problem: prove that the set of national numbers Q, equipped with the two binary operation of addition and meltiplication, forms a field.

Ans: To prove that the set of mational numbers Q, coupped with the standard operations of addition (+) and multipliation ('), froms a field, we must werely that if satisfies' the 11 fields arxinoms.

the set of mational numbers is desimed as:

let x, y, 2 be arbitrary madinal markus in Q.

Axiom of Addition;

1. closers und oden addition

the meet of two national metabons is a pertional metabon.

let $x = \frac{a}{b}$ and $y = \frac{e}{d}$ when, $a, e, \in 2$ and $b, d \in \mathbb{Z}[5]$ $x + y = \frac{a}{b} + \frac{d}{d} = \frac{ed + be}{bd}$

Since $a, b, e, d \in \mathbb{Z}$, then $ad+be \in \mathbb{Z}$ and $bd \in \mathbb{Z}$. Also Since $b \neq 0$ and $d \neq 0$, then $bd \neq 0$. Mos $x+y \in \mathbb{Q}$.

1. Aprociativity of Addition:

the grouping of numbers not affect them som.

(X+4)+7- X1/412) (x+y)+z=x+(y+z)

this property is inherited from the acrocitivity of addition in the instruments (2).

3. Commutativity of addition:

the ondon of numbers does not offered their sum. X+Y = 7+ 2

this proparty is also inhunted from the commutativity of addition in z.

4. Additive Identity (zeno Elements):

the exists a unique clemente, e, in Q such that fon all x ∈ Q:

the additive identity is 0 = = 0, sime x+0=x.

5. Additive invers;

for any $a = \frac{x}{y} \in \mathbb{Q}$ considur, $-a = \frac{-x}{y}$

Sime x i ar interen, - x is elso ar intern, si, -a t a

$$a + (-a) = \frac{x}{y} + \frac{-x}{y} = \frac{0}{y} = 0$$

thus, very alimets is a way are addition irwas.

Asions of mult pricutions:

6. alvans anden multiplication: the product of $\frac{1}{4}$ two patiends multiplications is a patiental multiplication of $\frac{1}{4}$ two patients multiplications is a patiental multiplication of $\frac{1}{4}$ two patients $\frac{1}{4}$ and $\frac{1}{4}$ $\frac{1}{4$

Sime a, b, e, d te sum a e & 2 and bd & 2. Sime b to and d \$ 0, the bod \$ 0, they x, y & a

2. Associtivity of multipleations: the groupsing of markens of markens of mot of heart three product.

this proposity is inhumited from the associationity of multipulseation or z.

8. Commutatively of multiprication: the order of numbers does not ofhere here produce. $x \cdot y = y \cdot \eta$ this proposety is inhumbered from the commitatively of smultplusticer in z.

9. multiplie a Live i dont de (comity alements):

there exists a unique oclumets, 1, in a such that for all n & Q;

2.1=2

the mettipliseetine identity i 1= - = 1,

5, ne 2.1 = 2

10. multiplicative inverse:

For any non-2000 $a = \frac{4}{5} \notin a$, we much have $2 \neq 0$, consider the external $a = \frac{4}{2}$.

Since $2 \neq 0$.

a-1 is a well-detimed particular mountains. a, a = 1 = \frac{n}{y}. \frac{y}{n} = \frac{ny}{y^2} = 1

a maltiplietive invuse.

Arion of ontombetion.

11. De Junchediuty;

Multipliention is dudundution over addition.

2. (4+2) = (n.4) + (r.2)

the propulary is inhumerted from the distriction -

Sime the set of Rational mumbury Q Satrifice all eleven field oxioms with Propert to addition and multiplication. So (Q, +, .) is a field.