, tipping over $\neq$ classical control stability definition

static stability criterion: keep the CoM projection inside the support polygon



QUADRUPED



BIPED

$\Rightarrow$ we want to be able to control CoM

$\Rightarrow$ first let's start from the base

Outline

Quasi-Static Control

Projection-based Approach

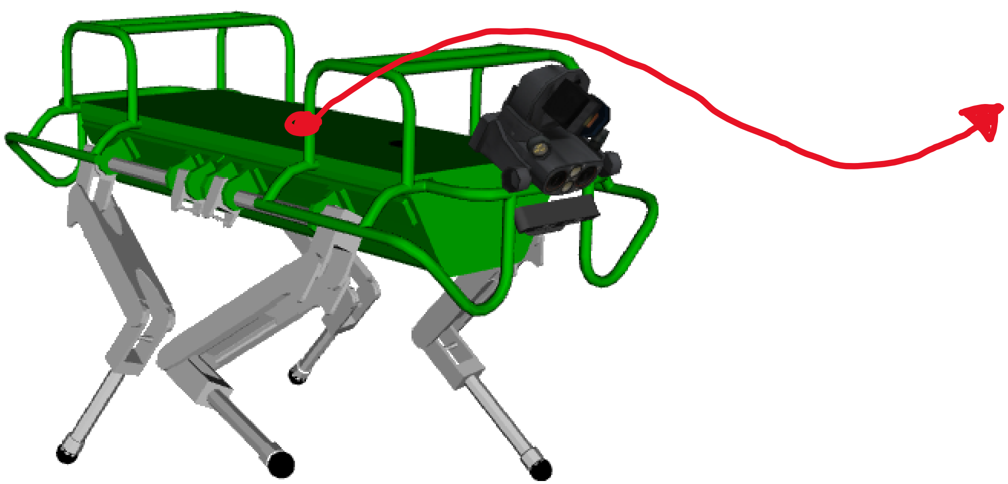
QP-Based Approach

Friction Constraints

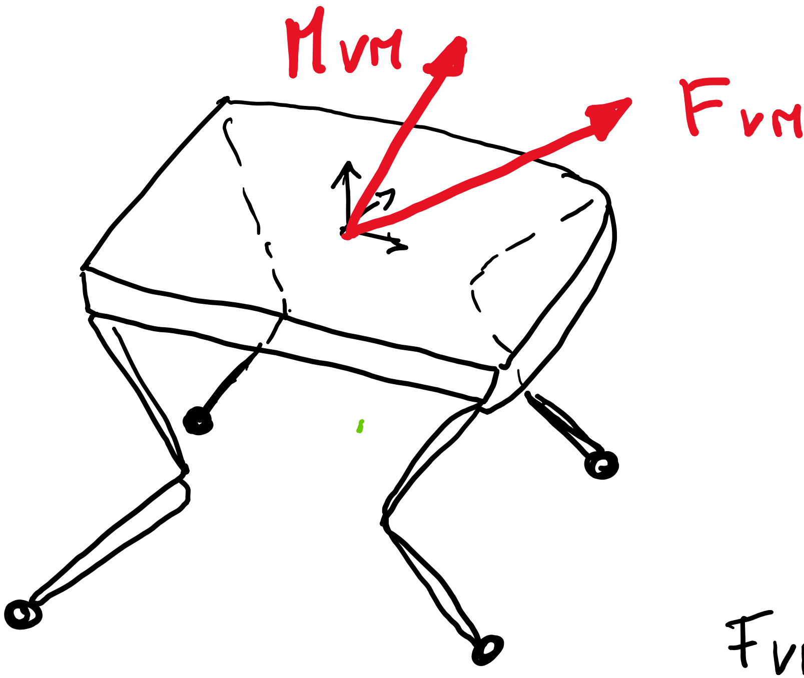
Different Regularizations

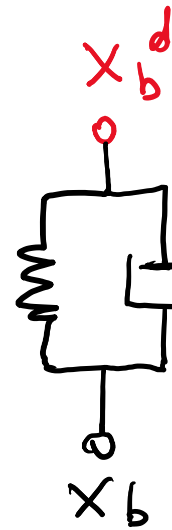
QUASI - STATIC APPROACH

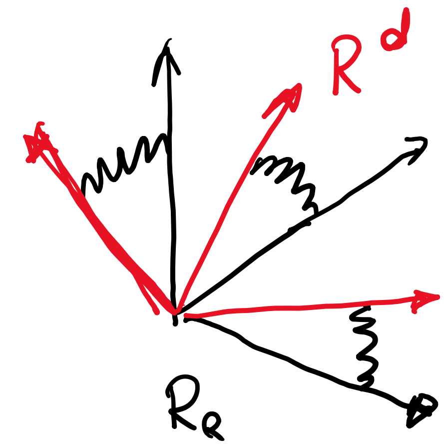
- Applicability : slow motions, quasi static motions
- assumptions : neglects $M, \ddot{q}, h$
- computes contact forces / Torques to produce the same effect of VIRTUAL FORCES / MOMENTS at the base frame (e.g. emulate virtual springs / dampers attached to the base)
- These virtual elements track a reference Trajectory coming from a planner



DESIRED WRENCH COMPUTATION







$$M_{VM} = -K_\theta e_O$$

$$F_{VM} = K_p (x_b^d - x_b)$$

DESIRED WRENCH:

$$W^d = \begin{bmatrix} F_{VM} \\ M_{VM} \end{bmatrix} \in \mathbb{R}^6$$

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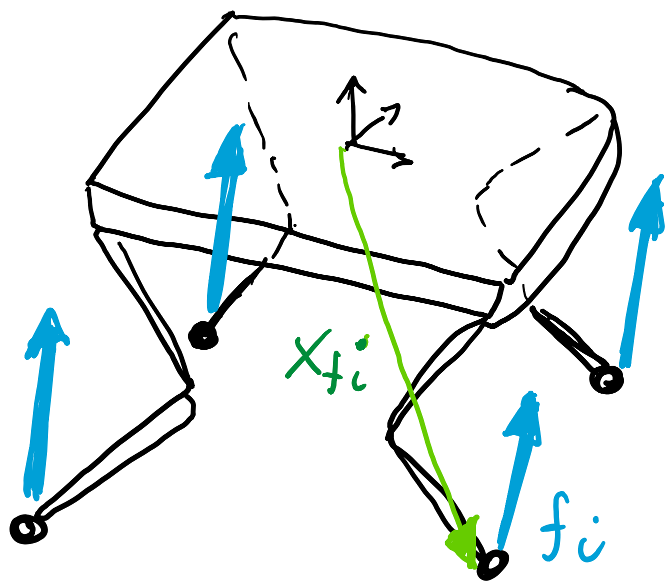
VIRTUAL MODELS

- introduced by PRATT in 1997
- extends the concept of virtual work considering as end-effector the base frame
- exploits the J_b^T matrix for the mapping between base wrench and contact forces



$$\boxed{w^d = J_{cb}^T f^d} \quad (1)$$

- quasi-static assumption

$$\boxed{\tau \simeq -J_{cq}^T f + g} \quad (2)$$

how does J_b^T look like?

it is called "contact wrench matrix". From statics:

$$\sum f_i = F_{VM}$$

$$\sum X_{f_i} \times f_i = M_{VM}$$

$$[v]_\times = \begin{bmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{bmatrix}$$

3 legs on ground

$$\underbrace{\begin{bmatrix} I_1 & I_2 & I_3 \\ [X_{f_1}]_\times & [X_{f_2}]_\times & [X_{f_3}]_\times \end{bmatrix}}_{J_b^T} \underbrace{\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}}_{f} = \underbrace{\begin{bmatrix} F_{VM} \\ M_{VM} \end{bmatrix}}_{W^d}$$

$\Rightarrow$ first we map W^d into contact forces:

$$f = (J_b^T)^\# W^d$$

$\Rightarrow$ Then we map into torques:

$$\boxed{\tau = -J_{cJ}^T f = -J_{cJ}^T (J_{cb}^T)^\# W^d}$$

$\hookrightarrow$ f is a contact force not end-effector force

⊕ very simple

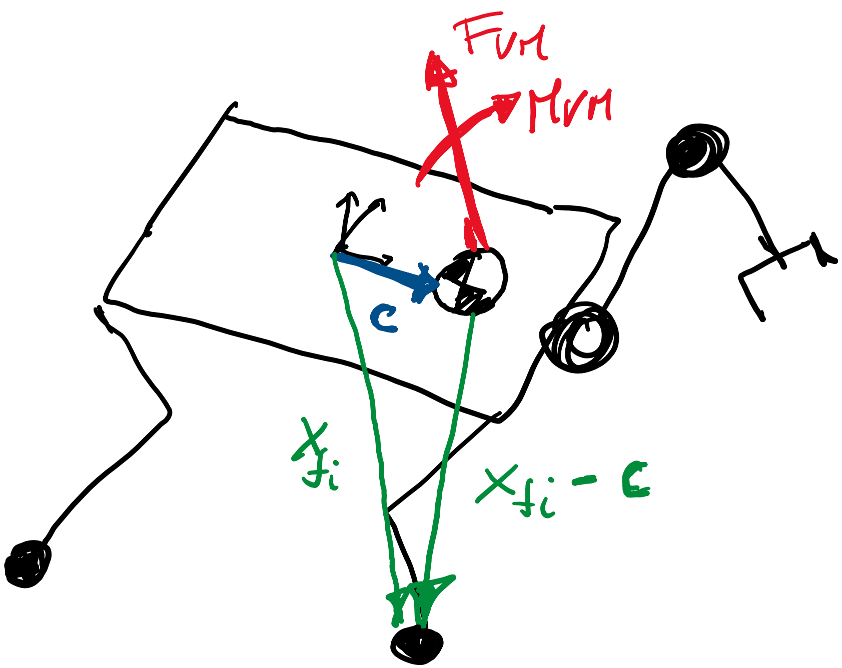
⊖ No guarantee on contact unilaterality (I can have $f_z < 0$) or slippage constraints

- we can exploit force redundancy getting different solutions by setting a weighting matrix in the pseudo-inverse.

⊖ we close the loop on the base but locomotion stability depends on com

DERIVATION FOR COM

- mandatory if you have decentralized loads (e.g. an arm on the quadruped)
- consider the feet position w.r.t com when computing J_b^T (e.g. equilibrium of moments about com)



$$\sum f_i = F_{VM}$$

$$\sum (x_{f_i} - c) \times f_i = M_{VM}$$

$$J_{com}^T = \begin{bmatrix} I_1 & \cdots & I_c \\ [x_{f1} - c]_\times & \cdots & [x_{fc} - c]_\times \end{bmatrix}$$

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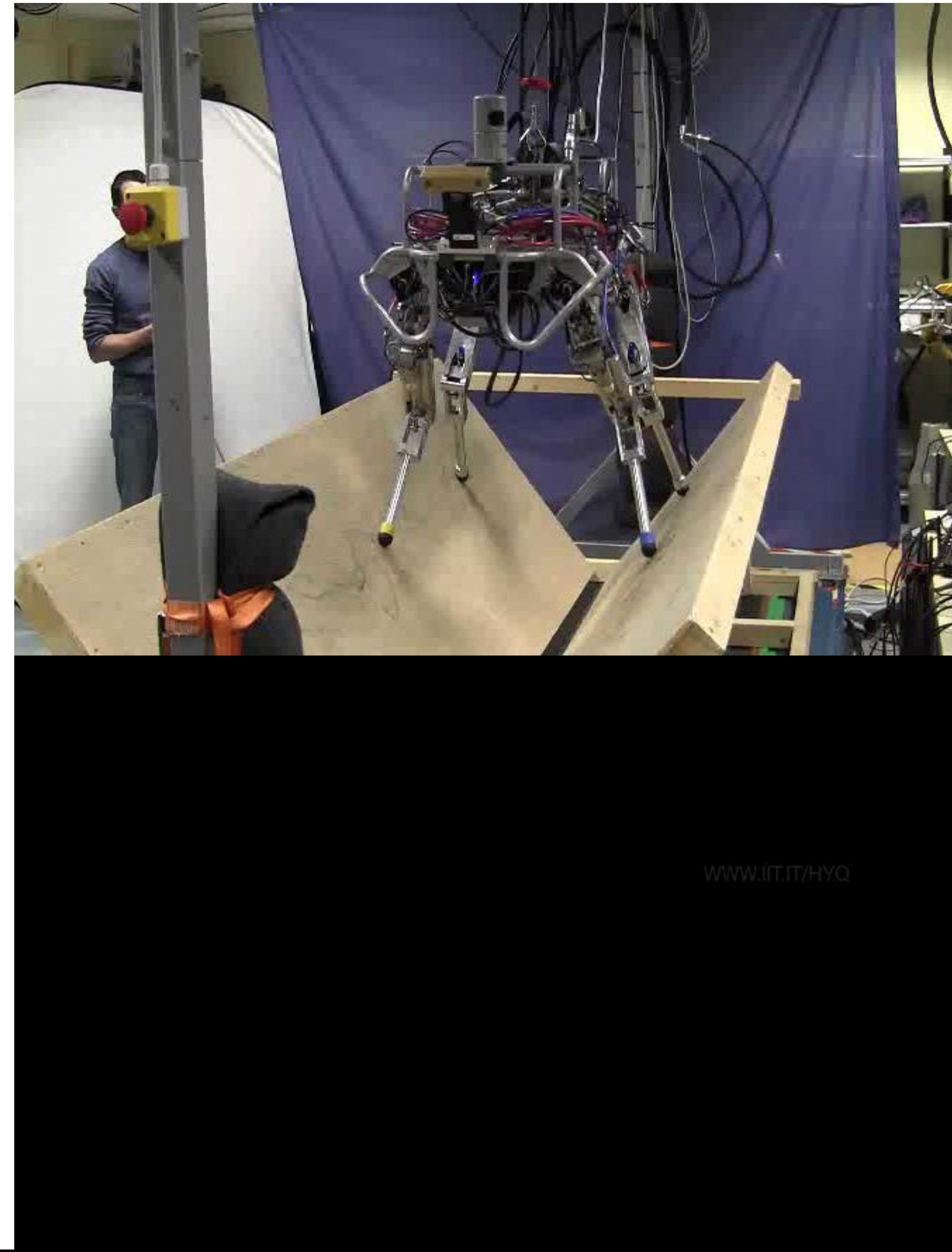
Chimney-climb

No Optimization of contact forces



Optimization of contact forces

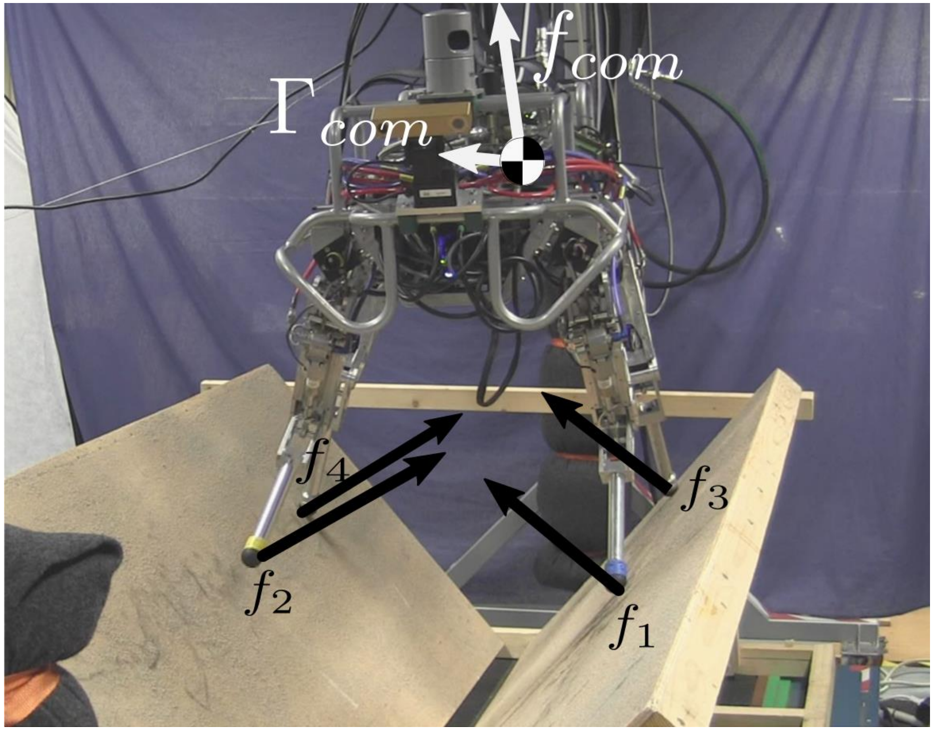
Walking on slippery terrain

No Optimization of contact forces

Optimization of contact forces

QUASI STATIC QP

Idea: cast the control problem as an optimization problem, with contact forces as decision variables



FORMULATION OF OPTIMIZATION PROBLEM

COST : $\| W - W^d \|_2^2$

$$f^* = \underset{f \in \mathbb{R}^k}{\arg\min} \| J_b^T f - W^d \|^2 + f^T W f$$

s.t.

$$\boxed{\text{The physics of the contact must be satisfied}}$$

Note : f are the contact forces only of the legs in <u>stance</u>

J_b^T changes dimension when the contact state changes

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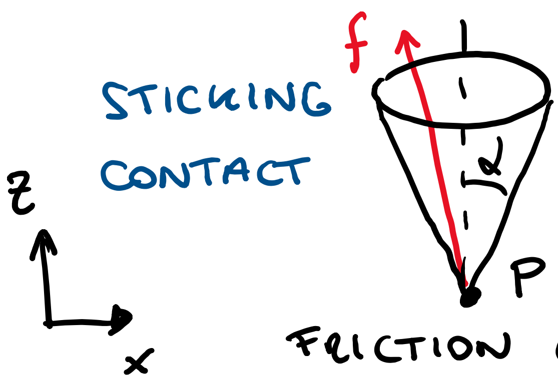
RIGID CONTACT MODEL

Physics puts limits on:

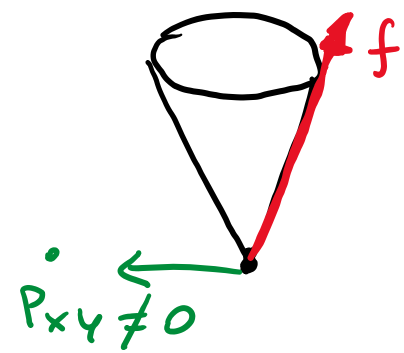
① FORCES TANGENTIAL TO THE CONTACT SURFACE [wieber]

COULOMB FRICTION MODEL



$\alpha = 2\tan(\mu)$

SLIDING CONTACT



FRICTION CONE

$\| f_{xy} \| \leq \mu \, f_z$

$\dot{P}_{xy} = 0$

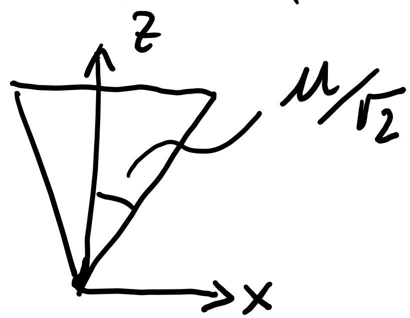
$f_{xy} = -\mu \, f_z \dfrac{\dot{P}_{xy}}{\|\dot{P}_{xy}\|}$

$\dot{P}_{xy} \neq 0$

⇒ FRICTION CONE CONSTRAINTS

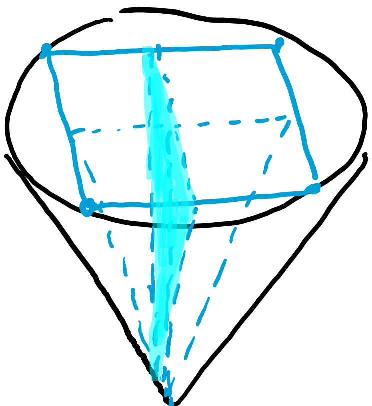
PYRAMID APPROXIMATION OF FRICTION CONES

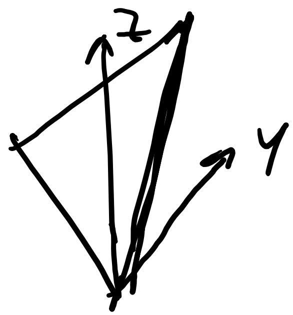
- Linear constraints formulation



$$f_x \le \mu f_z$$

$$-f_x \le \mu f_z$$





$$f_y \le \mu f_z$$

$$-f_y \le \mu f_z$$

$$f_x = t_x^T f$$
$$f_y = t_y^T f$$
$$f_z = n^T f$$

$$\Longrightarrow \underbrace{\begin{bmatrix} t_x^T - \mu n^T \\ -t_x^T - \mu n^T \\ t_y^T - \mu n^T \\ -t_y^T - \mu n^T \end{bmatrix}}_{A_{fc}} f \le \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(B) FORCES ORTHOGONAL TO THE CONTACT SURFACE

$f_z > 0$ if $p_z = 0$ $\Rightarrow$ unilaterality condition:

$f_z = 0$ if $p_z > 0$

- solid in contact push one another but don't pull
- no penetration

$\Rightarrow$ UNILATERAL CONSTRAINTS

⊕ Implicitely enforced by friction cone

⊖ can be encoded as COMPLEMENTARITY CONDITION

$f_z \cdot p_z = 0 \quad p_z \geq 0, \; f_z \geq 0$

↳ either p_z or f_z is zero → Pathological if both p_z and f_z are decision variables

LOADING - UNLOADING OF LEG

- To avoid Torque discontinuities when you chang contact state it is useful To first unload The leg To swing

- in This case is useful To explicitly set a constraint on f_z

$$\boxed{n^T f \leq f_{max}}$$

QUASI-STATIC QP

$$f^* = \underset{f \in \mathbb{R}^k}{\arg\min} \, (J_{cb}^T f - W^d)^T S (J_{cb}^T f - W^d) + \underbrace{f^T W f}_{\text{REGULARIZATION}}$$

$$\text{s.t.} \quad A_{fc} f \leq 0$$

- Then, as in the projection case, compute the torques

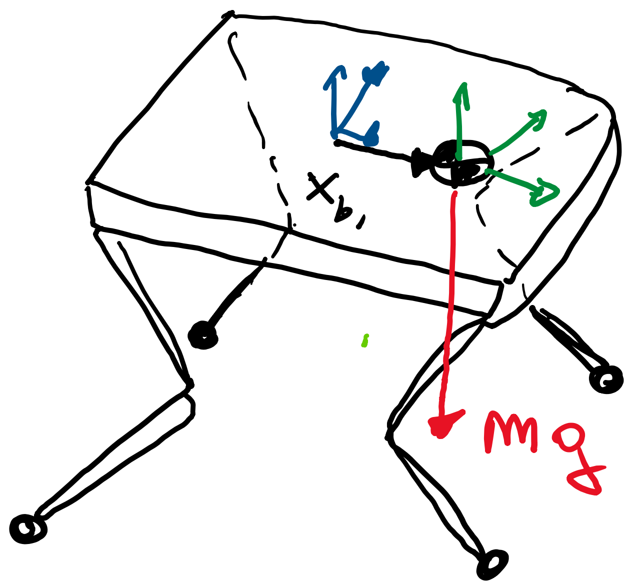
$$\tau = -J_{cg}^T f^* + g_J \rightarrow \text{leg gravity compensation}$$

- The S matrix can be used to weight differently the wrench directions in case of underactuation ($W^d \neq W$)

GRAVITY COMPENSATION WRENCH

- you should add a gravity compensation term to W^d

$$W^d = W_{fBk} + W_g \to \begin{cases} \begin{bmatrix} mg \\ x_{b,com} \times mg \end{bmatrix} & \text{base frame} \\ \begin{bmatrix} mg \\ 0 \end{bmatrix} & \text{com frame} \end{cases}$$



QUASI-STATIC QP-REGULARIZATION

$$f^* = \arg\min_{f \in \mathbb{R}^k} (J_{cb}^T f - W^d)^T S (J_{cb}^T f - W^d) + f^T W f$$

$$\text{s.t.} \quad A_{fc} f \leq 0$$

REGULARIZATION

- wherever J_{cb}^T is not square, $\exists$ a Null-space there is not an unique solution and the Hessian $J_{cb} S J_{cb}^T$ is positive semi-definite
- we use a regularization term $f^T W f$ to make the Hessian of the QP positive definite.

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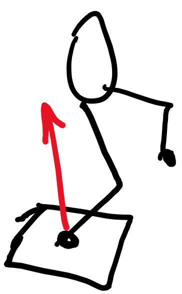
RECALL ON FORCE REDUNDANCY

- we can apply torques on n DOFs
  - part of them will generate motions
  - part will generate internal forces (no motion)

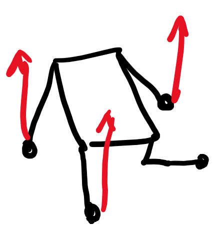
$\downarrow$

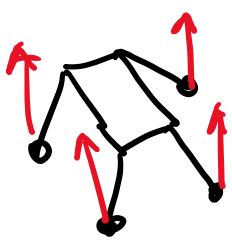
This depends on kind of contact constraints and rank of J_b



- rank $(J_b^T) = 6$, $k = 6$ $\Rightarrow$ no redundancy

- rank $(J_b^T) = 6$, $k > 6$ $\Rightarrow$ force redundancy





Depending on The weighting matrix W we can get different solutions:

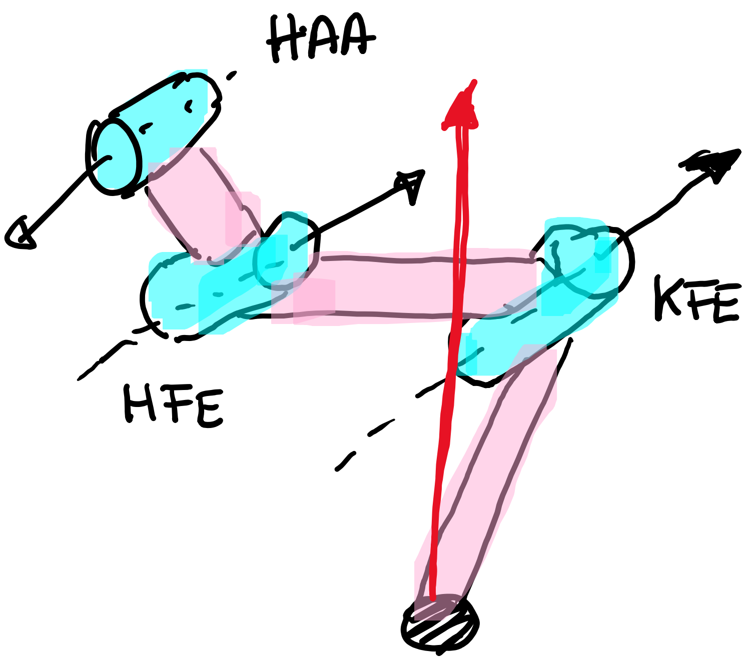
Ⓐ $W = I_{k \times k}$ minimum norm solution of The force vector

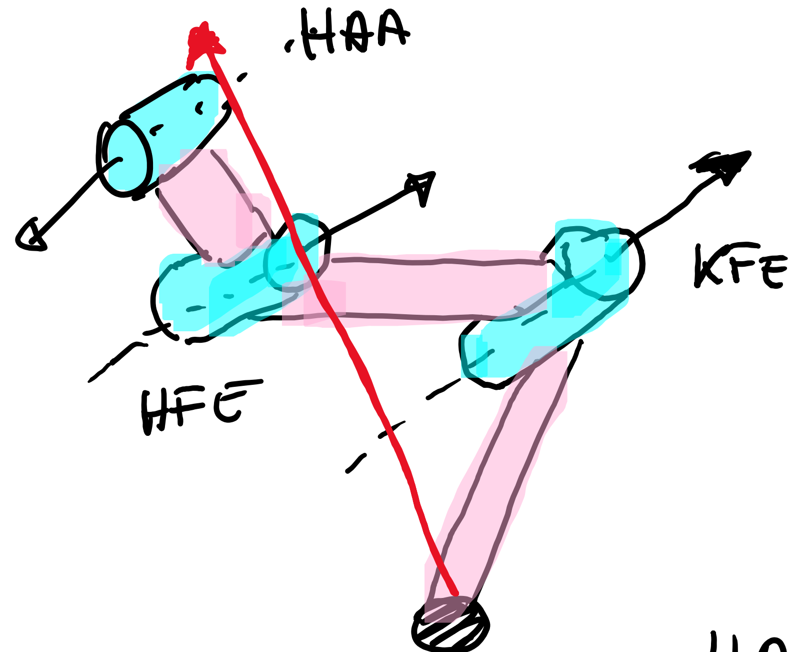
Ⓑ $W = J_{cq} W_\tau J_{cq}^T$ torque minimization

$$f^T W f = \underbrace{f^T J_{cq}}_{\tau^T} W_\tau \underbrace{J_{cq}^T f}_{\tau} = \tau^T W_\tau \tau$$

we can decide To penalize $\underline{\underline{\text{selective}}}$ ly the Torque in some joints with W_τ

- The contact forces will try to have a line of action passing through the join axes





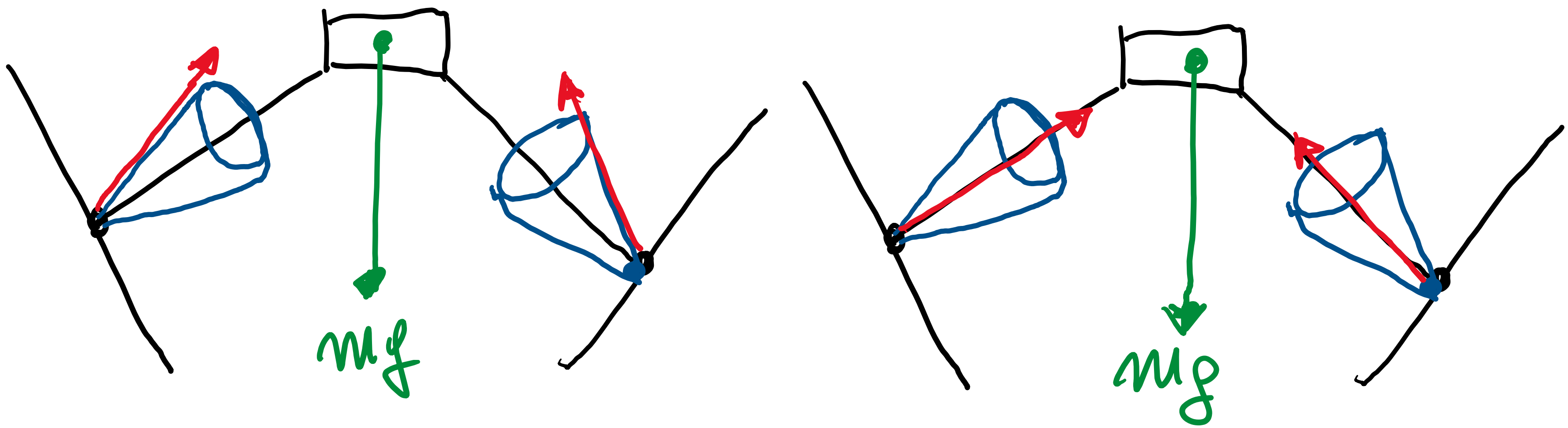
$$W_{z_{LEG}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

↳ KFE

HAA

$$W_{z_{LEG}} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

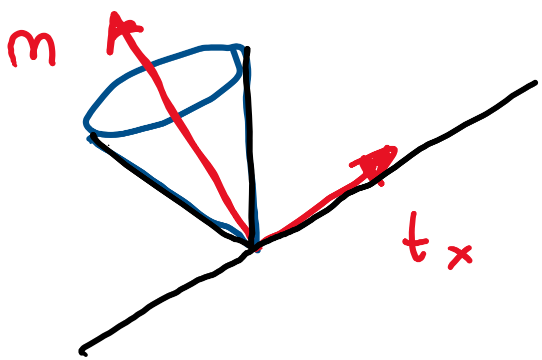
(c) MAXIMIZE CONTACT ROBUSTNESS



Not robust contact — robust contact

- having the contact force in the middle of the cone improves robustness in face of uncertainties on:
  - friction coefficient
  - estimation of the terrain normal (VISION)

- We normalize the contact forces in the contact frame C



$$ {}_wR_c = [\, t_x \,\vdots\, t_y \,\vdots\, n \,] $$

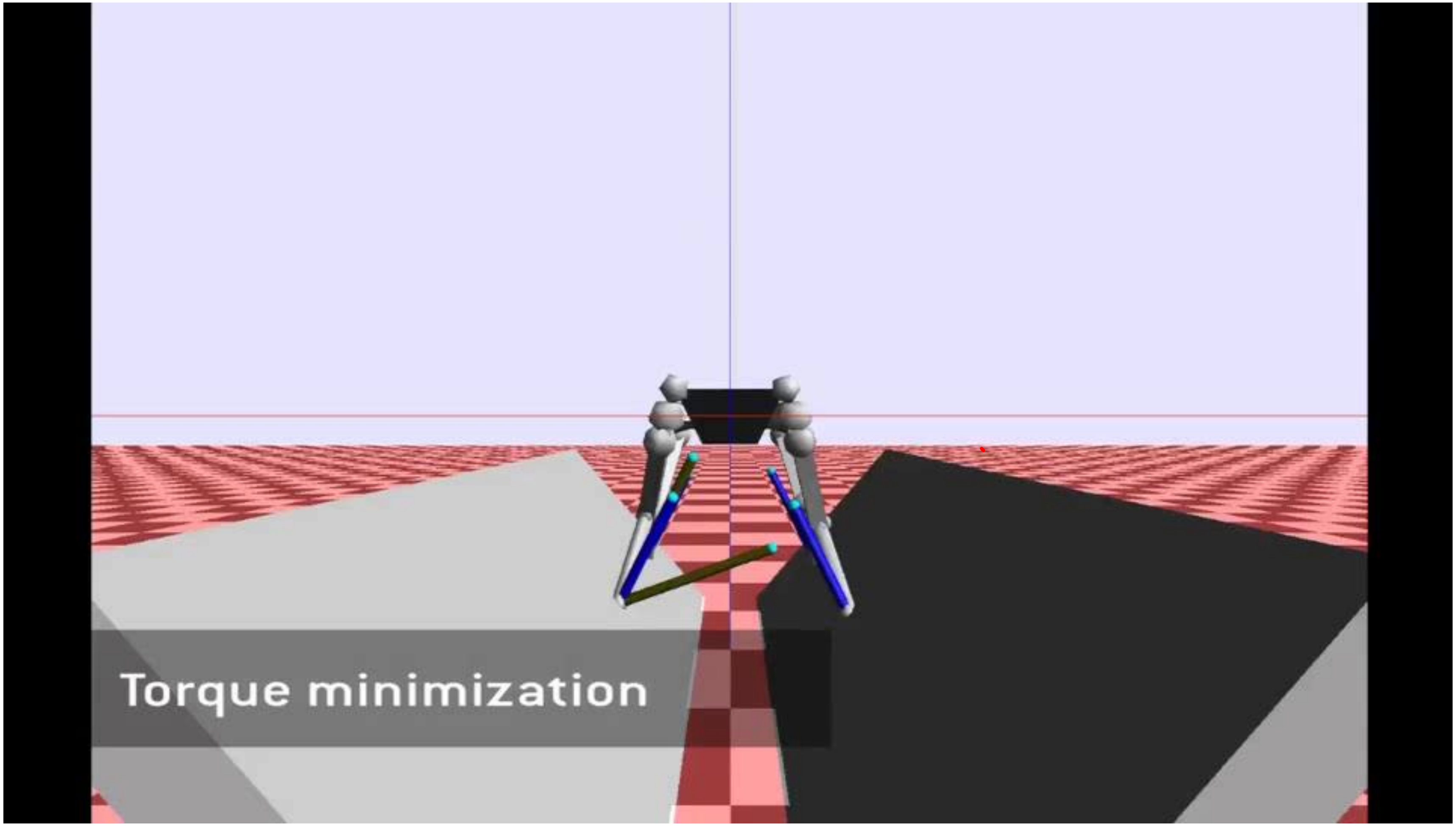
$$ W_{LEG_i} = {}_wR_{c_i} \, W_f \, {}_wR_{c_i}^T $$

$$ W_f = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

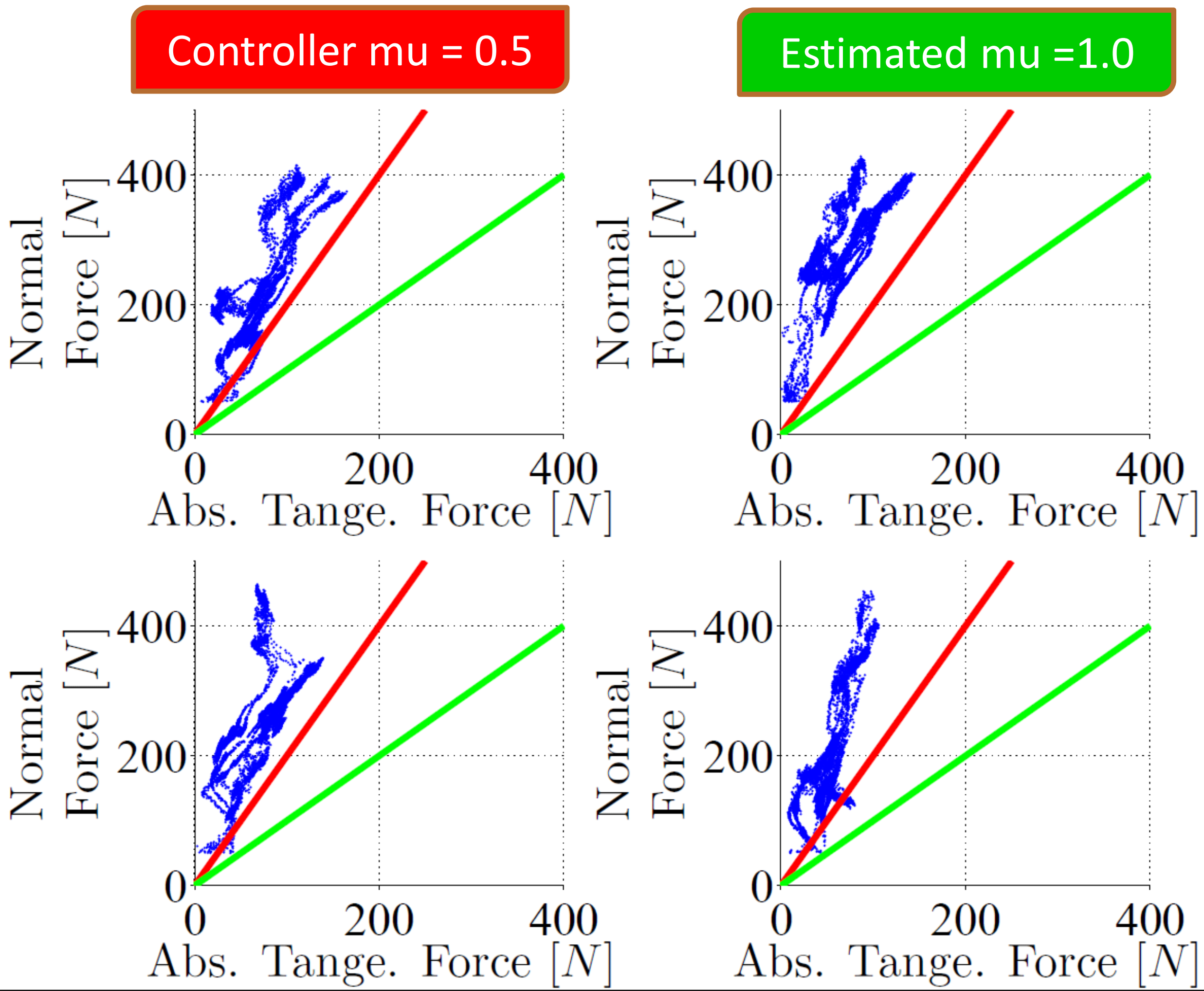
penalize Tangential components

$$ W = \begin{bmatrix} W_{LEG1} & 0 & 0 & 0 \\ 0 & W_{LEG2} & 0 & 0 \\ 0 & 0 & W_{LEG3} & 0 \\ 0 & 0 & 0 & W_{LEG4} \end{bmatrix} $$

Simulation of different regularizations



Experimental results: friction cones



Conservative friction coefficient ensures better tolerance to **uncertainties** in the **normal direction** of the surface

tolerance to slope estimation errors of up to 18deg

References:

- Modeling and Control of Legged Robots, P.Wieber, 2015: https://groups.csail.mit.edu/robotics-center/public_papers/Wieber15.pdf
- Virtual Model Control: An Intuitive Approach for Bipedal Locomotion, J.Pratt, 2001.
- High-slope terrain locomotion for torque-controlled quadruped robots**,** M. Focchi, Autonomous Robots, 2017