Floating Base Dynamics

Reparametrization in Centroidal Coordinates

Recall on Centroidal Quantities

Full Diagonalization of Inertia Matrix

Newton-Euler Equations

MOTIVATION

- we want to control com → include it in the representation of the robot position

→ involves a change of state variables that will symplify the dynamics in the new state

SYSTEM STATE TRANFORM FOR CENTROIDAL DYNAMICS

$$\begin{bmatrix} M_b(q) & M_{bJ}(q) \\ M_{Jb}(q) & M_J(q)\end{bmatrix}\begin{bmatrix}\ddot q_b\\ \ddot q_J\end{bmatrix} + \begin{bmatrix} h_b(q,\dot q)\\ h_J(q,\dot q)\end{bmatrix} = \begin{bmatrix}0\\ \tau_J\end{bmatrix} + \begin{bmatrix} J_{c_b}^T(q)\, f \\ J_{c_J}^T(q)\, f\end{bmatrix}$$

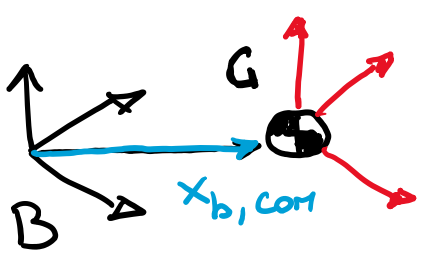
The first 6 rows correspond to the underactuated dynamics of the robot

If we rewrite this equation parameterizing using com velocity

$\dot x_b \rightarrow \dot x_{com}$

$\omega_b \rightarrow \omega$

$$\underbrace{\begin{bmatrix}\dot x_{com}\\ \omega^b\\ \dot q_J\end{bmatrix}}_{\dot q_G} = \begin{array}{c} 3\\ 3\\ n\end{array} \underbrace{\begin{bmatrix} I & -[x_{b,com}]_\times & J_{b,com}(q)\\ 0 & I & 0\\ 0 & 0 & I\end{bmatrix}}_{{}_G T_B} \underbrace{\begin{bmatrix}\dot x_b\\ \omega^b\\ \dot q_J\end{bmatrix}}_{\dot q_B}$$



! $\dot q_B \in \mathbb{R}^{n+6} \neq \dot q_b \in \mathbb{R}^6$

G frame :
- origin in com
- aligned with B

${}_G T_B$: maps into the new frame G of the floating base

[TRAVERSARO 2017]

$$\dot{q}_G = {}_G T_B \, \dot{q}_B$$

$$\ddot{q} = {}_G T_B \, \ddot{q}_B + \dot{T}_B \, \dot{q}_B = {}_G T_B \, \ddot{q}_B + {}_G \dot{T}_B \, {}_G T_B^{-1} \, \dot{q}_G$$

expressing base coordinates

(1) $$\boxed{\dot{q}_B = {}_G T_B^{-1} \, \dot{q}_G}$$

(2) $$\boxed{\ddot{q}_B = {}_G T_B^{-1} \left[\ddot{q}_G - {}_G \dot{T}_B \, {}_G T_B^{-1} \, \dot{q}_G \right]}$$

- end effector velocity is also invariant wrt change of floating base frame

$$\dot{x}_f = J_B \dot{q}_B = J_G \dot{q}_G \quad \oplus \quad \dot{q}_G = {}_G T_B \, \dot{q}_B$$

$$\Downarrow$$

$$J_B = J_G \, {}_G T_B$$

$$\Downarrow$$

$$(3) \quad \boxed{J_B^T = {}_G T_B^{\,T} \, J_G^T}$$

→ The Jacobian changes because we changed the floating base

- Torque is invariant too:

$$(4) \quad \boxed{{}_G T_B \begin{bmatrix} 0 \\ \tau \end{bmatrix} = \begin{bmatrix} 0 \\ \tau \end{bmatrix}}$$

- Replacing (1), (2), (3), (4) into dynamics we perform the change of variables $\dot{q}_B \to \dot{q}_G$
- for light notation $T = {}_G T_B$

$$M_B T^{-1}\ddot{q}_G - M_B T^{-1}\dot{T}\,T^{-1}\dot{q}_G + \overbrace{C_B T^{-1}\dot{q}_G + G_B}^{h_B} = \begin{bmatrix} 0 \\ \tau \end{bmatrix} + T^{T} J_G^{T} f$$

- left-multiply by T^{-T}

$$\underbrace{T^{-T} M_B T^{-1}}_{M_G}\ddot{q}_G + \underbrace{\left[-T^{-T} M_b T^{-1}\dot{T}\,T^{-1} + T^{-T} C_B T^{-1} \right]}_{C_G}\dot{q}_G + \underbrace{T^{-T} G_B}_{G_G} = S^T \tau + \cancel{T^{-T}}\,\cancel{T^{T}}\, J_G^{T} f$$

$$\boxed{M_G\,\ddot{q}_G + h_G = S^T \tau + J_G^{T} f}$$

DYNAMICS WITH COM AS FLOATING BASE

PROPERTIES OF M_G:

[OTT 2011]

$$M_G = \begin{bmatrix} m I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & \bar{M}(q) \end{bmatrix}$$

$\rightarrow$ TOTAL ROBOT MASS

$n+3 \times n+3$

$\hookrightarrow \in \mathbb{R}$

- M_G is BLOCK DIAGONAL $\rightarrow$ com acceleration $\ddot{x}_c$ independent from joint accelerations $\ddot{q}_J$
- we didn't change ω, we still have couplings at the angular level

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TOTAL MOMENTUM

In euclidean algebra you need reference points (and frames) to express vectors.

${}_W h \in \mathbb{R}^6$: total momentum = sum of all the linear and angular momenta of each rigid body, expressed w.r.t the origin of frame W (and in the axes of W)

- linear forces → affect both linear/angular momentum
- moments → affect angular momentum

⇒ To be summed they should be projected to the same frame W

frame W conveys 2 informations:

- origin - about this point we compute angular momentum that needs a reference point
- axes - we take the chance to express the vectors in these axes

- The total momentum of the system can be also expressed w.r.t other frames.
- if we select a frame G attached to CoM and aligned with W the total momentum is called CENTROIDAL MOMENTUM

CENTROIDAL MOMENTUM

$$
{}_G h = \begin{bmatrix} m \dot{x}_{G} \\ {}_G K \end{bmatrix}
$$

$\dot{x}_G$ → CoM velocity expressed in G

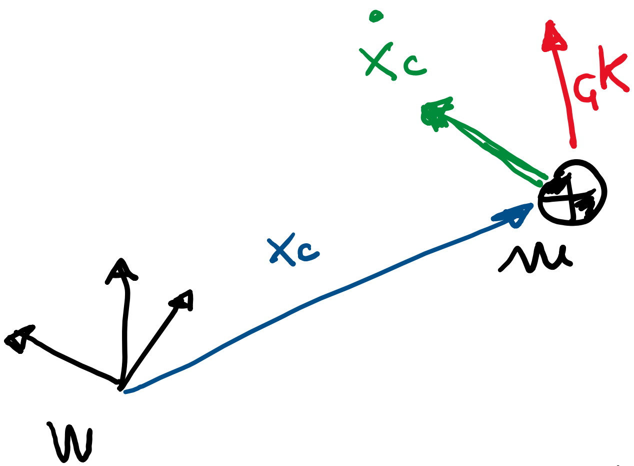
${}_G K$ → angular momentum expressed in G

$$
K_G = \sum_{k}^{\text{LINKS}} (x_{com,k} - x_{com}) \times m_k \dot{x}_k + L_k \omega_k
$$

KÖNIG THEOREM

L_k → inertia of link k about COM_k

NOTE :



$$
{}_W h = \begin{bmatrix} m \dot{x}_c \\ {}_G K + x_c \times m \dot{x}_c \end{bmatrix}
$$

${}_G K$ ↑ SPIN ANGULAR MOMENTUM

$x_c \times m \dot{x}_c$ ↑ ORBITAL ANGULAR MOMENTUM

of a particle m of speed $\dot{x}_c$

- The only difference between ${}_W h$ and ${}_G h$ is that ${}_W h$ contains within its angular part also the angular momentum of CoM (orbital)

NON CONVEXITY OF CENTROIDAL MOMENTUM

if centroidal momentum is used in Trajectory optimization for com there is a bi-linear non-convex Term:

$$K_G = {}_W K - x_c \times (m \dot{x}_c)$$

↓ cross product of 2 decision variables

bilinear ≜ fixing either x_c or $\dot{x}_c$ the function becomes linear (eg. replace upper-bound ...)

LOCKED SPATIAL VELOCITY

[SACCON 2017]

For a system of bodies, the locked velocity is the velocity the base link should move (while considering the links "locked" to get the same value of the momentum.

$$ {}_B h = M_b \underbrace{\begin{bmatrix} \dot{x}_b \\ \omega_b \end{bmatrix}}_{\dot{q}_b} + M_{bJ} \dot{q}_J = M_b V_{Lock} $$

$$ \boxed{V_{Lock} = \dot{q}_b + M_b^{-1} M_{bJ} \dot{q}_J} $$

AVERAGE SPATIAL VELOCITY

[ORIN 2013]

(1) $\boxed{V_{AVE} = {}_G X_B \, V_{LOCK}}$

$\hookrightarrow$ motion transform

in humanoid literature is common to define it as function of centroidal momentum (${}_G h$) and centroidal inertia (${}_G M_b$)

(2) $\boxed{V_{Ave} = {}_G M_b^{-1} \, {}_G h}$

$A(q)$ centroidal momentum matrix

$${}_G h = \underbrace{{}_G X_B^* \begin{bmatrix} M_b & M_{bJ} \end{bmatrix}}_{A(q)} \dot{q}_B$$

$$V_{Ave} = {}_G M_b^{-1} \, {}_G X_B^* \, M_b \, \dot{q}_b + \underbrace{{}_G M_b^{-1} \, {}_G X_B^* \, M_{bJ}}_{S_{B,G}} \dot{q}_J$$

with ${}_G M_b^{-1} = {}_G X_B \, M_b^{-1} \, {}_G X_B^*$

(3) $\boxed{V_{Ave} = {}_G X_B \, \dot{q}_b + S_{B,G} \, \dot{q}_J = \begin{bmatrix} {}_G X_B & S_{B,G} \end{bmatrix} \dot{q}_B}$

- it can be proved (1) and (3) are equivalent

- if we analize (2) closer:

$$\boxed{V_{ave} = {}_G M_b^{-1} \; {}_G h} = \begin{bmatrix} \dot{x}_{com} \\ \bar{\omega} \end{bmatrix}$$

- centroidal spatial inertia is block diagonal

$${}_G M_b = \begin{bmatrix} m I & 0 \\ 0 & {}_G L \end{bmatrix}$$

$\sum {}_G L_i$, ${}_G L_i$ rotational inertia of link i projected to COM

- ${}_G h$ is the sum of link spatial momenta

$${}_G h = \sum {}_G I_i \begin{bmatrix} \dot{x}_{ci} \\ \omega_i \end{bmatrix}$$

spatial velocity of link i

$$\boxed{\bar{\omega} = {}_GL^{-1} \sum {}_GL_i \omega_i}$$ angular part of (2)

Average angular velocity $\triangleq$ angular velocity of the entire system assuming internal joints are fixed

- analoguous to formula

$$\boxed{\dot{x}_{COM} = \frac{1}{m} \sum m_i \dot{x}_{COM,i}}$$

! abuse of terms because $\bar{\omega}$ is not defined as the angular velocity of an orientation frame.

! $\nexists\ R \in SO(3)$ such that $\dot{R} = \bar{\omega} \times R$

$\bar{\omega}$ cannot be integrated [wieber 2005]

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using definition of average angular velocity we can define a new parametrization of generalized velocities:

$$\underbrace{\begin{bmatrix} \dot{x}_{com} \\ \bar{\omega} \\ \dot{q}_J \end{bmatrix}}_{\dot{q}_{\bar{G}}} = \underbrace{\begin{bmatrix} ? & ? & ? \\ 0 & 0 & I \end{bmatrix}}_{{}_{\bar{G}}T_B} \underbrace{\begin{bmatrix} \dot{x}_b \\ \omega_b \\ \dot{q}_J \end{bmatrix}}_{\dot{q}_B}$$

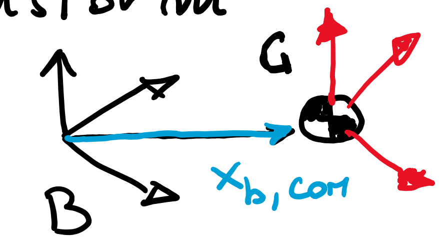
AVG ANGULAR VELOCITY $\rightarrow \bar{\omega}$

Recall:

$$V_{Ave} = \begin{bmatrix} \dot{x}_{com} \\ \bar{\omega} \end{bmatrix} = \begin{bmatrix} {}_{G}X_B & \vdots & {}_{G}M_b^{-1} \, {}_{G}X_b^{*} \, M_{bJ} \end{bmatrix} \underbrace{\begin{bmatrix} \dot{q}_b \\ \dot{q}_J \end{bmatrix}}_{\dot{q}_B}$$

$${}_G X_B = \begin{bmatrix} I_{3\times3} & -[x_{b,com}]\times \\ 0 & I_{3\times3} \end{bmatrix}$$

Motion Transform



$$\underbrace{\begin{bmatrix} \dot{x}_{com} \\ \bar{\omega} \\ \dot{q}_J \end{bmatrix}}_{\dot{q}_{\bar{G}}} = \begin{array}{c} \\ 3 \\ 3 \\ n \end{array} \underbrace{\begin{array}{c} \begin{array}{ccc} 3 & 3 & n \end{array} \\ \left[\begin{array}{cc|c} I_{3\times3} & -[x_{B,com}]\times & {}_G M_b^{-1}\, {}_G X_B^{*} M_{bJ} \\ 0 & I_{3\times3} & \\ \hline 0 & 0 & I_{n\times n} \end{array}\right] \end{array}}_{{}_{\bar{G}}T_B} \underbrace{\begin{bmatrix} \dot{x}_b \\ \omega_b \\ \dot{q}_J \end{bmatrix}}_{\dot{q}_B}$$

$$\boxed{{}_{\bar{G}}T_B = \left[\begin{array}{c|c} {}_G X_B & {}_G M_b^{-1}\, {}_G X_B^{*} M_{bJ} \\ \hline 0_{n\times6} & I_{n\times n} \end{array}\right]}$$

- using the trasformation $T = {}_{\bar{G}}T_B$ we can obtain in a similar way as before:

$$\boxed{M_{\bar{G}}\, \ddot{q}_{\bar{G}} + C_{\bar{G}}\, \dot{q}_{\bar{G}} + G_{\bar{G}} = S^T \tau + J_{\bar{G}}^T f_{\bar{G}}}$$

where

- $$\boxed{M_{\bar{G}} = T^{-T} M_B T^{-1}}$$
- $$\boxed{C_{\bar{G}} = -T^{-T} M_b T^{-1} \dot{T} T^{-1} + T^{-T} C_B T^{-1}}$$
- $$\boxed{G_{\bar{G}} = T^{-1} G_B}$$

ANALYSIS OF $M_{\bar{G}}$

$$M_{\bar{G}} = \begin{array}{c} \begin{array}{ccc} \ddot{x}_c & \dot{\bar{\omega}} & \ddot{q}_J \end{array} \\ \begin{bmatrix} m I_{3\times 3} & 0_{3\times 3} & 0_{3\times n} \\ 0_{3\times 3} & {}_G L & 0_{3\times n} \\ 0_{3\times 3} & 0_{3\times 3} & {}_{\bar{G}}M_J \end{bmatrix} \end{array}$$

TOTAL MASS

LOCKED 3D TENSOR OF INERTIA EXPRESSED AT G

CENTROIDAL JOINT MASS MATRIX

- $M_{\bar{G}}$ has block DIAGONAL structure (we will check this in the lab)
- ${}_G L$, ${}_{\bar{G}}M_J$ can be computed with ABA, CRBA algorithms and also...

FACT 1: floating base accelerations are INDEPENDENT from joint accelerations

FACT 2: gravity term is CONSTANT and influences only the acceleration of the com

$$ {}_{\bar{G}}G = \begin{bmatrix} -m\bar{g} \\ 0_{3\times 1} \\ 0_{n\times 1} \end{bmatrix} \rightarrow \text{is independent from } q! $$

if $\dot{M} - 2C$ skew sim also some coriolis/centrifugal terms vanish [TRAVERSARO 17]

$$ {}_{\bar{G}}C = \begin{bmatrix} 0_{3\times 3} & 0_{3\times(n+3)} \\ 0_{(3+n)\times 3} & \bar{C} \end{bmatrix} = \begin{bmatrix} 0_{3\times 3} & 0_{3\times 3} & 0_{3\times n} \\ 0_{3\times 3} & \# & \# \\ 0_{n\times 3} & \# & \# \end{bmatrix} $$

$\hookrightarrow \in \mathbb{R}^{(3+n)\times(3+n)}$

SPARSITY OF FLOATING BASE DYNAMICS

FACT 3: due to the block diagonal structure the UNDERACTUATED and ACTUATED parts can be DECOUPLED

$$
\begin{array}{c} 3 \\ 3 \\ n \end{array}
\begin{bmatrix} m I_{3\times3} & 0 & 0 \\ 0 & {}_G L & 0 \\ 0 & 0 & {}_G M \end{bmatrix}
\begin{bmatrix} \ddot{x}_{com} \\ \dot{\bar{\omega}} \\ \ddot{q}_j \end{bmatrix}
+ \begin{bmatrix} 0 \\ \# \\ \# \end{bmatrix}
+ \begin{bmatrix} m g \\ 0 \\ 0 \end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \\ \tau \end{bmatrix}
+ \begin{bmatrix} {}_G J_{cb}^T \\ {}_G J_{cj}^T \end{bmatrix} f
$$

because (3 contacts): ${}_G J_{cb}^T = \begin{bmatrix} I_{3\times3} & I_{3\times3} & I_{3\times3} \\ [x_{f_1} - x_{com}]_\times & [x_{f_2} - x_{com}]_\times & [x_{f_3} - x_{com}]_\times \end{bmatrix}$

$$
\begin{cases}
\overbrace{m\,\ddot{x}_{com}}^{\dot{\ell}} = \sum_{i=1}^{c} f_i - m g \\[2ex]
\underbrace{{}_G L\,\dot{\bar{\omega}} + \bar{\omega} \times {}_G L\,\bar{\omega}}_{\dot{k}} = \sum_{i=1}^{c} (x_{f_i} - x_{com}) \times f_i
\end{cases}
$$

NEWTON EQUATION

EULER EQUATION

FACT 4: The first 6 rows of the under-actuated dynamics correspond to the **NEWTON-EULER** equations of the whole system (ie robot simplified as a single rigid body (with **variable** inertia))

- They can be obtained also by balancing forces and moments acting on the robot as a whole.

$\Rightarrow$ for an articulated robot these equations are called <u>**CENTROIDAL DYNAMICS**</u>

$$\boxed{L_{com}(q),\ \dot{L}_{com} \neq 0}$$

COMMON APPROXIMATIONS

- once centroidal Trajectory is computed we get $\bar{w}$
- To compute actuation Torques often the approximation $\bar{w} \simeq w_b$ is done.
- This is valid if the inertia of the base link (i.e. Trunk) is bigger than the ones of the other links.

Floating Base Dynamics

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Recall on Centroidal Quantities

Full Diagonalization of Inertia Matrix

Newton-Euler Equations

- They describe the motion of a <u>rigid body</u> with forces / moments applied at COM

- They represent conservation of linear / angular momentum

• NEWTON EQUATION :

variation of linear momentum = external forces balance

$$\frac{d}{dt}(\ell) = \frac{d}{dt}(m\dot{x}_{com}) = \boxed{m\ddot{x}_{com} = \sum f_i - mg}$$

↑ TOTAL MASS

↑ CONTACT FORCES

we need them to move in a direction different from gravity !

⇒ from f I can integrate and obtain $x_{com}(t)$

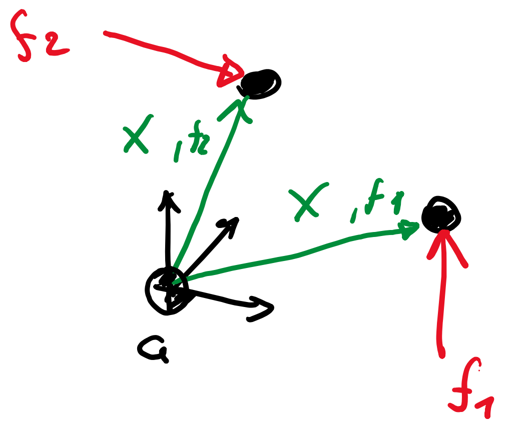
⇒ Dynamics is LINEAR

(ORIGINAL) EULER EQUATION

- Variation of angular momentum about point G (pivot point) = external moments balance about point G

appl. point of f_i

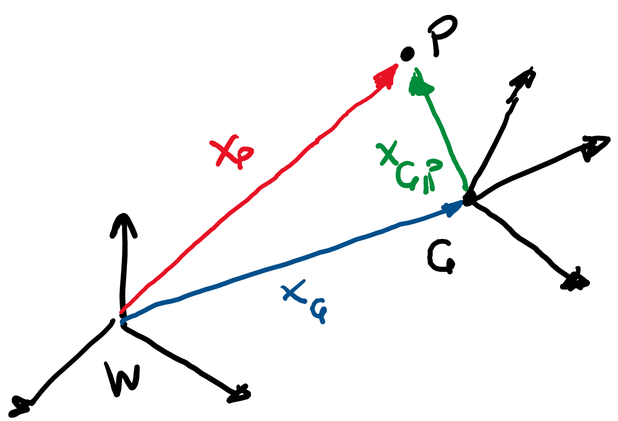
$$ {}_G\dot{k} = \underbrace{\sum_{i=1}^{c} (x_f - x_{com}) \times f_i}_{\text{external moments}} = M_{ext} $$



How does ${}_G\dot{k}$ look like?

DERIVATIVE OF A POINT IN A ROTATING FRAME

- in rotating frame the derivative of a vector has 2 components
  1. from the explicit time dependence (change of magnitude)
  2. from the frame own rotation (change of direction)



$$ {}_w x_P = {}_w x_G + \overbrace{{}_w R_G \cdot {}_G x_{G,P}}^{{}_w x_{P,G}} $$

$$ {}_w \dot{x}_P = {}_w \dot{x}_G + {}_w R_G \, {}_G \dot{x}_{G,P} + \underbrace{{}_w \dot{R}_G}_{{}_w\omega \times {}_w R_G} \, {}_G x_{G,P} $$

$$ {}_w \dot{x}_P = {}_w \dot{x}_G + \underbrace{{}_w \dot{x}_{G,P}}_{1} + \underbrace{{}_w \omega \times {}_w x_{G,P}}_{2} $$

${}_w\omega$ = angular velocity of frame W

${}_w \dot{x}_G$: linear velocity of frame G

1: magnitude variation of $x_{P,G}$

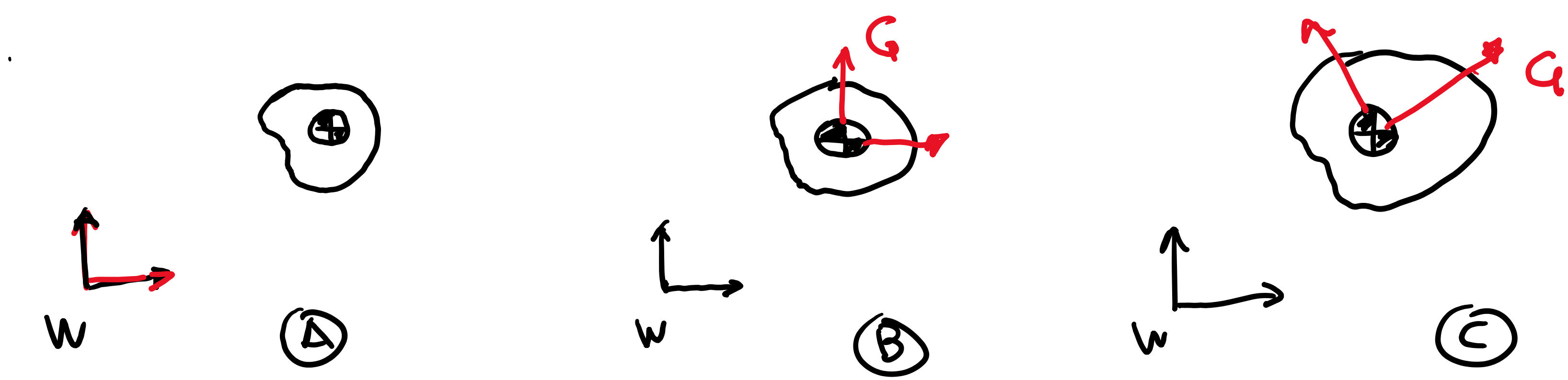
2: variation due to change of rotating frame

DERI ATION OF THE EULER EQUATION

$$\frac{d}{dt}(K) = \frac{d}{dt}({}_{G}L\,\omega) = {}_{G}L\,\dot{\omega} + \frac{d}{dt}({}_{G}L)\,\omega$$

distribute differentiation

- this is true for any reference frame !
- ${}_{G}L$ is the inertia about the COM point



inertia changes with time

inertia changes with time

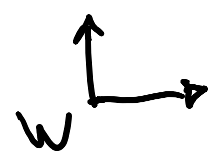
constant inertia

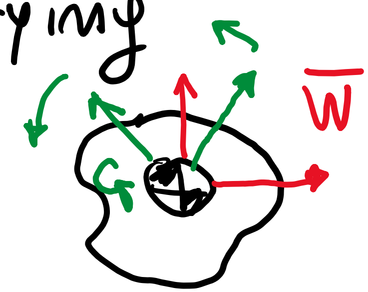
- if we express it in a rotating frame is CONSTANT (frame G rotating with body, so has angular velocity ω)

- if we express it in an inertial frame $\bar{W}$ (attached at com) is time-varying

$$ {}_{\bar{W}}L = {}_{W}R_{G} \; {}_{G}L \; {}_{W}R_{G}^{T} $$

$$ {}_{\bar{W}}\dot{L} \neq 0 $$





INERTIAL FRAME DERIVATION (B)

$$R = {}_{\bar{W}}R_G$$

$$\frac{d}{dt}\left({}_{\bar{W}}L\,\omega\right) = \frac{d}{dt}\left(R_G L R^T \omega\right) = {}_{\bar{W}}M_{ext}$$

$\dot{R} = \omega \times R$

$\dot{R}^T = R^T[\omega]_\times^T$

$= -R^T[\omega]_\times$

SKEW SYM $A^T = -A$

$$= R_G L R^T \dot{\omega} + \underbrace{\left[\dot{R}_G L R^T + R_G L \dot{R}^T\right]}_{{}_{\bar{W}}\dot{L}}\omega$$

$$= \underbrace{R_G L R^T}_{{}_{\bar{W}}L}\dot{\omega} + \left[\omega \times \underbrace{R_G L R^T}_{{}_{\bar{W}}L} - \underbrace{R_G L R^T}_{{}_{\bar{W}}L}\,\omega\times\right]\omega$$

$$= {}_{\bar{W}}L\dot{\omega} + \omega \times \left({}_{\bar{W}}L\omega\right) - \cancel{{}_{\bar{W}}L\,\omega \times \omega}$$

$= 0$

Cross product of // vectors

$${}_{W}L\dot{\omega} + \omega \times ({}_{W}L\omega) = {}_{W}M$$

changes with Time

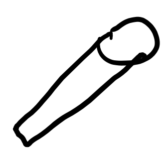
[MURRAY Sec. 2.4]

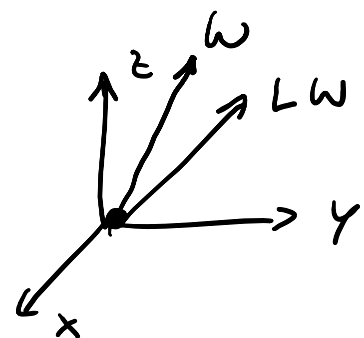
$$\boxed{{}_{\bar{w}}L\,\dot{\omega} + \omega \times {}_{\bar{w}}L\,\omega = {}_{\bar{w}}M_{ext}}$$

B

- Newton equation: mass is invariant wrt reference frame
- here we have the extra term $\omega \times {}_{\bar{w}}L\,\omega$ That it is due to the fact the inertia changes with the reference frame
- in general ω and $= L\omega$ are not aligned so the term $\omega \times L\omega \neq 0$. In case of a free fall This is The term responsible of the rotation of The body

$$L\dot{\omega} = -\,\omega \times L\omega$$





ROTATING FRAME DERIVATION ©

+ use constant ${}_G L$ that does not change

$${}_W\omega = {}_W R_G \, {}_G\omega$$

$${}_W M = {}_W R_G \, {}_G M$$

CONSTANT !

$$\frac{d}{dt}\left({}_W L \, {}_W\omega\right) = \frac{d}{dt}\left({}_W R_G \, {}_G L \, {}_G\omega\right) = {}_W R_G \, {}_G L \, {}_G\dot{\omega} + {}_W\dot{R}_G \left({}_G L \, {}_G\omega\right) = {}_W R_G \, {}_G M$$

it can be proof that:

$${}_W\dot{R}_G = {}_W\omega \times {}_W R_G = {}_W R_G \times {}_G\omega = {}_W R_G \left[{}_G\omega\right]_\times$$

left multiplying by ${}_W R_G^T$

$$\cancel{{}_W R_G^T \, {}_W R_G} \, {}_G L \, {}_G\dot{\omega} + \cancel{{}_W R_G^T \, {}_W R_G} \, {}_G\omega \times \left({}_G L \, {}_G\omega\right) = {}_G M_{ext}$$

$$\boxed{{}_6L \; {}_6\dot{\omega} + {}_6\omega \times {}_6L \; {}_6\omega = {}_6M_{ext}}$$ C

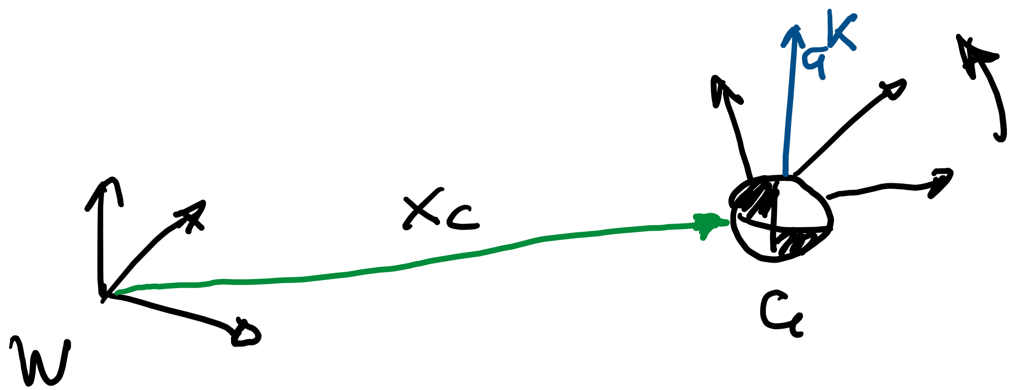
we still have This Term because of The rotating frame

if we express Newton equation in rotating frame (not adviced) we will have a similar Term

− all vectors are expressed in The rotating frame, I should compute ${}_6\omega$, ${}_6M_{ext}$

GENERALIZED EULER EQUATION (A)

- originally Euler equations are written in frames attached to COM (B, C)
- conservation of angular momentum holds also if we write it in a frame W not centered at COM



very complex with euclidean algebra

but... we have an articulated robot, not a rigid body...

NON HOLONOMY OF ANGULAR MOMENTUM DYN.

in absence of external forces the momentum conserves (DYNAMICS PRINCIPLE)

$$\begin{cases} m\dot{x}_c = \text{const} \\ {}_G L\, \omega = \text{const} \quad (\text{about } G) \end{cases}$$

GESTICULATION

if we differentiate: $0 = {}_G L\,\dot{\omega} + {}_G\dot{L}\,\omega = \dot{K}$

2 contributions to the rate of change of angular momentum

Angular acc. of whole robot body

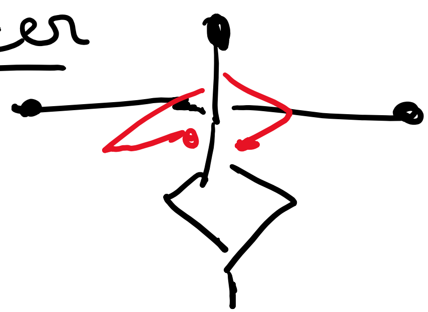
internal movements of the joints

A constraint is said holonomic if a constraint on velocities implies a constraint on positions

e.g. $\dot{x}_f = 0 \to \int \to x_f = \text{const}$

If $\omega = \text{const}$ (constraint on ang. velocity) is non-holonomic because the orientation is not obtained by simply integrating ω but this can be changed by controlling the joints of the system.

Dancer



Falling cat

- moving joints can influence the orientation due to the non-holonomic coupling between the rotation of the system and the joint configuration q

$I(q)\,\omega = \text{cost}$

→ orientation R is not obtained by $\dot{R} = \bar{\omega} \times R$ because also q plays a role!

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