Experimental Error Analysis

AERO 214

Every number appearing in an engineering report reflects an experimental measurement that must carry the ***unit*** of measurement and the ***uncertainty*** or error of the measurement. It is expected that you follow the guidelines listed below in the next reports for your AERO 213 lab.

Types of Error:

All measurements have errors and are mostly categorized as arising from three sources:

a) **Careless errors:**
These are due to mistakes in reading scales or careless setting of markers introduced by the experimenter. They can be eliminated by repetition of readings by one or two observers.

b) **Systematic errors:**
These are due to built-in errors in the instrument either in design or calibration that affect the *accuracy* of the measurement. Systematic errors are one sided errors, because, in the absence of other types of errors, repeated measurements yield results that differ from the true or accepted value by the same amount. The accuracy of measurements subject to systematic errors ***cannot*** be improved by repeating those measurements. Systematic errors *cannot* easily be analyzed by statistical analysis.

c) **Random errors:**
These always lead to a spread or distribution of results on repetition of a particular measurement and affect the *precision* of a measurement. Random errors are two-sided, because, in the absence of other types of errors, repeated measurements yield results that fluctuate above and below the true or accepted value. Measurements subjected to random errors differ from each other due to random, unpredictable variations in the measurement process. The precision of measurements subjected to random errors can be improved by repeating those measurements. Random errors are easily analyzed by statistical analysis.

Accuracy vs. Precision:

Precision is affected mainly by Random errors
Accuracy is affected by systematic errors

"Accuracy" deals with how close a measured value to an accepted or "true" value is. "Precision" deals with how reproducible experimental measurements are. An experiment can yield high precision but inaccurate results due to a wrongly calibrated dial, for example.

Calculating Experimental Error

When reporting the results of an experiment, the report must describe the accuracy and precision of the experimental measurements.

d) **Significant figures**

Recorded values should reflect the precision of an instrument. The least significant digit in a measurement depends on the smallest unit which can be measured using the instrument. Therefore the precision of the measurement is limited by the smallest unit measurable by the device. Commonly, any measurement is reported to a precision equal to 1/2 of the smallest unit on the measuring instrument, unless the instrument manufacturer indicates otherwise. For example, the digital angleometer reads 10.4; the precision of the angle measurement is ±½ of 0.1 degrees or ±0.05 degrees.

Significant digits included in the measurement beyond the precision of the device are absolutely useless and if appeared in an engineering report, show lack of engineering professionalism.

e) **Propagation of errors**

When measured values are used to calculate other values, the uncertainties in the measured values cause uncertainties in the calculated values. If a variable z is a function of two measured variables x and y associated with uncertainties Δx and Δy,

$$z = z(x, y)$$

The uncertainty Δz can be given by

$$\Delta z = \sqrt{\left(\frac{\partial z}{\partial x}\Delta x\right)^2 + \left(\frac{\partial z}{\partial y}\Delta y\right)^2}$$

For instance, if you are calculating the area

$$A = xy, \qquad \Delta A = A\sqrt{\left(\frac{\Delta x}{\mathrm{x}}\right)^2 + \left(\frac{\Delta y}{\mathrm{y}}\right)^2}$$

Significant Figures

The number of **significant figures** is the number of digits whose values are known with certainty.

The **numerical values of the experimental results** must be written according to specific rules. The number of significant figures that should be used in stating a result is inseparably connected with the accuracy with which the result is known.

(1) The number of significant figures in the experimental uncertainty is limited to one or (when the experimental uncertainty is small, e.g., ± 0.15) to two significant figures. You should not use more than two significant digits when stating the experimental uncertainty.

(2) Now the best estimate (usually the average value) and its uncertainty (experimental error) must always have the same number of digits after the decimal point, even if the uncertainty does not contain the same number of significant figures as the best estimate. If the uncertainty has more number of places after the decimal as compared to the best estimate, adding it to (or subtracting it from) the best estimate will leave the best estimate with more number of decimal places than your apparatus is capable of measuring. For example, 2.4 ± 0.16 implies that the result lies in the range 2.24 – 2.56. But the apparatus can measure only up to a precision of one place after the decimal. Hence the correct way to express the answer is 2.4 ± 0.2.

If the uncertainty has less number of places after the decimal than the best estimate (could be because of the restriction of one or two significant figures), then your best estimate is rounded off appropriately. This is because even if your apparatus has a higher resolution (i.e., it can display more places after the decimal), it does not have that accuracy (as proved by the error having less number of places after the decimal). Thus 2.456 ± 0.12 should be written as 2.46 ± 0.12.

(3) The value of the experimental uncertainty and rules (1) and (2) determine the number of significant digits in the best estimate (usually the average value) of the measured value.

(4) The number "zero" presents problems in significant figures. Zeros before the first non-zero number are not significant. Zeros after numbers after the decimal place are significant. Zeros before the decimal place may or may not be significant.

Each of the following numbers has three significant figures:

2.34 234 -0.234 0.00234 $-2.34*10^5$ $2.34*10^{-4}$

2.30 -0.230 $2.30*10^{-7}$ 0.000230 2.03 $2.30*10^{4}$

Each of the following numbers has <u>two significant figures</u>:

$2.3*10^{2}$ 2.3 -0.23 0.0023 $-2.3*10^{5}$ $2.3*10^{-4}$

(5) Writing an integer number (e.g., 350 m) presents a problem, because it does not clearly defines the number of significant figures (two significant figures or three?). <u>Use the scientific notation</u>, such as $3.50*10^{2}$ m, to indicate that three significant figures are known.

6) Both <u>measurements and experimental uncertainties must have the same units</u> (always the case unless specified as relative error that is always dimensionless and usually written as a percentage).

<u>Please, compare the following examples</u>:

Correct:	**Wrong:**
0.2 ± 0.3	0.20 ± 0.321
0 ± 2	0.05 ± 2.5
3.14 ± 0.01	3.14 ± 0.002
3.142 ± 0.002	3.14213 ± 0.002
3.14 ± 0.02	3.142133 ± 0.023523
$(2.34 \pm 0.15)*10^{-4}$	$(2.34 \pm 0.152)*10^{-4}$

(7) Do not mix notations - that is, do not write a measured value in scientific notation and its error as a decimal or vice versa.

(8) When multiplying or dividing measurement figures, the final answer may not have more significant figures than the *least* number of significant figures in the figures being multiplied or divided. This simply means that an answer cannot be more accurate than the least accurate measurement entering calculation, and that you cannot improve the accuracy of a measurement by doing a calculation (even if you have a 10-digit, scientific calculator). To minimize rounding errors, you may use more significant figures during intermediate calculations and adjust the significant figures only for the final results.

<u>Examples</u>: $3.5 * 22.3 = 78$ *not* 78.05, $6.2 / 833 = 0.0074$ *not* 0.007442977

Adding or subtracting: $42.4 - 41.62 = 0.8$, $4256 - 24.7 = 4231$, $33.8 + 15.63 = 49.4$