a

I would begin by using a simple model where every state had equal probability to follow any other state, and where I assumed that the classifier always was correct (H1 would always be interpreted as RIGHT etc). Training data for the discrete output would then be generated by me by using the robot, and in addition to giving the hand gestures to the robot I would note down how the robot would react and what my intention really was. By generating enough data I could calculate the probabilities of the robot misinterpreting my commands and create a discrete output matrix.

Using the same test data I could generate a state-transition probability matrix, by calculating the probabilities for each state following each other state in my test data.

To estimate the old model I would compare it to my new model created with my test data.

b

$P(x_2 = \text{LEFT}) = a_{\text{RIGHT RIGHT}} * a_{\text{RIGHT LEFT}} + a_{\text{RIGHT LEFT}} * a_{\text{LEFT LEFT}} + a_{\text{RIGHT STOP}} * a_{\text{STOP LEFT}} + a_{\text{RIGHT FORWARD}} * a_{\text{FORWARD LEFT}} = 0.45 * 0.09 + 0.09 * 0.55 + 0.10 * 0.12 + 0.36 * 0.09 = 0.1344$

Answer: $P(x_2 = \text{LEFT}) = 0.1344$

The probability of the second observation being o_2 is equal to the sum of the probabilities of each available state being second multiplied by the probability of the observation being H_2 for the corresponding state.

$P(x_2 = \text{RIGHT}) = a_{\text{RIGHT RIGHT}} * a_{\text{RIGHT RIGHT}} + a_{\text{RIGHT LEFT}} * a_{\text{LEFT RIGHT}} + a_{\text{RIGHT STOP}} * a_{\text{STOP RIGHT}} + a_{\text{RIGHT FORWARD}} * a_{\text{FORWARD RIGHT}} = 0.45 * 0.45 + 0.09 * 0.08 + 0.10 * 0.08 + 0.36 * 0.09 = 0.2501$

$P(x_2 = \text{LEFT}) = 0.1344$

$P(x_2 = \text{STOP}) = a_{\text{RIGHT RIGHT}} * a_{\text{RIGHT STOP}} + a_{\text{RIGHT LEFT}} * a_{\text{LEFT STOP}} + a_{\text{RIGHT STOP}} * a_{\text{STOP STOP}} + a_{\text{RIGHT FORWARD}} * a_{\text{FORWARD STOP}} = 0.45 * 0.10 + 0.09 * 0.10 + 0.10 * 0.60 + 0.36 * 0.15 = 0.168$

$P(x_2 = \text{FORWARD}) = a_{\text{RIGHT RIGHT}} * a_{\text{RIGHT FORWARD}} + a_{\text{RIGHT LEFT}} * a_{\text{LEFT FORWARD}} + a_{\text{RIGHT STOP}} * a_{\text{STOP FORWARD}} + a_{\text{RIGHT FORWARD}} * a_{\text{FORWARD FORWARD}} = 0.45 * 0.36 + 0.09 * 0.27 + 0.10 * 0.20 + 0.36 * 0.67 = 0.4475$

$P(o_2 = H_2) = P(x_2 = \text{RIGHT}) * b_{\text{RIGHT}}(H_2) + P(x_2 = \text{LEFT}) * b_{\text{LEFT}}(H_2) + P(x_2 = \text{STOP}) * b_{\text{STOP}}(H_2) + P(x_2 = \text{FORWARD}) * b_{\text{FORWARD}}(H_2) = 0.2501 * 0.05 + 0.1344 * 0.62 + 0.168 * 0.20 + 0.4475 * 0.04 = 0.147333$

Answer: $P(o_2 = H_2) = 0.147333$

The probability of o_2 being H_2 given that x_2 is LEFT can be read directly from the output probability matrix.

Answer: $P(o_2 = H_2 \mid x_2 = \text{LEFT}) = 0.62$

c

The most likely sequence is: RIGHT STOP STOP FORWARD RIGHT STOP STOP FORWARD RIGHT STOP.

I solved it using the Viterbi algorithm (look at the file viterbi.c to see how i solved it).