Outline

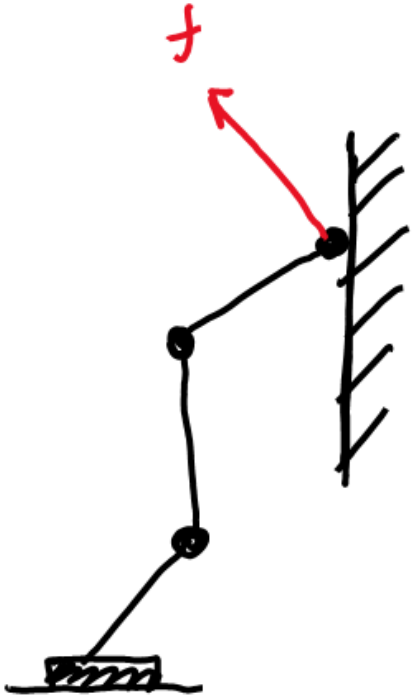
Dynamics of manipulator in contact

Type of contacts

Rigid contact model

Soft contact model

MODELING OF MANIPULATORS IN CONTACT



DYNAMIC EQUATION

$$(1) \quad M(q)\ddot{q} + h(q,\dot{q}) = \tau + J(q)^T f$$

$$J(q) = \frac{\partial x_c(q)}{\partial q}$$

→ FORWARD KINEMATICS MAPPING JOINT ANGLES INTO END EFFECTOR POSITIONS

TYPES OF CONTACTS

$k = 3$ contact point

only linear forces

3 kinematic constraints

$k = 6$ contact surface

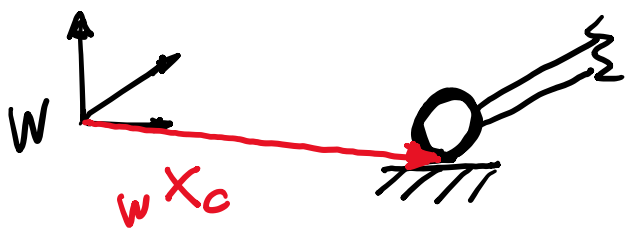
linear forces / moments

6 kinematic constraints

- affects Jacobian matrix: $J(q) \in \mathbb{R}^{k \times n}$
- if There are multiple contacts

$$J_1^T f_1 + J_2^T f_2 \cdots \Rightarrow \underbrace{\begin{bmatrix} J_1^T \\ J_2^T \end{bmatrix}}_{J^T} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

MODELING CONTACTS : RIGID CONTACTS



position of contact does not change (in inertial frame W)

$${}^{W}x_c(t) = \text{const.}$$

$${}^{W}\dot{x}_c(t) = 0 \Rightarrow J\dot{q} = 0 \quad \text{zero contact velocity}$$

$$\boxed{{}^{W}\ddot{x}_c(t) = 0 \Rightarrow J\ddot{q} + \dot{J}\dot{q} = 0} \quad \text{zero contact accel.}$$

if we replace $\ddot{q} = -J^{-1}(\dot{J}\dot{q})$ into (1) we get an expression to estimate contact forces without need of contact sensors

$$f = (J^{-T}MJ^{-1})(-\dot{J}\dot{q}) + J^{-T}(h - \tau)$$

$$\boxed{f = (J^{-T}MJ^{-1})\left[-\dot{J}\dot{q} + JM^{-1}(h - \tau)\right]}$$

- Need the model of the robot. Model error become estimation errors in the forces

OPTIMIZATION ANALOGY

The contact forces can be seem "Lagrange multipliers" that enter the dynamic equation to enforce the contact constraint (equality)

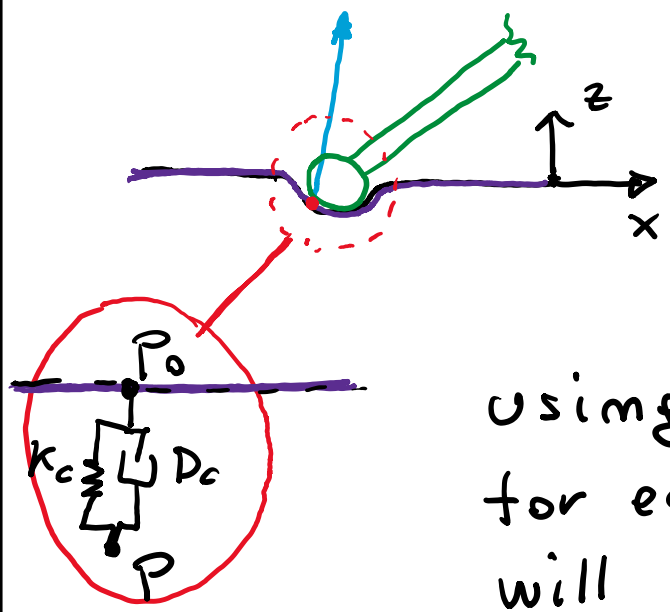
$$M\ddot{q} + h - \tau = 0$$ unconstrained dynamics

$$M\ddot{q} + h - \tau - J^T f = 0$$

↳ forces to make ${}_w\ddot{x}_c = 0$

SOFT CONTACT FORCES

world is not infinitely rigid: we model the force as function of the penetration of the ground



P_0 = initial contact point

P = current position of point of the robot that made contact

using linear spring / damper model for environment, the contact force will obey this equation:

K_c = contact stiffness (e.g. $[10^3, 10^6]$)

D_c = contact damping (e.g. $\approx 2\sqrt{K}$)

$$f = K_c(P_0 - P) - D_c \dot{P}$$

$$M\ddot{q} + h = \tau + J^T f$$