

AY: 2022-2023

Exam | AI-ECUE122

27/01/23 (09:00→10:30)

M1-S1: Dept. of Electrical Engineering

Teacher: A. Mhamdi

Time Limit: 1½ h

This document contains 5 pages numbered from 1/5 to 5/5. As soon as it is handed over to you, make sure that it is complete. The 4 tasks are independent and can be treated in the order that suits you.

The following rules apply:

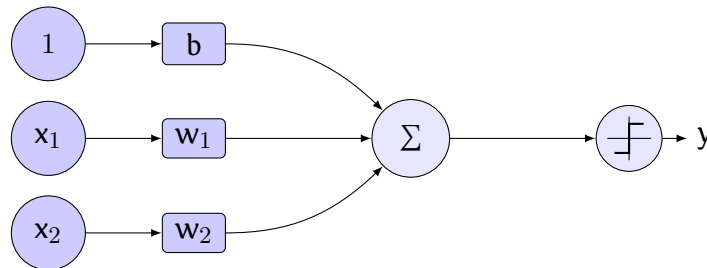
- ❶ A handwritten double-sided A4 sheet is permitted.
- ❷ Any electronic material, except basic calculator, is prohibited.
- ❸ Mysterious or unsupported answers will not receive full credit.
- ❹ Label all relevant aspects of the graph, if you are asked to draw one.
- ❺ Hand in your answer sheets at the end of the exam.



Task N°1

⌚ 20mn | (4 points)

We consider the vastly simplified model of real neuron, also known as **Threshold Logic Unit**. The processing element sums the weighted inputs $w_1x_1 + w_2x_2$, add a bias b and then applies a non linear activation function. The output transmits +1 if and only if the input is positive. Otherwise, it transmits -1.



Use bipolar data instead of binary data for the inputs x_1 and x_2 , i.e. ± 1 . Weights and bias are all set initially to zero: $w_1 = w_2 = b = 0$.

Consider the problem approximating an \vee (**OR**) gate, according to Hebbian learning rule. Reproduce and fill in the following table on your answer sheet.

| x_1 | x_2 | b | y | Δw_1 | Δw_2 | Δb | w_1 | w_2 | b |
|-------|-------|-----|-----|--------------|--------------|------------|-------|-------|-----|
| -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 |
| -1 | 1 | 1 | 1 | -1 | 1 | 1 | 0 | 2 | 0 |
| 1 | -1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |

Task N°2

⌚ 15mn | (4 points)

Perform the following arithmetic operations.

(a) (1 point) $[0, 1] + [-6, 5] = \underline{\underline{[-6, 6]}}$

(b) (1 point) $[0, 1] - [-6, 5] = \underline{\underline{[-5, 7]}}$

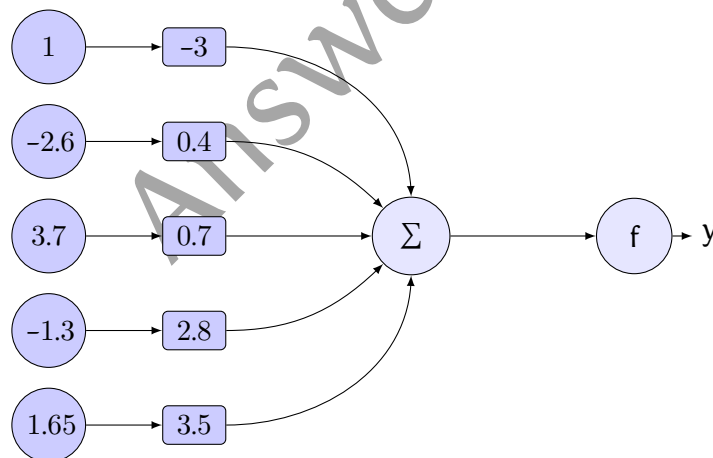
(c) (1 point) $[3, 4] \times [2, 2] = \underline{\underline{[6, 8]}}$

(d) (1 point) $[4, 10] \div [1, 2] = \underline{\underline{[2, 10]}}$

Task N°3

⌚ 25mn | (6 points)

- (a) (2 points) Compute the output being fired by the following neuron.
f designates the tanh function.



The output is given by:

$$y = \tanh(-3 \times 1 - 2.6 \times 0.4 + 3.7 \times 0.7 - 1.3 \times 2.8 + 1.65 \times 3.5) = \tanh(0.685)$$

It yields:

$$y = 0.594760307$$

- (b) (1 point) What is the machine learning library we used in Julia to train artificial neural networks.

Flux

- (c) (1 point) Given the code snippet as shown in Fig. 1, p. 4. Explain why are there 75 trainable parameters in `model`.

Total number of trainable parameters is

$$5 \times 8 + 8 + 8 \times 3 + 3 = 75$$

- (d) (1 point) What does the factor $w_{3,1}^{(1)}$ refer to, and what is its value.

$w_{3,1}^{(1)}$ designates the synaptic weight that connects the first input to the third neuron of the hidden layer. Its value is:

$$w_{3,1}^{(1)} = 0.503822$$

- (e) (1 point) What is the value of synaptic weight between neuron #7 of the hidden layer and neuron #2 of the output layer.

The synaptic weight is denoted by $w_{2,7}^{(2)}$, and is given by:

$$w_{2,7}^{(2)} = -0.568683$$

```

julia> using Flux

julia> model = Chain(
    Dense(5 => 8, relu),
    Dense(8 => 3, σ)
)

Chain(
  Dense(5 => 8, relu),          # 48 parameters
  Dense(8 => 3, σ),            # 27 parameters
)                               # Total: 4 arrays, 75 parameters, 556 bytes.

julia> model.layers[2].weight
3x8 Matrix{Float32}:
-0.313452 -0.20532  0.0164611  0.40596 ... -0.205025 -0.486443  0.6834
-0.565272 -0.422708 -0.409977  0.716418 -0.0062125 -0.568683  0.0565138
 0.201109 -0.372519  0.0448279 -0.493097 -0.611313  0.0418072 -0.476514

julia> model.layers[1].weight
8x5 Matrix{Float32}:
-0.675119  0.581365  0.510767 -0.0869667 -0.226824
-0.543659 -0.553984 -0.111358 -0.0142636 -0.117261
 0.503822  0.432929  0.535884 -0.536289  0.509142
-0.296016 -0.551115  0.562242 -0.500802 -0.640758
-0.431213 -0.358273  0.502375 -0.0316917  0.394754
 0.597894  0.651843  0.254814 -0.211738  0.211448
-0.525809 -0.504301  0.631113  0.113009 -0.510733
 0.132268  0.08442  0.0403786 -0.184404 -0.552655

julia>

```

Fig. 1. Julia REPL

Task №4

⌚ 30mn | (6 points)

Suppose we have three fuzzy predicates: \mathcal{A} , \mathcal{B} and \mathcal{C} described by these trapezoidal fuzzy sets:

$$\mathcal{A} \Pi(1, 2, 5, 7)$$

$$\mathcal{B} \Pi(4, 12, 15, 16)$$

$$\mathcal{C} \Pi(7, 8, 9, 12)$$

x and y are fuzzy variables, each one ranges between 0 and 16. Given the following three rules:

$$\mathcal{R}_1 \ (x \text{ is } \mathcal{A} \vee x \text{ is } \mathcal{B}) \wedge \neg(y \text{ is } \mathcal{C}) \rightarrow u = 9$$

$$\mathcal{R}_2 \ \neg(x \text{ is } \mathcal{B}) \wedge (y \text{ is } \mathcal{C}) \rightarrow u = 3$$

$$\mathcal{R}_3 \ (x \text{ is } \mathcal{B} \vee x \text{ is } \mathcal{C}) \wedge \neg(y \text{ is } \mathcal{A}) \rightarrow u = 16$$

Compute the degree of satisfaction for $x = 6$ & $y = 10$.

$$\mathfrak{R}_1 (\mu_{\mathcal{A}}(6) \max \mu_{\mathcal{B}}(6)) \min (1 - \mu_{\mathcal{C}}(10)) = (0.5 \max 0.12) \min (1 - 2/3) = 1/3$$

$$\mathfrak{R}_2 (1 - \mu_{\mathcal{B}}(6)) \min \mu_{\mathcal{C}}(10) = (1 - 1/4) \min 2/3 = 2/3$$

$$\mathfrak{R}_3 (\mu_{\mathcal{B}}(6) \max \mu_{\mathcal{C}}(6)) \min (1 - \mu_{\mathcal{A}}(y)) = (1/4 \max 0) \min (1 - 0) = 1/4$$

The final result is:

$$u^{\star} = \frac{1/3 \times 9 + 2/3 \times 3 + 1/4 \times 16}{1/3 + 2/3 + 1/4} = 7.2$$

Answers