湖南大學
HUNAN UNIVERSITY

研究生期末大作业

课程名称：　有限元方法及应用

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一．线性三角形单元

在不规则形状物体受到外力作用时，可能会在某些部位造成应力集中，导致该部位易产生损坏，若能提前分析出应力大的部位，便能及时更改部件形状，以免造成不必要的损失。

下图为一受单一力的薄平板结构，平板长宽均为 2，右下角为 1*1 的缺口，板厚为 0.2。杨氏模量为 210Gpa，泊松比为 0.3。

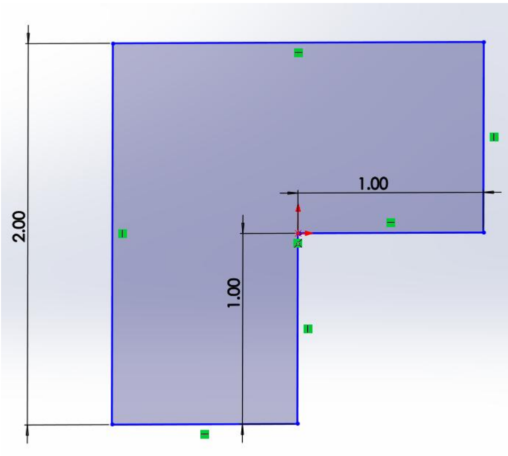


图 1.平板结构

在划分不同数量表格及不同的受力下可得到下图结果

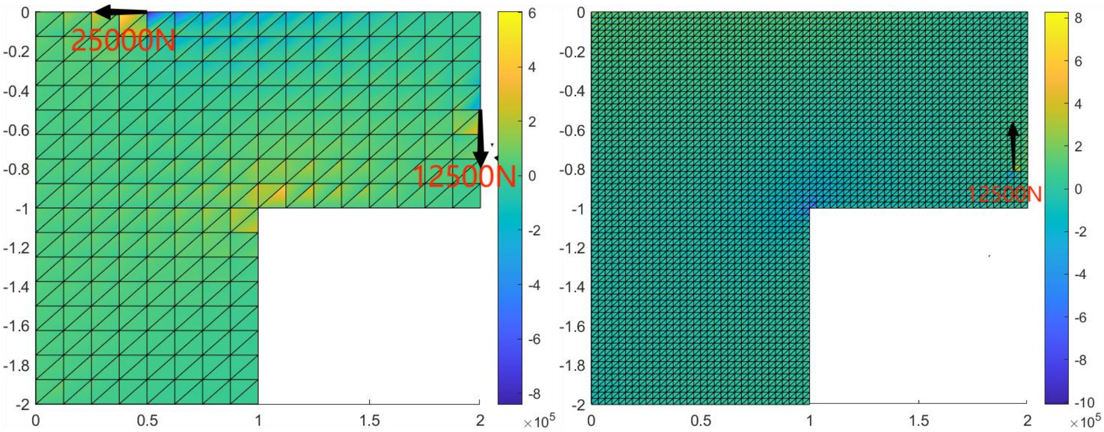


图 2.线性三角形单元所得应力分布

二、线性四面体元

若平板长宽高相近，便不可忽略某一方向的尺寸，若采用线性三角形单元计算便会出现较大误差，得到的结果也会不对，例如下图

该三维结构长宽高皆为2，右下角为1*1*2的缺口，其中，杨氏模量为210Gpa，泊松比为 0.3。

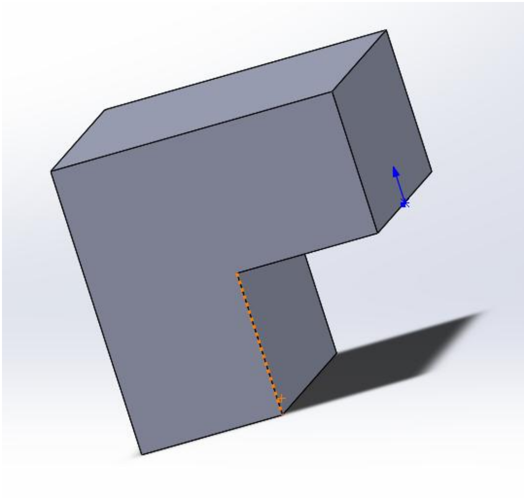


图 3. 三维结构

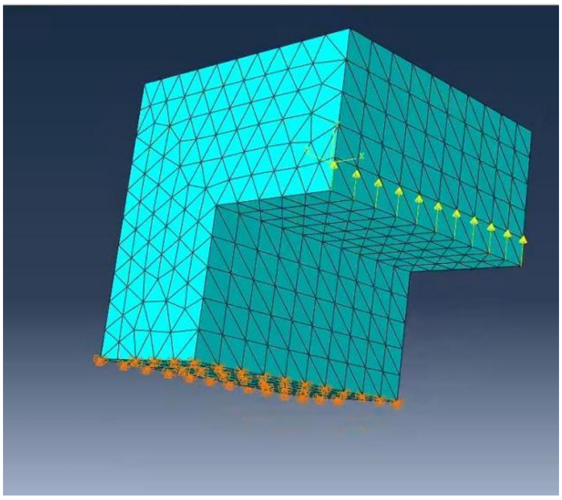
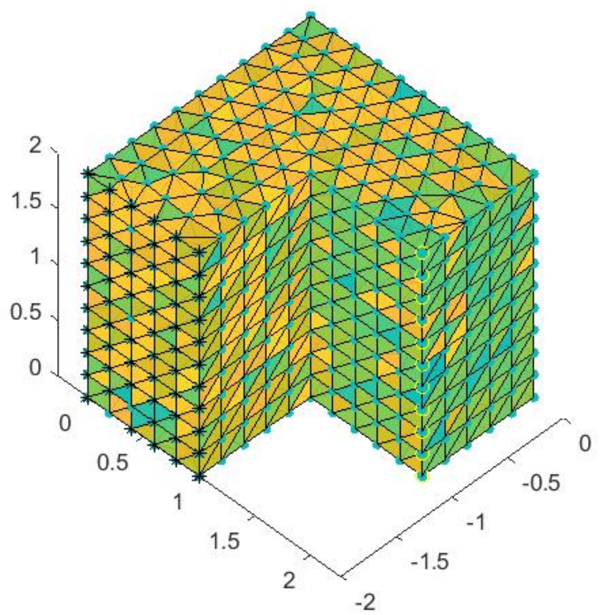


图 4.abaqus 网格划分及施加载荷、边界条件结果

此时运用线性四面体元可得到如下结果：



文件夹中文本文件 xd、yd、zd 分别为节点在 x、y、z 方向的位移分量

四. 附录代码

1.线性三角形单元

Main 1

```
first_time=cputime;
format long
%--------------------------------------------
%input data for control parameters
%--------------------------------------------
lengthx=2;                    %length of x-axis side of problem
lengthy=2;                    %length of y-axis side of proble
h=0.2;                        %thickness of problem
emodule=2.1*10^11;                 %elastic modulus
poisson=0.3;                  %Poisson's ratio
fload=-12500;                    % the total load
lx=16;                        % number of element in x-axis
ly=16;                          % number of element in y-axis
nel=2*0.75*lx*ly;                  % number of element
nnel=3;                       %number of nodes per element
ndof=2;                       %number of dofs per node
nnode=(lx/2+1)*(ly+1)+(lx/2)*(ly/2+1);      %total number of nodes in system
sdof=nnode*ndof;            %total system dofs
edof=nnel*ndof;             %degrees of freedom per element
x0=[];
for i=1:lx+1
    if i<=(lx/2+1)
         for j=1:ly+1
             x0=[x0; (i-1)*lengthx/lx        -(j-1)*lengthy/ly];
         end
    else
        for j=1:(ly/2+1)
            x0=[x0; (i-1)*lengthx/lx        -(j-1)*lengthy/ly];
        end
    end
end
nodes=[];
for i=1:lx
    if i<lx/2+1
        for j=1:ly
```

```
            nodes=[nodes; (ly+1)*(i-1)+j (ly+1)*i+j (ly+1)*(i-1)+j+1;];
            nodes=[nodes; (ly+1)*i+j (ly+1)*i+j+1 (ly+1)*(i-1)+j+1;];
        end
    elseif i==lx/2+1
            for j=1:ly/2
                nodes=[nodes; (ly+1)*(i-1)+j
(ly/2+1)*(i-lx/2-1)+j+153   (ly+1)*(i-1)+j+1;];
                nodes=[nodes; (ly/2+1)*(i-lx/2-1)+j+153 (ly/2+1)*(i-lx/2-1)+j+154
(ly+1)*(i-1)+j+1;];
            end
    else
        for j=1:ly/2
            nodes=[nodes; (ly/2+1)*(i-lx/2-2)+j+153 (ly/2+1)*(i-lx/2-1)+j+153
(ly/2+1)*(i-lx/2-2)+j+154;];
            nodes=[nodes; (ly/2+1)*(i-lx/2-1)+j+153 (ly/2+1)*(i-lx/2-1)+j+154
(ly/2+1)*(i-lx/2-2)+j+154;];
        end
    end
end
bcdof=[];
bcval=[];
for i=1:ly+1
        bcdof=[bcdof 1+2*(i-1) 2+2*(i-1)];
        bcval=[bcval   0     0];
end
ff=sparse(sdof,1);              %system force vector
k=sparse(edof,edof);            %initialization of element matrix
kk=sparse(sdof,sdof);           %system matrix
disp=sparse(sdof,1);            %system displacement vector
eldisp=sparse(edof,1);          %element displacement vector
stress=zeros(nel,3);            %matrix containing stress components
strain=zeros(nel,3);            %matrix containing strain components
index=sparse(edof,1);           %index vector
kinmtx=sparse(3,edof);          %kinematic matrix
matmtx=sparse(3,3);         %constitutive matrix
matmtx=fematiso(1,emodule,poisson);       %constitutive matrice
for iel=1:nel           %loop for the total number of element
    nd(1)=nodes(iel,1);             %1st connected node for (iel)-th element
    nd(2)=nodes(iel,2);             %2nd connected node for (iel)-th element
    nd(3)=nodes(iel,3);             %3rd connected node for (iel)-th element
    x1=x0(nd(1),1); y1=x0(nd(1),2);        %coord values of 1st node
    x2=x0(nd(2),1); y2=x0(nd(2),2);        %coord values of 2nd node
    x3=x0(nd(3),1); y3=x0(nd(3),2);        %coord values of 3rd node
    index=feeldof(nd,nnel,ndof);          %extract system dofs for the element
```

```
    area=abs(0.5*(x1*y2+x2*y3+x3*y1-x1*y3-x2*y1-x3*y2)); %area of triangula
    area2=area*2;
     dhdx=(1/area2)*[(y2-y3) (y3-y1) (y1-y2)];          %derivatives w.r.t x
     dhdy=(1/area2)*[(x3-x2) (x1-x3) (x2-x1)];          %derivatives w.r.t y
    kinmtx2=fekine2d(nnel,dhdx,dhdy);                  %kinematic matrice
    k=kinmtx2'*matmtx*kinmtx2*area*h;                  %element stiffness matrice
    kk=feasmb_2(kk,k,index);                            %assemble element
matrics
end
kk1=kk;
%-----------------------------------------------------------------------
%     force vector
%-----------------------------------------------------------------------
ff(442,1)=fload;
ff(137,1)=2*fload;
%------------------------
%   apply boundary condition
%------------------------
[kk,ff]=feaplyc2(kk,ff,bcdof,bcval);
%-------------------------
%   solve the matrix equation
%-------------------------
%disp=kk\ff;
[LL UU]=lu(kk);
utemp=LL\ff;
disp=UU\utemp;
EU=0.5*disp'*kk1*disp;
%--------------------------------------
%   element stress computation
%--------------------------------------
energy=0;
Patch_xy = zeros(6,nel);
for ielp=1:nel                  % loop for the total number of elements
     nd(1)=nodes(ielp,1); % 1st connected node for (iel)-th element
     nd(2)=nodes(ielp,2); % 2nd connected node for (iel)-th element
     nd(3)=nodes(ielp,3); % 3rd connected node for (iel)-th element

    x1=x0(nd(1),1); y1=x0(nd(1),2);% coord values of 1st node
    x2=x0(nd(2),1); y2=x0(nd(2),2);% coord values of 2nd node
    x3=x0(nd(3),1); y3=x0(nd(3),2);% coord values of 3rd node
    Patch_xy(:,ielp)=[x1;x2;x3;y1;y2;y3];
    xcentre=(x1+x2+x3)/3; ycentre=(y1+y2+y3)/3;

    index=feeldof(nd,nnel,ndof);% extract system dofs associated with element
```

```
    %------------------------------------------------------
    %   extract element displacement vector
    %------------------------------------------------------
   for i=1:edof
        eldisp(i)=disp(index(i));
    end
    area=abs(0.5*(x1*y2+x2*y3+x3*y1-x1*y3-x2*y1-x3*y2));   % area of triangule
    area2=area*2;
    dhdx=(1/area2)*[(y2-y3) (y3-y1) (y1-y2)];   % derivatives w.r.t. x-axis
    dhdy=(1/area2)*[(x3-x2) (x1-x3) (x2-x1)];   % derivatives w.r.t. y-axis
    kinmtx2=fekine2d(nnel,dhdx,dhdy);              % compute kinematic matrix
    estrain=kinmtx2*eldisp;                   % compute strains
    estress=matmtx*estrain;                   % compute stresses
    for i=1:3
        strain(ielp,i)=estrain(i);              % store for each element
        stress(ielp,i)=estress(i);              % store for each element
    end
    energy=energy+0.5*estrain'*matmtx*estrain*area;
end
neigh_node = cell(nnode,1);
indneigh=zeros(1,nnode);
for i=1:nel
    for j=1:3
        indneigh(nodes(i,j))=indneigh(nodes(i,j))+1;
        neigh_node{nodes(i,j)}(indneigh(nodes(i,j)))=i;
    end
end
stress_node=zeros(3,nnode);
for inode=1:nnode
      numel= indneigh(inode);
      for i=1:numel
            ind_nel= neigh_node{inode}(i);
           for j=1:3
                 stress_node(j,inode)=stress_node(j,inode)+stress(ind_nel,j);
           end
    end
    stress_node(:,inode)=stress_node(:,inode)/numel;
end
%---------------------------------------------------------
% Data output
%---------------------------------------------------------
fid_out=fopen('result_beam02.plt','w');

fprintf(fid_out,'TITLE="test case governed by poisson equation"\n');
```

```
fprintf(fid_out,'VARIABLES="x" "y" "u" "v" "sigax"   "sigmay" "sigmaxy"\n');
fprintf(fid_out,'ZONE T="flow-field", N= %8d,E=%8d,ET=TRIANGLE,
F=FEPOINT\n',nnode,nel);
disp=full(disp);
for i=1:nnode
        fprintf(fid_out,'%16.6e%16.6e%16.6e%16.6e%16.6e%16.6e%16.6e\n',x0(i,1),x
0(i,2),disp(2*i-1),disp(2*i),stress_node(1,i),stress_node(2,i),stress_node(3,i));
end
for i=1:nel
       fprintf(fid_out,'%8d%8d%8d\n',nodes(i,1),nodes(i,2),nodes(i,3));
end
scatter(x0(:,1), x0(:,2),50,disp(1:2:end),"filled");
colorbar
clf
hold on
patch(Patch_xy(1:3,:), Patch_xy(4:6,:),stress.');
colorbar
```

Main 2

```
first_time=cputime;
format long
%-------------------------------------------
%input data for control parameters
%-------------------------------------------
lengthx=2;                  %length of x-axis side of problem
lengthy=2;                  %length of y-axis side of proble
h=0.2;                      %thickness of problem
emodule=2.1*10^11;                 %elastic modulus
poisson=0.3;                %Poisson's ratio
fload=-12500;                  % the total load
lx=64;                      % number of element in x-axis
ly=64;                       % number of element in y-axis
nel=2*0.75*lx*ly;                % number of element
nnel=3;                     %number of nodes per element
ndof=2;                     %number of dofs per node
nnode=(lx/2+1)*(ly+1)+(lx/2)*(ly/2+1);      %total number of nodes in system
sdof=nnode*ndof;            %total system dofs
edof=nnel*ndof;             %degrees of freedom per element
x0=[];
for i=1:lx+1
    if i<=(lx/2+1)
        for j=1:ly+1
            x0=[x0; (i-1)*lengthx/lx        -(j-1)*lengthy/ly];
```

```
            end
      else
            for j=1:(ly/2+1)
                  x0=[x0; (i-1)*lengthx/lx         -(j-1)*lengthy/ly];
            end
      end
end
nodes=[];
for i=1:lx
      if i<lx/2+1
            for j=1:ly
                  nodes=[nodes; (ly+1)*(i-1)+j (ly+1)*i+j (ly+1)*(i-1)+j+1;];
                  nodes=[nodes; (ly+1)*i+j (ly+1)*i+j+1 (ly+1)*(i-1)+j+1;];
            end
      elseif i==lx/2+1
                  for j=1:ly/2
                        nodes=[nodes; (ly+1)*(i-1)+j
(ly/2+1)*(i-lx/2-1)+j+2145   (ly+1)*(i-1)+j+1;];
                        nodes=[nodes; (ly/2+1)*(i-lx/2-1)+j+2145 (ly/2+1)*(i-lx/2-1)+j+2146
(ly+1)*(i-1)+j+1;];
                  end
      else
            for j=1:ly/2
                  nodes=[nodes; (ly/2+1)*(i-lx/2-2)+j+2145 (ly/2+1)*(i-lx/2-1)+j+2145
(ly/2+1)*(i-lx/2-2)+j+2146;];
                  nodes=[nodes; (ly/2+1)*(i-lx/2-1)+j+2145 (ly/2+1)*(i-lx/2-1)+j+2146
(ly/2+1)*(i-lx/2-2)+j+2146;];
            end
      end
end
bcdof=[];
bcval=[];
for i=1:ly+1
            bcdof=[bcdof 1+2*(i-1) 2+2*(i-1)];
            bcval=[bcval   0     0];
end
ff=sparse(sdof,1);              %system force vector
k=sparse(edof,edof);              %initialization of element matrix
kk=sparse(sdof,sdof);              %system matrix
disp=sparse(sdof,1);              %system displacement vector
eldisp=sparse(edof,1);            %element displacement vector
stress=zeros(nel,3);             %matrix containing stress components
strain=zeros(nel,3);             %matrix containing strain components
index=sparse(edof,1);              %index vector
```

```
kinmtx=sparse(3,edof);        %kinematic matrix
matmtx=sparse(3,3);       %constitutive matrix
matmtx=fematiso(1,emodule,poisson);       %constitutive matrice
for iel=1:nel          %loop for the total number of element
    nd(1)=nodes(iel,1);            %1st connected node for (iel)-th element
    nd(2)=nodes(iel,2);            %2nd connected node for (iel)-th element
    nd(3)=nodes(iel,3);            %3rd connected node for (iel)-th element
    x1=x0(nd(1),1); y1=x0(nd(1),2);       %coord values of 1st node
    x2=x0(nd(2),1); y2=x0(nd(2),2);       %coord values of 2nd node
    x3=x0(nd(3),1); y3=x0(nd(3),2);       %coord values of 3rd node
    index=feeldof(nd,nnel,ndof);          %extract system dofs for the element
    area=abs(0.5*(x1*y2+x2*y3+x3*y1-x1*y3-x2*y1-x3*y2)); %area of triangula
    area2=area*2;
     dhdx=(1/area2)*[(y2-y3) (y3-y1) (y1-y2)];          %derivatives w.r.t x
     dhdy=(1/area2)*[(x3-x2) (x1-x3) (x2-x1)];          %derivatives w.r.t y
    kinmtx2=fekine2d(nnel,dhdx,dhdy);                  %kinematic matrice
    k=kinmtx2'*matmtx*kinmtx2*area*h;                  %element stiffness matrice
    kk=feasmb_2(kk,k,index);                               %assemble element
matrics
end
kk1=kk;
%-----------------------------------------------------------------------
%     force vector
%-----------------------------------------------------------------------
ff(6258,1)=-fload;
%ff(5000,1)=2*fload;
%-----------------------
%   apply boundary condition
%-----------------------
[kk,ff]=feaplyc2(kk,ff,bcdof,bcval);
%------------------------
%   solve the matrix equation
%------------------------
%disp=kk\ff;
[LL UU]=lu(kk);
utemp=LL\ff;
disp=UU\utemp;
EU=0.5*disp'*kk1*disp;
%--------------------------------------
%   element stress computation
%--------------------------------------
energy=0;
Patch_xy = zeros(6,nel);
for ielp=1:nel               % loop for the total number of elements
```

```
     nd(1)=nodes(ielp,1); % 1st connected node for (iel)-th element
     nd(2)=nodes(ielp,2); % 2nd connected node for (iel)-th element
     nd(3)=nodes(ielp,3); % 3rd connected node for (iel)-th element

    x1=x0(nd(1),1); y1=x0(nd(1),2);% coord values of 1st node
    x2=x0(nd(2),1); y2=x0(nd(2),2);% coord values of 2nd node
    x3=x0(nd(3),1); y3=x0(nd(3),2);% coord values of 3rd node
    Patch_xy(:,ielp)=[x1;x2;x3;y1;y2;y3];
    xcentre=(x1+x2+x3)/3; ycentre=(y1+y2+y3)/3;

    index=feeldof(nd,nnel,ndof);% extract system dofs associated with element
    %-------------------------------------------------------
    %   extract element displacement vector
    %-------------------------------------------------------
   for i=1:edof
        eldisp(i)=disp(index(i));
    end
    area=abs(0.5*(x1*y2+x2*y3+x3*y1-x1*y3-x2*y1-x3*y2));   % area of triangule
    area2=area*2;
    dhdx=(1/area2)*[(y2-y3) (y3-y1) (y1-y2)];   % derivatives w.r.t. x-axis
    dhdy=(1/area2)*[(x3-x2) (x1-x3) (x2-x1)];   % derivatives w.r.t. y-axis
    kinmtx2=fekine2d(nnel,dhdx,dhdy);              % compute kinematic matrix
    estrain=kinmtx2*eldisp;                   % compute strains
    estress=matmtx*estrain;                   % compute stresses
    for i=1:3
        strain(ielp,i)=estrain(i);             % store for each element
        stress(ielp,i)=estress(i);             % store for each element
    end
    energy=energy+0.5*estrain'*matmtx*estrain*area;
end
neigh_node = cell(nnode,1);
indneigh=zeros(1,nnode);
for i=1:nel
    for j=1:3
        indneigh(nodes(i,j))=indneigh(nodes(i,j))+1;
        neigh_node{nodes(i,j)}(indneigh(nodes(i,j)))=i;
    end
end
stress_node=zeros(3,nnode);
for inode=1:nnode
      numel= indneigh(inode);
      for i=1:numel
            ind_nel= neigh_node{inode}(i);
          for j=1:3
```

```
                stress_node(j,inode)=stress_node(j,inode)+stress(ind_nel,j);
            end
      end
      stress_node(:,inode)=stress_node(:,inode)/numel;
end
%------------------------------------------------------------
% Data output
%------------------------------------------------------------
fid_out=fopen('result_beam01.plt','w');

fprintf(fid_out,'TITLE="test case governed by poisson equation"\n');
fprintf(fid_out,'VARIABLES="x" "y" "u" "v" "sigax"   "sigmay" "sigmaxy"\n');
fprintf(fid_out,'ZONE T="flow-field", N= %8d,E=%8d,ET=TRIANGLE,
F=FEPOINT\n',nnode,nel);
disp=full(disp);
for i=1:nnode
       fprintf(fid_out,'%16.6e%16.6e%16.6e%16.6e%16.6e%16.6e%16.6e\n',x0(i,1),x
0(i,2),disp(2*i-1),disp(2*i),stress_node(1,i),stress_node(2,i),stress_node(3,i));
end
for i=1:nel
      fprintf(fid_out,'%8d%8d%8d\n',nodes(i,1),nodes(i,2),nodes(i,3));
end
scatter(x0(:,1), x0(:,2),50,disp(1:2:end),"filled");
colorbar
clf
hold on
patch(Patch_xy(1:3,:), Patch_xy(4:6,:),stress.');
colorbar

function [kk,ff]=feaplyc2(kk,ff,bcdof,bcval)
 n=length(bcdof);
 sdof=size(kk);
 for i=1:n
      c=bcdof(i);
      for   j=1:sdof
            kk(c,j)=0;
      end
 kk(c,c)=1;
     ff(c)=bcval(i);
 end

function [kk]=feasmb_2(kk,k,index)
edof = length(index);
for i=1:edof
```

```
      ii=index(i);
      for j=1:edof
           jj=index(j);
           kk(ii,jj)=kk(ii,jj)+k(i,j);
      end
end

function [index]=feeldof(nd,nnel,ndof)
 k=0;
    for i=1:nnel
           start = (nd(i)-1)*ndof;
           for j=1:ndof
                    k=k+1;
                    index(k)=start+j;
         end
    end

function [kinmtx2]=fekine2d(nnel,dhdx,dhdy)
 for i=1:nnel
         i1=(i-1)*2+1;
         i2=i1+1;
         kinmtx2(1,i1)=dhdx(i);
         kinmtx2(2,i2)=dhdy(i);
         kinmtx2(3,i1)=dhdy(i);
         kinmtx2(3,i2)=dhdx(i);
 end

function [matmtrx]=fematiso(iopt,elastic,poisson)
if   iopt==1             % plane stress
 matmtrx= elastic/(1-poisson*poisson)* ...
      [1   poisson 0; ...
       poisson   1   0; ...
       0   0   (1-poisson)/2];
elseif     iopt==2            % plane strain
 matmtrx= elastic/((1+poisson)*(1-2*poisson))* ...
    [(1-poisson)   poisson 0;
    poisson   (1-poisson)   0;
    0   0   (1-2*poisson)/2];
 elseif   iopt==3        % axisymmetry
 matmtrx= elastic/((1+poisson)*(1-2*poisson))* ...
    [(1-poisson)   poisson   poisson   0;
    poisson   (1-poisson)     poisson   0;
    poisson   poisson   (1-poisson)     0;
    0      0      0     (1-2*poisson)/2];
```

```
elseif   iopt==4           % three-dimension
 matmtrx= elastic/((1+poisson)*(1-2*poisson))* ...
   [(1-poisson)   poisson   poisson    0    0     0;
   poisson   (1-poisson)    poisson    0    0     0;
   poisson   poisson   (1-poisson)     0    0     0;
   0     0     0     (1-2*poisson)/2    0     0;
   0     0     0     0     (1-2*poisson)/2    0;
   0     0     0     0     0    (1-2*poisson)/2];
end
return

first_time=cputime;
format long
%-------------------------------------------
%input data for control parameters
%-------------------------------------------
lengthx=4;                  %length of x-axis side of problem
lengthy=2;                  %length of y-axis side of proble
emodule=1.0;                  %elastic modulus
poisson=0.0;                %Poisson's ratio
fload=-1;                 % the total load
lx=16;                     % number of element in x-axis
ly=8;                      % number of element in y-axis
nel=2*lx*ly;               % number of element
nnel=3;                    %number of nodes per element
ndof=2;                    %number of dofs per node
nnode=(lx+1)*(ly+1);      %total number of nodes in system
sdof=nnode*ndof;          %total system dofs
edof=nnel*ndof;            %degrees of freedom per element
x0=[];
for i=1:lx+1
    for j=1:ly+1
          x0=[x0; (i-1)*lengthx/lx        -0.5*lengthy*(1+(lx+1-i)/lx)*(1-(j-1)/ly)];
    end
end
nodes=[];
for i=1:lx
    for j=1:ly
        nodes=[nodes; (ly+1)*(i-1)+j (ly+1)*i+j (ly+1)*(i-1)+j+1;];
        nodes=[nodes; (ly+1)*i+j (ly+1)*i+j+1 (ly+1)*(i-1)+j+1;];
    end
end
bcdof=[];
bcval=[];
```

```
for i=1:ly+1
          bcdof=[bcdof 1+2*(i-1) 2+2*(i-1)];
          bcval=[bcval   0     0];
end
ff=sparse(sdof,1);              %system force vector
k=sparse(edof,edof);            %initialization of element matrix
kk=sparse(sdof,sdof);           %system matrix
disp=sparse(sdof,1);            %system displacement vector
eldisp=sparse(edof,1);          %element displacement vector
stress=zeros(nel,3);            %matrix containing stress components
strain=zeros(nel,3);            %matrix containing strain components
index=sparse(edof,1);           %index vector
kinmtx=sparse(3,edof);          %kinematic matrix
matmtx=sparse(3,3);        %constitutive matrix
matmtx=fematiso(1,emodule,poisson);        %constitutive matrice
for iel=1:nel          %loop for the total number of element
    nd(1)=nodes(iel,1);             %1st connected node for (iel)-th element
    nd(2)=nodes(iel,2);             %2nd connected node for (iel)-th element
    nd(3)=nodes(iel,3);             %3rd connected node for (iel)-th element
    x1=x0(nd(1),1); y1=x0(nd(1),2);        %coord values of 1st node
    x2=x0(nd(2),1); y2=x0(nd(2),2);        %coord values of 2nd node
    x3=x0(nd(3),1); y3=x0(nd(3),2);        %coord values of 3rd node
    index=feeldof(nd,nnel,ndof);           %extract system dofs for the element
area=0.5*(x1*y2+x2*y3+x3*y1-x1*y3-x2*y1-x3*y2); %area of triangula
    area2=area*2;
     dhdx=(1/area2)*[(y2-y3) (y3-y1) (y1-y2)];          %derivatives w.r.t x
     dhdy=(1/area2)*[(x3-x2) (x1-x3) (x2-x1)];          %derivatives w.r.t y
    kinmtx2=fekine2d(nnel,dhdx,dhdy);                  %kinematic matrice
    k=kinmtx2'*matmtx*kinmtx2*area;                  %element stiffness matrice
    kk=feasmb_2(kk,k,index);                                 %assemble element
matrics
end
kk1=kk;
%----------------------------------------------------------------------
%     force vector
%----------------------------------------------------------------------
ff(sdof,1)=fload;
%------------------------
%   apply boundary condition
%------------------------
[kk,ff]=feaplyc2(kk,ff,bcdof,bcval);
%-------------------------
%   solve the matrix equation
%-------------------------
```

```
%disp=kk\ff;
[LL UU]=lu(kk);
utemp=LL\ff;
disp=UU\utemp;
EU=0.5*disp'*kk1*disp;
%--------------------------------------
%   element stress computation
%--------------------------------------
energy=0;
for ielp=1:nel                 % loop for the total number of elements
      nd(1)=nodes(ielp,1); % 1st connected node for (iel)-th element
      nd(2)=nodes(ielp,2); % 2nd connected node for (iel)-th element
      nd(3)=nodes(ielp,3); % 3rd connected node for (iel)-th element

     x1=x0(nd(1),1); y1=x0(nd(1),2);% coord values of 1st node
     x2=x0(nd(2),1); y2=x0(nd(2),2);% coord values of 2nd node
     x3=x0(nd(3),1); y3=x0(nd(3),2);% coord values of 3rd node

     xcentre=(x1+x2+x3)/3; ycentre=(y1+y2+y3)/3;

     index=feeldof(nd,nnel,ndof);% extract system dofs associated with element
     %-------------------------------------------------------
     %   extract element displacement vector
     %-------------------------------------------------------
    for i=1:edof
          eldisp(i)=disp(index(i));
     end
     area=0.5*(x1*y2+x2*y3+x3*y1-x1*y3-x2*y1-x3*y2);   % area of triangule
     area2=area*2;
     dhdx=(1/area2)*[(y2-y3) (y3-y1) (y1-y2)];   % derivatives w.r.t. x-axis
     dhdy=(1/area2)*[(x3-x2) (x1-x3) (x2-x1)];   % derivatives w.r.t. y-axis
     kinmtx2=fekine2d(nnel,dhdx,dhdy);              % compute kinematic matrix
     estrain=kinmtx2*eldisp;                   % compute strains
     estress=matmtx*estrain;                   % compute stresses
     for i=1:3
          strain(ielp,i)=estrain(i);              % store for each element
          stress(ielp,i)=estress(i);              % store for each element
     end
     energy=energy+0.5*estrain'*matmtx*estrain*area;
end
neigh_node = cell(nnode,1);
indneigh=zeros(1,nnode);
for i=1:nel
     for j=1:3
```

```
            indneigh(nodes(i,j))=indneigh(nodes(i,j))+1;
            neigh_node{nodes(i,j)}(indneigh(nodes(i,j)))=i;
      end
end
stress_node=zeros(3,nnode);
for inode=1:nnode
         numel= indneigh(inode);
         for i=1:numel
                  ind_nel= neigh_node{inode}(i);
                for j=1:3
                         stress_node(j,inode)=stress_node(j,inode)+stress(ind_nel,j);
                end
      end
      stress_node(:,inode)=stress_node(:,inode)/numel;
end
%------------------------------------------------------------
% Data output
%------------------------------------------------------------
fid_out=fopen('result_beam01.plt','w');

fprintf(fid_out,'TITLE="test case governed by poisson equation"\n');
fprintf(fid_out,'VARIABLES="x" "y" "u" "v" "sigax"   "sigmay" "sigmaxy"\n');
fprintf(fid_out,'ZONE T="flow-field", N= %8d,E=%8d,ET=TRIANGLE,
F=FEPOINT\n',nnode,nel);
disp=full(disp);
for i=1:nnode
          fprintf(fid_out,'%16.6e%16.6e%16.6e%16.6e%16.6e%16.6e%16.6e\n',x0(i,1),x
0(i,2),disp(2*i-1),disp(2*i),stress_node(1,i),stress_node(2,i),stress_node(3,i));
end
for i=1:nel
         fprintf(fid_out,'%8d%8d%8d\n',nodes(i,1),nodes(i,2),nodes(i,3));
end
```

2. 线性四面体元

Main

```
clear
clc
format short
lengthx=2;                    %length of x-axis side of problem
lengthy=2;                    %length of y-axis side of proble
lengthz=2;
%h=0.2;                        %thickness of problem
elastic=2.1*10^11;                  %elastic modulus
poisson=0.3;                 %Poisson's ratio
iopt=4;
Nodes = load("node.txt"); %total number of nodes in system
Load = load("load.txt")';
bdn = load("boundary.txt")';
elements = load("element.txt");
nnode=length(Nodes);%total number of nodes in system
nel=length(elements);% number of element
fload=[0,1250,0];                  % the total load
%lx=16;                         % number of element in x-axis
%ly=16;                           % number of element in y-axis
%lz=16;
%nel=5*0.75*lx*ly*lz;                   % number of element
nnel=8;                       %number of nodes per element
ndof=3;                       %number of dofs per node
%nnode=(lx/2+1)*(ly+1)*(lz+1)+(lx/2)*(ly+1)*(lz/2+1);       %total number of nodes
in system
sdof=nnode*ndof;              %total system dofs
edof=nnel*ndof;              %degrees of freedom per element
B=zeros(6,12,nel);
K=zeros(sdof,sdof);

%Assemble tetrahedral units
for i=1:nel
      element_nodes_index = elements(i,2:5);
      four_nodes_matrix = Nodes(element_nodes_index,2:end);
      x1=four_nodes_matrix(1,1);
      y1=four_nodes_matrix(1,2);
      z1=four_nodes_matrix(1,3);
      x2=four_nodes_matrix(2,1);
      y2=four_nodes_matrix(2,2);
      z2=four_nodes_matrix(2,3);
      x3=four_nodes_matrix(3,1);
      y3=four_nodes_matrix(3,2);
```

```
        z3=four_nodes_matrix(3,3);
        x4=four_nodes_matrix(4,1);
        y4=four_nodes_matrix(4,2);
        z4=four_nodes_matrix(4,3);
        [k,
b,D]=TetrahedronElementStiffness(elastic,poisson,x1,y1,z1,x2,y2,z2,x3,y3,z3,x4,y4,z
4);
        B(:,:,i)= b;
        for j = 1:4
                row_ = element_nodes_index(j);
                for m = 1:4
                        col_ = element_nodes_index(m);
                        K((3*row_-2):row_*3,(3*col_-2):col_*3) = ...
                                K((3*row_-2):row_*3,(3*col_-2):col_*3)...
                                + k((3*j-2):3*j,(3*m-2):3*m);
                end
        end
end
F = zeros(sdof,1);
nLoad = length(Load);
for i = 1 :ndof
        F((Load-1)*ndof+i,1) = fload(i)/nLoad;
end
Constrain=constrain(bdn,ndof);
K_constrain = K;
F_constrain = F;
K_constrain(Constrain,:) = [];
F_constrain(Constrain,:) = [];
K_constrain(:,Constrain) = [];
U_ = K_constrain\F_constrain;
%Return node displacement
U=nodedisplacement(U_,bdn);
%Post-processing calculation of cell node stress
snodes = zeros(nnode, 10);
xd = U(1:3:end);
yd = U(2:3:end);
zd = U(3:3:end);
for i = 1:nel
        elements_nodes_index = elements(i,2:5);
        u = zeros(12,1);
        for index = 1:4
                u((index-1)*3+1:index*3,:) = [xd(elements_nodes_index(index));...
                                                yd(elements_nodes_index(index));...
                                                zd(elements_nodes_index(index));];
```

```
        end
        sigma = (D*B(:,:,i)*u)';
        for index = 1:4
            node_index = elements_nodes_index(index);
            snodes(node_index,1:6) = snodes(node_index,1:6) + sigma;
            snodes(node_index, 10) = snodes(node_index, 10) +1;
        end
end
snodes(:,1:6) = snodes(:,1:6)./snodes(:,10);
for i = 1:nnode
        sigma = snodes(i,1:6);
        snodes(i,7:9) =TetrahedronElementPStresses(sigma);
end
figure1 = figure();
hold on;
%figure2 = figure();
scatter3(Nodes(:,2),Nodes(:,3),Nodes(:,4),20,snodes(:,1),'fill');
%figure3 = figure();
scatter3(Nodes(:,2),Nodes(:,3),Nodes(:,4),20,snodes(:,2),'fill');
%figure4 = figure();
scatter3(Nodes(:,2),Nodes(:,3),Nodes(:,4),20,snodes(:,3),'fill');
tetramesh(elements(:,2:5), Nodes(:,2:4));
hold on
%figure5 = figure();
scatter3(Nodes(:,2),Nodes(:,3),Nodes(:,4),"g.");
axis equal;
%绘制边界节点
Boundary_nodes_coor = Nodes(bdn,2:4);
scatter3(Boundary_nodes_coor(:,1),Boundary_nodes_coor(:,2),Boundary_nodes_coor
(:,3),"black*");
%绘制加载节点
Load_nodes_coor = Nodes(Load,2:4);
scatter3(Load_nodes_coor(:,1),Load_nodes_coor(:,2),Load_nodes_coor(:,3),"yo");
view([45,45])

function[K,B,D]=TetrahedronElementStiffness(E,NU,x1,y1,z1,x2,y2,z2,x3,y3,z3,x4,
y4,z4)
xyz=[1 x1 y1 z1;1 x2 y2 z2;1 x3 y3 z3;1 x4 y4 z4];
V=det(xyz)/6;
mbeta1=[1 y2 z2;1 y3 z3;1 y4 z4];
mbeta2=[1 y1 z1;1 y3 z3;1 y4 z4];
mbeta3=[1 y1 z1;1 y2 z2;1 y4 z4];
mbeta4=[1 y1 z1;1 y2 z2;1 y3 z3];
mgamma1=[1 x2 z2;1 x3 z3;1 x4 z4];
```

```
mgamma2=[1 x1 z1;1 x3 z3;1 x4 z4];
mgamma3=[1 x1 z1;1 x2 z2;1 x4 z4];
mgamma4=[1 x1 z1;1 x2 z2;1 x3 z3];
mdelta1=[1 x2 y2;1 x3 y3;1 x4 y4];
mdelta2=[1 x1 y1;1 x3 y3;1 x4 y4];
mdelta3=[1 x1 y1;1 x2 y2;1 x4 y4];
mdelta4=[1 x1 y1;1 x2 y2;1 x3 y3];
beta1=-1*det(mbeta1);
beta2=det(mbeta2);
beta3=-1*det(mbeta3);
beta4=det(mbeta4);
gamma1=det(mgamma1);
gamma2=-1*det(mgamma2);
gamma3=det(mgamma3);
gamma4=-1*det(mgamma4);
delta1=-1*det(mdelta1);
delta2=det(mdelta2);
delta3=-1*det(mdelta3);
delta4=det(mdelta4);
B1=[beta1 0 0;0 gamma1 0;0 0 delta1;gamma1 beta1 0;0 delta1 gamma1;delta1 0
beta1];
B2=[beta2 0 0;0 gamma2 0;0 0 delta2;gamma2 beta2 0;0 delta2 gamma2;delta2 0
beta2];
B3=[beta3 0 0;0 gamma3 0;0 0 delta3;gamma3 beta3 0;0 delta3 gamma3;delta3 0
beta3];
B4=[beta4 0 0;0 gamma4 0;0 0 delta4;gamma4 beta4 0;0 delta4 gamma4;delta4 0
beta4];
B=[B1 B2 B3 B4]/(6*V);
D=(E/((1+NU)*(1-2*NU)))*[1-NU NU NU 0 0 0;NU 1-NU NU 0 0 0;NU NU 1-NU
0 0 0;0 0 0 (1-2*NU)/2 0 0;0 0 0 0 (1-2*NU)/2 0;0 0 0 0 0 (1-2*NU)/2];
K=V*B'*D*B;

function y=TetrahedronElementPStresses(sigma)
s1=sigma(1)+sigma(2)+sigma(5);
s2=sigma(1)*sigma(2)+sigma(1)*sigma(3)+sigma(2)*sigma(3)-sigma(4)*sigma(4)-si
gma(5)*sigma(5)-sigma(6)*sigma(6);
ms3=[sigma(1) sigma(4) sigma(6);sigma(4) sigma(2) sigma(5);sigma(6) sigma(5)
sigma(3)];
s3=det(ms3);
y=[s1;s2;s3];

function U=nodedisplacement(U_,Boundary_nodes)
U = U_;
for i = 1:length(Boundary_nodes)
```

```
    index = Boundary_nodes(i);
    forward_ = U(1:(index-1)*3,:);
    backward_ = U((index-1)*3+1:end,:);
    U = [forward_;0;0;0;backward_];
end

function cons=constrain(Boundary_nodes,ndof)
con_dofs = zeros(length(Boundary_nodes),3);
for i = 1:ndof
    con_dofs(:,i) = (Boundary_nodes-1)*ndof+i;
end
switch ndof
    case 1
        cons = con_dofs(:,1);
    case 2
        cons = [con_dofs(:,1);con_dofs(:,2)];
    case 3
        cons = [con_dofs(:,1);con_dofs(:,2);con_dofs(:,3)];
end
```