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who has worked for the year 2020 in the Computer Laboratory.

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Teacher In-Charge

Head of Department

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Examiner

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PRACTICAL NO. 1

TOPIC :- Limits & continuity

1. $\lim\limits_{x\to a} \dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$

$$= \lim_{x\to a} \frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$= \lim_{x\to 0} \frac{a+2x-3x}{3a+x-4x} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}-\sqrt{3x}}$$

$$= \lim_{x\to a} \frac{a-x}{3a-3a} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$= \lim_{x\to a} \frac{1}{3} \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$= \frac{1}{3} \frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$$

$$= \frac{1}{3} \frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$= \frac{1}{3} \frac{2\sqrt{a}+2\sqrt{a}}{2\sqrt{3a}}$$

$$\frac{2\sqrt{a}}{3} = \frac{2\sqrt{a}}{3\sqrt{3a}}$$

2. $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$= \lim_{y \to 0} \frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}}$

$= \lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y} \times \sqrt{a+y} + \sqrt{a}}$

$= \lim_{y \to 0} \frac{\cancel{y}}{\cancel{y}\sqrt{a+y} \times \sqrt{a+y} + \sqrt{a}}$

$= \lim_{y \to 0} \frac{1}{\sqrt{a} \times \sqrt{a} + \sqrt{a}}$

$= \frac{1}{\sqrt{a}\, 2\sqrt{a}} = \frac{1}{2a}$

3. $\lim_{x \to \pi/6} \left[\frac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$

$\Rightarrow$ Put $h = x - \pi/6$ $\quad x \to \pi/6$

$x = h + \pi/6$

$h \to 0$

$= \lim_{h \to 0} \frac{\cos(h+\frac{\pi}{6}) - \sqrt{3}\sin(h+\frac{\pi}{6})}{\pi - 6(h+\pi/6)}$

$= \lim_{h \to 0} \frac{(\cos h \cdot \cos\pi/6 - \sin h \cdot \sin\pi/6) - \sqrt{3}(\sin h \cos\frac{\pi}{6} + \cos h \sin\pi/6)}{\cancel{\pi} - 6h - \cancel{\pi}}$

$= \lim_{h \to 0} \frac{(\frac{\sqrt{3}}{2}\cosh - \sin h \times \frac{1}{2} - \sqrt{3}\sin h \times \frac{\sqrt{3}}{2} - \cos h \times \frac{\sqrt{3}}{2}}{-6h}$

$= \lim_{h \to 0} \frac{-\frac{\sin h}{2} - \sin h \times \frac{3}{2}}{-6h}$

$= \lim_{h \to 0} \frac{-\frac{2\sin h}{2}}{-6h}$

$= \lim_{h \to 0} \frac{-2\sin h}{-6h} = \lim_{h \to 0} \frac{1}{3}\frac{\sin h}{h}$

$= \frac{1}{3}\lim_{h \to 0} \frac{\sin h}{h} = \frac{1}{3}$

4) $\lim_{h \to \infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}$

By rationalizing numerator & Denominator.

$\lim_{x \to \infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}}$

$= \lim_{x \to \infty} \frac{(x^2+5-x^2+3)}{(x^2+3-x^2-1)} \frac{(\sqrt{x^2+3} + \sqrt{x^2+1})}{\sqrt{x^2+5} + \sqrt{x^2-3}}$

$$= \lim_{x \to \infty} \frac{48}{72} \frac{(\sqrt{x^2+3} + \sqrt{x^2+1})}{\sqrt{x^2+5} + \sqrt{x^2-3}}$$

$$= 4 \lim_{x \to \infty} \frac{x\left(\sqrt{1+\frac{3}{x^2}} + \sqrt{1+\frac{1}{x^2}}\right)}{x\left(\sqrt{1+\frac{5}{x^2}} + \sqrt{1-\frac{3}{x^2}}\right)}$$

$$= 4 \times \frac{\sqrt{1}+\sqrt{1}}{\sqrt{1}+\sqrt{1}}$$

$$= 4.$$

5. Examine the continuity of the following function at given points

(i) $f(x) = \left.\begin{matrix} \frac{\sin 2x}{\sqrt{1-\cos 2x}} & \text{for } 0 < x \leq \pi/2 \\ & \text{for } \frac{\pi}{2} < x < \pi \end{matrix}\right\}$ at $x = \frac{\pi}{2}$

$$f\left(\frac{\pi}{2}\right) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$$

$$= \frac{\sin \pi}{\sqrt{1-\cos \pi}}$$

$$= 0.$$

$$LHL \cdot \lim_{x \to \frac{\pi}{2}} f(x)$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\pi - 2x}$$

put $x - \frac{\pi}{2} = h$ $\quad x \to \pi/2$

$x = h + \pi/2 \quad h \to 0.$

$$\therefore \lim_{h \to 0} \frac{\cos(\pi/2 + h)}{\pi - 2(\pi/2 + h)}.$$

$$= \lim_{h \to 0} \frac{-\sin h}{\pi - \pi + 2h}$$

$$= \lim_{h \to 0} \frac{-\sin h}{2h}$$

$$= \frac{-1}{2} \lim_{h \to 0} \frac{\sin h}{h}.$$

$$= \frac{-1}{2}.$$

$$RHL = \lim_{x \to \frac{\pi}{2}^+} f(x)$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$$

$$= \lim_{x \to \pi/2} \frac{\sqrt{2}\, 2 \cdot \sin x \cdot \cos x}{\sqrt{2} \sin x}$$

$$= \sqrt{2} \lim_{x \to \pi/2} \cos \pi/2.$$

$= \sqrt{2} \cdot 0$.

$= 0$.

$\therefore$ LHL $\neq$ RHL.

$\therefore f$ is not continous at $x = \pi/2$

(ii) $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < 3 < 3$
$\quad = x+3$ $\quad 3 \leq x < 6$ $\quad$ at $x=3$
$\quad = \frac{x^2-9}{x+3}$ $\quad 6 \leq x < 9$ $\quad x=6$

At $x=3$

$f(3) = x+3 = 3+3 = 6$

LHL $= \lim_{x \to 3^-} f(x) = \lim_{x \to 3} x+3$

$= 3+3$

$= 6$.

RHL $\lim_{x \to 3^+} f(x) = \lim_{x \to 3} \frac{x^2-9}{x-3}$

$= \lim_{x \to 3} \frac{(x-3)(x+3)}{(x-3)}$

$= \lim_{x \to 3} (x+3)$

$= 3+3$

$= 6$.

LHL $=$ RHL $= f(3)$

$\therefore f$ is continous at $x=3$.

At $x=6$.

$f(6) = \frac{x^2-9}{x+3} = \frac{(x-3)(x+3)}{(x+3)} = 6-3 = 3$

LHL $= \lim_{x \to 6^-} \frac{x^2-9}{x+3}$

$= \lim_{x \to 6^-} \frac{(x-3)(x+3)}{(x+3)}$

$= \lim_{x \to 6} x-3$

$= 6-3$

$= 3$

RHL $= \lim_{x \to 6^+} f(x)$

$= \lim_{x \to 6} x+3$

$= 6+3$

$= 9$

$\therefore$ LHL $\neq$ RHL

$\therefore f$ is not continous at $x=6$.

6(i) $f(x) = \frac{1-\cos 4x}{x^2} \quad x \neq 0 \\ = k \quad x = 0 \Big\}$ at $x = 0$

$\Rightarrow$ $\because f$ is continous at $x = 0$

$\lim_{x \to 0} f(x) = f(0)$

$\therefore \lim_{x \to 0} \frac{1-\cos 4x}{x^2} = k$

$\lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = k$

$\Rightarrow 2 \lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$

$2(2)^2 = k$

$\boxed{k = 8}$

(ii) $f(x) = (\sec^2 x)^{\cot^2 x} \quad x \neq 0 \\ = k \quad x = 0 \Big\}$ at $x = 0$

$\Rightarrow$ f is continous at $x = 0$

$\therefore \lim_{x \to 0} f(x) = k$

$\lim_{x \to 0} (\sec^2 x)^{\cot^2 x} = k$

$\lim_{x \to 0} (1 + \tan^2 x)^{\frac{1}{\tan^2 x}} = k$

$\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$

$e = k$

$k = e$

(iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x} \quad x \neq \pi/3 \\ = k \quad , x = \pi/3 \Big\}$ at $x = \pi/3$

$\Rightarrow$ f is continous at $x = \pi/3$

$\therefore \lim_{x \to \pi/3} f(x) = f(\pi/3)$

$\Rightarrow \lim_{x \to \pi/3} \frac{\sqrt{3} - \tan x}{\pi - 3x} = k$

put $x - \frac{\pi}{3} = h \cdot x = \frac{\pi}{3} + h$, as $x \to \pi/3$, $h \to 0$

$\lim_{h \to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

$= \lim_{h \to 0} \frac{\sqrt{3} - \left(\frac{\tan\frac{\pi}{3} + \tan h}{1 - \tan\pi/3 \cdot \tan h}\right)}{\pi - \pi - 3h}$

$= \lim_{h \to 0} \frac{\sqrt{3}(1 - \tan\pi/3 \tan h) - \tan\pi/3 - \tan h}{(-3h)(1 - \tan\pi/3 \tan h)}$

$= \lim_{h \to 0} \frac{\sqrt{3}(1 - \sqrt{3}\tan h) - \sqrt{3} - \tan h}{-3h(1 - \sqrt{3}\tan h)}$

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$= \lim_{h \to 0} \frac{\sqrt{3} - 3\tan h - \sqrt{3} - \tan h}{-3h(1-\sqrt{3}\tan h)}$

$= \lim_{h \to 0} \frac{-4\tan h}{-3h(1-\sqrt{3}\tan h)}$

$= \lim_{h \to 0} \frac{4}{3} \lim_{h \to 0} \left(\frac{\tan h}{h}\right)\left(\frac{1}{1-\sqrt{3}\tan h}\right)$

$= \frac{4}{3}\left(\frac{1}{1-\sqrt{3}(0)}\right)$

$= \frac{4}{3}.$

7) (i) $f(x) = \frac{1-\cos 3x}{x\tan x}$ $\quad x \neq 0$

$= 9. \quad x = 0$

Sol^n: $\lim f(x) = \frac{1-\cos 3x}{x\tan x}$

$\lim_{x \to 0} = \frac{2\sin^2 3/2x}{x\tan x}$

$\lim_{x \to 0} = \frac{2\sin^2 3/2x}{x\tan x}$

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$\lim_{x \to 0} \frac{\frac{2\sin^2 \frac{3x}{2}}{x^2} \times x^2}{\frac{x \cdot \tan x}{x^2} \times x^2}$

$= 2\lim_{x \to 0} \frac{\left(\frac{3}{2}\right)^2}{1} = \frac{2 \times 9}{4} = \frac{9}{2}$

$\lim_{x \to 0} f(x) = \frac{9}{2} \quad y = f(0).$

$\therefore f$ is not continous at $x = 0.$

Redefine function

$f(x) = \begin{cases} \frac{1-\cos 3x}{x\tan x} & x \neq 0 \\ \frac{9}{2} & x = 0 \end{cases}$

Now $\lim_{x \to 0} f(x) = f(0)$

f has removable discontinuity at $x = 0.$

(ii) $f(x) = \left.\begin{matrix} \frac{(e^{3x}-1)\sin x^\circ}{x^2} & x \neq 0 \\ = \pi/6 & x = 0 \end{matrix}\right\}$ at $x = 0$

$\lim_{x \to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$

$\lim_{x \to 0} \frac{e^{3x}-1}{x} \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}.$

$$\lim_{x\to 0} \frac{3e^{3x}-1}{3\,3x} \quad \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{18}\right)}{x}$$

$$3\lim_{x\to 0} \frac{e^{3x}-1}{3x} \quad \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3\log_e \frac{\pi}{180} = \frac{\pi}{6} = f(0).$$

f is continous at $x=0$.

8. $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ $x = 0$.

is continous at $x=0$.

$\therefore$ Given f is continous at $x=0$.

$$\lim_{x\to 0} f(x) = f(0)$$

$$= \lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$= \lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$\lim_{x\to 0} \frac{(e^{x^2}-1) + (1-\cos x)}{x^2}.$$

$$\lim_{x\to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \frac{1-\cos x}{x^2}$$

$$\log_e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$$

$$\log_e + 2\lim_{x\to 0} \left(\frac{\sin x/2}{x}\right)^2$$

Multiply with 2 in Num & denominator

$$= \frac{1}{1} + 2 \times \frac{1}{4} = \frac{3}{2} = f(0)$$

9. $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $x \neq \pi/2$

f(0) is continous at $x = \pi/2$.

$$\lim_{x\to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$\lim_{x\to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x\,(\sqrt{2} + \sqrt{1+\sin x}}$$

$$= \lim_{x\to \pi/2} \frac{1+\sin x}{1-\sin^2 x(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x\to \pi/2} \frac{\cancel{1+\sin x}}{(1-\sin x)\cancel{(1+\sin x)}(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x\to \pi/2} \frac{1}{(1-\sin x)(\sqrt{2} + 1+\sin x)}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$= \frac{1}{2(2\sqrt{2})} = \frac{1}{4\sqrt{2}} \quad \therefore f(\pi/2) = \frac{1}{4\sqrt{2}}$$

AV 6/12/19

PRACTICAL - 02

TOPIC : Derivative

Q1. Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable

(i) $\cot x$

$f(x) = \cot x$

$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$

$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$

$= \lim_{x \to a} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$

$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h(\tan(a+h)\tan a)}$

Formula : $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$\tan A - \tan B = \tan(A-B)(1 + \tan A \tan B)$

$= \lim_{h \to 0} \frac{\tan(a-a-h) - (1 + \tan A + \tan(a+h))}{h \times \tan(a+h)\tan a}$

$= \lim_{h \to 0} -\frac{\tan h}{h} \times \frac{1 + \tan a \tan(a+h)}{\tan(a+h)\tan a}$

$= -1 \times \frac{1 + \tan^2 a}{\tan^2 a}$

$= -\frac{\sec^2 a}{\tan^2 a}$

$= \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$

$= -\operatorname{cosec}^2 a$

$\therefore Df(a) = -\operatorname{cosec}^2 a$

$\therefore f$ is differentiable $\forall a \in R$

(ii) $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$

$= \lim_{x \to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x - a}$

$= \lim_{x \to a} \frac{\sin a - \sin x}{(x-a)\sin a \cdot \sin x}$

put $x - a = h$

$x = a + h$

as $x \to a, h \to 0$

$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$

Formula :

$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$Df(2^-) = 4$

RHD :

$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$

$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$

$= \lim_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$

$= 2+2 = 4$

$Df(2^+) = 4$

f is differentiable at $x=2$

Q3 If $f(x) = 4x+7$, $x<3$
$= x^2+3x+1$, $x \geq 3$ at $x=3$ then
find f is differentiable or not .

Solution :

RHD :

$Df(3^+) = \lim_{x \to 3^+} \frac{f(x)-f(3)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3\times3+1)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$

$= \lim_{x \to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$

$= \lim_{x \to 3^+} \frac{(x-3)(x+6)}{(x-3)}$

$= \lim 3+6 = 9$

$Df(3^+) = 9$.

LHD : $Df(3^-) = \lim_{x \to 3^-} \frac{f(x)-f(3)}{x-3}$

$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3}$

$= \lim_{x \to 3^-} \frac{4x-12}{x-3}$

$= \lim_{x \to 3^-} \frac{4(x-3)}{(x-3)}$

$Df(3^-) = 4$

RHD $\neq$ LHD

∴ f is not differentiable at $x=3$

Q4 If $f(x) = 8x-5$, $x<2$
$= 3x^2-4x+7$ $x \geq 2$ at $x=2$ then
find f is differentiable or not

Solution :

$f(2) = 8\times2-5 = 16-5 = 11$

RHD

$Df(2^+) = \lim_{x \to 2^+} \frac{f(x)-f(2)}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2-4x+7-11}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2-6x+2x-4}{x-2}$

$= \lim_{x \to 2^+} \frac{3x(x-2)+2(x-2)}{x-2}$

$$= \lim_{x\to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 3\times 2+2 = 8$$

$Df(2^+) = 8$

LHD:

$Df(2^-)$

$$= \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8x-5-11}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8x-16}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8(x-2)}{(x-2)}$$

$$= 8$$

$Df(2^-) = 8$

LHD = RHD

$\therefore f$ is differentiable at $x=3$

A.L.
20/12/17

PRACTICAL - 3

TOPIC : Application of Derivative.

1) Find the intervals in which function is increasing or decreasing.

i) $f(x) = x^3-5x-11$

ii) $f(x) = x^2-4x$

iii) $f(x) = 2x^3+x^2-20x+4$

iv) $f(x) = x^3-27x+5$

v) $f(x) = 69-24x-9x^2+2x^3$

2) Find the intervals in which function is concave upwards and downwards.

i) $y = 3x^2-2x^3$

ii) $y = x^4-6x^3+12x^2+5x+7$

iii) $y = x^3-27x+5$

iv) $y = 69-24x-9x^2+2x^3$

v) $y = 2x^3+x^2-20x+4$

Solution:

Q1

1) $f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\because f$ is increasing iff $f'(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

$+ \quad -\sqrt{5/3} \quad - \quad \sqrt{5/3} \quad +$ $\quad \therefore x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore 3(x^2 - 5/3) < 0$

$\therefore (x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$

$+ \quad -\sqrt{5/3} \quad - \quad \sqrt{5/3} \quad +$ $\quad x \in (-\sqrt{5/3}, \sqrt{5/3})$

2) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\because f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$\therefore 2(x-2) > 0$

$\therefore x - 2 > 0$

$x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x-2) < 0$

$\therefore x - 2 < 0$

$x \in (-\infty, 2)$

3) $f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$\therefore 2(3x^2 + x - 10) > 0$

$\therefore 3x^2 + x - 10 > 0$

$\therefore 3x^2 + 6x - 5x - 10 > 0$

$\therefore 3x(x+2) - 5(x+2) > 0$

$\therefore (x+2)(3x-5) > 0$

$+ \quad -2 \quad - \quad 5/3 \quad +$ $\quad x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$\therefore 2(3x^2 + x - 10) < 0$

$\therefore 3x^2 + x - 10 < 0$

$\therefore 3x^2 + 6x - 5x - 10 < 0$

$\therefore 3x^2(x+2) - 5(x+2) < 0$

$\therefore (x+2)(3x-5) < 0$

$+ \quad -2 \quad - \quad 5/3 \quad +$ $\quad x \in (-2, 5/3)$

Subject : Roll No. Table No.

4) $f(x) = x^3 - 27x + 5$

$\therefore f'(x) = 3x^2 - 27$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore (x-3)(x+3) > 0$

$+ \quad - \quad +$

$-3 \quad 3$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore (x-3)(x+3) < 0$

$+ \quad - \quad +$

$-3 \quad 3$

$\therefore x \in (-3, 3)$

5) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$\therefore x^2 - 4x + x - 4 > 0$

$\therefore x(x-4) + 1(x-4) > 0$

$\therefore (x-4)(x+1) > 0$

$+ \quad - \quad +$

$-1 \quad 4$

$x \in (-\infty, -1) \cup (4, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 - 18x - 24 < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

$\therefore x^2 - 4x + x - 4 < 0$

$\therefore x(x-4) + 1(x-4) < 0$

$\therefore (x-4)(x+1) < 0$

$+ \quad - \quad +$

$-1 \quad 4$

$\therefore x \in (-1, 4)$

Q2)

1) $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6x - 12x$

f is concave upward if $f''(x) > 0$

$\therefore (6 - 12x) > 0$

$\therefore 12(6/12 - x) > 0$

$x - 1/2 > 0$

$x > 1/2$

$\because f''(x) > 0$

$\therefore x \in (1/2, \infty)$

$f''(x)$ is concave downwards iff, $f''(x) < 0$

$6(1 - 2x) < 0$

$1 - 2x < 0$

$-2x < -1$

$2x > 1$

$x > 1/2$

$x \in (1/2, \infty)$

2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - \cancel{18}36x + 24$

f is concave upward if $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

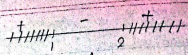
$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 3x + 2 > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore x(x-2) - 1(x-2) > 0$

$\therefore (x-2)(x-1) > 0$



$x \in (-\infty, 1) \cup (2, \infty)$

3) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x \in (0, \infty)$

4) $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0 \quad \therefore x > 3/2 \quad \therefore x \in$

$\therefore x \in (3/2, \infty)$

$f''(x)$ is concave downward iff

$f''(x) < 0$

$-18x + 12x < 0$

$12x < 18$

$x > \frac{18}{12}$

$x \in (-\infty, 3/2)$

5) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 12(x + 2/12) > 0$

$\therefore (x + 1/6) > 0$

$\therefore x < -1/6$

$\therefore f''(x) \not> 0$

$\therefore$ There exist no interval.

$f''(x)$ is concave downward iff

$f''(x) < 0$

$2(6x + 1) < 0$

$6x + 1 < 0$

$6x < -1$

$x < -1/6$

$x \in (-\infty, -1/6)$

20/12/19

PRACTICAL - 4

TOPIC : Application of Derivative & Newton's Method.

Q.1 Find maximum & minimum value of following function

i) $f(x) = x^2 + \frac{16}{x^2}$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 + 1$ in $[\frac{-1}{2}, 4]$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

Q.2 Find the root of following equation by Newton's method

i) $f(x) = x^3 - 3x^2 - 55x + 95$ (take $x_0 = 0$).

ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$.

Answers :-

Q1 (i) $f(x) = x^2 + 16/x^2$

$f'(x) = 2x - \frac{32}{x^3}$

Now consider

$f'(x) = 0$

$\therefore 2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = 32/2$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/2^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = 2$.

$\therefore f(2) = 2^2 + \frac{16}{2^2}$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$f''(-2) = 2 + \frac{96}{-2^4}$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = -2$

$\therefore$ Function reaches minimum value at $x = 2$ and $x = -2$.

ii) $f(x) = 3-5x^3+3x^5$
$f'(x) = -15x^2+15x^4$
Consider,
$f'(x)=0$
$-15x^2+15x^4=0$
$15x^4 = 15x^2$
$x^2=1$
$x=\pm 1$

$\therefore f''(x) = -30x+60x^3$
$f''(1) = -30+60$
$= 30>0$
$\therefore f$ has minimum value at $x=1$
$\therefore f(1) = 3-(5(1)^3+3(1)^5$
$= 6-5$
$= 1$
$f''(-1) = -30(-1)+60(-1)^3$
$= 30-60$
$= -30<0$
$\therefore f$ has maximum value at $x=-1$

$f(-1) = 3-5(-1)^3+3(-1)^5$
$= 3+5-3$
$= 5$

f has the maximum value 5 at $x=-1$ and has the minimum value 1 at $x=1$

iii) $f(x) = x^3-3x^2+1$
$\therefore f'(x) = 3x^2-6x$

Consider,
$f'(x)=0$
$3x^2-6x=0$
$3x(x-2)=0$
$3x=0$ or $x-2=0$
$x=0$ or $x=2$
$f''(x) = 6x-6$
$f''(0) = 6(0)-6$
$= -6<0$
$\therefore f$ has maximum value at $x=0$
$f(0) = (0)^3-3(0)^3+1$
$= 1$

$f''(2) = 6(2)-6$
$= 12-6$
$= 6>0$

f has minimum value at $x=2$

$f(2) = (2)^3-3(2)^2+1$
$= 8-3(4)+1$
$= 9-12$
$= -3$

$\therefore f(x)$ has maximum value at 1 at $x=0$ and f has minimum value -3 at $x=2$

8

$F(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$
$= 0.0050 - 0.0879 - 9.6416 + 9.5$
$= 0.0011$

$F'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$
$= 0.0879 - 1.0272 - 55$
$= -55.9393$

$\therefore x_3 = x_2 - \frac{F(x_2)}{F'(x_2)}$
$= 0.1712 + \frac{0.0011}{55.9393}$
$= 0.1712$

$\therefore$ The root of the equation is 0.1712.

ii) $F(x) = x^3 - 4x - 9$ $\qquad$ $[2,3]$ - given.
$F'(x) = 3x^2 - 4$.

$F(2) = 2^3 - 4(2) - 9$
$= 8 - 8 - 9$
$= -9$

$F(3) = 3^3 - 4(3) - 9$
$= 27 - 12 - 9$
$= 6$.

Let $x_0 = 3$ be the initial approximation,
By newton's method,

$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$

$x_1 = x_0 - \frac{F(x_0)}{F'(x_0)}$
$= 3 - \frac{6}{23}$ ---- $\left[\begin{array}{l} F'(x) = 3x^2 - 4 \\ F'(3) = 27 - 4 = 23 \end{array}\right]$
$= 2.7392$

$F(x_1) = (2.7392)^3 - 4(2.7392) - 9$
$= 20.5528 - 10.9568 - 9$
$= 0.596$

$F'(x) = 3(2.7392)^2 - 4$
$= 22.5096 - 4$
$= 18.5096$

$x_2 = x_1 - \frac{F(x_1)}{F'(x_1)}$
$= 2.7392 - \frac{0.596}{18.5096}$
$= 2.7071$

$F(x_2) = (2.7071)^3 - 4(2.7071) - 9$
$= 19.8386 - 10.8284 - 9$
$= 0.0102$

$F'(x_2) = 3(2.7071)^2 - 4$
$= 21.9851 - 4$
$= 17.9851$

$\therefore x_3 = x_2 - \frac{F(x_2)}{F'(x_2)}$
$= 2.7071 - \frac{0.0102}{17.9851}$
$= 2.7071 - 0.0056$
$= 2.7015$

$F(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= 19.7158 - 10.806 - 9$
$= -0.0901$

$F'(x_3) = 3(2.7015)^2 - 4$
$= 21.8943 - 4$
$= 17.8943$

$x_4 = 2.7015 + \frac{0.0901}{17.8943}$
$= 2.7015 + 0.0050$
$= 2.7065$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1, 2]$
$F'(x) = 3x^2 - 3.6x - 10$
$F(1) = (1)^3 - 1.8(1) - 10(1) + 17$
$= 1 - 1.8 - 10 + 17$
$= 6.2$

$F(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$
$= 8 - 7.2 - 20 + 17$
$= -2.2$

Let $x_0 = 2$ be initial approximation
By Newton's Method,
$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$

$x_1 = x_0 - \frac{F(x_2)}{F'(x_2)}$

$= 2 - \frac{2.2}{5.2}$
$= 2 - 0.4230$
$= 1.577$
$F(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$
$= 3.9219 - 4.4764 - 15.77 + 17$
$= 0.6755$

$F'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$
$= 7.4608 - 5.6772 - 10$
$= -8.2164$

$\therefore x_2 = x_1 - \frac{F(x_1)}{F'(x_1)}$
$= 1.577 + \frac{0.6755}{8.2164}$
$= 1.577 + 0.0822$
$= 1.6592$
$F(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$
$= 4.5677 - 4.9553 - 16.592 + 17$
$= 0.0204$
$F'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$
$= 8.2588 - 5.97312 - 10$
$= -7.7143$

$x_3 = x_2 - \frac{F(x_2)}{F'(x_2)}$

$= 1.6592 + \frac{0.0204}{7.7143}$

$= 1.6592 + 0.0026$

$= 1.6618$

$F(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.0004$

$F'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - \frac{F(x_3)}{F'(x_3)}$

$= 1.6618 + \frac{0.0004}{7.6977}$

$= 1.6618$

$\therefore$ The root of equation is 1.6618.

PRACTICAL - 5

TOPIC : Integration

Q1. Solve the following integration

(i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$ (ii) $\int (4e^{3x}+1)dx$ (iii) $\int (2x^2 - 3\sin x + 5\sqrt{x})dx$

(iv) $\int \frac{x^3+3x+4}{\sqrt{x}}$ (v) $\int t^7 \sin(2t^4)dt$

(vi) $\int \sqrt{x}(x^2-1)dx$ (vii) $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$.

(viii) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}$ (ix) $\int e^{\cos^2 x}\sin 2x\,dx$

(x) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right)dx$

Answers

i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$I = \int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{dx}{\sqrt{x^2+2x+1-4}}$

$= \int \frac{dx}{\sqrt{(x+1)^2-(2)^2}}$

comparing with $\int \frac{dx}{\sqrt{x^2-a^2}}$ $\quad x^2 = (x+1)^2$, $a^2 = (2)^2$

$I = \log |x + \sqrt{x^2-a^2}| + C$

$= \log |x+1 + \sqrt{(x+1)^2-(2)^2}| + C$

(ii) $\int (4e^{3x}+1)\, dx$

$I = \int (4e^{3x}+1)\, dx$

$= \int 4e^{3x} dx + \int 1\, dx$

$= \frac{4e^{3x}}{3} + x + C$

(iv) $\int \frac{x^3+3x+4}{\sqrt{x}} dx$

$I = \int \frac{x^3+3x+4}{\sqrt{x}} dx$

$= \int \left(\frac{x^3}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{4}{\sqrt{x}}\right) dx$

$= \int \left(\frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\right) dx$

$= \int \left(x^3 \cdot x^{-1/2} + 3x \cdot x^{-1/2} + 4x^{-1/2}\right) dx$

$= \int x^{5/2} dx + 3\int x^{1/2} dx + 4\int x^{-1/2} dx$

$= \frac{2}{7} x^{7/2} + 3 \cdot \frac{2}{3} x^{3/2} + 4^{1/2} \cdot 2 + C$

$= \frac{2}{7} x^{7/2} + 2x^{3/2} + 8\sqrt{x} + C$

(v) $\int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

$I = \int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

$= 2\int x^2 dx - 3\int \sin x\, dx + 5\int \sqrt{x}\, dx$

$= \frac{2x^3}{3} + 3\cos x + 5 \times \frac{2}{3} x^{3/2} + C$

$= \frac{2}{3} x^3 + 3\cos x + \frac{10x^{3/2}}{3} + C$

Batch No. 3

Subject : Maths

Roll

Table No.

(viii) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}$

$I = \int \frac{\cos x}{\sqrt[3]{\sin^2 x}} dx$

Let $\sin x = t$

$\cos x\, dx = dt$

$I = \int \frac{dt}{\sqrt[3]{t^2}}$

$\therefore I = \int \frac{dt}{t^{2/3}}$

$= \int t^{-2/3} dt$

$= 3 t^{1/3} + c$

$= 3 (\sin x)^{1/3} + c$

$= 3\sqrt[3]{\sin x} + c$

ix) $\int e^{\cos^2 x} \cdot \sin 2x\, dx$

$I = \int e^{\cos^2 x} \sin 2x\, dx$

Let $\cos^2 x = t$

$-2\cos x \sin x\, dx = dt$

$-2\sin 2x\, dx = dt$

$I = -\int -\sin 2x\, e^{\cos^2 x} dx$

$= -\int e^t dt$

$= -e^t + c$

Resubstitute $t = \cos^2 x$

$\therefore I = -e^{\cos^2 x} + c$

(x) $\int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1}\right) dx$

$I = \int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1}\right) dx$

Let,

$x^3 - 3x^2 + 1 = t$

$\therefore (3x^2 - 6x) dx = dt$

$3(x^2 - 2x) dx = dt$

$(x^2 - 2x) dx = \frac{dt}{3}$

$\therefore I = \int \frac{1}{t} \frac{dt}{3}$

$= \frac{1}{3} \int \frac{dt}{t}$

$= \frac{1}{3} \log t + c$

Resubstituting $t = x^3 - 3x^2 + 1$

$\therefore I = \frac{1}{3} \log (x^3 - 3x^2 + 1) + c$

17/01/2020

PRACTICAL NO. 6.

TOPIC :- Application of integration & Numerical integration.

1. Find the length of the following curve.

1) $x = t - \sin t \quad y = 1 - \cos t \quad t \in [0, 2\pi]$

$$l = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$x = t - \sin t \quad \therefore \frac{dx}{dt} = 1 - \cos t$

$y = 1 - \cos t \quad \therefore \frac{dy}{dt} = 0 - (-\sin t) = \sin t$

$$l = \int_0^{2\pi} \sqrt{(1-\cos t)^2 + (\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt = \int_0^{2\pi} \sqrt{2(1-\cos t)} = \int_0^{2\pi} \sqrt{2 \cdot 2\sin^2 t/2} = \sqrt{4\sin^2 t/2}$$

$$= \int_0^{2\pi} 2\left|\sin \frac{t}{2}\right| dt \quad \because \sin^2 \frac{t}{2} = \frac{1-\cos t}{2}$$

$$= \int_0^{2\pi} 2\sin \frac{t}{2}\, dt$$

$$= \left[-2\cos\left(\frac{t}{2}\right)\cdot 2\right]_0^{2\pi} = 2\left[-\cos\left(\frac{t}{2}\right)\right]_0^{2\pi} \cdot 2$$

$$= 4[-1, -1]$$

$$= 8.$$

2. $y = \sqrt{4 - x^2} \quad x \in [-2, 2]$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$y = \sqrt{4 - x^2} \quad \therefore \frac{dy}{dx} = \int_{-2}^{2} \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$

$$= 2\int_0^2 \sqrt{1 + \frac{x^2}{4-x^2}}\, dx \qquad \int_2^2 \frac{1+x^2}{4-x^2}$$

$$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx \qquad \int \sqrt{\frac{4}{4-x^2}}$$

$$= 4\left(\sin^{-1}(x/2)\right)_0^2$$

$$= 2\pi. \qquad \frac{du}{dx} = 2\pi.$$

3. $y = x^{3/2}$ in $[0, 4]$

$f'(x) = \frac{3}{2} x^{1/2}$

$[f'(x)]^2 = \frac{9}{4} x$

$$l = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

$$= \int_0^4 \sqrt{1 + \frac{9}{4}x}\, dx$$

$$= \int_0^4 \sqrt{1 + [f'(x)]^2}\, dx$$

$$= \int_0^4 \sqrt{1 + \frac{9}{4}x}\, dx$$

$$= \int_0^4 \sqrt{\frac{4+9x}{4}}\, \cancel{dx}$$

$$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\, dx$$

$$= \frac{1}{2}\left[\frac{4+9x^{1/2+1}}{1/2+1}\right]_0^4 dx$$

$$= \frac{1}{27}\left[4+9x\right]^{3/2}\Big]_0^4$$

$$= \frac{1}{27}\left[(4+0)^{3/2} - (4+36)^{3/2}\right]$$

$$= \frac{1}{27}(4)^{3/2} - (40)^{3/2}.$$

4) $x = 3\sin t$, $y = 3\cos t$ $t \in [0, 2\pi]$

$\frac{dx}{dt} = 3\cos t$ $\frac{dy}{dt} = -3\sin t$

$L = \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin)^2}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$

$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\, dt$

$= \int_0^{2\pi} \sqrt{9(1)}\, dt$

$= \int_0^{2\pi} 3\,dt = 3\int_0^{2\pi} dt$

$= 3(t)_0^{2\pi}$

$= 3(2\pi - 0)$

$= 6\pi$.

Q.2 using simpson's rule solve the following.

i) $\int_0^2 e^{x^2}\, dx$ with $n = 4$

$a = 0$ $b = 2$ $n = 4$.

$h = \frac{2-0}{4} = \frac{1}{2} = 0.5$

x	0	0.5	1	1.5	2
y	1	1.2840	2.7183	9.4877	54.5982
	y_0	y_1	y_2	y_3	y_4

$\int_0^2 e^{x^2} dx = \frac{0.5}{3}[(y_0 + y_4) + 4(y_1 + y_3) + 2(y_2 + x)]$

$= \frac{0.5}{3}[(1 + 54.5982) + 4(1.2840 + 9.4877) + 2(2.7183)]$

$= \frac{0.5}{3}[55.5982 + 43.0866 + 5.436]$

$= 17.3535$

ii) $\int_0^4 x^2\, dx$ $h = 4$.

$h = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16
	y_0	y_1	y_2	y_3	y_4

$\int_0^4 x^2 dx = \frac{h}{3}[(y_0 + y_4) + 4(y_1 + y_3) + 2(y_2)]$

$= \frac{1}{3}[(0 + 16) + 4(1 + 9) + 2(4)]$.

$= \frac{1}{3}[16 + 4(10) + 8]$.

$= \frac{1}{3}[16 + 40 + 8]$.

$= 64/3$

$= 21.333$

iii) $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $h = 6$.

$\cancel{h} = \frac{\frac{\pi}{3} - 0}{6} = \frac{\pi}{18}$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$6\pi/18$
y	0	1.4167	0.4585	0.7071	0.8017	0.752	0.9306
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$\int_0^{\pi/3} \sqrt{\sin x} = h/3[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(x_2 + y_4)]$

$= \frac{\pi/18}{3}[(0.4167 + 0.9306) + 4(0.4167 + 0.7071 + 0.9752) +$
$2(0.5848 + 0.8017)$

2

$= \frac{\pi}{54}[1.3473 + 4(1.999) + 2(1.3865)]$

$= \frac{\pi}{54}[1.3473 + 7.996 + 2.773]$

$= \frac{\pi}{54} \times 12.1163$

$= 0.7049$

Q.1.5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$, $y = [1,2]$.

$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2} = \frac{y^4-1}{2y^2}$

$= \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\, dy$

$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$

$= \frac{1}{2}\int y^2 dy + \frac{1}{2}\int y^{-2} dy$

$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-1}}{1}\right]_1^2$

$= \frac{1}{2}\left[\cancel{\frac{8}{3} - \frac{1}{2}} - \frac{1}{3} + 1\right]$

$= 2\frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right] = \frac{17}{12}$

PRACTICAL · 07

TOPIC : Differential equation

Q.1. solve the following differential equation

1) $x\frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + P(x)y = Q(x)$

Integration formula.

$(I.F) = e^{\int P dx}$

$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$P(x) = \frac{1}{x}$ $Q(x) = \frac{e^x}{x}$

$IF = e^{\int P(x) dx}$

$= e^{\int \frac{1}{x} dx}$

$= e^{\log x}$

$= x$

$y(I.F) = \int Q(x)(I.F) dx + c$

$= \int \frac{e^x}{x} \cdot x \cdot dx + c$

$= \int e^x dx + c$

$xy = e^x + c$

2) $e^x \frac{dy}{dx} + \cancel{2e^x y = 1}$

$\frac{dy}{dx} + \frac{2e^x}{e^x}y = \frac{1}{e^x}$ ($\div$ by e^x)

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$ $\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2$ $Q(x) = e^{-x}$

$\int P(x) dx$

$I.F. = e^{\int 2 dx}$

$= e^{2x}$

$$Y(I \cdot F) \int Q(x)(I \cdot F)\,dx + C$$
$$Y \cdot e^{2x} = \int e^{-x} + 2x\,dx + C$$
$$= \int e^{x}\,dx + C$$
$$Y \cdot e^{2x} = e^{x} + C.$$

3 $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y.$

$$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y.$$
$$\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$$
$$P(x) = 2(x) \quad Q(x) = \frac{\cos x}{x^2}$$
$$I.F = e\int P(x)\,dx$$
$$= e\int 2/x\,dx$$
$$= e^{2/x}\,dx$$
$$= \ln x^2$$
$$Y(I \cdot F) = \int Q(x)(I \cdot F)\,dx + C$$
$$= \int \frac{\cos x}{x^2} - x^2\,dx + C$$
$$= \int \cos x + C$$
$$x^2 y = \sin x + C$$

4) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ ($\div$ by x on both side)

$$P(x) = 3/x \quad Q(x) = \sin x / x^3$$
$$= e\int P(x)\,dx$$
$$= e\int 3/x\,dx$$
$$= e^{3/x}x$$
$$= e^{\ln x^3}$$
$$= x^3$$
$$y(I \cdot F) = \int Q(x)(I \cdot F)\,dx + C.$$
$$= \int \frac{\sin x}{x^3} - x^3\,dx + C$$
$$= \int \sin x\,dx + C$$
$$x^3 y = -\cos x + C$$

5. $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$$
$$P(x) = 2 \quad Q(x) = 2x/e^{2x} = 2xe^{-2x}$$
$$I.F = e\int P(x)\,dx$$
$$= e\int 2\,dx$$
$$= e^{2x}$$
$$y(I \cdot F) = \int Q(x)(I \cdot F)\,dx + C$$
$$= \int 2xe^{-2x} \cdot e^{2x}\,dx + C$$
$$= \int 2x\,dx + C$$
$$ye^{2x} = x^2 + C.$$

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6. $\sec^2 x \tan y dx + \sec^2 y \tan x \cdot dy = 0$

$\sec^2 x \tan y \, dx = -\sec^2 y . \tan x dy.$

$\frac{\sec^2 x dx}{\tan x} = -\frac{\sec^2 y dy}{\tan y}$

$\int \frac{\sec^2 x dx}{\tan x} = -\int \frac{\sec^2 y \, dy}{\tan y}$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\log|\tan x - \tan y| = c$

$\tan x . \tan y = c.$

7. $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$

Differentiating on both sides

$x - y + 1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$\frac{1-dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v.$

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v.$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v \, dv = \int dx$

$\tan v = x + c$

$\tan(x+1-1) = x$

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viii) $\frac{du}{dx} = \frac{2x+3y-1}{6x+9y+6}$

put $2x+3y = v$

$2 + 3\frac{du}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+1}\right)$

$\frac{dv}{dx} = \frac{v-1}{v+1} + 2.$

$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$

$\frac{dv}{dx} = \frac{3v+3}{v+2} = \frac{3(v+1)}{(v+2)}$

$= \int \frac{(v+2)}{(v+1)} dv = 3dx$

$\therefore \int \frac{v+1}{v} dx + \int \frac{1}{v+1} dv = 3x$

$v + \log|x| = 3x + c$

$2x + 3y + \log|2x+3y+1| = 3x + c$

$3y = x - \log|2x+3y+1| + c$

10/01/2020

PRACTICAL - 8

TOPIC :- Euler's Method.

1. $\frac{dy}{dx} = y + e^x - 2$, $y(0) = 2$, $h = 0.5$ Find $(y(2))$
2. $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$, $h = 0.2$ find $y(1)$
3. $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$, $h = 0.2$ find $y(1)$
4. $\frac{dy}{dx} = 3x^2 + 1$, $y(1) = 2$, Find $y(2)$ for $h = 0.5$, $h = 0.25$
5. $\frac{dy}{dx} = \sqrt{xy} + 2$. $y(1) = 1$ Find $y(1.2)$ with $h = 0.2$

Answers :-

$\frac{dy}{dx} = y + e^x - 2$

$F(x,y) = y + e^x - 2$, $y_0 = 2$, $x_0 = 0$, $h = 0.5$

n	xn	yn	F(xn, yn)	yn+1
0	0	2	1	2.5
1	0.5	2.5	2.487	3.57435
2	1	3.5743	4.2925	5.3615

$$\boxed{y_{n+1} = y_n + h.f(x_n, y_n)}$$

n	xn	yn	F(xn, yn)	yn+1
3	1.5	5.3615	7.8431	9.28305
4	2	9.2831		

∴ By Euler's formula,

$y(2) = \underline{9.2831}$

$\frac{dy}{dx} = 1 + y^2$

$f(x,y) = 1 + y^2$ $y_0 = 0$, $x_0 = 0$ $h = 0.2$.

using Eulers iteration formula,

$y_{n+1} = y_n + hf(x_n, y_n)$

a

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1605	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8530	1.2942
5	1	1.2942		

3. $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0)=1$ $x_0=0$ $h=0.2$

using Euler's iteration formula,

$y_{n+1} = y_n + hf(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0895
2	0.4	1.0895	0.6059	1.1211
3	0.6	1.1211	0.7316	1.1463
4	0.8	1.1463	0.8354	1.16708
5	1	1.16708		

∴ By Euler's formula
$y(1) = 1.16708$

4. $\frac{dy}{dx} = 3x^2+1$ $y_0=2$, $x_0=1$, $h=0$
For $h=0.5$
using Euler's iteration formula,
$y_{n+1} = y_n + hf(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	4.9	28.5
2	2	28.6		

By Euler's formula,
$y(2) = 28.5$

For $h=0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.3594
3	1.75	6.3594	10.1815	8.9048
4	2	8.9048		

By Euler's formula
$y(2) = 8.9048$

5) $\frac{dy}{dx} = \sqrt{xy} + 2 \quad y_0 = 1, \; x_0 = 1, \; h = 0.2$

Using Euler's iteration formula,

$y_{n+1} = y_n + hf(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

$\therefore$ By Euler's formula,

$y(1.2) = 1.6$

PRACTICAL - 9

TOPIC: Limits & partial order derivative.

1. Evaluate the following limits.

i) $\lim\limits_{(x,y)\to(-4,-1)} \frac{x^3 - 3y + y^2 - 1}{xy + 5}$ ii) $\lim\limits_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

iii) $\lim\limits_{(x,y,z)\to(1,1,1)} \frac{x^2 - y^2 \cdot z^2}{x^3 - x^2yz}$

2) Find f_x, f_y for each of the following f

i. $f(x,y) = xye^{x^2+y^2}$ ii) $f(x,y) = e^x \cos y$

iii. $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

3) Using definition find values of f_x, f_y at $(0,0)$ for $f(x,y) = \frac{2x}{1+y^2}$

4) Find all second order partial derivatives of f. Also verify whether $f_{xy} = f_{yx}$

i) $f(x,y) = \frac{y^2 - xy}{x^2}$ ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

iii) $f(x,y) = \sin(xy) + e^{x+y}$

5. Find the linearization of $f(x,y)$ at given point

i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

ii) $f(x,y) = 1 - x + y\sin x$ at $(\frac{\pi}{2}, 0)$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$

Answers

Q1

i. $\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$

At $(-4,-1)$, Denominator $\neq 0$

By applying limit

$= \frac{(-4)^3-3(-1)+(-1)^2-1}{-4(-1)+5}$

$= \frac{-64+3+1-1}{4+5}$

$= \frac{-61}{9}$

ii. $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

At $(2,0)$ Denominator $\neq 0$

By applying limit:

$= \frac{(0+1)((2)^2+0-4(2))}{2+0}$

$= \frac{1(4+0-8)}{2}$

$= \frac{-4}{2}$

$= -2$

iii. $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

At $(1,1,1)$, Denominator $= 0$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{(x-yz)(x+yz)}{x^2(x-yz)}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$

on applying limit

$= \frac{1+1(1)}{(1)^2}$

$= 2$

$f(x,y) = xye^{x^2+y^2}$

$f(x) = \frac{d}{dx}(f(x,y))$

$= \frac{d}{dx}(xye^{x^2+y^2})$

$= ye^{x^2+y^2}(2x)$

$f(x) = 2xye^{x^2+y^2}$

$fy = \frac{d}{dy}(f(x,y))$

$= \frac{d}{dy}(xye^{x^2+y^2})$

$= xe^{x^2+y^2}(2y)$

$fy = 2yxe^{x^2+y^2}$

(ii) $f(x,y) = e^{x}\cos y$

$f(x) = \frac{d}{dx}(f(x,y))$

$= \frac{d}{dx}(e^{x}\cos y)$

$fx = e^{x}\cos y$

$fy = \frac{d}{dy}(f(x,y))$

$= \frac{d}{dy}(e^{x}\cos y)$

$fy = -e^{x}\sin y$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f(x) = \frac{d}{dx}(f(x,y))$

$= \frac{d}{dx}(x^3y^2 - 3x^2y + y^3 + 1)$

$f(x) = 3x^2y^2 - 6xy$

$fy = \frac{d}{dy}(f(x,y))$

$= \frac{d}{dy}(x^3y^2 - 3x^2y + y^3 + 1)$

$fy = 2x^3y - 3x^2 + 3y^2$

) $f(x,y) = \frac{2x}{1+y^2}$

$fx = \frac{d}{dx}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{d}{dx}(2x) - 2x\frac{d}{dx}(1+y^2)}{(1+y^2)^2}$

$= \frac{2+2y^2-0}{(1+y^2)^2}$

$= \frac{2(1+y^2)}{(1+y^2)(1+y)^2}$

$= \frac{2}{1+y^2}$

at $(0,0)$

$= \frac{2}{1+0}$

$= 2$

$fy = \frac{d}{dy}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{d}{dx}(2x) - 2x\frac{d}{dx}(1+y^2)}{(1+y^2)^2}$

$= \frac{1+y^2(0) - 2x(2y)}{(1+y)^2} = \frac{-4xy}{(1+y^2)^2}$

At (0,0)

$$= \frac{-4(0)(0)}{(1+0)^2}$$

$$= 0$$

4. (i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$$f_x = \frac{x^2 \frac{d}{dx}(y^2 - xy) - (y^2 - xy)\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2 y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{d}{dx}\left(\frac{-x^2 y - 2x(y^2 - xy)}{x^4}\right)$$

$$= \frac{x^4\left(\frac{d}{dx}(-x^2 y - 2xy^2 + 2x^2 y)\right) - (-x^2 y - 2xy + 2x^2 y)\frac{d}{dx}(x^4)}{(x^4)^2}$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2 y - 2xy + 2x^2 y)}{x^6} \quad — (1)$$

$$f_{yy} = \frac{d}{dy}\left(\frac{2y - x}{x^2}\right) \quad — (2)$$

$$= \frac{2 - 0}{x^2} = \frac{2}{x^2}$$

$$f_{xy} = \frac{d}{dy}\left(\frac{-x^2 y - 2xy^2 + 2x^2 y}{x^4}\right) —$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4} \quad — (3)$$

$$f_{yx} = \frac{d}{dx}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{x^2 \frac{d}{dx}(2y - x) - (2y - x)\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4} \quad — (4)$$

From 3 & 4

$f_{xy} = f_{yx}$

(ii) $f(x,y) = x^3 + 3x^2 y^2 - \log(x^2 + 1)$

$$f_x = \frac{d}{dx}(x^3 + 3x^2 y^2 - \log(x^2 + 1))$$

$$= 3x^2 + 6xy^2 - \frac{2x}{x^2 + 1}$$

$$f_y = \frac{d}{dy}(x^3 + 3x^2 y^2 - \log(x^2 + 1))$$

$$= 0 + 6x^2 y - 0$$

$$= 6x^2 y$$

$$f_{xx} = 6x + 6y^2 - \left(\frac{x^2 + 1\frac{d(2x)}{dx} - 2x\frac{d(x^2+1)}{dx}}{(x^2+1)^2}\right)$$

$$= 6x + 6y^2 - \left(\frac{2(x^2 + 1) - 4x^2}{(x^2 + 1)^2}\right) \quad — (1)$$

$$f_{yy} = \frac{d}{dy}(6x^2 y) = 6x^2 \quad — (2)$$

$$f_{xy} = \frac{d}{dy}\left(3x^2 + 6xy^2 - \frac{2x}{x^2 + 1}\right)$$

$$= 0 + 12xy - 0$$

$$= 12xy \quad — (3)$$

$$f_{yx} = \frac{d}{dx}(6x^2 y)$$

$$= 12xy \quad — (4)$$

From (3) & 4

$\therefore f_{xy} = f_{yx}$

iii) $f(x,y) = \sin(xy) + e^{x+y}$

$f_x = y\cos(xy) + e^{x+y}(1)$
$= y\cos(xy) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}(1)$
$= x\cos(xy) + e^{x+y}$

$f_{xx} = \frac{d}{dx}\left(y\cos(xy) + e^{x+y}\right)$
$= -y\sin(xy)\cdot(y) + e^{x+y}(1)$
$= y^2\sin(xy) + e^{x+y}$ — (1)

$f_{yy} = \frac{d}{dy}\left(x\cos(xy) + e^{x+y}\right)$
$= -x\sin(xy)(x) + e^{x+y}(1)$
$= -x^2\sin(xy) + e^{x+y}$ — (2)

$f_{xy} = \frac{d}{dy}\left(y\cos(xy) + e^{x+y}\right)$
$= -y^2\sin(xy) + \cos(xy) + e^{x+y}$ — (3)

$f_{yx} = \frac{d}{dx}\left(x\cos(xy) + e^{x+y}\right)$
$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$ — (4)

from (3) & (4)
$f_{xy} \neq f_{yx}$

Q.5
(i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$
$= \frac{x}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$

$f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$
$= \frac{y}{\sqrt{x^2+y^2}}$

f_y at $(1,1) = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$
$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$
$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$
$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$
$= \cancel{\sqrt{2}} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \cancel{\frac{2}{\sqrt{2}}}$
$= \frac{x+y}{\sqrt{2}}$

ii) $f(x,y) = 1 - x + y\sin x$ at $\left(\frac{\pi}{2}, 0\right)$

$f\left(\frac{\pi}{2}, 0\right) = 1 - \frac{\pi}{2} + 0 = 1 - \frac{\pi}{2}$

$f_x = 0 - 1 + y\cos x$
f_x at $\left(\frac{\pi}{2}, 0\right) = -1 + 0$
$= -1$

$f_y = 0 - 0 + \sin x$
f_y at $\left(\frac{\pi}{2}, 0\right) = \sin\frac{\pi}{2} = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$
$= 1 - \frac{\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1(y-0)$
$= 1 - \cancel{\frac{\pi}{2}} - x + \cancel{\frac{\pi}{2}} + y$
$= 1 - x + y.$

III $f(x,y) = \log x + \log y$ at $(1,1)$
$f(1,1) = \log(1) + \log(1) = 0$
$f(x) = \frac{1}{x} + 0$ $\qquad$ $fy = 0 + \frac{1}{y}$

fx at $(1,1) = 1$ $\qquad$ fy at $(1,1) = 1$

$\therefore L(x,y) = f(a,b) + fx(a,b)(x-a) + fy(a,b)(y-b)$
$= 0 + 1(x-1) + 1(y-1)$
$= x - 1 + y - 1$
$= x + y - 2$

PRACTICAL - 10

TOPIC : Directional derivative, gradien vector & maxima minima Tangetit & normal vectors $\qquad$ 68

Q1 Find the directional derivative of the following function at given points & in the direction of given vector.

1) $f(x,y) = x + 2y - 3$, $a = (1,-1)$, $u = 3i - j$
2) $f(x,y) = y^2 - 4x + 1$, $a = (3,4)$ $u = i + 5j$
3) $f(x,y) = 2x + 3y$, $a = (1,2)$, $u = 3i + 4j$

Q2 Find gradient vector for the following function at given point

1) $f(x,y) = x^y + y^x$, $a(1,1)$
2) $f(x,y) = (\tan^{-1})\cdot y^2$ $a(1,-1)$
3) $f(x,y) = xyz - e^{x+y+z}$ $a(1,-1,0)$

Q3 Find the equation of tangent & normal to each of the following curves at given points

1. $x^2\cos y + e^{xy} = 2$ at $(1,0)$
2. $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

Q4 Find the equation of tangent & normal to each of the following curves at given points line to each of the following surface

1. $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$
2. $3xyz - x - y + z = -4$ at $(1,-1,2)$

Q5 Find the local maxima & minima for the following function

1. $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$
2. $f(x,y) = 2x^4 + 3x^2y - y^2$
3. $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

11. $f(x,y) = x + 2y - 3$ $\quad a(1,-1)$, $u = 3i - j$

Here,

$u = 3i - j$ is not a unit vector.

$\bar{u} = 3\bar{i} - \bar{j}$

$|\bar{u}| = \sqrt{10}$

$\therefore$ unit vector along u is $\frac{\bar{u}}{|\bar{u}|} = \frac{1}{\sqrt{10}}(3\hat{i} - \hat{j})$

$= \frac{1}{\sqrt{10}}(3, -1)$

$= \left(\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}}\right)$

Now,

$f(a + hu) = f\left((1,-1) + h\left(\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}}\right)\right)$

$= f\left(1 + \frac{3h}{\sqrt{10}}, -1 - \frac{h}{\sqrt{10}}\right)$

$= \cancel{-4 + \frac{3h}{\sqrt{10}}} \; -4 \;\; 1 + \frac{3h}{\sqrt{10}} + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 - 2 - 3 + \frac{3h}{\sqrt{10}} - \frac{2h}{\sqrt{10}}$

$= -4 + \frac{h}{\sqrt{10}}$

$\therefore D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \frac{-4 + \frac{h}{\sqrt{10}} - (-4)}{h}$

$= \lim_{h \to 0} \frac{h}{h\sqrt{10}}$

$= \frac{1}{\sqrt{10}}$

ii) $f(x,y) = y^2 - 4x + 1$, $a(3,4)$ $u = i + 5j$

Here,

$u = i + 5j$ is not a unit vector

$\bar{u} = i + 5j$

$|\bar{u}| = \sqrt{26}$

$\therefore$ unit vector along u is $\frac{\bar{u}}{|\bar{u}|} = \frac{1}{\sqrt{26}}(i + 5j)$

$= \frac{1}{\sqrt{26}}(1, 5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

Now,

$f(a+hu) = f\left((3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)\right)$

$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$

$= \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$

$= \lim_{h \to 0} \frac{25h^2}{26} + \frac{36}{\sqrt{26}}$

$= \lim_{h \to 0} h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)$

$= \frac{25(0)}{26} + \frac{36}{\sqrt{26}}$

$= \frac{36}{\sqrt{26}}$

iii) $f(x,y) = 2x + 3y$. $a = (1,2)$, $u = 3i + 4j$

Here,

$u = 3i + 4j$ is not a unit vector

$\bar{u} = 3\bar{i} + 4\bar{j}$

$|\bar{u}| = \sqrt{25} = 5$

unit vector along $u = \frac{\bar{u}}{|\bar{u}|} = \frac{1}{5}(3\bar{i} + 4\bar{j})$

$= \frac{1}{5}(3, 4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

Now

$f(a+hu) = f((1,2) + h(\frac{3}{5}, \frac{4}{5}))$

$= f(1+\frac{3h}{5}, \frac{2+4h}{5})$

$= 2(1+\frac{3h}{5}) + 3(2+\frac{4h}{5})$

$= 2+\frac{6h}{5} + 6 + \frac{12h}{5}$

$= 8 + \frac{18h}{5}$

$\therefore D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h\to 0} \frac{8 + \frac{18h}{5} - 8}{h}$

$= \lim_{h\to 0} \frac{18h}{5h}$

$= \frac{18}{5}$

Q2.

i) $f(x,y) = x^y + y^x$ $\quad a(1,1)$

$f_x = y(x^{y-1}) + y^x \log y$

$f_y = x(y^{x-1}) + x^y \log x$

$\nabla f(x,y) = (F_x, F_y)$

$= (yx^{y-1} + y^x \log y, xy^{x-1} + x^y \log x)$

$\nabla f(x,y)$ at $(1,1)$

$= (1(1)^0 + 1^1 \log(1), 1(1)^{(1-1)} + 1 \log 1)$

$= (1,1)$

$f(x,y) = (\tan^{-1}x) \cdot y^2$, $a(1,-1)$

$f_x = y^2(\frac{1}{1+x^2}) = \frac{y^2}{1+x^2}$

$f_y = 2y \tan^{-1}x$

$\nabla f(x,y) = (f_x, f_y)$

$= (\frac{y^2}{1+x^2}, 2y \tan^{-1}x)$

$\nabla f(x,y)$ at $(1,-1)$

$= (\frac{(-1)^2}{1+(1)^2}, 2(-1) \tan^{-1}(1))$

$= (\frac{1}{2}, \frac{-2\pi}{4})$

$= (\frac{1}{2}, \frac{-\pi}{2})$

iii) $f(x,y,z) = xyz - e^{x+y+z}$. at $(1,-1,0)$

$fx = yz - e^{x+y+z}$
$fy = xz - e^{x+y+z}$
$fz = xy - e^{x+y+z}$

$\nabla f(x,y,z) = (fx, fy, fz)$
$= (yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z})$

$\nabla f(x,y,z)$ at $(1,-1,0)$

$= (-1(0) - e^{1-1+0}, 1(0) - e^{1-1+0}, 1(-1) - e^{1-1+0})$
$= (0-1, 0-1, -1, -1)$
$= (-1, -1, -2)$

Q3
(i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f(xy) = x^2 \cos y + e^{xy} - 2$
$fx = 2x \cos y + y e^{xy}$
$fy = -x^2 \sin y + x e^{xy}$

$(x_0, y_0) = (1,0)$
fx at $(1,0) = 2(1) \cos 0 + 0$
$= 2$
fy at $(1,0) = -(1)^2 \sin(0) + 1(e)^{1(0)}$
$= 1$

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$fx(x - x_0) + fy(y - y_0) = 0$
$2(x-1) + 1(y-0) = 0$
$2x - 2 + y = 0$
$2x + y - 2 = 0$ ⟶ Equation of Tangent

Now,
For Equation of Normal;
$bx + ay + d = 0$
$x + 2y + d = 0$
$(1) + 2(0) + d = 0$ At $(1,0)$
$1 + d = 0$
$d = -1$
$x + 2y - 1 = 0$ ⟶ Equation of Normal

(ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$
$f(x,y) = x^2 + y^2 - 2x + 3y + 2$
$fx = 2x + 0 - 2 + 0 + 0$ fx at $(2,-2) = 2(2) - 2$
$= 2$
$fy = 0 + 2y + 0 + 3 + 0$ fy at $(2,-2) = 2(-2) + 3$
$= 2y + 3$ $= -1$

Equation of Tangent
$fx(x - x_0) + fy(y - y_0) = 0$
$2(x-2) + (-1)(y+2) = 0$
$2x - 4 + -y - 2 = 0$
$2x - y - 6 = 0$ ⟶ Equation of Tangent

For equation of Normal,

$\therefore$ $bx + ay + d = 0$

$-x + 2y + d = 0$

$-2 + 2(-2) + d = 0$ at $(2, -2)$

$-2 - 4 + d = 0$

$d = +6$

$\therefore -x + 2y + 6 = 0$ ⟶ Equation of Normal

Q4.

i) $x^2 - 2yz + 3y + xz = 7$ at $(2, 1, 0)$

$f(x, y, z) = x^2 - 2yz + 3y + xz - 7$

$f_x = 2x - 0 + 0 + z - 0$
$= 2x + z$

$f_y = -2z + 3 + 0 - 0$

$f_z = 0 - 2y + 0 + x - 0$
$= -2y + x$

$\therefore f_x$ at $(2, 1, 0) = 2(2) + 0 = 4$

f_y at $(2, 1, 0) = -2(0) + 3 = 3$

f_z at $(2, 1, 0) = -2(1) + 2 = 0$

Equation of tangent,

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$4(x - 2) + 3(y - 1) + 0(z - 0) = 0$

$4x - 8 + 3y - 3 = 0$

$\therefore 4x + 3y - 11 = 0$ — Equation of tangent

Equation of Normal,

$\dfrac{x - x_0}{f_x} = \dfrac{y - y_0}{f_y} = \dfrac{z - z_0}{f_z}$ — Equals

$\dfrac{x - 2}{4} = \dfrac{y - 1}{3} = \dfrac{z - 0}{0}$ — Equation of Normal

(ii) $3xyz - x - y + z = -4$ at $(1, -1, 2)$

$f(x, y, z) = 3xyz - x - y + z + 4$

$f_x = 3yz - 1 - 0 + 0 + 0$
$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$
$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 - 0$
$= 3xy + 1$

f_x at $(1, -1, 2) = 3(-1)(2) - 1 = -7$

f_y at $(1, -1, 2) = 3(1)(2) - 1 = 5$

f_z at $(1, -1, 2) = 3(1)(-1) + 1 = -2$

$\therefore$ Equation of Tangent

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$-7(x - 1) + 5(y + 1) + (-2)(z - 2) = 0$

$-7x + 7 + 5y + 5 + -2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$ — Equation of Tangent

Equation of Normal

$\dfrac{x - x_0}{f_x} = \dfrac{y - y_0}{f_y} = \dfrac{z - z_0}{f_z}$

$\dfrac{x - 1}{-7} = \dfrac{y + 1}{5} = \dfrac{z - 2}{-2}$ — Equation of normal

Q5 (i) $f(x,y) = 3x^2+y^2-3xy+6x-4y$.

$\therefore f_x = 6x+0-3y+6-0$

$= 6x-3y+6$ —— 1

$f_y = 2y-3x+0-4$

$= 2y-3x-4$ —— 2

$f_x = 0$

$6x-3y+6=0$

$3(2x-y+2) =$

$2x-y+2=0$

$2x-y=-2$ —— 3

$f_y = 0$

$2y-3x-4=0$

$2y-3x=4$ —— 4

Multiplying 3 by 2 and subtracting 4 from 3

$4x - 2y = -4$

$-2y - 3x = 4$

$7x = 0$

$x = 0$

Substituting value of x in 3

$2(0) - y = -2$

$0 - y = 2$

$y = 2$

Critical points are $(0,2)$

Now,

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

$rt - s^2 = 12-9$

$= 3 > 0$

Here, $r > 0$ and $rt - s^2 > 0$.

$\therefore f$ has minimum at $(0,2)$

$\therefore f(0,2) = 3(0)^2+(2)^2-3(0)(2)+6(0)-4(2)$

$= 0+4-0-10+-8$

$= -4$.

$f(x,y) = 2x^4+3x^2-y^2$

$f_x = 8x^3+6xy-0$

$= 8x^3+6xy$.

$f_y = 0+3x^2-2y$

$= 3x^2-2y$

Now,

$f_x = 0$

$8x^3+6xy=0$

$2x(4x^2+6xy)=0$

$4x^2+6y=0$ —— 1

$f_y = 0$

$3x^2-2y=0$

$3x^2-2y=0$

$3x^2-2y=0$ —— 2

Multiply in 1 by 3 and 2 by 4 and subtract 2 from 1

$12x^2+18xy=0$

$-12x^2-8y=0$

$24y=0$

$y=0$ —— 3

Substituting (3) in (4)

$3x^2 - 2(0) = 0$

$3x^2 = 0$

$x^2 = 0$

$x = 0$ —— (4)

Critical points are (0,0)

Now,

$r = fxx = 24x^2 + 6y$

$t = fyy = -2$

$s = fxy = 6x$

$rt - s^2 = (24x^2 + 6y)(-2) - (6x)^2$

$= -48x^2 - 12y - 36x^2$

$= -84x^2 - 12y$

At (0,0)

$r = 24(0)^2 + 6(0)$

$= 0$

$s = 6(0) = 0$

$rt - s^2 = -84(0)^2 - 12(0) - 0$

$r = 0$ and $rt - s^2 = 0$

Nothing can be said

(iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$fx = 2x - 0 + 2 + 0 - 0$

$= 2x + 2$

$fy = -2y + 0 + 8 - 0$

$= -2y + 8$

$fx = 0$

$2x + 2 = 0$

$2(x+1) = 0$

$x + 1 = 0$

$x = -1$

$fy = 0$

$-2y + 8 = 0$

$-2(y-4) = 0$

$y - 4 = 0$

$y = 4$

Critical points are (-1,4)

$\therefore r = fxx = 2$

$t = fyy = -2$

$s = fxy = 0$

$rt - s^2 = 2(-2) - 0^2$

$= -4 < 0$

Here $r > 0$ and $rt - s^2 < 0$

$\therefore$ Nothing can be said.

A.R. 07/01/2020