Game Theory 2

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Assignment 1

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1.1 a)

A has 1 type. Types of T are different firm values (x) distributed over [0.1].
A_A bids Y to take over T, Y distributed over [0, ∞].
$A_T =$ reject or accept Y
T will only accept Y if

$$Y \geq X$$

A beliefs X is drawn from CDF $F(X) = X(2 - X)$, T knows own X
Strategy A: Bid

$$\Rightarrow Y[0, \infty]$$

Strategy T: to accept or reject Y $\Rightarrow$ T only accepts if:

$$Y \geq X$$

Pay offs:

$$U_A(Y, Accept) = 2X - Y$$

$$U_A(Y, Reject) = 0$$

$$U_T(Y, Accept) = Y$$

$$U_T(Y, Reject) = X$$

b)

$$E(X) = \int_{-\infty}^{\infty} x * f(x)\, dx$$

$$F(X) = X(2 - X) = X^2 + 2X$$

$$f(X) = F' = -2x + 2$$

$$E(X) = \int_{0}^{1} x(-2x + 2)dx$$

$$= \int_0^1 -2x^2 + 2dx =$$

$$= \frac{-2}{3}X^3 + X^2$$

$$= \frac{-2}{3} * 1 + 1^2 = \frac{1}{3}$$

c)

Firm value is distributed over

$$[0.1]$$

T will only accept Y if:

$$\Rightarrow Y \geq X$$

A can bid below or higher than 1, if bid is equal or higher than 1 T will always accept (max firm value=1)

$$\Rightarrow E(X|X \leq Y)$$

is the expected X of firm that accepts Y.

$$\Rightarrow E(X|X \leq Y) = \frac{\int_0^Y x * f(x)dx}{F(Y)}$$

$$f(x) = -2x + 2$$

$$= \int_0^Y x(-2x+2)dx =$$

$$= \int_0^Y (-2x^2+2)dx =$$

$$= [\frac{-2}{3}X^3 + X^2]_{X=0}^{X=Y}$$

$$= \frac{-2}{3}Y^3 + Y^2$$

$$F(Y) = Y(2-Y)$$

$$E(X|X \leq Y) = \frac{-2\frac{-2}{3}Y^3 + Y^2}{-Y^2 + 2Y}$$

The expected value of a firm if it accepts is

$$= E(X|X \leq Y) = \frac{-2\frac{-2}{3}Y^3 + Y^2}{-Y^2 + 2Y}$$

if $Y \leq 1$, if $Y \geq 1$ all types will accept, expected value of all types is

$$\Rightarrow \frac{1}{3}$$

Pay off A if $Y \leq 1$:

$$= 2x - Y = 2(\frac{-2\frac{-2}{3}Y^3 + Y^2}{-Y^2 + 2Y}) - Y$$

which is ≤ 0 on interval [0,1].
Payoff A if $Y \geq 1$

$$= 2 * \frac{1}{3} - Y \leq 0$$

Nash equilibrium is for A to bid 0.

d)

1.2 a)

$$\pi = [V((a - q_1(L) - q_2(L) - C_L) \cdot q_1(L)) + (1 - V)((a - q_1(L) - q_2(H) - C_L) \cdot q_1($$

$$\pi_1'(L) \Rightarrow q_1(L) \frac{[a - q_2(L) - c_L]V + [a - q_2(H) - c_L]1 - V}{2}$$

$$\pi_1'(H) \Rightarrow q_1(H) = \frac{[a - q_2(L) - c_H]V + [a - q_2(H) - c_H]1 - V}{2}$$

$$\pi_1'(L) \Rightarrow q_2(L) \frac{[a - q_1(L) - c_L]V + [a - q_1(H) - c_L]1 - V}{2}$$

$$\pi_2'(H) \Rightarrow q_2(H) = \frac{[a - q_1(L) - c_H]V + [a - q_1(H) - c_H]1 - V}{2}$$

b)

A strategy for i is how much to produce given that j is expected to certain Q determined wheter it has low cost or high cost.

c)

Assuming symmetry of firm 1 & 2 $q_1(L) = q_2(L)$
and
$q_1(H) = q_2(H)$

$$\Rightarrow q(L) = \frac{a - (1+V)q(H) - c_L}{2+V}$$

$$\Rightarrow q(H) = \frac{a - Vq(L) - c_H}{3-V}$$

1.3 a)

With k=0.5 the game is a prisoners dilemma with the pure strategy Nash equilibrium (Fight, Fight).
With k=2 the game is a type of coordination game which is called chicken game. This game has two pure strategy Nash equilibria which are (Fight,Accomodate) and (Accomodate,Fight).

b)

Type:

$$\theta_1 = (k = 0.5, k = 2)$$

$$\theta_2 = (k = 0.5 \times p, k = 2 \times (1 - p))$$

Action:

$$A = Accomodate, Fight$$