PETROLEUM SOCIETY
CANADIAN INSTITUTE OF MINING, METALLURGY & PETROLEUM

PAPER 2006-048

CANADIAN INTERNATIONAL
PETROLEUM CONFERENCE

A Comprehensive Neural Network Model for Predicting Two-Phase Liquid Holdup Under Various Angles of Pipe Inclinations

M. MOHAMMADI
Petroleum University of Technology, Iran

This paper is to be presented at the Petroleum Society's 7th Canadian International Petroleum Conference (57th Annual Technical Meeting), Calgary, Alberta, Canada, June 13 – 15, 2006. Discussion of this paper is invited and may be presented at the meeting if filed in writing with the technical program chairman prior to the conclusion of the meeting. This paper and any discussion filed will be considered for publication in Petroleum Society journals. Publication rights are reserved. This is a pre-print and subject to correction.

Abstract

Accurate prediction of liquid holdup associated with multiphase flow is a critical element in the design and operation of modern production systems. This prediction is made difficult by the complexity of the distribution of the phases and the wide range of fluid properties encountered in production operations. Consequently, the performance of existing correlations is often inadequate in terms of desired accuracy and range of application. This investigation focuses on the development of a neural network model, a relatively new approach that has been successfully applied to a variety of complex engineering problems. 2292 data sets from five independent sources were used to develop a neural network model for predicting liquid holdup in two-phase flow at all inclinations from upward(+90 degrees) to downward(-90 degrees) flow. A three-layer back-propagation neural network has utilized. Seven parameters including inclination from horizontal, pipe diameter, gas and liquid superficial velocity, liquid viscosity, density and surface tension are used as inputs to the network. A detailed comparison with some empirical correlations which are applicable for whole range of inclinations reveals that the developed model provides better accuracy and predicts liquid holdup in terms of the lowest absolute average percent error (15.31), the lowest standard deviation (30.15) and the highest correlation coefficient (0.9962).

Introduction

Multiphase flow is defined as the concurrent flow of two or more phases, liquid, solid or gas, where the motion influences the interface between the phases. The prediction of liquid holdup in pipeline is very important to the petroleum industry. Liquid holdup, which is defined as the fraction of pipe occupied by liquid, must be predicted to properly design separation equipment and slug catchers in pipeline operations. Many correlations have been published for predicting this important parameter. The commonly used correlations are those of Eaton et al.(1), Guzhov et al.(2), Beggs and Brill(3), Minami and Brill(4), Gregory et al.(5), Mukherjee and Brill(6), Hughmark and Pressburg(7), Hughmark(8), Abdul-Majeed(9)(10), Xiao et al.(11), Baker et al.(12) and Gomez et al.(13). Some are very general while others only apply to a narrow range of conditions. Each of those correlations was developed for a specific orientation (vertical, horizontal or inclined upward or downward) and can't predict liquid holdup for other orientations correctly.

Many of these approaches begin with a prediction of flow pattern, with each flow pattern having an associated method of predicting liquid holdup. The liquid holdup prediction is used to determine a two-phase friction factor from which a pressure gradient is calculated. One of the problems with this approach is that it is dependent on the accuracy of flow pattern predictions and is subject to discontinuities in predictions made across flow pattern transition boundaries.(14) Comparative studies(15)(16) have shown that these models perform inconsistency as flow conditions change. This limitation makes difficult the task of selecting the most appropriate flow correlation.

Artificial neural networks (ANN) are parallel-distributed information processing models that can recognize highly complex patterns within available data. The recent development and success of applying ANNs to solve various difficult engineering problems has drawn the attention to its potential applications in the petroleum industry. The use of artificial intelligence in petroleum industry can be tracked back just almost twenty years.(17) The use of ANNs in solving many petroleum industry problems was reported in the literature by several authors. Recently, ANN has been applied in the multiphase flow area and achieved promising results compared with the conventional methods (correlations and mechanistic models). With regard to this field, a few researchers applied ANN technique to resolve some problems associated with multiphase flow problems including pressure drop(18), flow patterns identification(19)(20)(21) and liquid holdup.(22) Experiences show that empirical correlations and mechanistic models failed to provide a satisfactory and a reliable tool for estimating liquid holdup in multiphase flow.

Ternyik et al.(19) utilized neural networks to predict holdup and flow pattern in pipes under various angles of inclinations. They used a Kohonen type network to classify the four different correlations for flow patterns with all input data. The resulting classifications from the output layer in binary form were used as data input to a three-layer back-propagation neural network for predicting holdup. Holdup values predicted by the neural network had a value of 0.945 for the square of the correlation coefficient. While the results were encouraging, the complexity of the model (37 hidden nodes) relative to the number of training cases and the relatively small number of test cases (10%) may limit generalization. However, it is not possible to confirm this as the individual training/verification/testing errors were not reported and the neural network models were not evaluated with independent data sets.(22)

Another study done by Shippen and Scott(22) predicts liquid holdup for horizontal flow using a back-propagation neural network with eight hidden neurons which considers no-slip liquid holdup, diameter, gas and liquid superficial velocity, liquid viscosity, density and surface tension as input parameters. Holdup values predicted by their neural network using all data set had a value of 0.985 for the correlation coefficient. Osman(20) developed another three-layer back-propagation neural network which predicts liquid holdup for horizontal flow with a correlation coefficient of 0.9896. Although he has gained better results against conventional method but only four parameters (gas and liquid superficial velocity, pressure and temperature) were used as input to the network and lower range of liquid holdup was considered.

Present paper utilizes an artificial neural networks model for predicting the liquid holdup in gas-liquid two-phase flow using five independent sets of experimental data. This model is independent of flow pattern determination and uses an individual method for all conditions. Specific advantage of this model is covering all various angles of inclinations with wide range of flow and pipe conditions, simultaneously.

Artificial Neural Networks

Artificial neural networks are computing tools composed of many simple interconnected elements called neurons. These neurons are inspired by the biological nervous system. Neural networks have been trained to perform complex problems in various fields of applications which include visual processing, control systems, and speech synthesis. A neuron performs two simple tasks: (1) a weighted summation of its input array and (2) the application of a sigmoid function (S-shaped) to this summation to give an output which can serve as input to other neurons. An ANN has an input layer, an output layer, and one or more hidden layers. The input layer contains an array of variables into which the input data of the system is read from an external source. Similarly, the predicted data or results, which can be multiple vectors, are written in the output layer. Initially, the input layer receives the input and passes it to the first hidden layer for processing. The processed information from the first hidden layer is then passed to the other hidden layers for processing. Finally, the output layer receives information from the last hidden layer and sends the results to an external source. All the hidden layers have no direct connections to the outside world and the entire processing step is hidden from us.

The operations of an ANN are divided in two phases; training or ″learning″ phase and a cross-validation phase. The finding of set of suitable weights which minimizes the error between the predicted and the actual output is called the training of the network. The values of these weights are first set by using a random number generator. During training, the network error is computed using some suitable training algorithm and the weights of all the interconnections between neurons are adjusted based on the magnitude of the error and a parameter called the learning rate until the ANN ″learns″ the correct input-output behavior. The training phase is a time-consuming process, and may take days of computer time to obtain an adequate input-output performance. ANNs have many training algorithms. A common case is the back-propagation algorithm with momentum which is used here. The momentum procedure is used in order to help the search process not to get stuck in local minima. The fitting procedure from which the weights are determined has been performed using a least-squared minimization routine. In this routine, the sum of the square of the errors between the calculated and the experimental data is to be minimized. In general, the back-propagation uses the following steps:

(1) Enter a specific input, and calculate its corresponding output.
(2) If the sum-squared-error between this output and the desired output is acceptable, then stop.
(3) If step (2) is not true, minimize the error by adjusting the weights between the neurons in the following manner:
    (a) Begin at the output nodes and adjust their weights.
    (b) Propagate backward to the layer adjacent to the output layer by calculating errors and adjusting weights.
    (c) Continue going backward until all errors are calculated and weights are adjusted.

It should be noted that even though a good fit of a large number of data is obtained, there is no guaranty that the model can successfully predict data not contained in the learning data set. This is why after the training phase is completed; the cross-validation phase is required to ensure the accuracy and the generalization of the model. This phase is divided in two steps. First the ANN is subjected to data not seen during the training phase, and hopefully the output prediction by the ANN is acceptable. The most successful ANN architecture is the one which minimizes the prediction error on a data set for which it was not trained. Second, the ANN is subjected to intermediate data points seen during the training phase to insure that oscillation or ″over-fitting″ did not occur. The over-fitting problem occurs due to the use of more hidden layers than actually needed.

Data Acquisition and Analysis

This paper presents an artificial neural network model for prediction of the Liquid Holdup in multiphase flow at all ranges of pipe inclinations from minus 90 degrees to plus 90 degrees. The model was developed and tested using 2292 data sets collected from various study and experimental work covering a wide range of variables including inclination from horizontal, pipe diameter, gas and liquid superficial velocity, liquid viscosity, density and surface tension. Table 1 summarized the source experimental data conditions. These data sets were divided into training, cross validation and testing sets in the ratio of 4:1:1 (suggested by Haykin[(23)]).

All data sets were used to develop the network model. Each set of data collected from an individual study, randomly divided into three different groups: training, validation and testing so every group includes data from all study at various ranges. The training set is used to develop and adjust the weights and biases in the network; the validation set is used to ensure the generalization of the developed network during the training phase, and the testing set is used to examine the final performance of the network and compare the model performance with the correlations of Beggs and Brill[(3)] and Mukherjee and Brill.[(6)] These two correlations have selected because of their ability to predict liquid holdup at every range of inclinations. Statistical description of training, validation and testing data are given in Table 2 and Table 3, respectively. Table 3 shows a comparison of the errors based on training, validation and test cases. While the correlation coefficient is slightly lower for the test case, the overall error analysis among the cases shows comparable performance, indicating that the neural network did not suffer from over-fitting.

Liquid Holdup Network Architecture

The developed ANN model utilizes a multiple-layer feed forward network, which was selected due to its capabilities of representing non-linear functional mappings between inputs and outputs. The first layer consists of seven neurons representing the input parameters, the second (hidden) layer consists of ten neurons, and the third layer contains one neuron that represents the output value of the liquid holdup. A simplified schematic of the neural network is illustrated in Fig. 1. A logarithmic sigmoid function was used as transfer function. This topology is achieved after a series of optimization processes by monitoring the performance of the network until the best network structure was accomplished. The resulting weights and biases for the neural network are given in Table 4.

Results and Discussion

When neural network trained with train data group, the network performance was confirmed by introducing the validation data group to the network. After that, the neural network model becomes ready for testing and evaluation. To perform this, the test data group (382 data sets), which was not seen by the neural network during training, was used. To compare the performance and accuracy of the new models with other empirical correlations, two correlations including Beggs and Brill[(3)] and Mukherjee and Brill[(6)] were used. The statistical results of the comparison are given in Table 5. Several statistical parameters were used in the evaluation, namely, average percent error, APE; absolute average percent error, AAPE; average root mean square error, ARMS; the correlation coefficient; R and standard deviation, SD. Equations describing these error parameters are given in Appendix A.

Figures 2 through Figure 4 illustrate scatter diagrams of the estimated versus experimental liquid holdup values. These cross plots indicates the degree of agreement between the experimental and the estimated values. If the agreement is perfect, then all points should lie on the 45 degrees line on the plot. Figure 4 shows the tightest cloud of points around the 45 degrees line indicating an excellent agreement between the experimental and the calculated data. These results verified the success of neural networks to recognize the implicit relationships between input and output variables.

It should be noted that the excellent results predicted by the other two correlations may partially be due to the fact that the major portion of data used in developing and testing the network and also evaluating the performance of those correlations was used by respectful authors in developing their correlations. Despite this, the developed neural network model outperforms all the published correlations in terms of the lowest absolute average percent error (15.31), the lowest average root mean square error (0.0260), the highest correlation coefficient (0.9962) and finally, the lowest standard deviation (30.15).

Conclusion

1. A thee-layer back-propagation neural network model was presented for the prediction of the liquid holdup in gas-liquid two-phase flow which outperforms other empirical correlations, satisfactorily.
2. The present study clearly demonstrates the power of artificial neural network model in solving complicated engineering problems. The developed model could perform even better if more data were used for training.
3. The newly developed model can be used only within the range of used data. Caution should be taken beyond the range of used input variables. By broadening the range of input variables, developed model can adapt to make more accurate predictions over a wider range of conditions.
4. The developed model is independent from flow pattern determination and uses an individual method for all conditions.
5. As a future work, a sensitive analysis can be performed to determine the effect of each input variable on predicting the corresponding liquid holdup.

Acknowledgement

The authors would like to express their special thanks to the Gas Engineering Dept. at the Petroleum University of Technology, Ahwaz, Iran for their continuous support during the course of this project. I would also like to express sincere appreciation to Dr. M. Khoshbakht for his assistance and suggestions.

NOMENCLATURE

AAPE	=	average absolute percent relative error
APE	=	average percent relative error
ARMS	=	average root mean square error

d	=	pipe diameter, in.
e	=	error
E	=	relative error
H_L	=	liquid holdup, dimensionless
n	=	number of data
R	=	correlation coefficient
SD	=	standard deviation
V_{SG}	=	superficial gas velocity, ft/s
V_{SL}	=	superficial liquid velocity, ft/s
Greek Letters		
μ	=	viscosity, cp
ρ	=	density, lbm/ft^3
σ	=	surface tension, dynes/cm
Θ	=	inclination from horizontal, degree
Subscripts		
est	=	estimated
exp	=	experimental
L	=	liquid

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Appendix A; Statistical Error Parameters

Some criteria for comparing the performance and accuracy of the neural network model with other empirical correlations are used. The equations of statistical parameters used for comparison are given below:

1. Average Percent Relative Error: (APE)

$$APE = \frac{1}{n}\sum_{i=1}^{n} E_i \quad \text{(A-1)}$$

It is defined as the relative deviation from the experimental data; Where E is defined as follows:

$$E_i = \left[\frac{H_{L\exp} - H_{Lest}}{H_{L\exp}} \times 100\right]_i \quad i = 1,2,3,\ldots n \quad \text{(A-2)}$$

2. Average Absolute Percent Relative Error: (AAPE)

It measures the relative absolute deviation from the experimental values.

$$AAPE = \frac{1}{n}\sum_{i=1}^{n} |E_i| \quad \text{(A-3)}$$

3. Average Root Mean Square (ARMS) Error:

$$ARMS = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e_i^{\,2}} \quad \text{(A-4)}$$

ARMS measures the data dispersion around zero deviation, in which; e is the difference between experimental and estimated data:

$$e_i = \left[H_{L\exp} - H_{Lest}\right]_i \quad \text{(A-5)}$$

4. Correlation Coefficient: (R)

This represents the degree of success in reducing the standard deviation by regression analysis.

$$R = \frac{\sum_{i=1}^{n}\left(\left(H_{L\exp}\right)_i - \overline{H}_{L\exp}\right) \times \left(\left(H_{Lest}\right)_i - \overline{H}_{Lest}\right)}{\sqrt{\sum_{i=1}^{n}\left[\left(H_{L\exp}\right)_i - \overline{H}_{L\exp}\right]^2 \times \sum_{i=1}^{n}\left[\left(H_{Lest}\right)_i - \overline{H}_{Lest}\right]^2}} \quad \text{(A-6)}$$

Mean or average liquid holdup is defined as follows:

$$\overline{H}_L = \frac{1}{n}\sum_{i=1}^{n}\left(H_L\right)_i \quad \text{(A-7)}$$

5. Standard Deviation: (SD)

The standard deviation is a measurement of how widely values are dispersed from the average value (the mean) and is based on the entire population. A lower value of standard deviation indicates a smaller degree of scatter.

$$SD = \left[\frac{n\sum_{i=1}^{n} E_i^2 - \left(\sum_{i=1}^{n} E_i\right)^2}{n^2}\right]^{\frac{1}{2}} \quad \text{(A-8)}$$

The ARMS error is a measurement of scatter or lack of precision. It is relatively insensitive to errors in low ranges of liquid holdup and is strongly affected by errors in the high range. The APE is a measurement of the centering or average accuracy of the predictions, but is more strongly influenced by errors in the low range of liquid holdup. Like the ARMS error, the AAPE is also a measurement of the lack of precision; however it is very sensitive to errors associated with small measured values of liquid holdup.

The correlation coefficient, R, a more complex statistical parameter, is used to evaluate the degree of linear relationship between the measured and estimated holdup. A value of 1 indicates a perfect linear relationship, whereas a value of 0 means there is absolutely no linear dependence.

Investigator(s)	No. Tests	Pipe Diameter(in.)	Gas Superficial Velocity (ft/s)	Liquid Superficial Velocity (ft/s)	Orientation (degree)
Beggs[24]	584	1 and 1.5	0.64 to 160	0.007 to 5.98	-90 to +90
Minami and Brill[4]	600	1.5	0.01 to 100	0.094 to 12	-90 to +90
	150	1.5	0.01 to 100	0.094 to 12	-90 to +90
Mukherjee[25]	57	3.068	1.77 to 54.43	0.02 to 3.12	0
	54	3.068	1.56 to 49.13	0.02 to 2.92	0
Eaton[26]	238	2 and 4	0.8 to 73.37	0.036 to 6.92	0
Hideaki[27]	236	0.823 to 1.98	0.026 to 0.40	1.3 to 3.38	+90
Minagawa[28]	373	0.823 to 1.98	0.725 to 2.897	1.332 to 2.346	+90
All Sets	2292	0.823 to 4	0.01 to 160	0.007 to 12	-90 to +90
Investigator(s)	**Fluids (liquid/gas)**	**Liquid Viscosity (cp)**	**Liquid Surface Tension (Dynes/cm)**	**Liquid Density(lbm/ft^3)**	**Measured Liquid Holdup**
Beggs[24]	Water-air	0.89 to 1.60	68.45 to 70.63	62.1 to 62.6	0.017 to 0.856
Minami and Brill[4]	Kerosene-air	1.34 to 1.99	26 to 28.33	51.73 to 52.62	0.009 to 0.4354
	Water-air	0.58 to 0.92	68.31 to 72.10	62.4	0.008 to 0.4515
Mukherjee[25]	Kerosene-air	0.92 to 2.05	22.62 to 26.12	49.1 to 51.1	0.02 to 0.92
	Lube oil-air	20.2 to 44.4	33.72 to 37.51	52.6 to 54.1	0.03 to 0.99
Eaton[26]	Water-gas	0.71 to 1.33	61.56 to 66.55	62.9 to 63.6	0.006 to 0.732
Hideaki[27]	Water-air	0.909 to 1.34	72.1 to 74.4	62.241 to 62.428	0.865 to 0.9897
Minagawa[28]	Water-air	0.80 to 1.45	71.2 to 74.9	62.178 to 62.428	0.446 to 0. 824
All Sets		0.58 to 44.4	22.62 to 74.9	49.1 to 63.6	0.006 to 0.99

TABLE 1. Summary of used experimental data.

	Training Data (1528 sets)			Validation Data (382 sets)			Testing Data (382 sets)		
Property	Min	Max	Mean	Min	Max	Mean	Min	Max	Mean
Diameter, in.	0.823	4	1.54	0.823	4	1.72	0.823	4	1.70
Gas superficial velocity, ft/s	0.026	160	42.92	0.055	159.69	42.38	0.056	159.19	41.98
Liquid superficial velocity, ft/s	0.017	12	3.72	0.018	12	3.65	0.007	11.91	3.71
Liquid viscosity, cp	0.58	43.93	3.22	0.63	44.4	3.35	0.66	42.15	3.46
Liquid surface tension, dynes/cm	22.62	74.9	54.74	22.63	74.4	54.73	22.63	74.9	55.11
Liquid density, lbm/ft^3	49.1	63.6	58.40	49.18	63.6	58.38	49.1	63.57	58.36
Angle from horizontal, degree	-90	+90	23.21	-89	90	24.17	-89	90	23.57
Liquid holdup	0.006	0.989	0.346	0.009	0.987	0.348	0.013	0.983	0.358

TABLE 2. Statistical analysis of used data sets.

	Error parameter				
Data set	**APE**	**AAPE**	**ARMS**	**R**	**SD**
Training (1528 sets)	-9.42	16.42	0.0235	0.9970	30.30
Validation (382 sets)	-6.09	14.69	0.0317	0.9946	30.38
Test (382 sets)	-4.44	11.50	0.0305	0.9946	29.34
All data (2292 sets)	-8.03	15.31	0.0260	0.9962	30.15

TABLE 3. Statistical description of neural network performance.

	d	V_{SG}	V_{SL}	μ_L	σ_L	ρ_L	Θ	Bias	H_L
Hidden 1	-0.0160	-0.0440	-0.0477	-0.3462	0.0424	-0.1949	0.0689	8.1874	3.0325
Hidden 2	-0.0020	0.0049	-0.2551	0.4253	-0.0132	0.1317	-0.0086	-6.9857	-2.9264
Hidden 3	-0.0041	-2.5488	-0.0395	-0.4630	0.0019	0.0026	0.0365	-1.6980	5.2101
Hidden 4	0.0099	-0.0581	-0.0146	0.9766	-0.2011	0.0661	-0.0314	-3.3979	1.8537
Hidden 5	-0.0040	-0.0007	-1.6799	0.0132	0.0136	-0.2504	0.0717	10.7400	-2.8257
Hidden 6	0.0350	0.0365	0.0697	0.5622	-0.0039	0.2179	-0.0455	-10.4870	1.6088
Hidden 7	0.0829	-0.1011	-0.6377	1.2193	0.1230	-0.1221	-0.1259	0.7433	0.5882
Hidden 8	-0.0056	0.0589	-0.4616	0.0675	-0.1438	0.3572	-0.2289	-16.7280	-0.6771
Hidden 9	-0.0106	-0.0081	-0.0234	-0.3850	0.0039	0.1626	-0.0065	-5.9071	2.1947
Hidden 10	0.0042	-0.3919	0.0006	0.4049	0.0075	0.1809	-0.0440	-9.3709	3.4167
								Bias	-3.9882

TABLE 4. Liquid holdup network weights and biases.

Method	Error parameter				
	APE	AAPE	ARMS	R	SD
Beggs and Brill[3]	-4.34	20.04	0.0847	0.9755	31.60
Mukherjee and Brill[6]	-4.10	21.32	0.0631	0.9777	28.23
Neural network	-8.03	15.31	0.0260	0.9962	30.15

TABLE 5. Statistical comparison of neural network method performance with empirical correlations.

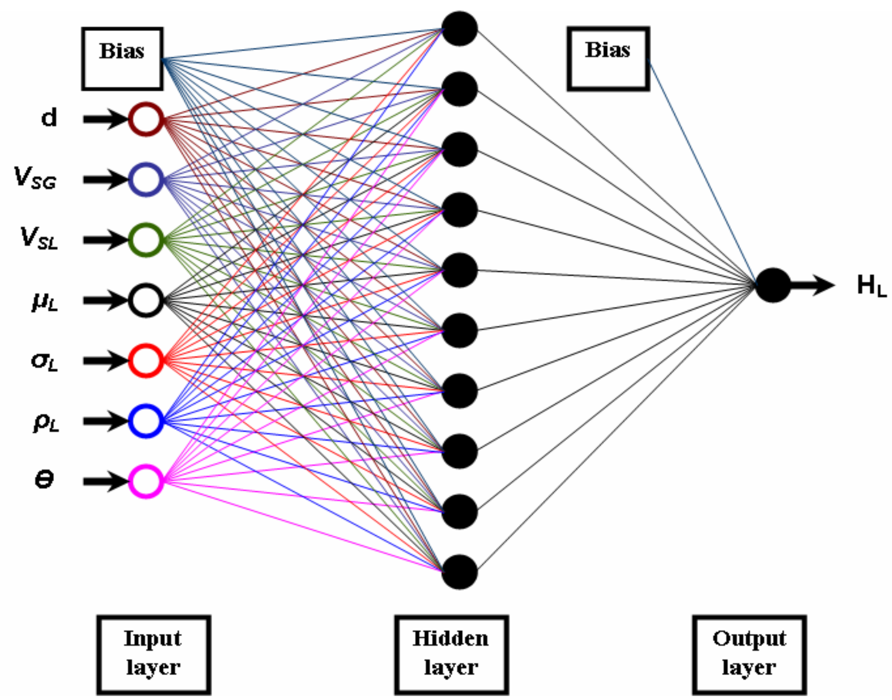

FIGURE 1. Schematic of the liquid holdup neural network.

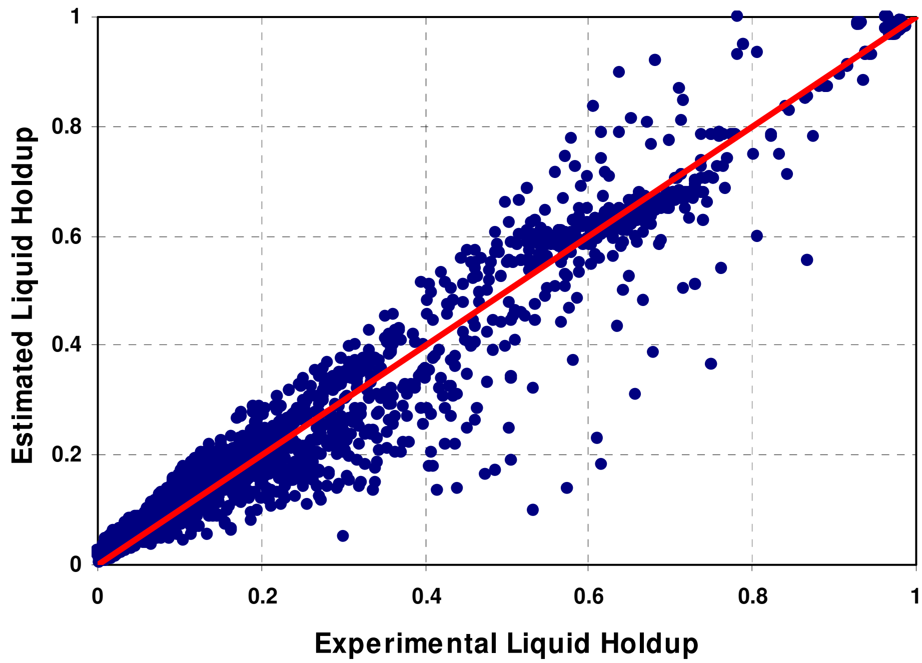

FIGURE 2. Cross plot of Beggs and Brill correlation.

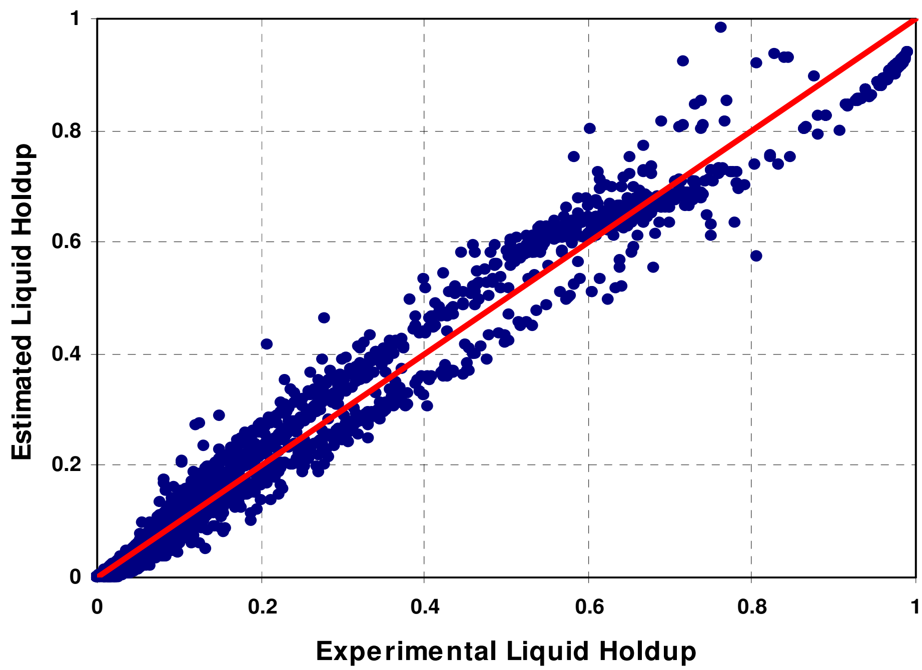

FIGURE 3. Cross plot of Mukherjee and Brill correlation.

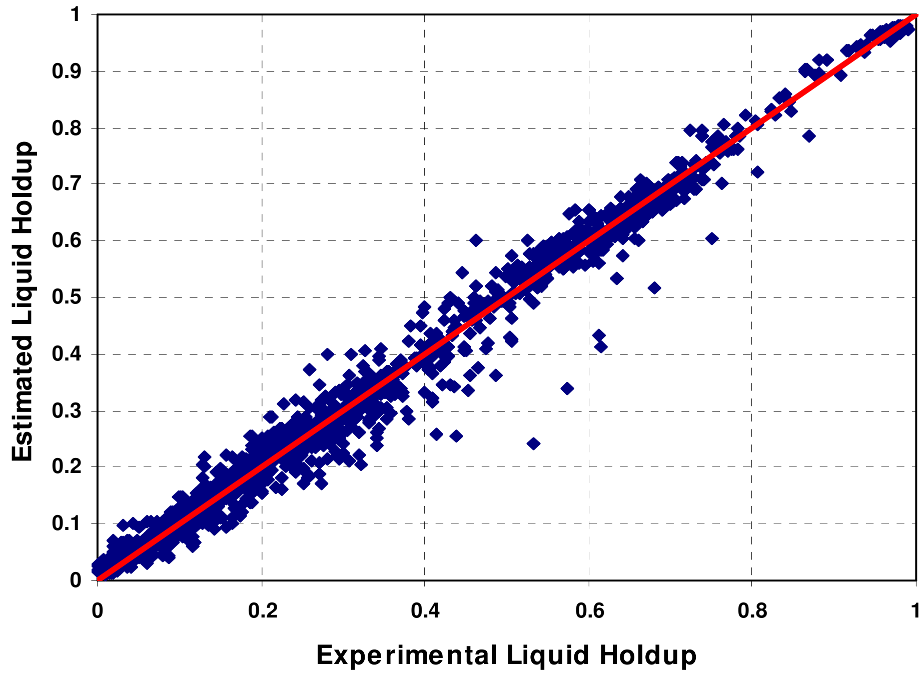

FIGURE 4. Cross plot of Neural Network model.