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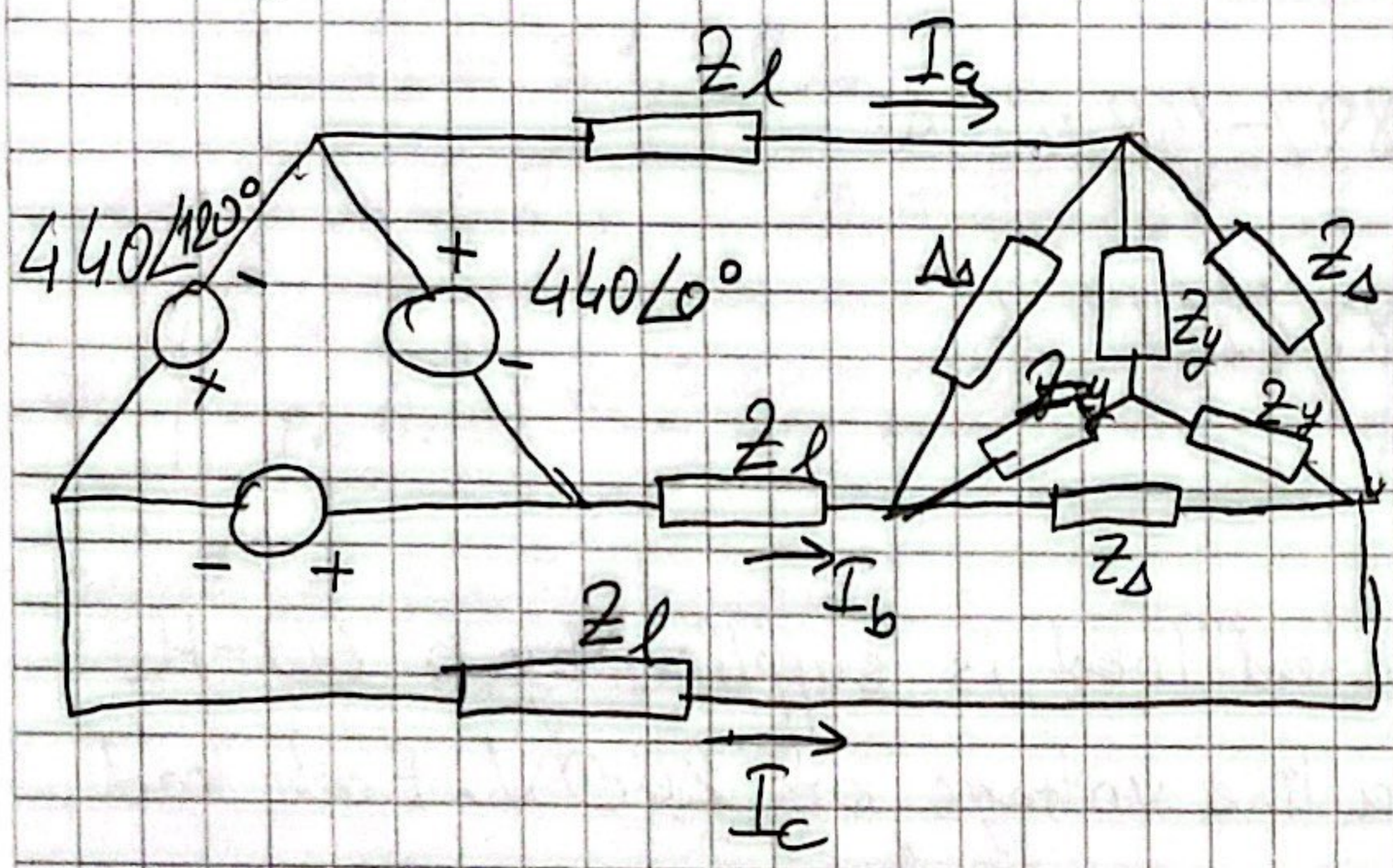
EE 214 - Circuit Theory 2 Homework - 3 Hiknet

Question 1:

Find the line currents I_a, I_b and I_c in the three-phase network of Fig. 12.53 below. Take $Z_\Delta = 12 - j15\,\Omega$, $Z_y = 4 + j6\,\Omega$, and $Z_l = 2\,\Omega$.



Solution:

Convert the Δ-connected source to a Y-connected source.

$$V_{an} = \frac{V_p}{\sqrt{3}}\angle -30^\circ = \frac{440}{\sqrt{3}}\angle -30^\circ = 254\angle -30^\circ$$

Convert the Δ-connected load to a Y-connected load.

$$Z = Z_y \parallel \frac{Z_\Delta}{3} = (4+j6) \parallel (4-j5) = \frac{(4+j6)(4-j5)}{8+j}$$

$$Z = 5{,}723 - j0{,}2153$$

UniNote

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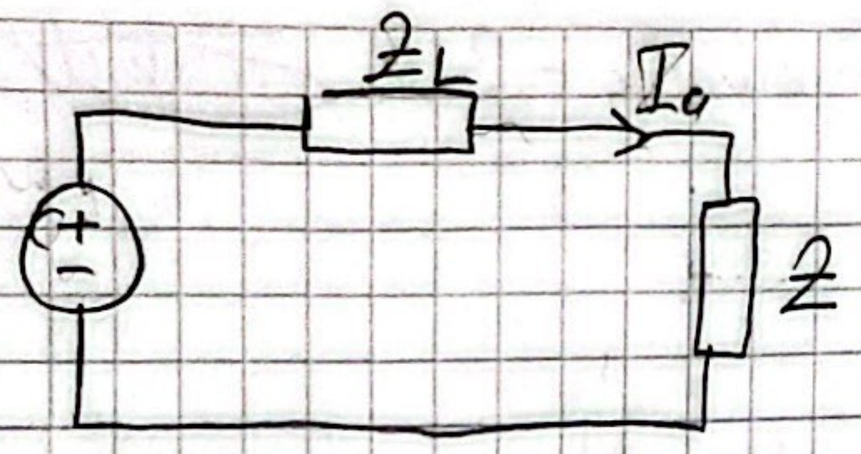

$$I_a = \frac{V_{an}}{Z_L + Z} = \frac{254\angle -30^\circ}{7{,}723 - j0{,}2153} = 32{,}88\angle -28{,}4^\circ \text{ A}$$

$$I_b = I_a \angle -120^\circ = 32{,}88\angle -148{,}4^\circ \text{ A}$$

$$I_c = I_a \angle 120^\circ = 32{,}88\angle 91{,}6^\circ \text{ A}$$

Question 2:

A balanced delta-connected load is supplied by a 60-Hz three-phase source with a line voltage of 240V. Each load phase draws 6 kW at a lagging power factor of 0,8. Find:

a) the load impedance per phase
b) the line current
c) the value of capacitance needed to be connected in parallel with each load phase to minimize the current from the source.

Solution:

Balanced delta connected load

$f = 60\,Hz$, $V_{Line} = 240\,V$

$P = 6\,kW$ at $0{,}8$ pf. lagging (each phase)

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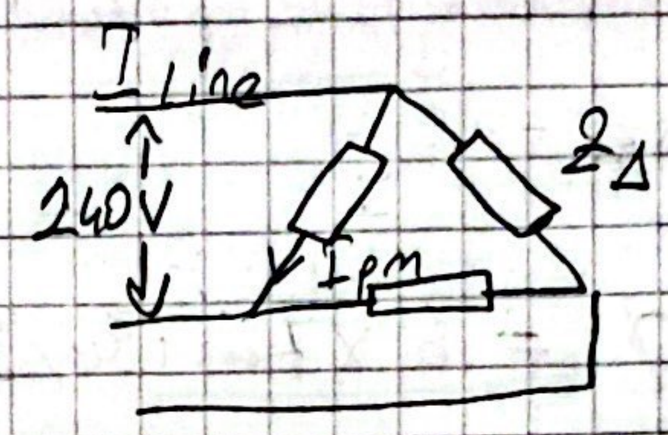
a) per phase load current

$$I_{ph(load)} = \frac{6 \times 10^3}{240 \times 0,8} \qquad I = \frac{P}{V\cos\phi}$$



$I_{ph}(load) = 31,28\,A$

$\cos\phi = 0,8 \qquad \phi = 36,869°$

So $\boxed{\vec{I}_{ph}(load) = 31,25\angle -36,869°\ A}$

negative angle because of lagging power factor

$$Z_{\substack{load\\ph}} = \frac{240}{31,25\angle -36,869} \qquad \left(Z_{ph} = \frac{V_{ph}}{I_{ph}}\right)$$

$\boxed{Z_{ph}(load) = 6,144 + j4,608\ \Omega}$

b) Line current = $\sqrt{3}$ phase current $\angle -30°$

$$I_{Line} = \sqrt{3}\,(31,25\angle -36,869) * 1\angle 30°$$

$\boxed{I_{Line} = 54,126\angle -66,869\ Amp}$

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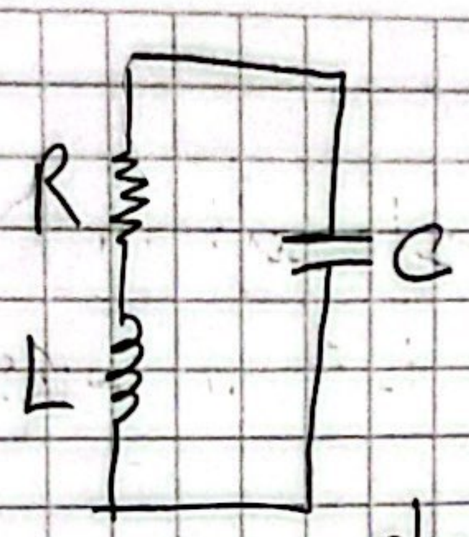
c)



improving the power factor by providing reactive power by capacitor.
reactive power consumed by inductor per phase

$$Q_2 = P \tan\theta = 6 \times \tan(36,869°) \times 10^3 = 4,5\,kVA$$

So capacitor must deliver all reactive power which is consumed by inductor per phase.

$$Q_C = 4,5\,kVA \;,\; \frac{V^2}{X_C} \;,\; X_C = \frac{240^2}{4,5 \times 10^3} = 12,8\,\Omega$$

$$\frac{1}{2\pi f C} = 12,8$$

$$C = \frac{1}{2\pi \times 60 \times 12,8} = 0,20723\,mF$$

$$\boxed{C = 207,23\,\mu F}$$

So, $C = 207,23\,\mu F$ required each phase to minimize the current.

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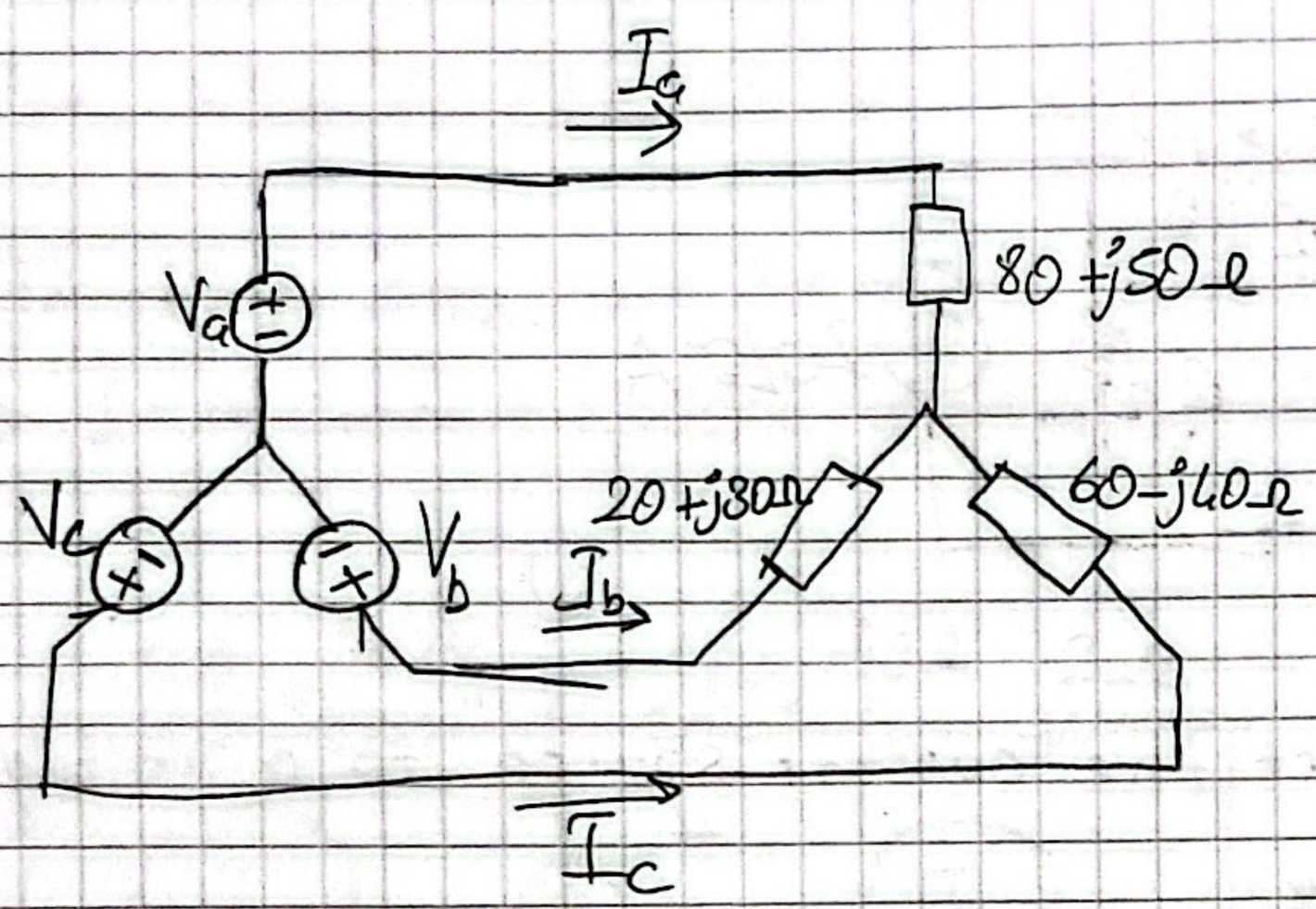
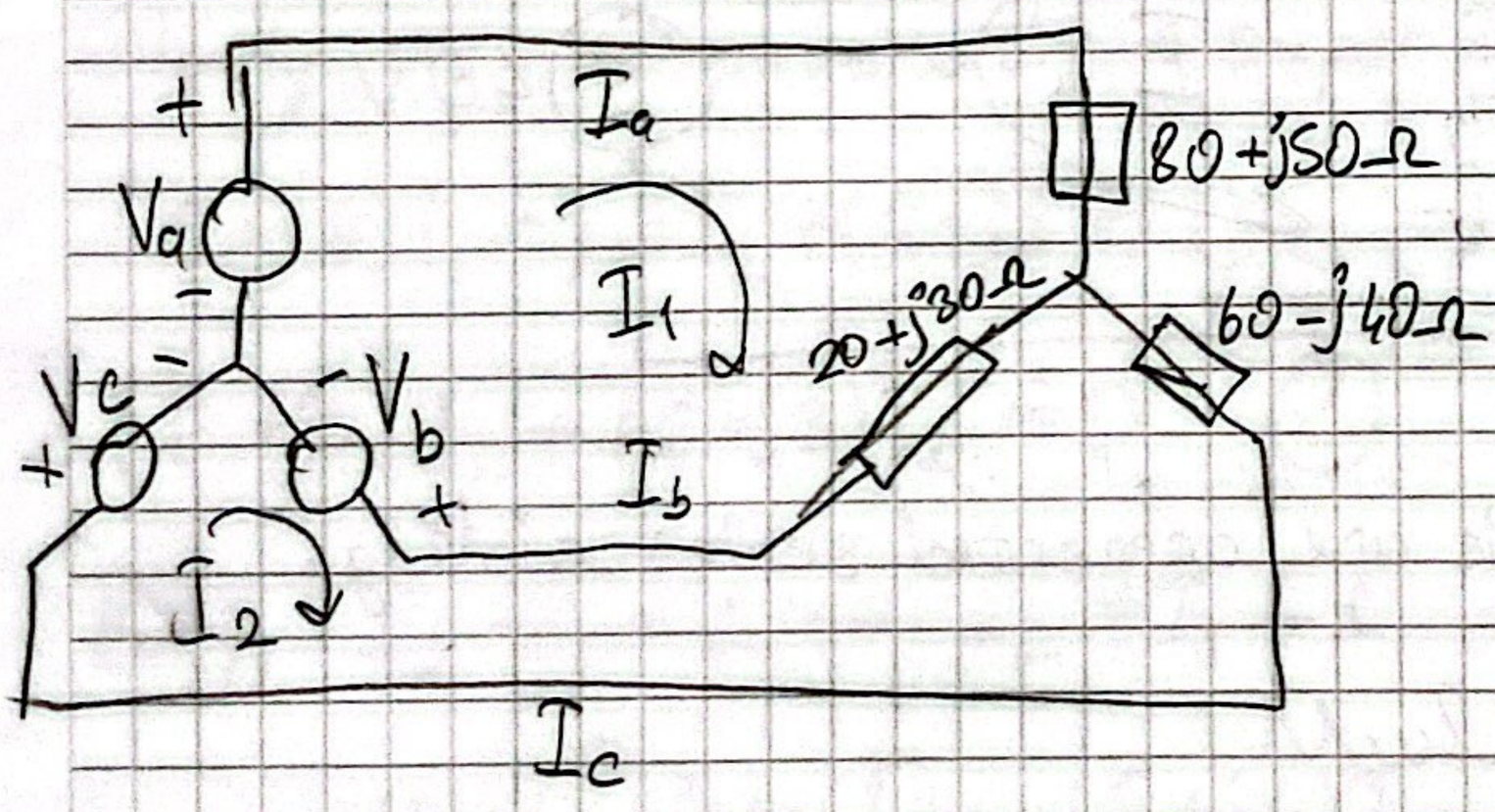
Question 3:

Determine the line currents for the three-phase circuit of Figure 12.64. Let $V_a = 110\angle 0°$, $V_b = 110\angle -120°$, $V_c = 110\angle 120°$ V



Solution:

We apply mesh analysis to the circuit shown below.



$(100+j80)I_1 - (20+j30)I_2 = V_a - V_b = 165+j95{,}263$ (1)

$-(20+j30)I_1 + (80-j10)I_2 = V_b - V_c = -j190{,}53$ (2)

Solving (1) and (2) gives $I_1 = 1{,}8616 - j0{,}6084$, $I_2 = 0{,}9088 - j1{,}72$

$I_a = I_1 = 1{,}9585\angle -18{,}1°$ A, $I_b = I_2 - I_1 = -0{,}528 - j1{,}1136 = 1{,}4656\angle -130{,}55°$ A

$(I_c = -I_2 = 1{,}947\angle 117{,}8°$ A$)$

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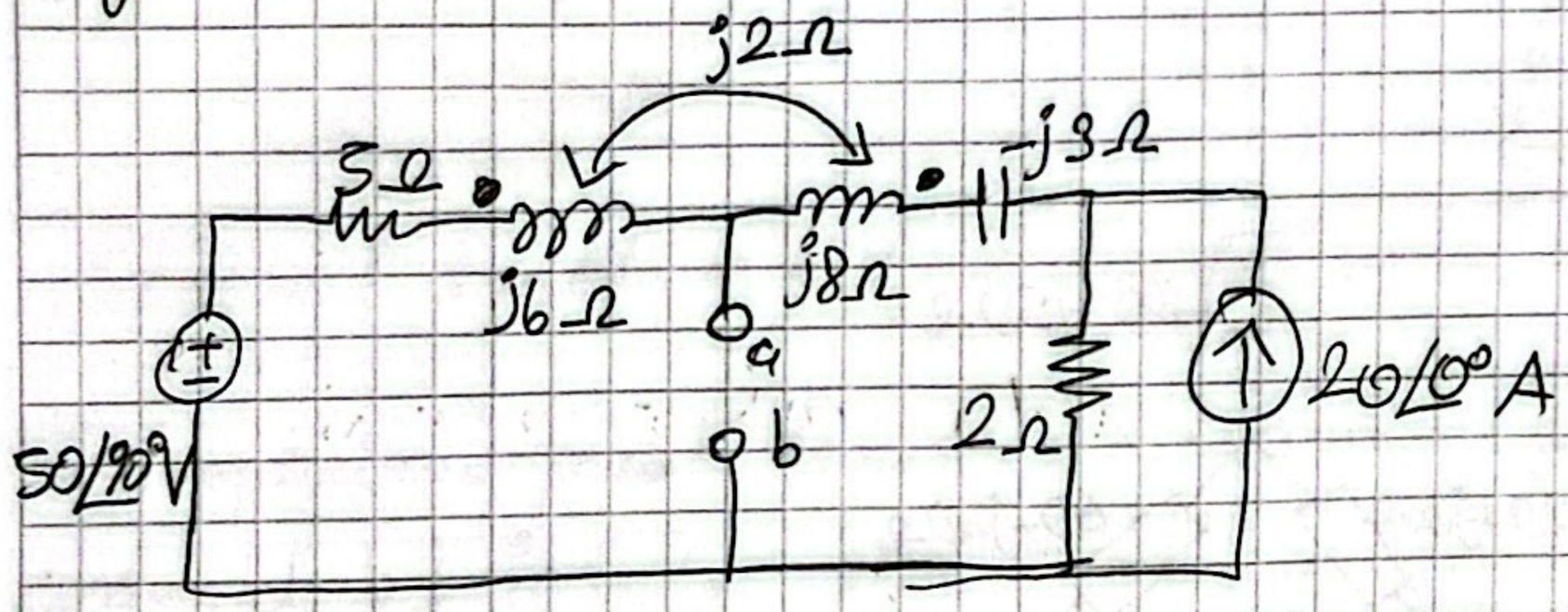
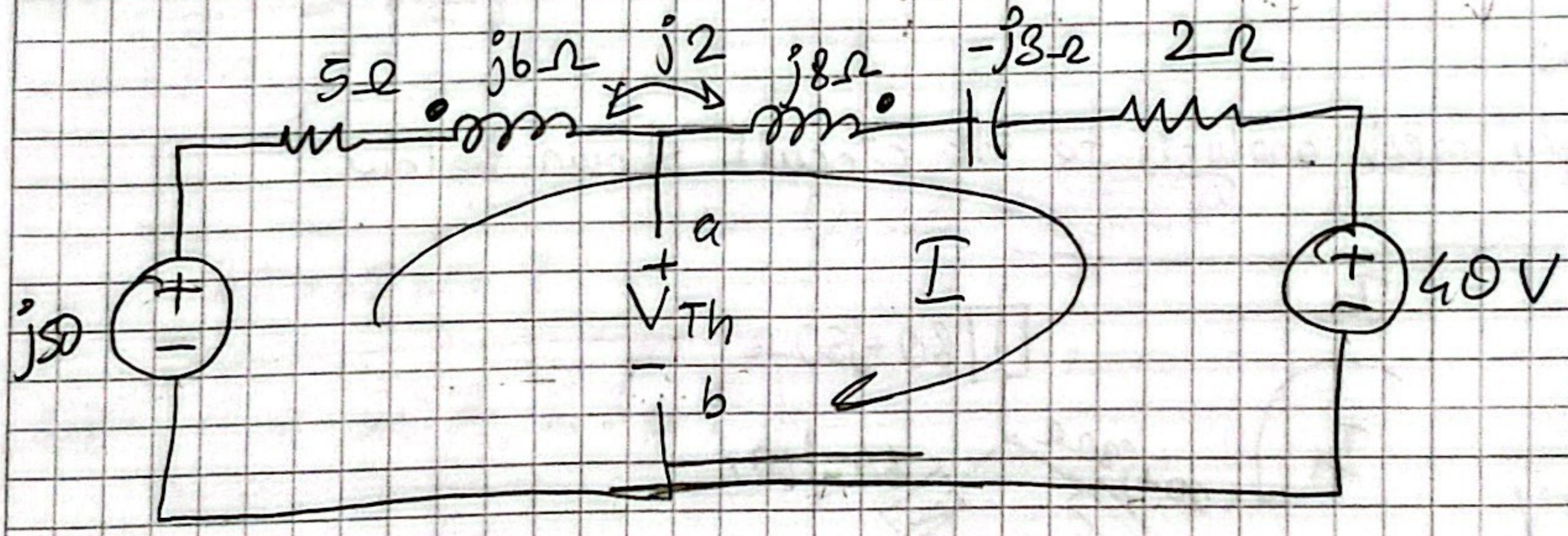
Question 4:

Obtain the Thevenin equivalent circuit for the circuit in Figure 13.83 at terminals a-b.



To obtain V_{Th}, convert the current source to a voltage source as shown below.



Note that the two coils are connected series aiding.

$$\omega L = \omega L_1 + \omega L_2 - 2\omega M$$

$$j\omega L = j6 + j8 - j4 = j10$$

Thus,

$$-j50 + (5 + j10 - j3 + 2)I + 40 = 0, \quad I = (-40 + j50)/(7 + j7)$$

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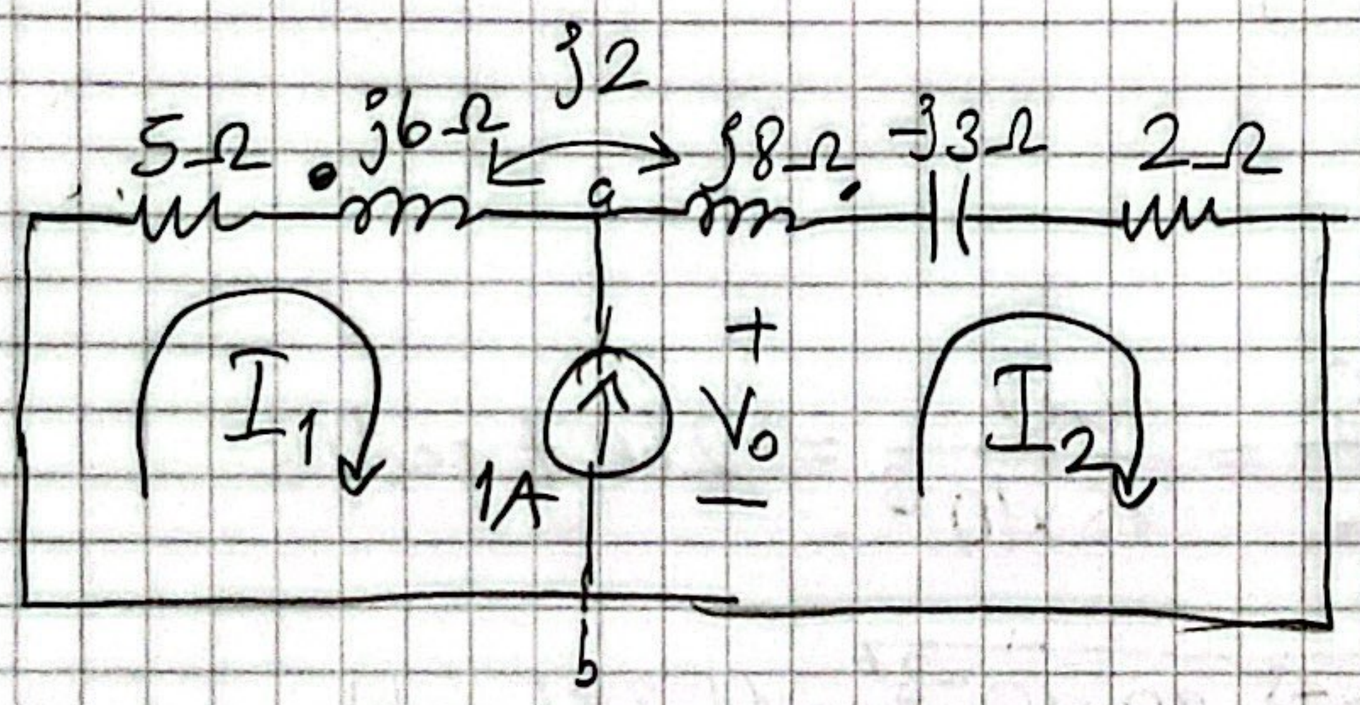
But, $-j50 + (5+j6)I - j2I + V_{Th} = 0$

$V_{Th} = j50 - (5+j4)I = j50 - (5+j4)(-40+j50)/(7+j7)$

$V_{Th} = 26,74 \angle 34,11^\circ$ V

To obtain Z_{Th}, we set all the sources to zero and insert a 1-A current source at the terminals a-b as shown below.



Clearly, we now have only a super mesh to analyze.

$(5+j6)I_1 - j2I_2 + (2+j8-j3)I_2 - j2I_1 = 0$

$(5+j4)I_1 + (2+j3)I_2 = 0$ (1)

But, $I_2 - I_1 = 1$ or $I_2 = I_1 - 1$ (2)

Substituting (2) into (1), $(5+j4)I_1 + (2+j3)(1+I_1) = 0$

$I_1 = -(2+j3)/(7+j7)$

Now, $(5+j6)I_1 - j2I_1 + V_o = 0$

$V_o = -(5+j4)I_1 = (5+j4)(2+j3)/(7+j7) = (-2+j23)/(7+j7)$
$= 2,332\angle 50^\circ$

$Z_{Th} = V_o/I = 2,332\angle 50^\circ\ \Omega$

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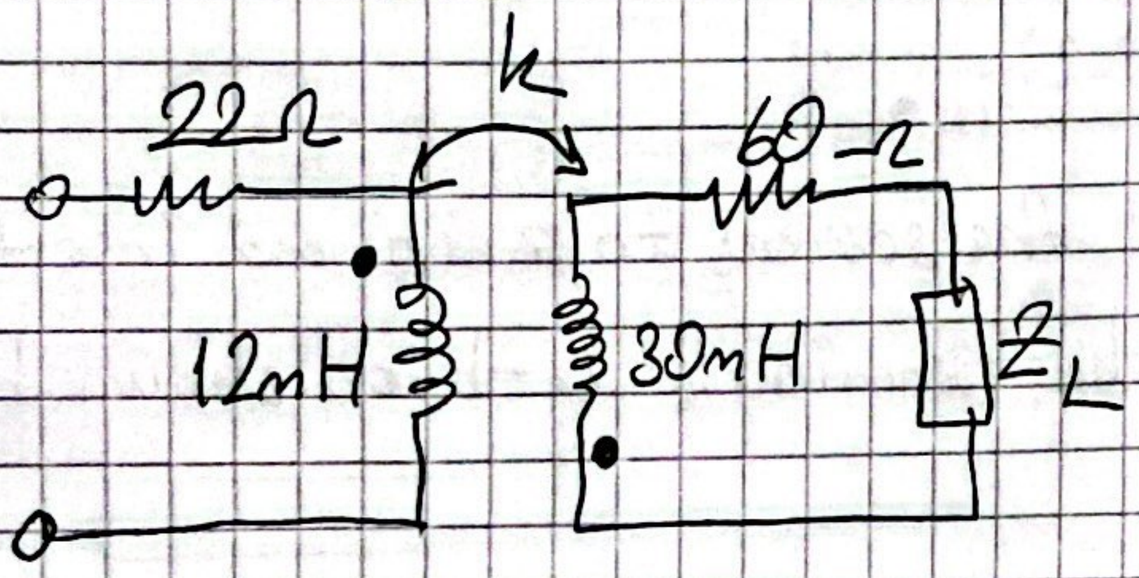
Question 5:

In the circuit of Figure 13.86, Z_L is a 15-mH inductor having an impedance of $j40\,\Omega$. Determine Z_{in} when $k=0.6$.



Solution:

$$j\omega L = j40 \longrightarrow \omega = \frac{40}{L} = \frac{40}{15\times10^{-3}} = 2667 \text{ rad/s}$$

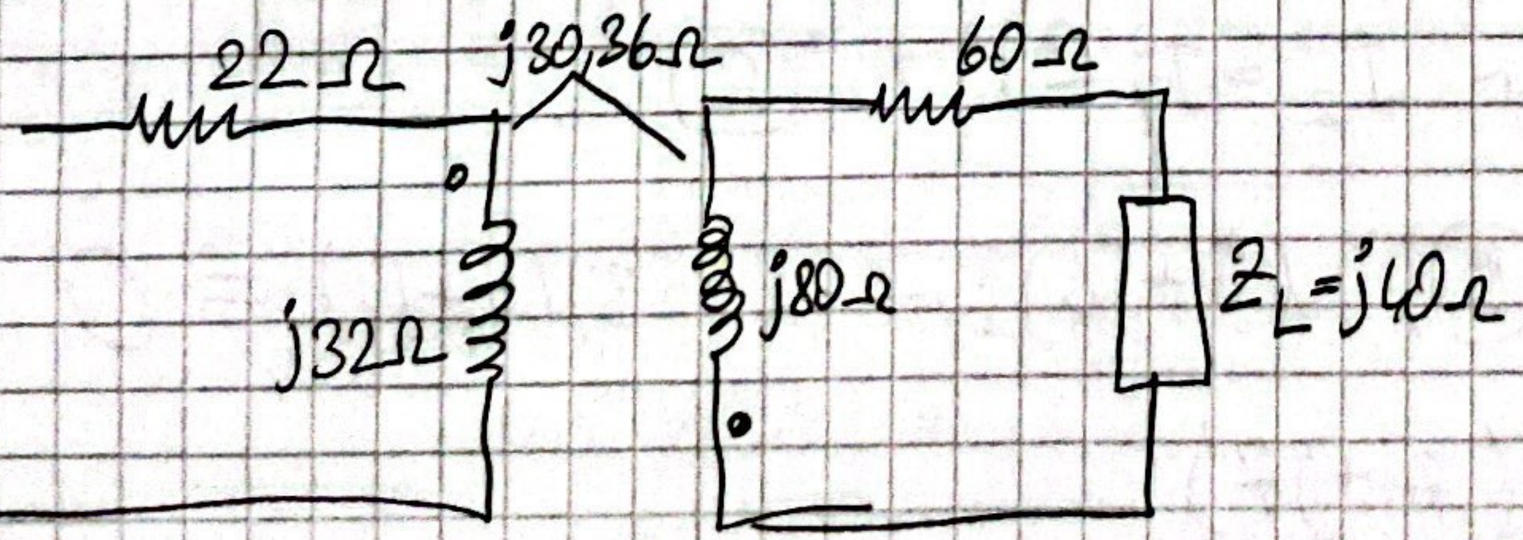
$$M = k\sqrt{L_1 L_2} = 0.6\sqrt{12\times10^{-3}\times30\times10^{-3}} = 11.384 \text{ mH}$$

If

$$15\text{mH} \longrightarrow 40\,\Omega$$

Then $12\text{mH} \longrightarrow 32\,\Omega$, $30\text{mH} \longrightarrow 80\,\Omega$, $11.384\text{mH} \longrightarrow 30.36\,\Omega$

The circuit becomes that shown below.



$$Z_{in} = 22 + j32 + \frac{\omega^2 M^2}{j80+60+j40} = 22 + j32 + \frac{(30.36)^2}{60+j120}$$

$$= 22 + j32 + \frac{921.7}{134.16\angle 63.43^\circ} = 22 + j32 + 6.87\angle -63.43^\circ = 22 + j32 + 3.073 - j6.144$$

$$= [25.07 + j25.86]\,\Omega$$

UniNote

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