Mathematical Modeling for Increasing Revenue in a Tutoring Business

Lehlohonolo Moloi

November 25, 2024

Introduction

Lehlohonolo Moloi's tutoring program operates on weekends, offering educational support in various subjects, focusing primarily on physical sciences and mathematics. The service is priced at R500 per learner, with discounts for multiple learners from the same household. Although the program accommodates about 20 learners per term, retention challenges and geographic constraints limit its growth. Furthermore, the absence of online tutoring and the reliance on in-person sessions restrict potential outreach. This report develops a mathematical model to represent revenue generation and explores strategies to optimize income and scalability while accounting for identified constraints.

Objective

The primary goal of this report is to create a mathematical model to predict and maximize revenue while considering constraints such as retention of students, availability of tutors, geographic reach and the potential transition to online tutoring.

Theoretical Framework

The model is based on the principle that revenue (R) is a function of the number of learners (N), the price per learner (P), and the dropout rate (D). The relationship is expressed as:

$$R = (N_{\text{in-person}} + N_{\text{online}}) \cdot P \cdot (1 - D)$$

where:

- R: Total revenue.
- $N_{\text{in-person}}$: Number of learners attending in-person sessions.
- N_{online}: Number of learners attending online sessions.
- P: Price charged per learner (R500).
- D: Dropout rate ($50\% = 0.5$).

Key Constraints

1. **Retention Rate:** A 50% dropout rate after one term reduces recurring revenue.
2. **Geographic Reach:** In-person learners are limited to a travel radius of 45 minutes.
3. **Tutor Capacity:** Only three tutors are available, which limits the number of learners that can be accommodated.
4. **Competition:** Other in-person tutoring services claim a significant share of the market.
5. **Technology Gap:** Online tutoring requires infrastructure development and adaptation.

Proposed Model Expansion

To address these constraints and improve revenue, the following adjustments are proposed:

1. **Retention Improvement:** Introduce incentives (e.g., performance rewards, feedback sessions) to improve retention rates. This will reduce D.
2. **Geographic Expansion via Online Tutoring:** Transition to online tutoring to remove geographic constraints. This introduces a new variable, N_{online}, which depends on the adoption rate of online sessions:

   $$N_{\text{online}} = \alpha \cdot N_{\text{total}}$$

   where α represents the fraction of learners who opt for online tutoring.
3. **Tutor Efficiency:** Maximize tutor efficiency by scheduling group sessions for common subjects. The capacity of a tutor (T) can be represented as:

   $$T = \frac{\text{Session Hours}}{\text{Learner Load Per Hour}}$$

4. **Marketing Optimization:** Enhance outreach through online platforms and partnerships with schools, increasing N_{total}.

Python Implementation and Visualization

The equations derived from this model can be implemented in Python to simulate revenue under different scenarios. For example:

- Plot revenue R versus the number of learners N_{total}.
- Simulate revenue growth by varying D (retention rate) and α (online adoption rate).
- Assess the impact of tutor capacity on total student intake.

Conclusion

This mathematical model provides a framework for analyzing and predicting the revenue potential of the Lehlohonolo Moloi tutoring program. Incorporating online tutoring, improving retention rates, and optimizing marketing efforts are pivotal strategies for increasing revenue and reaching underserved learners. Translating these strategies into Python simulations, the program can make data-driven decisions to achieve sustainable growth.