# Fixed Base Robots

# JOINT MOTION CONTROL

control : find the time history of torques to be sent to actuators in order to execute a task.

open loop control example : the output of trajectory generator $\theta^d, \dot{\theta}^d, \ddot{\theta}^d$ is passed to the robot model $\tau = M(\theta^d)\ddot{\theta}^d + C(\theta^d, \dot{\theta}^d) + G(\theta^d)$

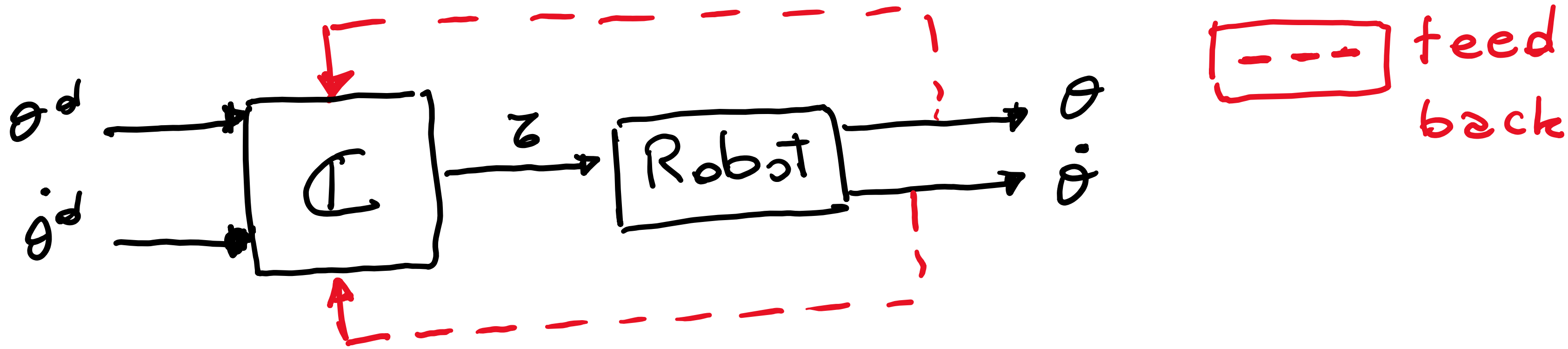
COMPUTED TORQUE

☹ PROBLEMS :

- disturbances
- model inaccuracies

$\Rightarrow$ tracking errors

# FEEDBACK CONTROL




INPUTS:

- position / velocity of joints: $\theta(t), \dot{\theta}(t)$ (sensor measurements)
- reference trajectory: $\theta^d(t), \dot{\theta}^d(t)$

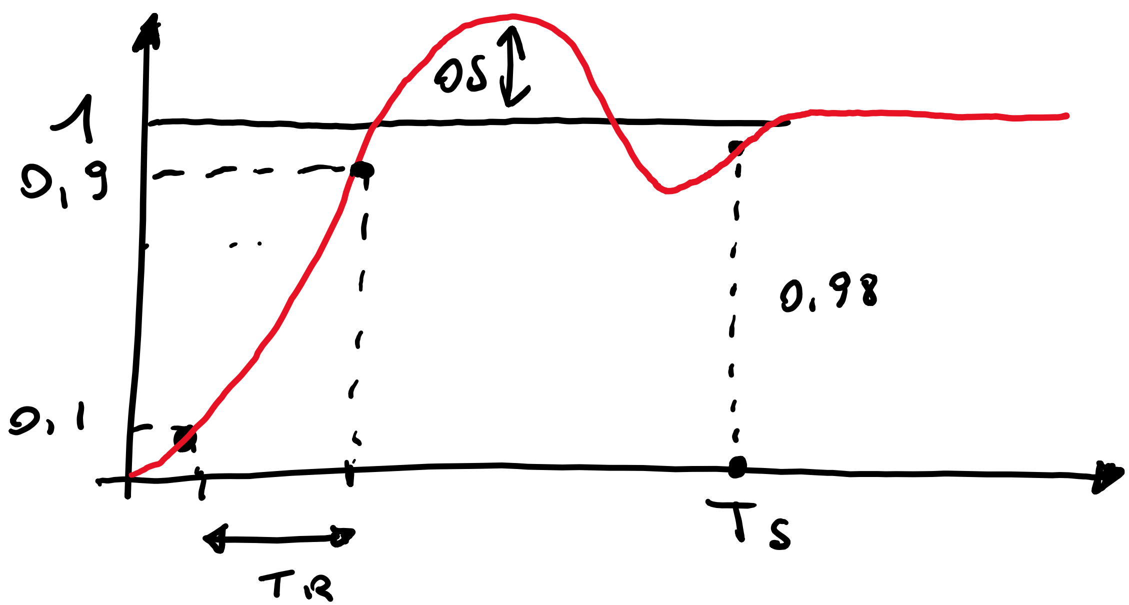
$\mathbb{C}$ can be:

- decentralized control (MOSTLY used)
- centralized control (accounts for dynamic interactions between joints)

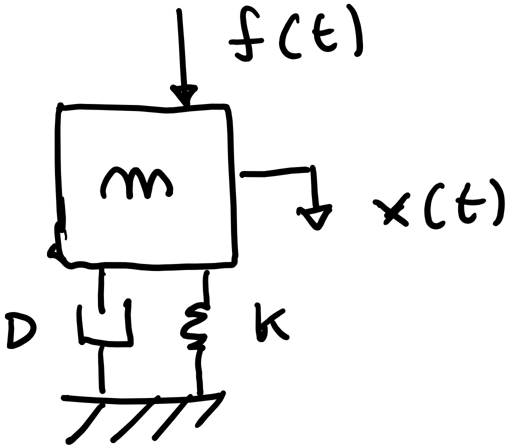
if $\theta^d = \text{const} \Rightarrow$ REGULATOR

# CONTROLLER SPECIFICATIONS

- frequency domain: closed loop BW
  (highest frequency sinusoid it can track)
- Time domain: raise time ($T_R$)
  settling time ($T_S$)
  overshoot (OS) %



# REVIEW ON SPRING-MASS DAMPERS




$$m\ddot{x} = f(t) - Kx - D\dot{x}$$

$$\Downarrow$$

$$m\ddot{x} + D\dot{x} + Kx = f(t)$$

2° ORDER LINEAR SYSTEM

(A) CASE $f(t) = 0$ : study of transient (natural dynamics)

$$x(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$

where $\lambda_1, \lambda_2$ are the roots of the characteristic equation $m\lambda^2 + D\lambda + K = 0$

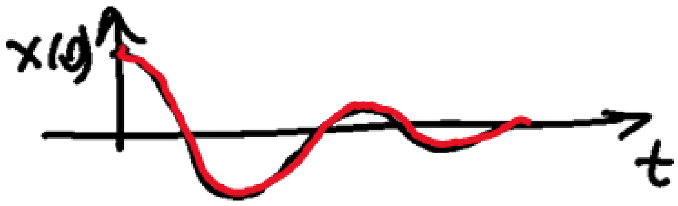
$$\lambda_{1,2} = \frac{-D \pm \sqrt{D^2 - 4mK}}{2m}$$

The system is always stable (poles have real part in left plane)

## 1 - CASE UNDERDAMPED

$D^2 - 4mk < 0 \Rightarrow$ solution: complex conjugate pair of roots

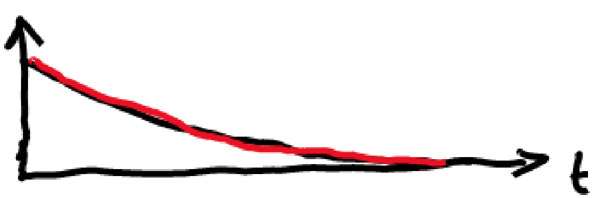
$D < 2\sqrt{mk}$




## 2 - CASE CRITICALLY DAMPED

$D^2 - 4mk = 0 \Rightarrow$ solution 2 real roots $s_1 = s_2$

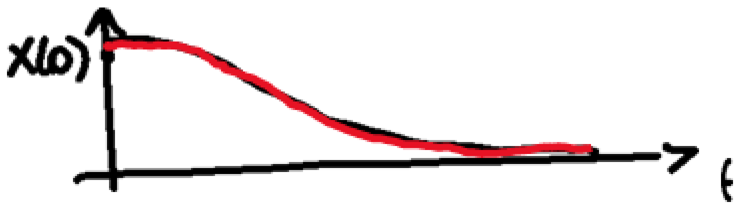
$D = 2\sqrt{mk}$



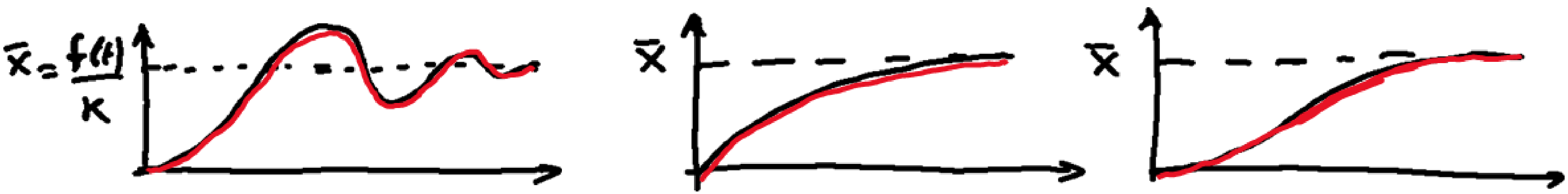
$\boxed{3-\text{CASE}}$ OVER DAMPED

$D^2 - 4mk > 0 \Rightarrow$ solution 2 real poles $s_1 \neq s_2$

$D > 2\sqrt{mk}$




③ CASE $f(t) \neq 0 = \text{cost}$, $x(0) = 0$




UNDER DAMPED CRITIC. DAMPED OVER DAMPED

# REMARK ON UNDERDAMPED 2° SYSTEMS

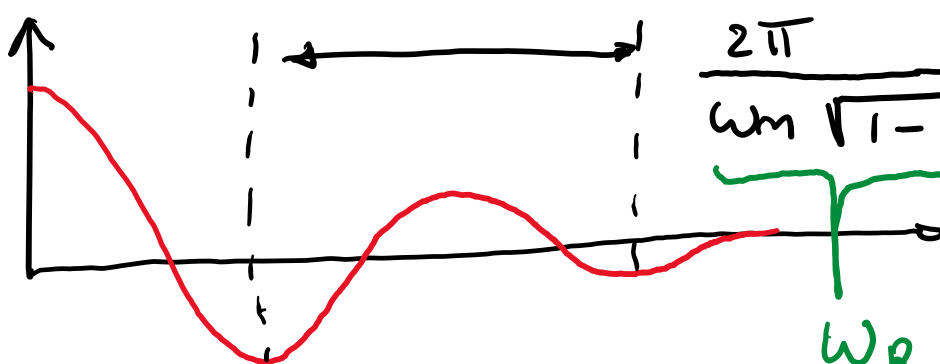
$$\ddot{x} + \frac{D}{m}\dot{x} + \frac{k}{m}x = 0 \Rightarrow \ddot{x} + 2\xi_n\omega_n\dot{x} + \omega_n^2 x = 0$$

$$\omega_n = \sqrt{\frac{k}{m}}$$ **Natural frequency**

$$\frac{D}{m} = 2\xi_n\omega_n \Rightarrow \xi_n = \frac{D}{2\omega_n m} = \frac{D}{2\sqrt{km}}$$ **Natural damping ratio**

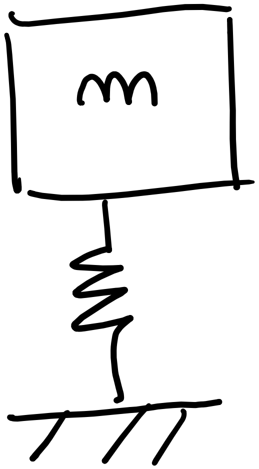
when $\xi_n = 1$, $D = 2\sqrt{km}$ CRITICAL DAMPING

Time response for $\xi < 1$ (UNDERDAMPED)

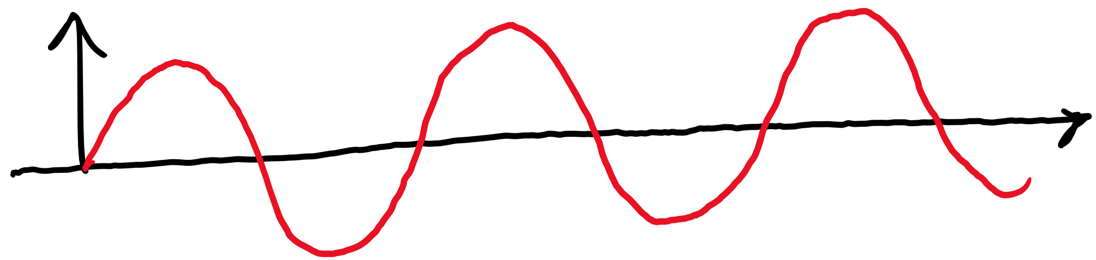


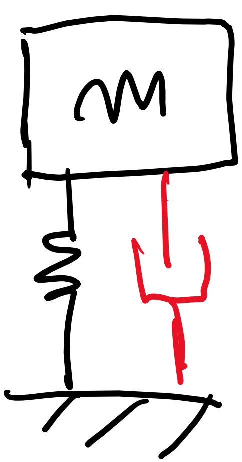
$$\frac{2\pi}{\underbrace{\omega_n\sqrt{1-\xi_n^2}}_{\omega_R}} > \frac{2\pi}{\omega_n} \quad \text{(NO DAMPING)}$$

$$\boxed{\omega_R < \omega_n}$$

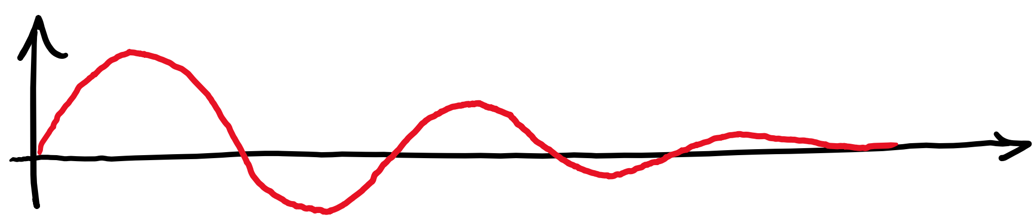



The system oscillates bouncing energy back and forth between mass (kinetic) and spring (potential)






unless there is a dissipative element



# Outline

PD  control (1 DoF)

PD  control + Feed-Forward (1 DoF)

PD  control + gravity compensation (1 DoF)

PID  control (1 DoF)

Little detour on 2 DoFs robot dynamics analysis

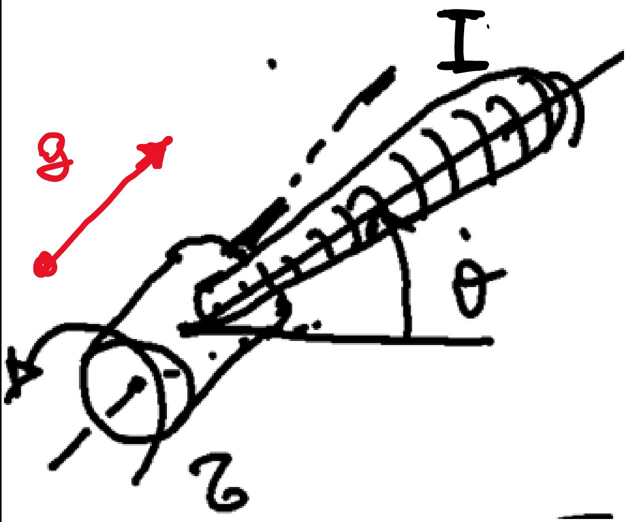
- Prismatic Joints
- Revolute Joints

Inverse Dynamics (2 DoFs)

# PD CONTROL

consider:

- 1 link moving on a plane ($\perp$ to gravity)
- 1 rotational joint




DYNAMICS: $I\ddot{\theta} = \tau$

no gravity $\Rightarrow$ only inertia with input torque

PD IDEA: create virtual spring / dampers to drive the link toward a desired position $\theta^d$

$$\boxed{\tau = K_p(\theta^d - \theta) + K_d(\dot{\theta}^d - \theta^d)}$$

## CLOSED LOOP

$$I\ddot{\theta} + K_d\dot{\theta} + K_p\theta = K_p\theta^d + K_d\dot{\theta}^d$$

at steady state: $\dot{\theta}, \ddot{\theta} = 0 \implies \theta = \theta^d$

since is a stable system, it converges exponentially to $\theta^d$

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## PD + FEED-FORWARD TERM

$$\boxed{\tau = I\ddot{\theta}^d + k_p(\theta^d - \theta) + K_d(\dot{\theta}^d - \dot{\theta})}$$

CLOSED LOOP

$$I\underbrace{(\ddot{\theta}^d - \ddot{\theta})}_{\ddot{e}} + K_d\underbrace{(\dot{\theta}^d - \dot{\theta})}_{\dot{e}} + k_p\underbrace{(\theta^d - \theta)}_{e} = 0$$

- The error dynamics converges to zero
- Transient depends on $I$, $k_p$, $K_d$

# TUNING PD GAINS

Knowing link inertia $I$ (from CAD) we can determine tracking performances by selecting $K_p$, $K_d$

EXAMPLE $I = 2.5\ kg\ m^2$

GOAL: settling time $\boxed{T_s = 2\ s}$ (98%)

2° order system: $1 - e^{-\xi_n \omega_n T_s} = 0{,}98$

$$e^{-\xi_n \omega_n T_s} = 0{,}02$$

$$\xi_n \omega_n T_s = \log \frac{1}{0{,}02} \cong 4$$

$$\boxed{T_s \cong \frac{4}{\xi_n \omega_n} = 4\tau}$$

critical damping: $\xi_n = 1 \Rightarrow \omega_n = \frac{4}{T_s} = \frac{4}{2} = 2\ rad/s$

$\omega_n = \sqrt{\frac{K_p}{I}} = 2 \Rightarrow \boxed{K_p = 4 \cdot I = 10\ N/m}$

$$\boxed{K_d = 2\sqrt{K_p I} = 2\sqrt{10 \cdot 2.5} \cong 10\ Nms/rad}$$

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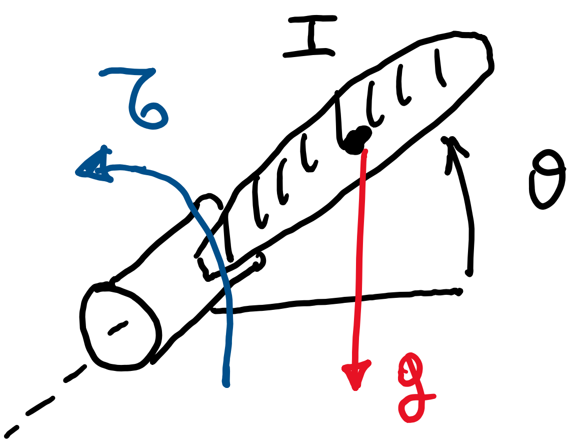
# PD + GRAVITY COMPENSATION




- on a vertical plane we have gravity

DYNAMICS:

$$I\ddot{\theta} + g(\theta) = \tau$$

- using only a PD, at equilibrium you have steady state error

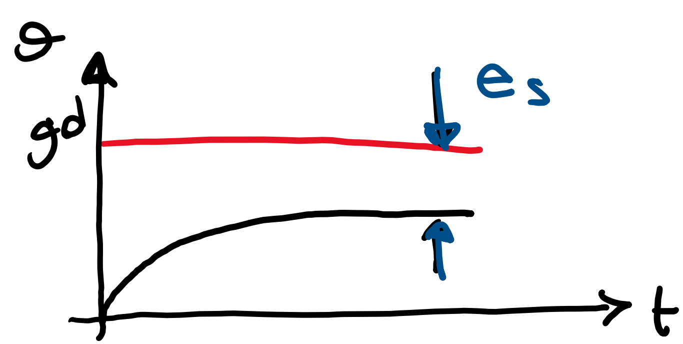
$$I\ddot{\theta} + g(\theta) = K_p(\theta^d - \theta) + K_d(\dot{\theta}^d - \dot{\theta})$$

$$\dot{\theta}, \ddot{\theta} = 0 \implies g(\theta) = K_p(\theta^d - \theta)$$

$$\theta = \theta^d - \frac{g(\theta)}{K_p}$$

$e_s$

steady-state error



gravity compensation $\Rightarrow$ zero error at equilibrium

$$\boxed{\tau = K_p(\theta^d + \theta) + K_d(\theta^d - \theta) + g(\theta)}$$

$$\dot{\theta}, \ddot{\theta} = 0 \Rightarrow \theta = \theta^d \Rightarrow e_s = 0$$

$\oplus$ stability is ensured $\forall K_p, K_d$ if they are positive

$\ominus$ $g(\theta)$ is configuration dependent and we need to recompute at each loop

Alternative... add integral action

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# PID CONTROL

- adding an integral action can remove steady-state errors in face of unknown constant disturbances

$$\tau = \underbrace{K_p(\theta^d - \theta)}_{P} + \underbrace{K_d(\dot\theta^d - \dot\theta)}_{D} + \underbrace{K_i\int(\theta^d - \theta)\,dt}_{I}$$

$$I\ddot\theta + f_{dist} + g = K_p\underbrace{(\theta^d - \theta)}_{e} + K_d(\dot\theta^d - \dot\theta) + K_i\int(\theta^d - \theta)\,dt$$

at steady-state: $\dot\theta, \ddot\theta = 0$

$$g + f_{dist} = K_p e + K_i\int e\,dt$$

$$\Downarrow \quad \frac{d}{dt}$$

$$0 = K_p\dot e + K_i e \implies e \text{ converges to zero}$$

The I component grows magnifying the error till the disturbance is not compensated

APPLICATION EXAMPLES


- unknown payload
- constant torque offset

Not good with Coulomb friction $\Rightarrow$ oscillations

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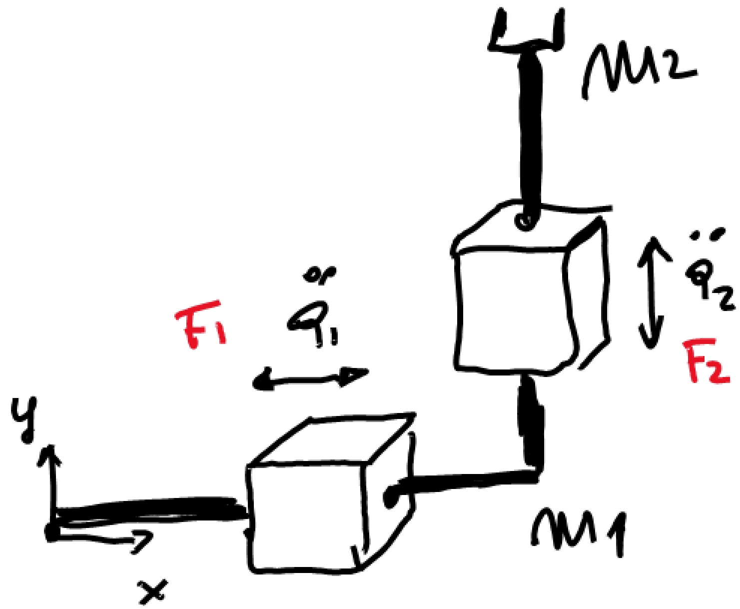
# 2 DOFS ROBOT ON A PLANE WITH PRISMATIC JOINTS

- on a plane $\Rightarrow$ no gravity
- prismatic joints at 90 DEGS

$q \in [q_1, q_2] \in \mathbb{R}^2$




DYNAMICS:

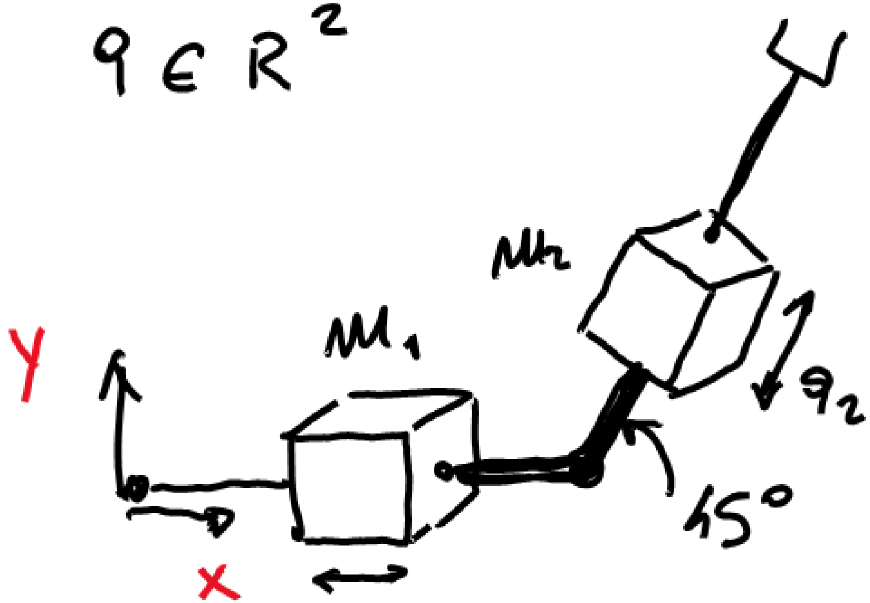
$$\begin{bmatrix} m_1 + m_2 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

CONSTANTS

- joint 1 feels the mass of link 1 ($m_1$) and link 2 ($m_2$)
- no inertial couplings

# PRISMATIC JOINTS <u>NOT</u> AT 90 DEGS

$q \in R^2$




DYNAMICS:

$$\begin{bmatrix} m_1 & m_{12} \\ m_{21} & m_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$m_1$, $m_2$, $m_{12}$ CONSTANT

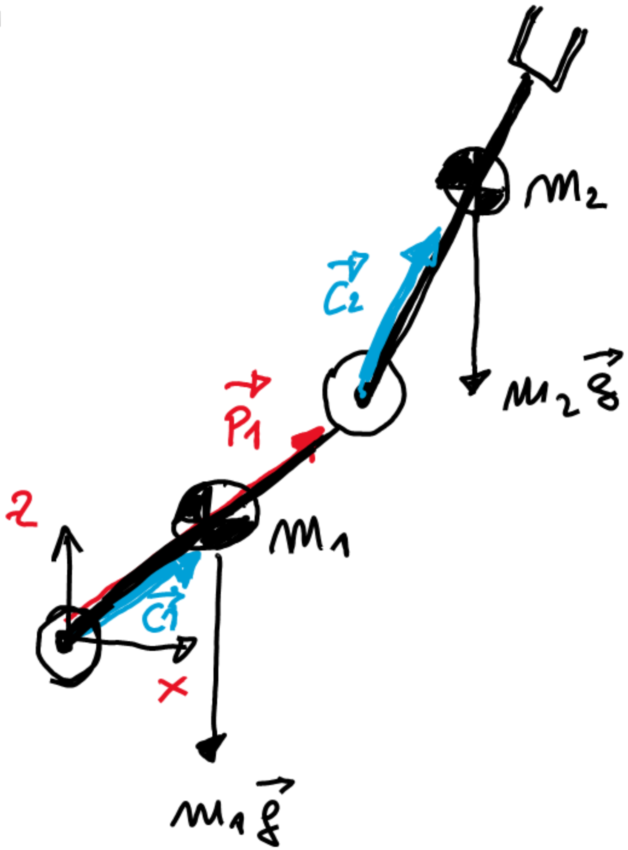
- There are inertial couplings $\Rightarrow$ the motion of a joint is affecting the other joint
- The inertia matrix is dense

$$m_1 \ddot{q}_1 + m_{12} \ddot{q}_2 = F_1$$

$\hookrightarrow$ inertial effect of JOINT 2 on JOINT 1

# 2 DOFS ROBOT WITH REVOLUTE JOINTS

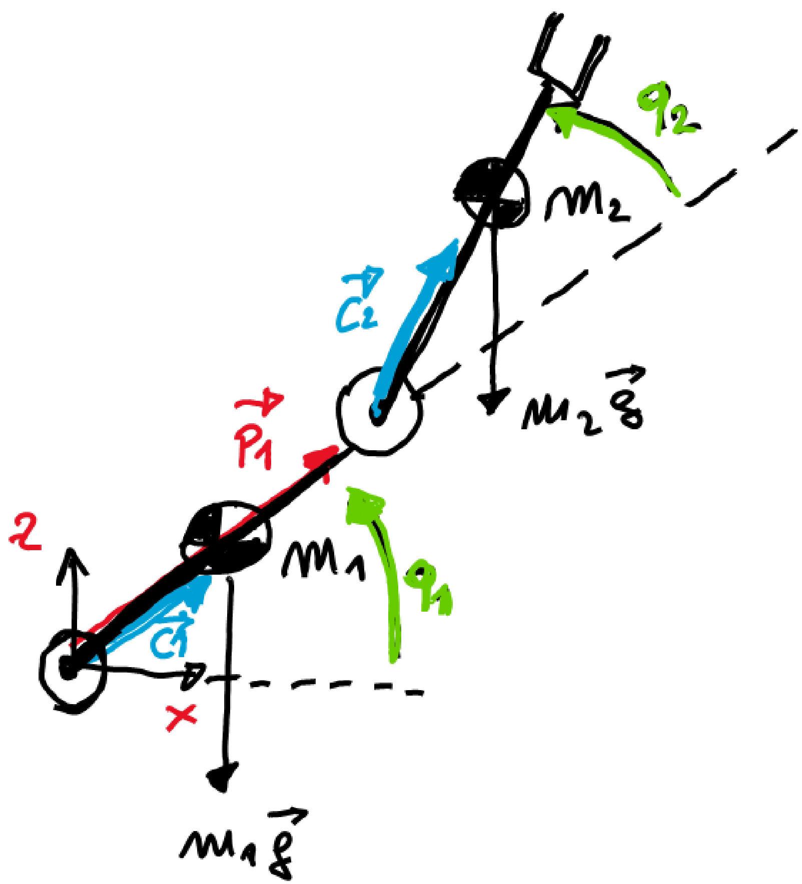
with revolute joints we have to consider also CORIOLIS / CENTRIFUGAL effects




DYNAMICS:

$$\underbrace{\begin{bmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{bmatrix}}_{M(q)} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \underbrace{\underbrace{\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}}_{\substack{\text{coriolis/} \\ \text{centrifugal} \\ \text{Torques} \\ C(q,\dot{q})\dot{q}}} + \underbrace{\begin{bmatrix} g_1 \\ g_2 \end{bmatrix}}_{\substack{\text{gravity} \\ \text{Torques} \\ G(q)}}}_{h(q,\dot{q})\ \text{BIAS FORCES}} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

# COMPUTATION OF GRAVITY TORQUES




$$\begin{cases} g_1 = \vec{C_1} \times m_1 \vec{g} + (\vec{P_1} + \vec{C_2}) \times m_2 \vec{g} \\ g_2 = \vec{C_2} \times m_2 \vec{g} \end{cases}$$

$$\begin{cases} g_1 = -\|\vec{C_1}\| \cos q_1 m_1 g \\ \qquad - \|\vec{C_1} + \vec{P_1}\| \cos(q_1 + q_2) m_2 g \\ g_2 = -\|\vec{C_2}\| \cos(q_1 + q_2) m_2 g \end{cases}$$

$$\Downarrow$$

$$\boxed{g(q)}$$

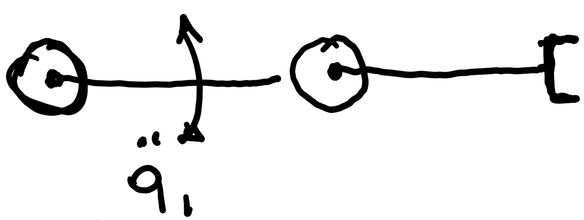
# INERTIA MATRIX FOR REVOLUTE JOINTS

$$\begin{bmatrix} I_{11}(q_1) & I_{12}(q_2) \\ I_{12}(q_2) & I_{22} \end{bmatrix}$$

$I_{22}$ → CONSTANT

- symmetric $M = M^T$
- positive definite $\forall$ eigs $> 0$
- configuration dependent $\boxed{M = M(q)}$

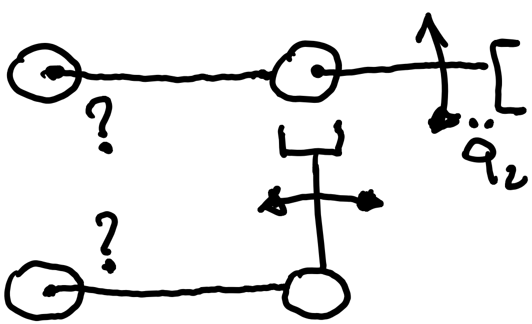
$I_{11}$ : inertia seen by JOINT 1




$I_{11}$ max

$I_{11}$ min

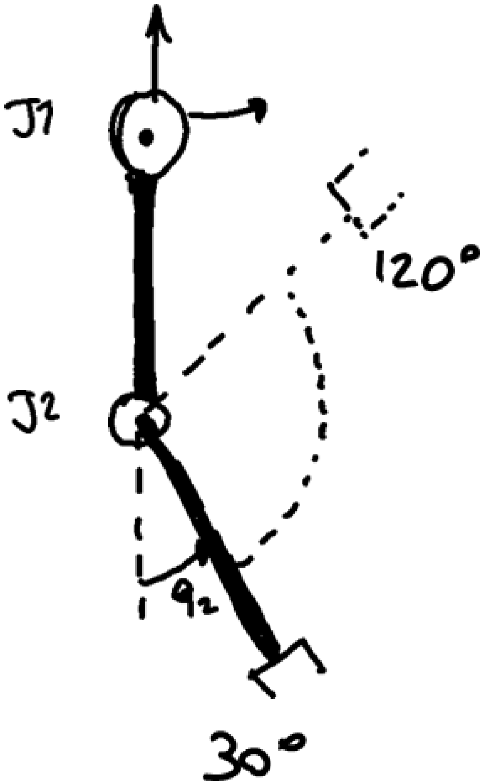
$I_{12}$ : influence from JOINT 2 on 1

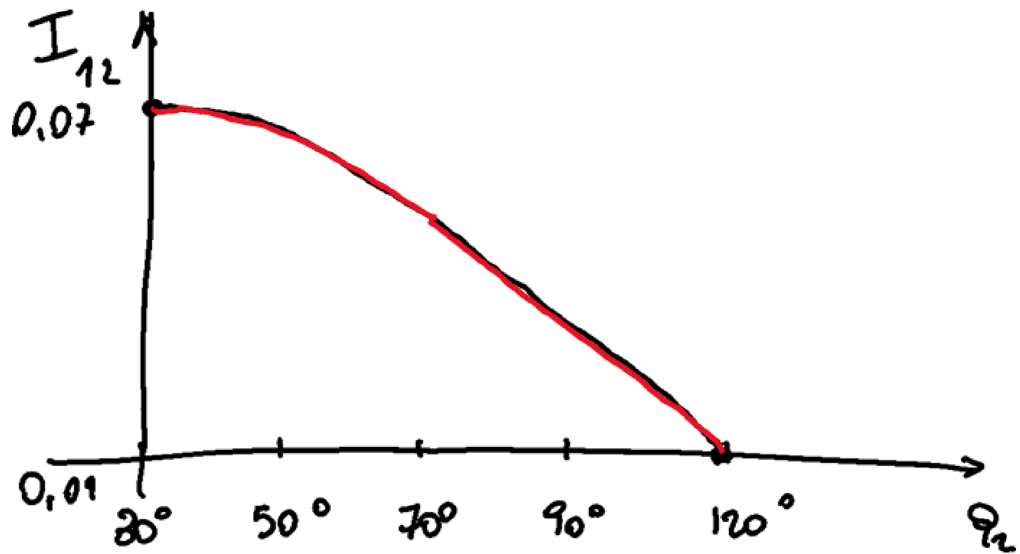



$I_{12}$ max (big)

$I_{12}$ min (small)

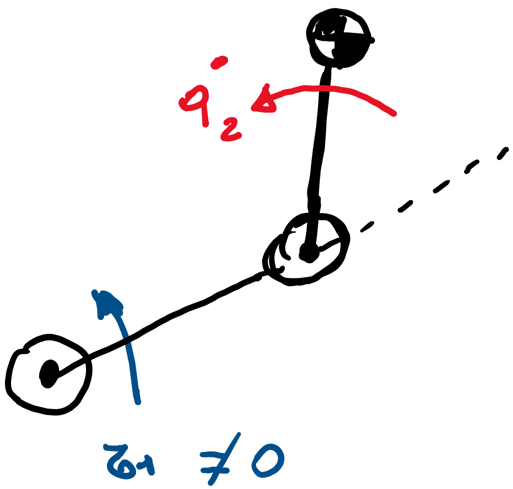
# COUPLED INERTIA ($I_{12}$) VARIATION

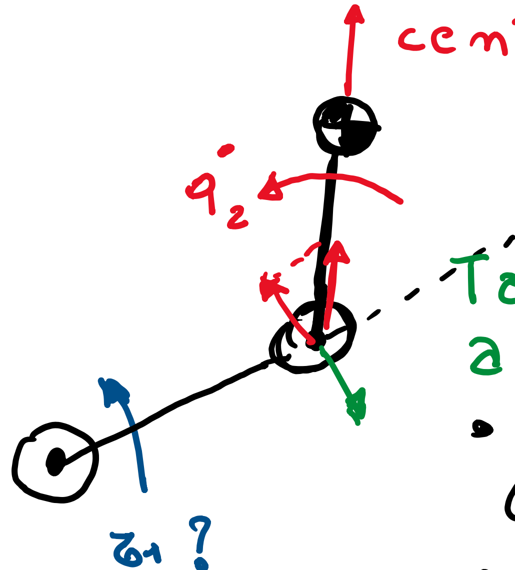


# CENTRIFUGAL TORQUES

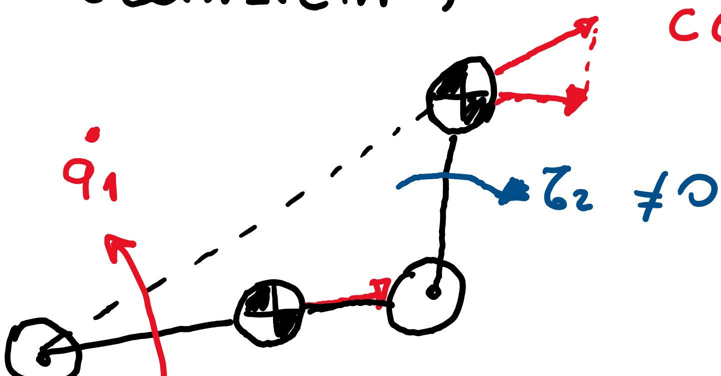
(A) Rotation of LINK 2, effect on LINK 1 / 2
(constant)




- influence on $\tau_1$
(max for $q_2 = 90°$)
- no effect on $\tau_2$

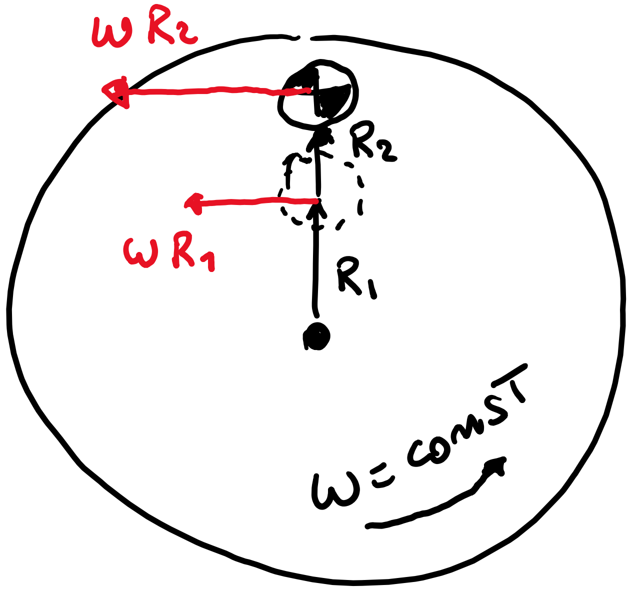
(B) Rotation of LINK 1, effect on LINK 1 / 2
(constant)

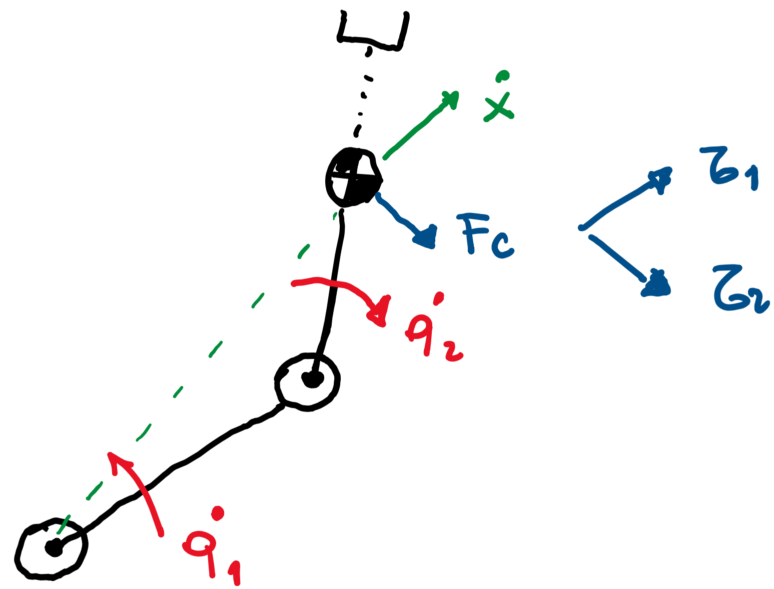



- no effect on $\tau_1$
- influence on $\tau_2$
(max for $q_2 = 90°$)

# CORIOLIS TORQUES

are related to the change of tangential velocity when the distance from rotation axis changes




- if $\dot{q}_1 = 0$ $\forall \dot{q}_2 = 0$
- if $\dot{q}_2 = 0$ $\forall \dot{q}_1 = 0$

} • $F_c$ depends on both $\dot{q}_1$ and $\dot{q}_2$

• $F_c$ affects $\tau_1$ and $\tau_2$

# STRUCTURE OF C MATRIX

**CENTRIFUGAL**

| | $\dot{q}_1$ | $\dot{q}_2$ |
|---|---|---|
| $\tau_1$ | $0$ | $\#(q_2)$ |
| $\tau_2$ | $\#(q_2)$ | $0$ |

$+$

**CORIOLIS**

| | $\dot{q}_1$ | $\dot{q}_2$ |
|---|---|---|
| $\tau_1$ | $\#(\dot{q}_2)$ | $\#(\dot{q}_1)$ |
| $\tau_2$ | $\#(\dot{q}_2)$ | $\#(\dot{q}_1)$ |

$\Downarrow$

C is dense and depends both on $q$ and $\dot{q}$

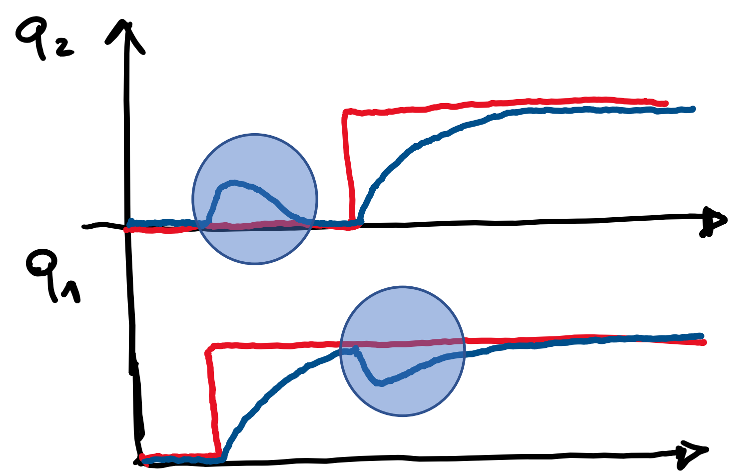
Let's put everything together:

$$\boxed{M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau}$$

DYNAMICS OF 2 DOFs ROBOT

- $M$, $C$, $G$ are configuration dependent
- $M$, $C$ have couplings
- using PID we can bring steady state error to zero, but ... the couplings affect the transient

Link 1 and 2 influence each other



EXAMPLE:

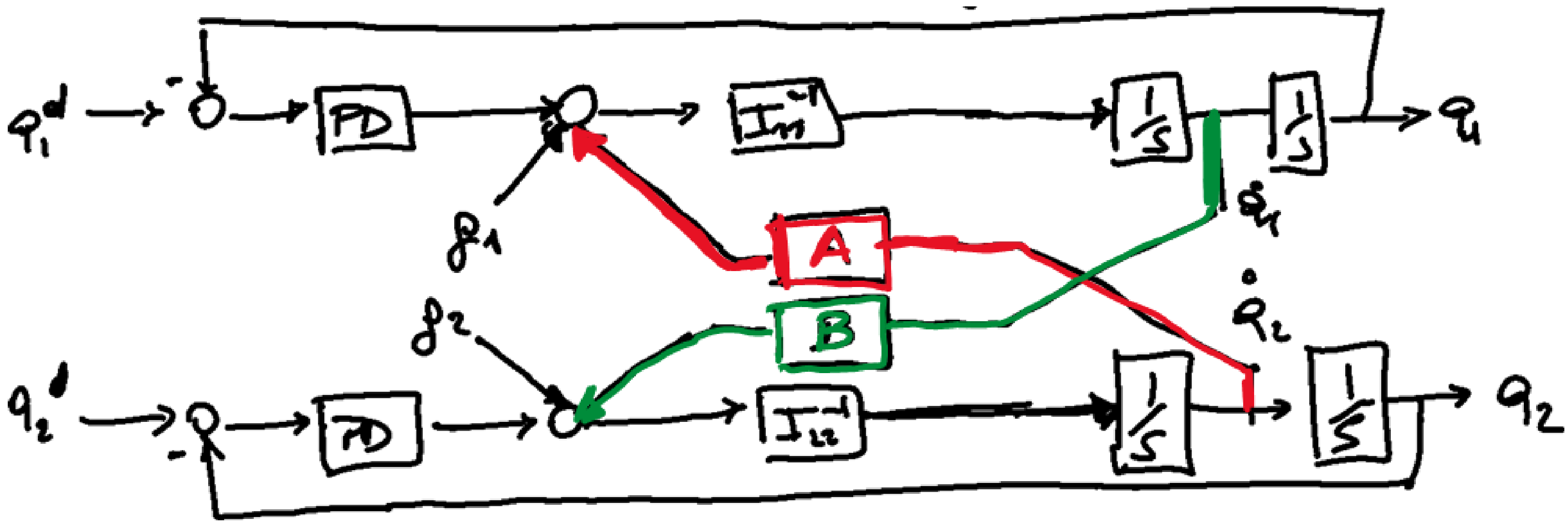
$$\begin{bmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 & I_{112} \\ -\frac{I_{112}}{2} & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1^{\,2} \\ \dot{q}_2^{\,2} \end{bmatrix} + \begin{bmatrix} I_{112} \\ 0 \end{bmatrix} \dot{q}_1 \dot{q}_2 + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

$$I_{11}\ddot{q}_1 + \underbrace{I_{12}\ddot{q}_2 + I_{112}\dot{q}_1\dot{q}_2 + I_{112}\dot{q}_2^{\,2}}_{A} + g_1 = \tau_1$$

$$I_{22}\ddot{q}_2 + \underbrace{I_{21}\ddot{q}_1 - \frac{I_{112}}{2}\dot{q}_1^{\,2}}_{B} + g_2 = \tau_2$$

A = influence of JOINT 2 on JOINT 1

B = influence of JOINT 1 on JOINT 2




- using DECENTRALIZED control we can reduce disturbances <u>only</u> increasing gains
- stability can be ensured $\forall$ $k_p$, $k_d$ as long as they are positive definite matrix
- becouse the system is non-linear stabitity should be proof with LIAPUNOV ANALYSIS

... I just crank up the gains...

- ⊖ unmodeled flexibilities can make the robot unstable
- ⊖ The robot becomes very rigid

Use non linear state feedback to "cancel" the coupled dynamics of the manipulator and achieve a linear system with low gains (soft)

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# RECALL FEED-BACK LINEARIZATION

$$\dot{x} = \alpha(x)\,u + \beta(x)$$

defining a control action as: $\boxed{u = \alpha(x)^{-1}(v - \beta(x))}$

I obtain a linearized system in the new control input $v$

$\boxed{\dot{x} = v}$ single integrator $\xrightarrow{v} \boxed{\frac{1}{s}} \rightarrow x$

- If we have a model of the system we can compensate for non linearities
- The system becomes LINEAR and DECOUPLED w.r.t. any input of the system (e.g. $v_i$ influences only $x_i$)

⊕ we can set ANY stabylizing controller in $v$

⊖ perfect cancellation is not possible due to model inaccuracies

for articulated robots feedback linearization is called:

INVERSE DYNAMICS ($\ddot{q} \rightarrow \tau$) CONTROLLERS

[ FORWAR DYNAMICS ($\tau \rightarrow \ddot{q}$) SIMULATORS ]

# INVERSE DYNAMICS

$$M\ddot{q} + h = \tau \qquad (h = C\dot{q} + G)$$

$$\oplus \quad \tau = Mv + h$$ $\Rightarrow$ model based state feed-back

$$\Rightarrow \quad M\ddot{q} + \cancel{h} = M\ddot{q}^{d} + \cancel{h}$$

$$\boxed{\ddot{q} = v}$$ $\Rightarrow$ Double integrator (relative degree 2)

$$v \rightarrow \boxed{\frac{1}{s}} \rightarrow \boxed{\frac{1}{s}} \rightarrow x$$

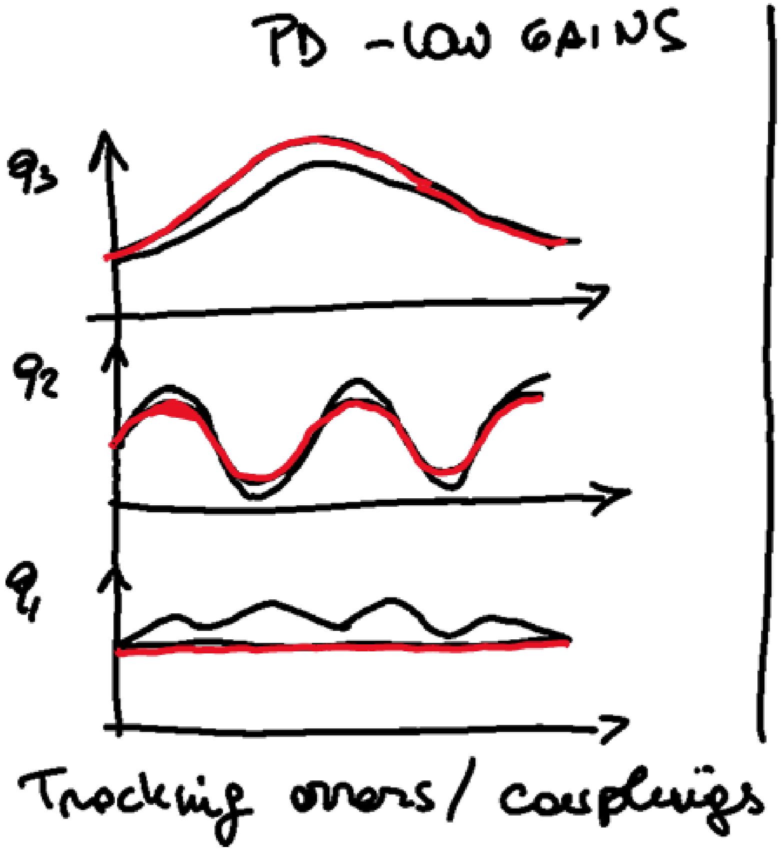
new input

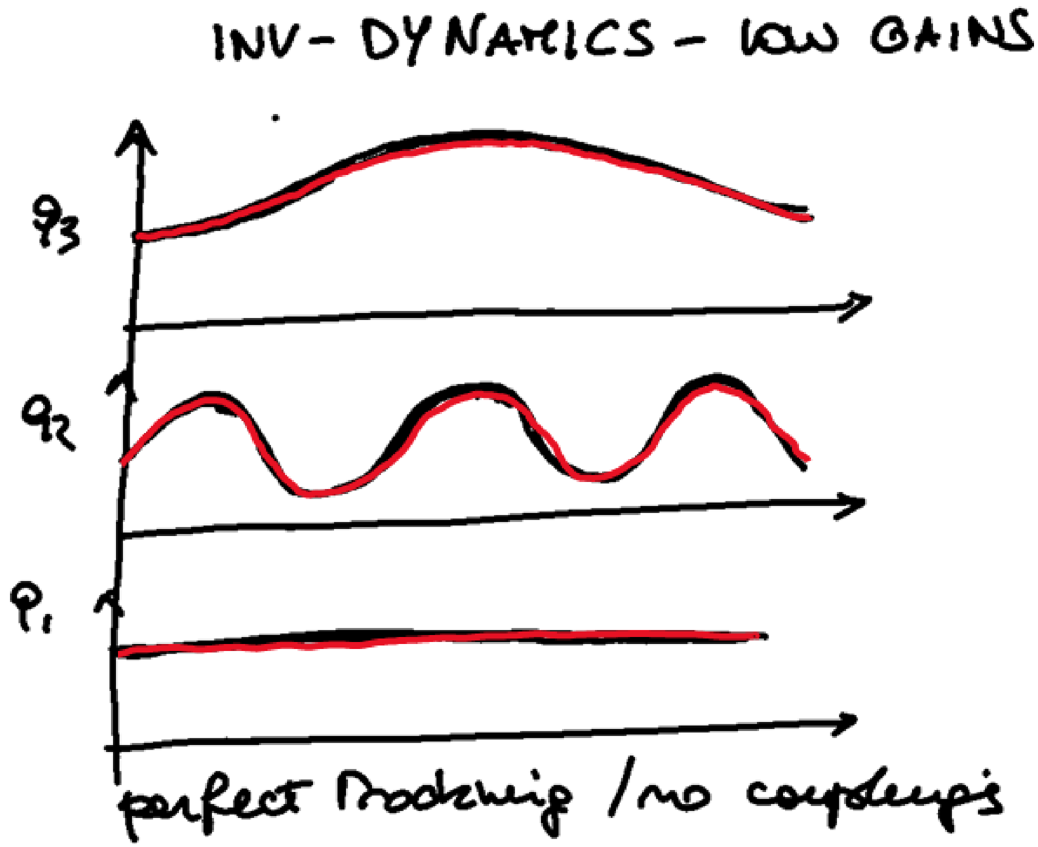
- The attractor is designed inside $v$

$$v = \ddot{q}^{d} + K_p(q^{d} - q) + K_d(\dot{q}^{d} - \dot{q})$$

FFWD

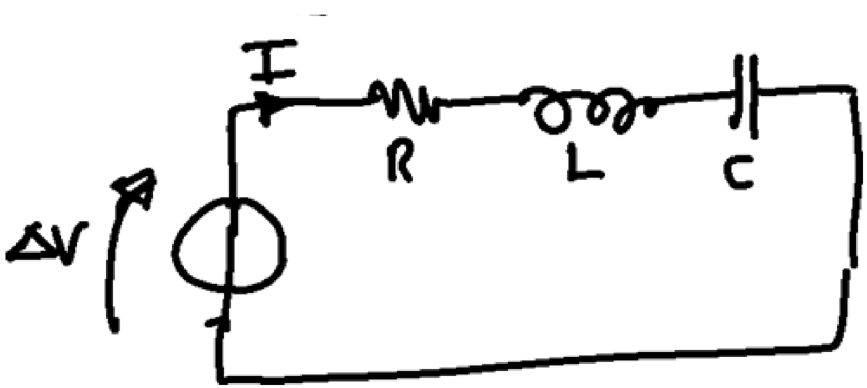
• Kp and Kd can be Tuned considering The system DECENTRALIZED



# ELECTRICAL ANALOGY

$$\left.\begin{array}{lll} I \leftrightarrow \dot{x} & & \text{FLOW} \\ \Delta V \leftrightarrow F & & \text{EFFORT} \end{array}\right\} \Rightarrow I \cdot \Delta V \,,\ \dot{x} \cdot F = \text{POWER}$$

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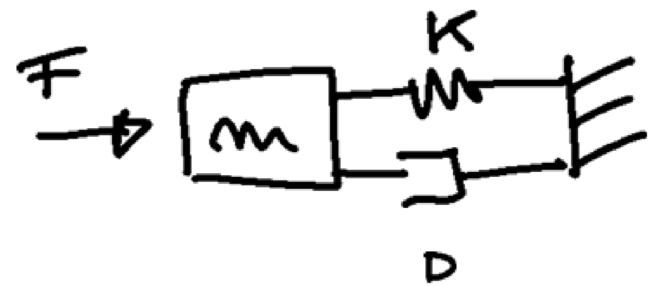

## ELECTRICAL

- RESISTANCE


$$\Delta V = R\,I$$

- INDUCTANCE


$$\frac{dI}{dt} L = \Delta V$$

accumulates magnetic field $\Phi$

- CAPACITOR


$$\frac{dV}{dt} C = I$$

accumulates charge $Q$

## MECHANICAL

- DAMPER

$$F = D\dot{x}$$

- MASS


$$\frac{d\dot{x}}{dt} M = F$$

accumulates kinetic energy

- SPRING

$$F = K x$$

$$\frac{dF}{dt} = K\dot{x}, \quad F = \int \dot{x}\, dt$$

accumulates potential energy

# References


- Robotics Modelling, Planning and Control - Siciliano, B., Sciavicco, L., Villani, L., Oriolo, G.