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Wie ich darauf achte, dass zumindest auf der Titelseite keine Fehler sind

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Kurzfassung

Validierung eines Wandmodels für Large Eddy Simulationen auf Basis der Lattice-Boltzmann Methode

15 zeilen

Abstract

Validation of a wall model for Large Eddy Simulations based on the Lattice Boltzmann Method

15 lines

Contents

Nomenclature

Latin Symbols	Unit	Description
$\mathbf{c}$	$\mathrm{m/s}$	Constant Velocity vector
c	$\mathrm{m/s}$	Constant Velocity
f	$\mathrm{kgs^3/m^6}$	Distribution function
E	$\mathrm{J/m^3}$	Energy Density
H	m	Channel half-height
Ma	-	Mach Number
p	Pa	Pressure
R	$\mathrm{J/kg/K}$	Specific gas Constant
Re	-	Reynolds Number
t	s	Time
T	K	Temperature
$\mathbf{u}$	$\mathrm{m/s}$	Macroscopic Velocity Vector
u	$\mathrm{m/s}$	Velocity in streamwise Direction
v	$\mathrm{m/s}$	Velocity in wallnormal Direction
w	$\mathrm{m/s}$	Velocity in spanwise Direction
$\mathbf{x}$	m	Vector of position
x	m	Coordinate in streamwise Direction
y	m	Coordinate in wallnormal Direction
z	m	Coordinate in spanwise Direction

Greek Symbols	Unit	Description
κ	-	Adiabatic Index
μ	$\mathrm{kg/m/s}$	Dynamic Viscosity
ν	$\mathrm{m^2/s}$	kinematic Viscosity
$\boldsymbol{\xi}$	$\mathrm{m/s}$	Microscopic Velocity
ρ	$\mathrm{kg/m^3}$	Density
Ω	$\mathrm{kgs^2/m^6}$	Collision Operator

Indices	Description
τ	Friction
b	Bulk
cp	Centerplane
f	Fluid
m	Mean
pwm	Plane-wise mean
rms	Root-mean square
s	Solid, Sound
w	Wall

Additional Symbols	Description
$\|\mathbf{a}\|$	Euclidian Norm
∇	Nabla Operator
Δ	Step
$\mathbf{a} \cdot \mathbf{b}$	Scalar Product, Matrix multiplication
a'	Fluctuation

Abbreviations	Description
BGK	Bhatnagar-Gross-Krook
DNS	Direct numerical Simulation
LBM	Lattice-Boltzmann-Method
LBE	Lattice-Boltzmann Equation
LES	Large-Eddy Simulation
MRT	Multiple Relaxation Times Operator
NSE	Navier-Stokes-Equations
pdf	Particle-distribution Function

1 Basic Windturbine Stuff

1.1 Description

1.2 Actuator Line Model

1.3 Greedy Control

2 Lattice Boltzmann Method

2.1 basics

To conduct the training of the agent, it needs to interact with the environment. In the case studied in this thesis the environment is a wind farm with multiple turbines. Therefore the fluid field within the wind park has to be calculated. The traditional approach would be to discretize the Navier-Stokes-Equations (NSE), however, solvers based on the NSE require many CPU-hours. A newer approach is the Lattice Boltzmann Method. The basics of its theory will be layed out in this chapter.
The Lattice Boltzmann Method (LBM) is based on a discretization of the Boltzmann Equation derived from the kinetic theory of gases. This description is often referred to as a mesoscopic description, since it lays in between the large scale of continuum theory and the microscopic scale of single particles. While the NSE describe the medium through density ρ, velocity $\mathbf{u}$ and pressure p, which will be referred to as the macroscopic quantities, the Boltzmann Equation considers the particle density function (pdf) f, which describes the distribution of particles in six dimensional phase space. It is therefore a function of space $\mathbf{x}$, microscopic velocity $\boldsymbol{\xi}$ and time. However, the macroscopic quantities can easily be recovered from the pdf by taking its zeroth and first moments in velocity space, as shown in (2.1) and (2.2). Similarly the energy can be found by the third moment. The pressure can not be recovered directly, instead it is calculated by a state equation such as the ideal gas law. The pdf tends towards an equilibrium, which is given by the Maxwell equilibrium. It can be described by only the macroscopic quantities. The Boltzmann Equation describes the change of the pdf over time. The change in time is governed by the collision operator Ω. It describes the change of f due to collisions of particles. In the Boltzmann Equaiton it is an integral operator, that accounts for all possible collisions. Therefore it is mathematically rather cumbersome which renders it unuseful for numerical discretization, thus in LBM another formulation for Ω is used, which is an active area of research in the LBM community[1]. The Boltzmann Equation is shown in (2.3). The second term in the second form is a convection term, while the third term corresponds to a forcing term. In comparison to the NSE the Boltzmann Equation lacks a diffusive term. Diffusion occurs only through the collision operator. [**Krueger**]

$$\rho(\mathbf{x},t) = \int_{-\infty}^{\infty} f(\mathbf{x},\boldsymbol{\xi},t)\mathrm{d}\boldsymbol{\xi} \tag{2.1}$$

$$\rho(\mathbf{x},t)\mathbf{u}(\mathbf{x},t) = \int_{-\infty}^{\infty} \boldsymbol{\xi} f(\mathbf{x},\boldsymbol{\xi},t)\mathrm{d}\boldsymbol{\xi} \tag{2.2}$$

$$\frac{Df(\mathbf{x},\boldsymbol{\xi},t)}{Dt} = \frac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial t} + \frac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial t} + \frac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial \boldsymbol{\xi}}\frac{\partial \boldsymbol{\xi}}{\partial t} = \Omega(f) \tag{2.3}$$

In order to use the Boltzmann Equation for numerical simulations it needs to be discretized in space and time and a collision operator has to be defined.

2.2 boundary conditions

3 Machine Learning for a Continuous Control Problem

3.1 Reinforcement Learning

3.1.1 Markov Decision Process

The mathematical formulation on which RL is based is the Markov Decision Process (MDP). Its two main components are the agent that given a state S_t takes an action A_t, and the environment, that responds to the action A_t with changing its state to S_{t+1} and giving feedback to the agent in form of a reward R_t. The interaction takes place at discrete time steps t and the sequence of state, action and reward is referred to as the trajectory. The dynamics of the MDP are described by the function p that is defined in (3.1). It defines the probability of the state s' with reward r occuring, given the state s and action a. Note that the following derivations will be constricted to finite MDPs, meaning that state and action space are discrete, however the concepts all are transferable to continuous action and state space.

$$p(s', r|s, a) \doteq Pr(S_t = s', R_t = r|S_{t-1} = s, A_{t-1} = a) \tag{3.1}$$

For a process to be a Markov Decision Process, the p must only depent on s and a. Therefore s must include all information necessary to determine the future behaviour of the environment. This is not limited to information currently present in the environment, when thinking of this in terms of the wind farm problem at hand, the state could include data about wind speeds at the current time but also from time steps in the past. This approach allows to model virtually any interaction as a MDP, simply by including every bit of information from the beginning of time into the state. Obviously this is not feasible and therefore a careful choice of the information in the state is necessary.
The goal of the learning process is to maximize the sum of the rewards in the long run. Therefore a new quantity is defined, the return G_t that includes not only R_t but also the rewards received in future time steps. While in many applications of RL, the process naturally comes to an end, referred to as the terminal state S_T, in problems of continuous control this is not the case. Therefore the timeline is broken up into episodes. This allows for a finite computation of G_t. A typical formulation of G_t, referred to as a discounted return is given in (3.2). It includes a discount rate γ, that emphasizes rewards in the near future. If $\gamma = 0$, $G_t = R_t$, if $\gamma = 1$, the return is the sum of all future rewards.

[8, p. 47- 57]

$$G_t \doteq R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \gamma^3 R_{t+3} ... = \sum_{t'=t}^{T} \gamma^{t'-t} R_{t'}, \quad \gamma \in [0, 1] \tag{3.2}$$

Now the goal of the learning process is defined, but not what to learn. There exist three possible answers to this question: a model of the environment, a value function or a policy. Also combinations of these components is possible. In the case of continuous control, most common approaches are model-free, meaning either learning a value function, a policy or both. Therefore model-based methods will not be discussed further and the reader is referred to the book by Sutton and Barto [8].

The policy π is the mapping from states to actions with a set of adjustable parameters. Under the policy π with its vector of parameters set to $\boldsymbol{\theta}$, the probability of Action $A_t = a$ given a state $S_t = s$ is denoted as $\pi_{\boldsymbol{\theta}}(a|s)$. The value function of a state s under a policy π is denoted as $v_\pi(s)$ and is the expected return if the agent acts according to π, starting from state s. Note that for convenience the parameters of π were be dropped. For MDPs it can be defined as (3.3).

$$v_\pi \doteq \mathrm{E}_\pi \left[G_t | S_t = s \right] = \mathrm{E}_\pi \left[\sum_{t'=t}^{T} \gamma^{t'-t} R_{t'} | S_t = s \right] \tag{3.3}$$

$$= \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r|s, a) \left(r + \gamma v_\pi(s') \right) \tag{3.4}$$

In the form of (3.4), the equation is referred to as the Bellmann equation and its unique solution is the value function v_π. Analogously, the action-value function is the expected reward of taking action a at state s under the policy π, denoted by $q_\pi(s, a)$. It is defined by (3.5). [8, p. 58-59]

$$q_\pi(s, a) \doteq \mathrm{E}_\pi \left[G_t | S_t = s, A_t = a \right] = \mathrm{E}_\pi \left[\sum_{t'=t}^{T} \gamma^{t'-t} R_{t'} | S_t = s, A_t = a \right] \tag{3.5}$$

3.1.2 Policy Gradient Methods

With a known value function, it is possible to construct a policy and by improving the value function, the policy can be improved. However, there exist advantages to directly improve the policy, especially for continuous state and action space. Policy gradient methods use the gradient of a perfomance measure $J(\boldsymbol{\theta})$ with respect to the parameters $\boldsymbol{\theta}$ of a policy. This gradient can be used in optimization algorithms, such as stochastic gradient descent [8, p. 201] or its extensions such as Adam [5]. In its basic form, SGD performs an update according to 3.6. The parameter α is called the

learning rate.

$$\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t + \alpha \nabla J(\boldsymbol{\theta}) \tag{3.6}$$

The policy gradient methods differ now only in the performance measure. The first such algorithm proposed is the REINFORCE algorithm [9]. Its definition of $\nabla J(\boldsymbol{\theta})$ is presented in (3.7). It follows from the policy gradient theorem and substituting all values of A and S with the actions and states from one trajectory. Thus all the values necessary for the computation of the gradient are known. [8, p.324-328]

$$\nabla J(\boldsymbol{\theta}) = \mathrm{E}_\pi \left[\sum_a \pi_{\boldsymbol{\theta}}(a|S_t) q_\pi(S_t, a) \frac{\nabla \pi_{\boldsymbol{\theta}}(a|S_t)}{\pi_{\boldsymbol{\theta}}(a|S_t)} \right] \tag{3.7}$$

$$= \mathrm{E}_\pi \left[G_t \frac{\nabla \pi_{\boldsymbol{\theta}}(A_t|S_t)}{\pi_{\boldsymbol{\theta}}(A_t|S_t)} \right] \tag{3.8}$$

While this algorithm makes a computation possible, it is inefficient. Therefore newer methods have been devised such as the Trust Region Policy Optimization (TRPO) [7] and the Proximal Policy Optimization (ppo) [6]. They are closely related and both propose a surrogate performance measure that limits the size of the gradient to ensure monotonic improvement in the case of TRPO and a close approximate in the case of ppo. However, the computation of the surrogate in ppo is simpler and more efficient, which helped policy gradient methods to become one of the most used algorithms in continuous control problems. One version of the surrogate performance measure, which is also referred to as objective, is given in (3.10). The probability ratio r_t compares the policy after the update to the policy before the update. Therefore $\pi_{\boldsymbol{\theta}}$ has to be approximated. $a_\pi(A_t|S_t)$ is an estimator of the advantage function, which is defined as $a_\pi(A_t|S_t) = q_\pi(A_t|S_t) - v_\pi(S_t)$. This estimator also has to be found, however, this is a regular optimization problem, which can be solved via SGD or Adam.

$$r_t(\boldsymbol{\theta}) \doteq \frac{\pi_{\boldsymbol{\theta}}(A_t|S_t)}{\pi_{\boldsymbol{\theta}_{old}}(A_t|S_t)} \tag{3.9}$$

$$J \doteq \mathrm{E}_\pi \left[\min \left(r_t(\boldsymbol{\theta}) a_\pi(A_t|S_t), \mathrm{clip}(r_t(\boldsymbol{\theta}), 1-\epsilon, 1+\epsilon) a_\pi(A_t|S_t) \right) \right] \tag{3.10}$$

$$\mathbf{a} \tag{3.11}$$

In addition to the ratio probability clipping, other regularizations can be added to the objective function, for example an l_2 regularization, that adds a penalty proportional to the size of $\boldsymbol{\theta}$.

Table 3.1: Activation functions commonly used in neural network

Name	Domain	Range	Definition
$\tanh$	$(-\infty, +\infty)$	$(-1, -1)$	$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^- x}$
σ	$(-\infty, +\infty)$	$(0, -1)$	$\sigma(x) = \frac{1}{1+e^{-x}}$
relu	$(-\infty, +\infty)$	$[0, +\infty)$	$\mathrm{relu}(x) = \max(x, 0)$

3.2 Artificial Neural Networks

3.2.1 Feed-forward networks

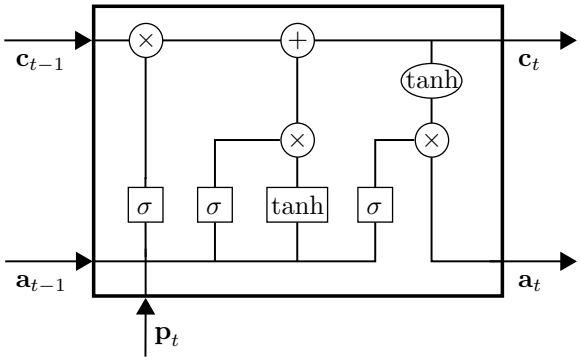
In the section above, a way to update parameters of the policy or value function were described, however, no description of the function itself was given. In principal any function with a set of parameters can be used, but usually, an artificial neural network (ANN) is used. ANNs are comprised of layers of neurons. More precisely a layer k of S neurons takes as input a vector $\mathbf{p}$ of length R. The output of the layer is a new vector a, wich then is the input for the next layer of the network. In a simple feed-forward layer output a_j^k of the j-th entry in a^k, the output of the k-th layer of the network, is computed according to (3.12). The entries $w_{i,j}^k$ of the matrix $\mathbf{W^k}$ are called the weights of the layer and the entries b_j^k of the vector $\mathbf{b}^k$ are called its bias. f is called the activation function, which can be chosen freely, but typical choices include the hyperbolic tangent (tanh), the sigmoid-function (σ) or the rectified linear unit (relu) function. A small overview over some of the key features these functions is given in Table 3.1. Thus a network of two layers with $\tanh$ as an activation function describes the function given in (3.13), with $\mathbf{p}$ being the input to the network and $\mathbf{a}$ its output and the activation function being applied element-wise to the vectors. [2, p. 2-2 - 2-12].

$$a_j^k = f(w_{i,j}^k p_i^k + b_j^k) \tag{3.12}$$

$$\mathbf{a} = \mathbf{tanh}(\mathbf{b^1} + \mathbf{W^1} \cdot \mathbf{tanh}(\mathbf{b^0} + \mathbf{W^0} \cdot \mathbf{p}) \tag{3.13}$$

3.2.2 Recurrent neural networks

It is obvious that a feed-forward network only produces an output influenced by the current activation, there can be no influence of previous activations for example in a time-series. Yet there exist many applications for which such a feature would be useful, for example the field of natural language processing [3] or transient physical processes. This led to the development of recurrent neural networks, in which the network includes has a hidden state, which is preserved. Especially the development of the long short-term memory (lstm) cell has had great impact on the success



of recurrent neural networks due to its ability to preserve hidden states over a longer period of time [4]. An lstm layer consists of one or multiple cells, with the input being the entire or parts of the activation. Each cell passes a vector called the cell state $\mathbf{c}$ as well as its output $\mathbf{a}$ to itself in the next time step. Thus a cell has as inputs at time t the cell state and output of the previous time step, $\mathbf{c}_{t-1}$ and $\mathbf{a}_{t-1}$, as well as the current activation $\mathbf{p}_t$. On the inside, the cell consists of a forget-gate, an input-gate and an output-gate. The forget-gate determines the influence of the cell-state, the input-gate determines the influence of the current time-step on the cell-state. The output-gate then computes the output based on the updated cell-state.

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Selbstständigkeitserklärung

Hiermit erkläre ich, dass ich die von mir am heutigen Tag der Professur für Strömungsmechanik eingereichte Interdisziplinäre Projektarbeit zum Thema

Validierung eines Wandmodels für Large Eddy Simulationen auf der Basis der Lattice-Boltzmann Methode

vollkommen selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt, sowie Zitate kenntlich gemacht habe.

Dresden, 06. Januar 2020

Henry Korb