DD2380 Artificial Intelligence: Homework 3, Part B, Answers

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Hidden Markov Models

a) Describe what procedures should be used for estimating the model and how training data would be generated.

You should use the robot as you're expecting to use it, in an environment where it's going to be used. From here, you act as you would. While doing this, you gather statistics for "what the robot percieves" when you for instance point LEFT. This will be the observation matrix. At the same time, you gather statistics of your own behaviour, because this is relevant for the transition matrix. When you're pointing forward, how common is it that this command is followed by another forward, for instance? These statistics, what commands follow others, is your transition matrix.

b) If we have no observation, what is the probability of the second state x_2 being LEFT? And the probability of second observation o_2 being H_2? Finally, what is the probability $P(o_2 = H_2 | x_2 = LEFT)$?

First, we iteratively calculate probabilities of $P(x_t = s)$ using

$$P(x_t) = \sum_{i \in S} P(x_{t-1} = i) * P(i \rightarrow s)$$

where $S = \{LEFT, RIGHT, STOP, FORWARD\}$, and s a state. That is, the probability of x_t being a certain state s, is the sum of the probabilities of x_{t-1} being each state multiplied by the transition probability between that state and s.

> *"It's 50% likely for me to get to a state, and 25% likely for me to enter this state from there. Therefore I'm 12.5% likely to get here from that state, and 87.5% likely to end up here, from another state, or outside."*

Note that $P(x_1)$ is the first row of the transition matrix. Because $x_0 = RIGHT$ is known, the probability of x_0 is simply the probability for the transition between $RIGHT$ and the state for that row.

The calculation of $P(x_2 = RIGHT)$ is given with $0.45 * 0.45 + 0.09 * 0.08 + 0.10 * 0.08 + 0.36 * 0.09 = 0.2501$ but was omitted from the table for readability.

The values, in order, are: $P(x_1 = RIGHT) * P(RIGHT \rightarrow RIGHT) + P(x_1 = LEFT) * P(LEFT \rightarrow RIGHT) + \{STOP\} + \{FWD\}$. The same step is repeated for each state.

state	$P(x_0)$	$P(x_1)$	$P(x_2)$
RIGHT	1.0	0.45	0.2501
LEFT	0.0	0.09	0.1344
STOP	0.0	0.10	0.168
FWD	0.0	0.36	0.4475

Similarly, we calculate the probability of observing H_2 from a state, and multiply by the chance of x_2 being that state. These calculations, for all possible states are summed up. Just like before, except we use the probability of observing H_2 instead of transitioning to a state.

$$P(o_2 = H_2) = 0.147333$$

c) Viterbi's algorithm

[RIGHT, FORWARD, FORWARD, FORWARD, STOP, STOP, STOP, STOP, STOP, STOP].

Source: ***viterbi.c***. *gcc -o viterbi viterbi.c; ./viterbi* to run.

d)With the sequence of observations described above, what is the sequence of most likely hidden states?

Source: ***alphabeta.c***. *gcc -o alphabeta alphabeta.c; ./alphabeta* to run.

e) With the sequence of observations described above and the most likely sequence in (c), what is the most likely o_{11}?

Source: ***viterbi.c***. Same as above.