Introductory Physics Lab

PLAB 193

How to find the uncertainty in the slope

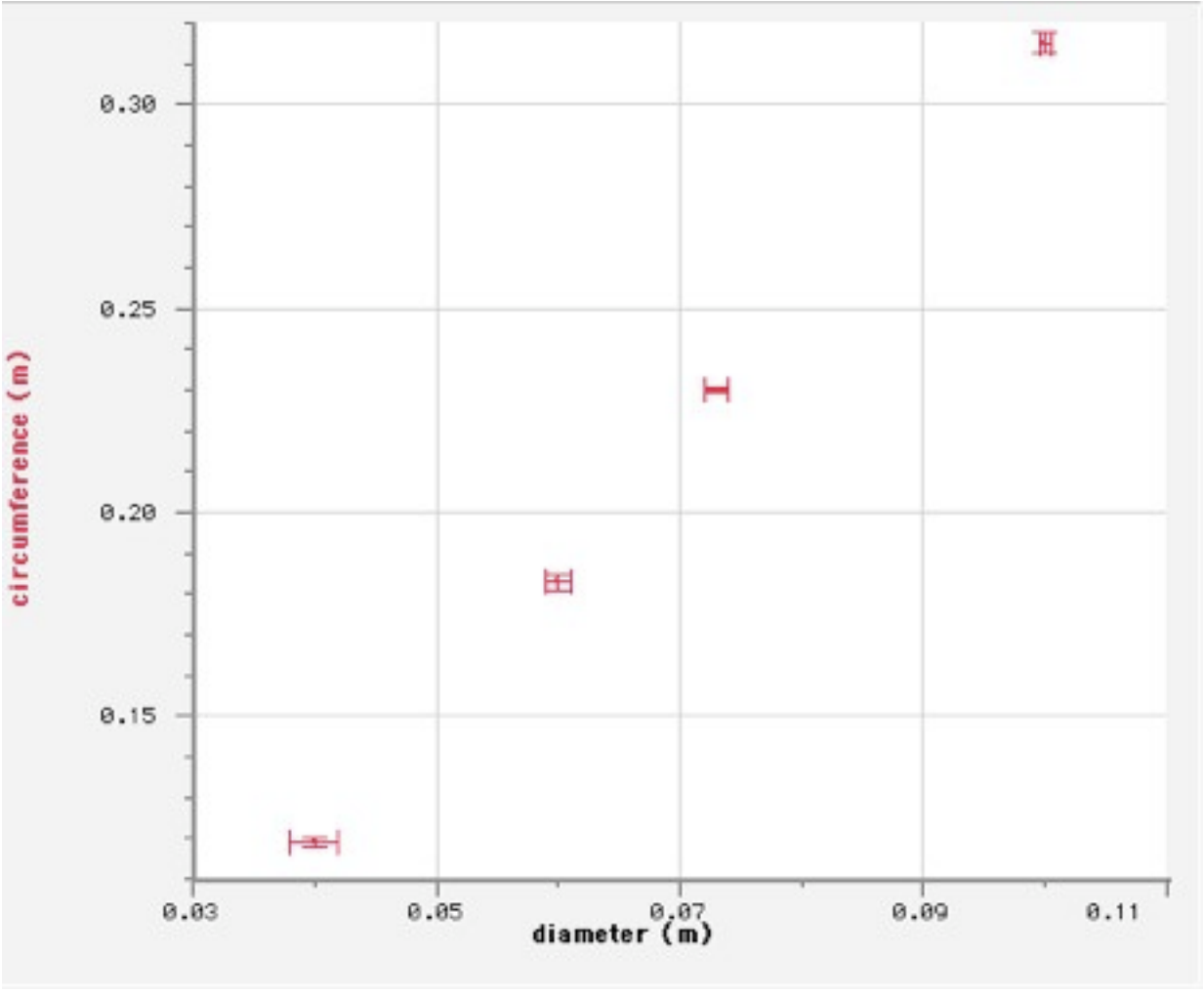
This is an issue that I have not really addressed much. However, it is important enough that I talk about it.

In many labs, you will collect data, make a graph, find the slope of a function that fits that data and use it for something. Well, what if need to find the uncertainty in the slope? How do you do that? There are a couple of ways you can do this, neither are absolutely correct. However, if you write a formal lab report and you find the slope you MUST find the uncertainty in it.

Here is some sample data. Suppose I measure the diameter and the circumference of several roundish objects. Here is my data.

diameter (m)	circumference (m)
0.100+/- 0.001	0.315+/-0.005
0.060+/-0.002	0.183+/-0.004
0.040+/-0.002	0.119+/-0.002
0.073+/-0.002	0.230+/-0.001

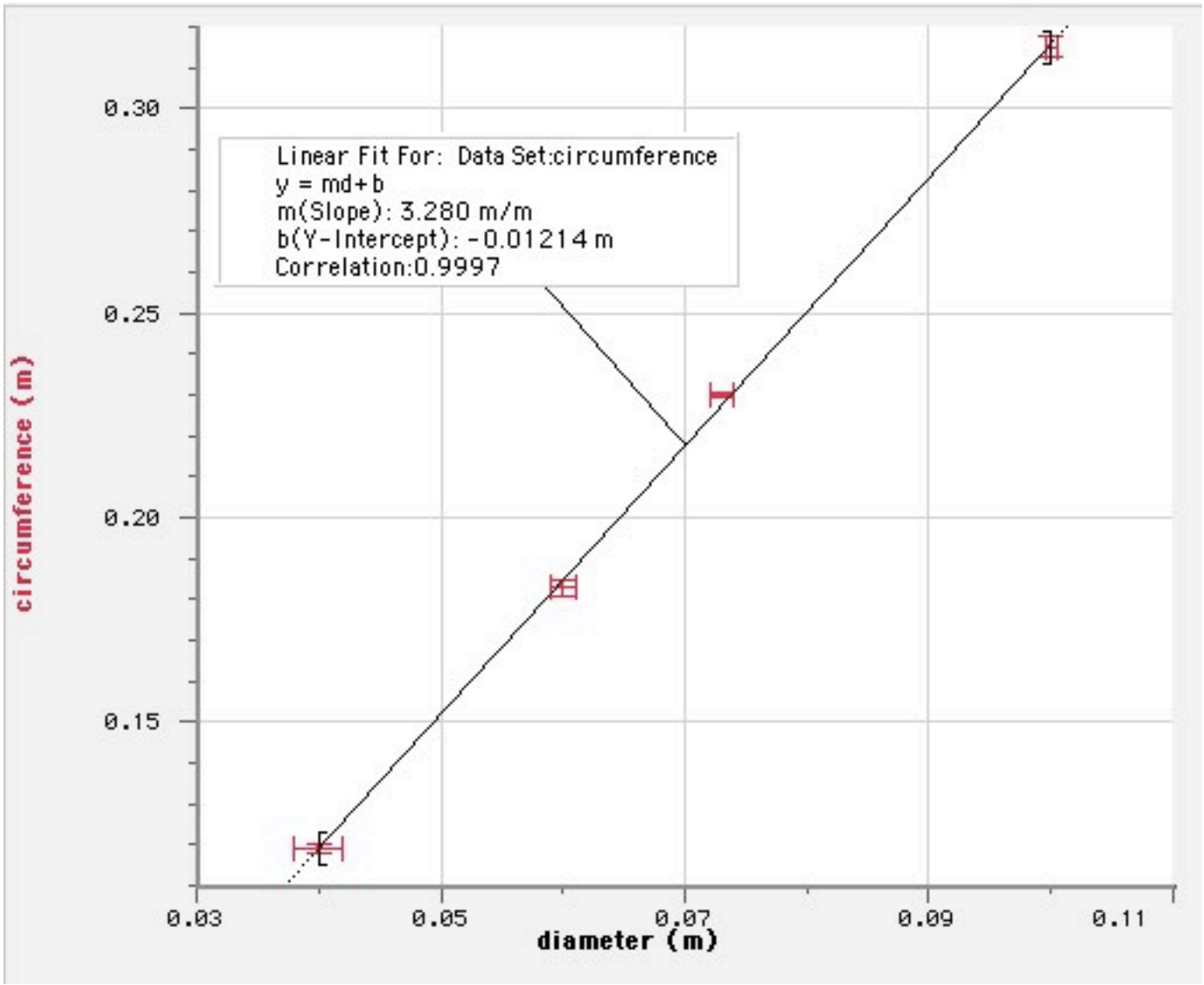
So, I want to plot this and find a functional relationship between these two. With error bars, this is what it should look like:



Now, I want to fit a linear function to this data. That should be ok, but what about the uncertainty?

Method 1 - use uncertainty of data points

I could get the ratio of C/d by just looking at each data point. This is not as good as the slope because the slope essentially uses all the data points at once. In this method, I am going to find the slope as normal. In Excel, you could fit a trendline. Or, you could draw a best fit line. Either way, I would get something like this (I did this in Logger Pro):



This gives a slope of 3.28 (compare to pi = 3.14). I could get a better slope if I required the fitting function to go through the origin (0,0), but I am not going to do that. In essence, the slope is:

$$\text{slope} = \frac{\Delta C}{\Delta d}$$

But, what if I just use one set of data points? Then I could use propagation of error as usual. This would give

$$\text{slope}_{\text{min}} = \frac{C_{\text{min}}}{d_{\text{max}}}$$

$$\text{slope}_{\text{max}} = \frac{C_{\text{max}}}{d_{\text{min}}}$$

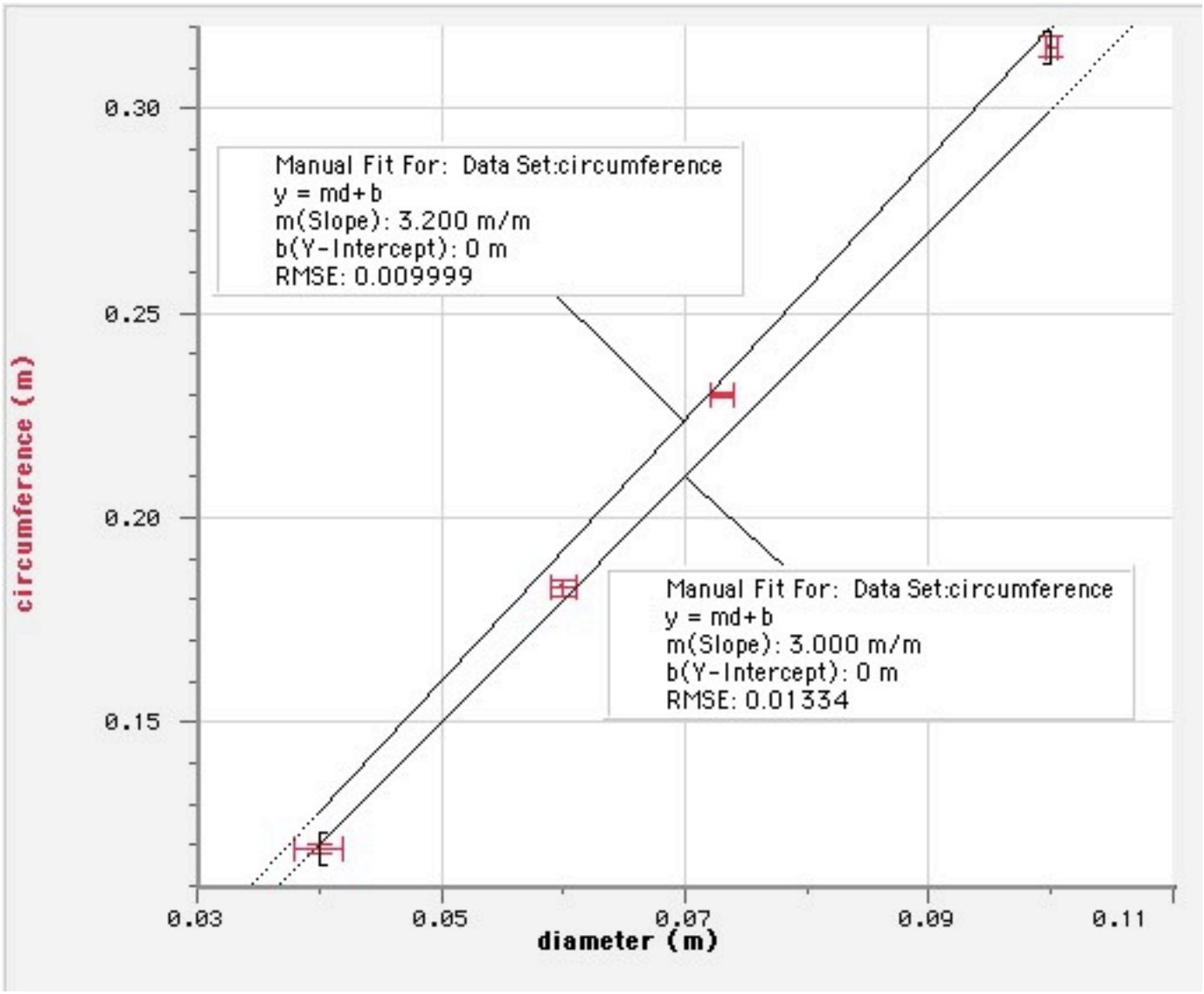
$$\Delta\text{slope} = \frac{\text{slope}_{\text{max}} - \text{slope}_{\text{min}}}{2}$$

Where the delta - slope represents the uncertainty in the slope. For this method, just pick the data pair with the largest uncertainty (to be safe) - although hopefully, it won't matter much. For this case, I will pick d= 0.06+/-0.002 m and C = 0.183 +/- 0.004 m. This would give an uncertainty in the slope of 0.2. I would write:

$$\text{slope} = 3.2 \pm 0.2$$ (there are no units - they canceled)

Method 2

The next method is better but a little tricky. Basically, if you draw a best fit line you could “wiggle” the line and still have it fit. This would give a minimum and a maximum slope. If you do the trendline in Excel, I am not sure how you would do this. If you do it by hand on graph paper, it would be easy. Here, I am going to manually fit two lines in Logger Pro. Here is the max slope that ‘looks’ like it fits.



This gives a min slope of 3.0 and a max of 3.2 for an uncertainty of +/- 0.1.
