In The Name Of God

Amirkabir University
of Technology

Flow Assurance
in Production Engineering

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Two Phase Flow Principles

Outline

1. ***Introduction***
2. ***The General Energy Equation***
3. ***Evaluation of Friction Losses***
4. ***Single Phase Flow***
5. ***Definitions of Variables Used In Two Phase Flow***
6. ***Modification of the Pressure Gradient Equation for two phase flow***
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1- Introduction

- Tile prediction of pressure gradients, liquid holdup and flow patterns occurring during the simultaneous flow of gas and liquid in pipes is necessary for design in the petroleum and chemical industries.

- Petroleum engineers encounter two-phase flow most frequently in well tubing and in flowlines.

- The flow may be vertical, inclined or horizontal and methods must be available for predicting pressure drop in pipes at any inclination angle.

1- Introduction

- Offshore producing has necessitated transporting both gas and liquid phases over long distances before separation.

- Two-phase flow occurs frequently in the chemical processing industry and the design of such facilities as condensers, heat exchangers, reactors and process piping requires methods to predict pressure drop, liquid holdup, and for heat transfer purposes, flow pattern.

2- The General Energy Equation

- The theoretical basis for many fluid flow equations is the general energy equation, an expression for the balance or conservation of energy between two points in a system.

- The energy balance simply states that the energy of a fluid entering a control volume, plus any shaft work done on or by the fluid, plus any heat energy added to or taken from the fluid, plus any change of energy with time in the control volume must equal the energy leaving the control volume. Figure 1 may be used to illustrate this principle.

2- The General Energy Equation

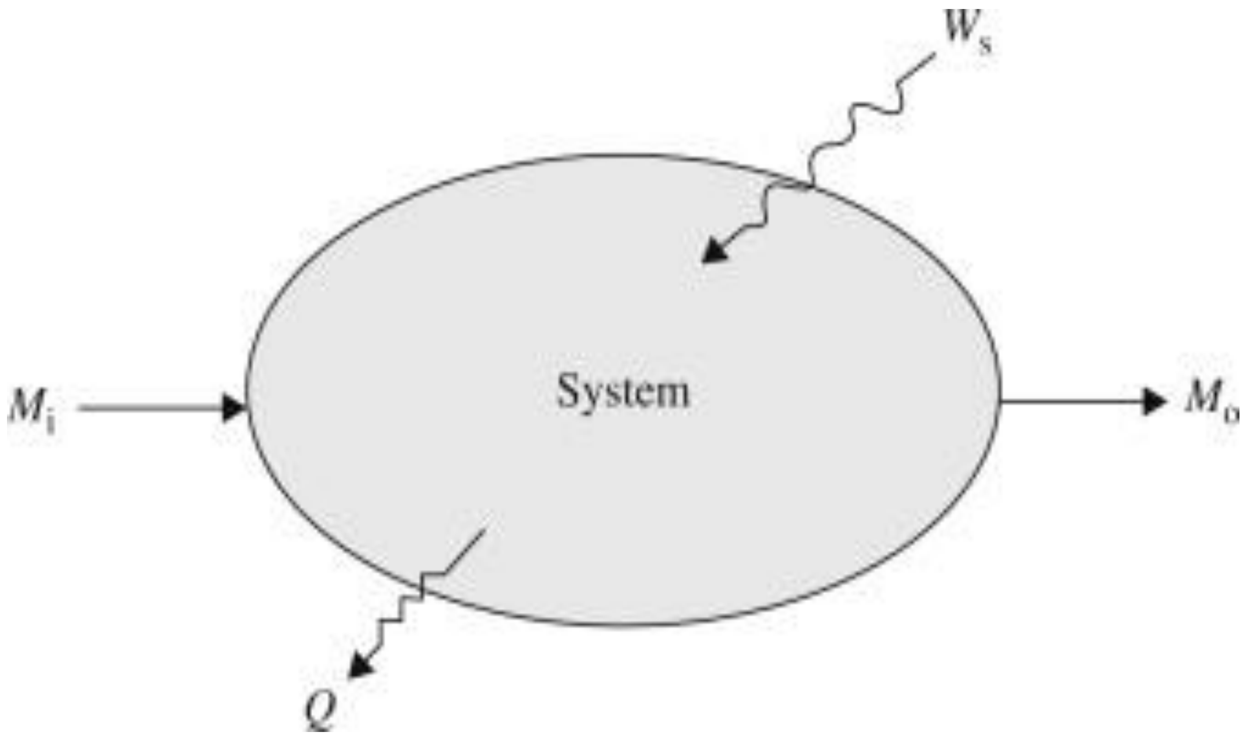


Figure 1: Flow System Control Volume.

2- The General Energy Equation

- By considering steady state system, the energy balance may be written as:

$$U_1' + p_1 v_1 + \frac{m v_1^2}{2 g_c} + \frac{m g z_1}{g_c} + q' + w_s' = U_2' + p_2 v_2 + \frac{m v_2^2}{2 g_c} + \frac{m g z_2}{g_c} \qquad (1)$$

$U' = Internal\ \ energy$

$pv = Energy\ of\ expansion\ \ or\ \ compression$

$\frac{mv^2}{2g_c} = Kinetic\ energy$

$\frac{m\,g\,z}{g_c} = Potential\ \ energy$

$q' = Heat\ \ energy\ \ added\ \ to\ \ fluid$

$W_s' = work\ \ done\ on\ the\ \ fluid$

$Z = Elevation\ above\ \ reference\ datum$

2- The General Energy Equation

- By assuming no work is done on the fluid, eq.2 can be obtain by using enthalpy-entropy relation as follow:

$$\frac{dp}{dL}+\frac{\rho v dv}{g_c dL}+\frac{g}{g_c}\rho \sin\phi+\rho\frac{dL_w}{dL}=0 \qquad (2)$$

- Where dL_w is the losses due to irreversibility, such as friction.

3- Evaluation of Friction Losses

- In horizontal pipe flow the energy losses or pressure drop is caused by change in kinetic energy and friction losses only.
- Since most of the viscous shear stress occurs at the pipe wall, the ratio of wall shear stress to kinetic energy per unit volume reflex the relative importance of wall shear stress to the total losses.
- This ratio forms a dimensionless group and defines a friction factor.

$$f' = \frac{\tau_w}{\rho v^2 / 2g_c} = \frac{2\tau_w g_c}{\rho v^2} \qquad (3)$$

3- Evaluation of Friction Losses

- To evaluate the wall shear stress, a force balance between pressure forces and wall shear stress can be formed Referring to Fig. 2.

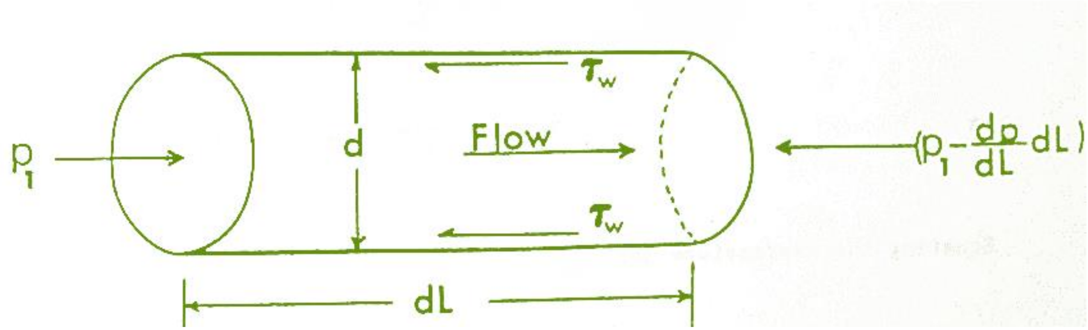


Figure 2. Force balance at pipe wall.

$$\left[p_1 - \left(p_1 - \frac{dp}{dL} dL\right)\right]\frac{\pi d^2}{4} = \tau_w (\pi d) dL \Rightarrow \tau_w = \frac{d}{4}\left(\frac{dp}{dL}\right)_f \qquad (4)$$

3- Evaluation of Friction Losses

- Substituting Eq. 4 into Eq. 3 results:

$$\text{Moody friction factor}: f_m = 4f' \Rightarrow \left(\frac{dp}{dL}\right)_f = \frac{f_m \rho v^2}{2 g_c d} \qquad (5)$$

- For laminar single phase flow ($N_{Re} \leq 2100$)

$$f_m = \frac{64\mu}{\rho v d} = \frac{64}{N_{Re}} \qquad (6)$$

- For Turbulent Single Phase Flow ($N_{Re} > 2100$)
  - ✓ Smooth wall:
    - Drew, Koo and McAdams:

$$f = 0.0056 + 0.5 N_{Re}^{-0.32} \qquad \text{For} \quad 3000 < N_{Re} < 3 \times 10^6 \qquad (7)$$

3- Evaluation of Friction Losses

- For Turbulent Single Phase Flow (N_{Re}>2100)
  - ✓ Smooth wall:
    - Blasius:

$$f = 0.316 N_{Re}^{-0.25} \qquad \text{For} \quad 3000 < N_{Re} < 1\times10^{5} \qquad (8)$$

  - ✓ Smooth-pipe friction factor (modified Blasius 1908):

$$f = 0.184 N_{Re}^{-0.2}$$

  - ➢ The original Blasius coefficients were 0.316 and −0.25

3- Evaluation of Friction Losses

- Partially Rough Wall (PRW)

  - ✓ Pipe roughness depends on:
    - Pipe material and Method of manufacture
    - The flowing fluid and Corrosive content
    - Hydrates, Paraffin or Asphaltene Deposits and Solids present

- The roughness is often affected by corrosive fluids and solid particles flowing in the pipe; thus, it is a function of operation time, fluids, and pipe material.

3- Evaluation of Friction Losses

❑ Turbulent Single Phase Flow ($N_{Re} > 2100$)

❖ Partially Rough Wall (PRW)

✓ Typical Values of pipe roughness:

Type of Pipe	Roughness ε (ft)
Plastic, Glass, etc.	0.0
New Tubing or Line Pipe	0.00005
Commercial Steel	0.00015
Dirty Well Tubing	0.00075

✓ Colebrook and White:

$$\frac{1}{\sqrt{f}} = 1.74 - 2\mathrm{Log}_{10}\left(\frac{2\varepsilon}{d} + \frac{18.7}{N_{Re}\sqrt{f}}\right) \qquad (9)$$

3- Evaluation of Friction Losses

- Turbulent Single Phase Flow ($N_{Re>}2100$)
  - Fully Rough Wall (FRW)
    - ✓Nikuradse:

$$\frac{1}{\sqrt{f}} = 1.74 - 2\mathrm{Log}_{10}\left(\frac{2\varepsilon}{d}\right) \qquad (10)$$

3- Evaluation of Friction Losses

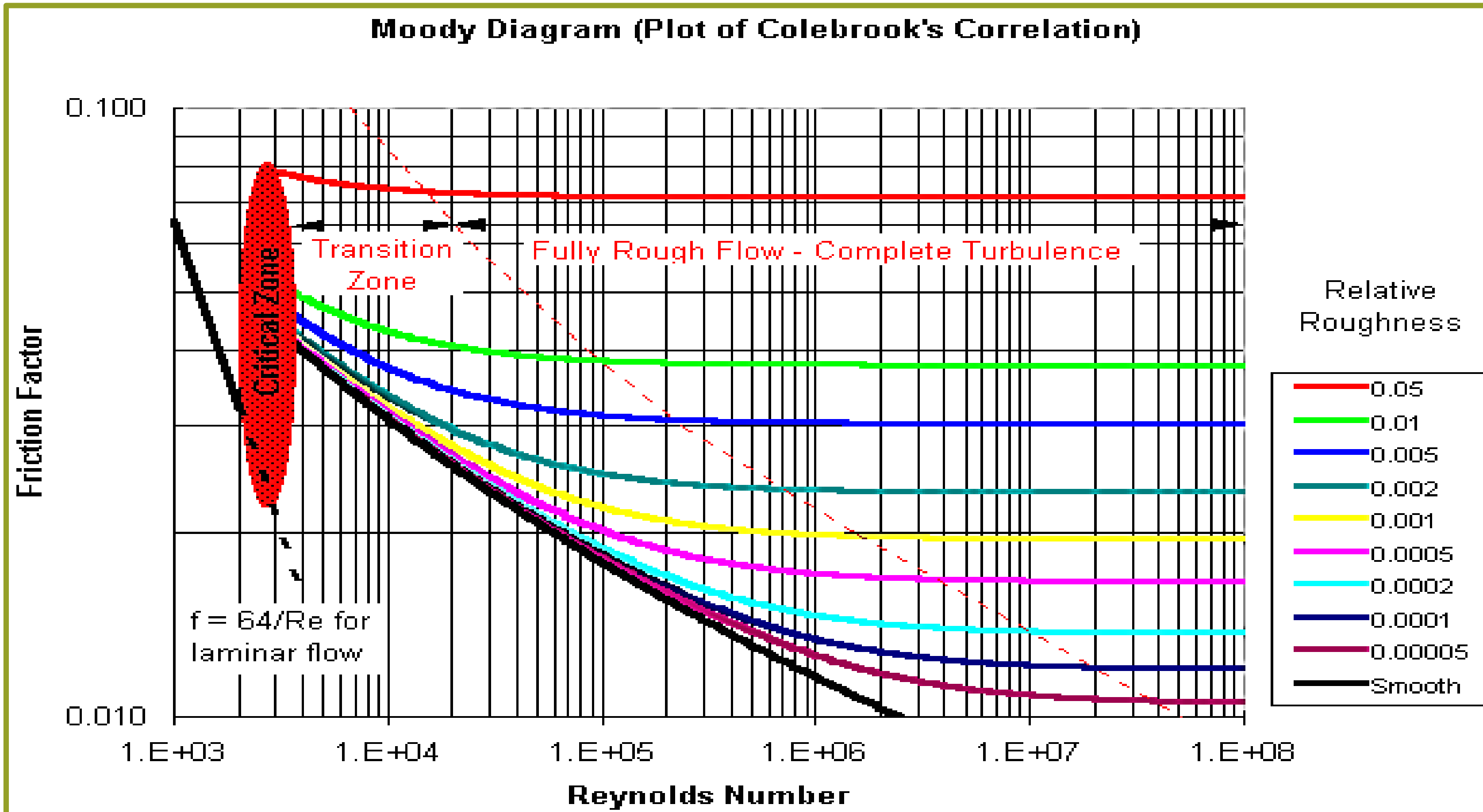


Figure 3: Moody diagram

3- Evaluation of Friction Losses

- Turbulent Single Phase Flow ($N_{Re>}$2100)
  - Fully Rough Wall (FRW)
    - ✓Nikuradse:

$$\frac{1}{\sqrt{f}} = 1.74 - 2Log_{10}(\frac{2\varepsilon}{d}) \qquad (10)$$

3- Evaluation of Friction Losses

- Turbulent Single Phase Flow ($N_{Re>}$2100)

  - Fully Rough Wall (FRW)

✓ An explicit friction factor equation was proposed by Jain and compared in accuracy to the Colebrook equation.

$$\frac{1}{\sqrt{f^{/}}} = 1.14 - 2log\left(\frac{\varepsilon}{d} + \frac{21.25}{N_{Re}{}^{0.9}}\right) \qquad (11)$$

3-1 Pressure Gradient

- The pressure gradient becomes:

$$\frac{dp}{dL}=\frac{g}{g_c}\rho\sin\phi+\frac{f\rho v^2}{2g_c d}+\frac{\rho v dv}{g_c dL} \qquad (12)$$

$$\frac{dp}{dL}=\left(\frac{dp}{dL}\right)_{ele}+\left(\frac{dp}{dL}\right)_{f}+\left(\frac{dp}{dL}\right)_{acc} \qquad (13)$$

$$\left(\frac{dp}{dL}\right)_{ele}=\text{Elevational Pressure Drop}$$

$$\left(\frac{dp}{dL}\right)_{f}=\text{Frictional Preessure Drop}$$

$$\left(\frac{dp}{dL}\right)_{acc}=\text{Acceleration Pressure Drop}$$

4- Single Phase Flow

- Some aspects of the pressure gradient equation as it applies to single phase flow are discussed in order to develop a thorough understanding of each component before modifying it for two-phase flow.

- Elevation Change Component:

  - ✓ This component is zero for horizontal flow only. It applies for compressible or incompressible, steady state or transient flow in both vertical and inclined flow. For downward flow the sine of the angle is negative and the hydrostatic pressure increases in the direction of flow.

4- Single Phase Flow

- Friction Loss Component:
  - ✓ This component applies for any type of flow at any pipe angle. It always causes a drop of pressure in the direction of flow.
- Acceleration Component:
  - ✓ This component applies for all transient flow conditions, but is zero for constant area, incompressible flow.

Example 1

- Calculate the total pressure gradient in a vertical gas well.

ID = 62 mm

Q_{SC} = 0.14 MMStd m^3/d

Z = 0.75

T = ۶۲ °C

P = 14.7 Mpa (2125 psi)

γ_g = 0.75

μ_g = 0.000018

ϵ = 0.0213 mm

Example 1

- Solution:

$$\rho = \frac{PM}{ZRT} = 149\frac{Kg}{m^3}$$

$$B_g = 352.3\frac{ZT}{P} = 0.00615\frac{m^3}{std\ m^3}$$

$$Q = Q_{sc} \times B_g = 861\ m^3/d$$

$$A = \frac{\pi d^2}{4}$$

$$v = \frac{Q}{A} = 3.32\frac{m}{s}$$

$$N_{Re} = \frac{\rho v d}{\mu} = 1700000\ (Turbulent)$$

$$\frac{\epsilon}{d} = 0.000344$$

Moody diagram: f = 0.0145

$$\frac{dP}{dL} = -\frac{f\rho v^2}{2d} - \rho g Sin\Theta = -192 - 1460 = 1652\frac{Pa}{m}$$

5- Definition of Variable Used in Two Phase Flow

- Calculation of pressure gradients requires values of flow conditions such as velocity and fluid properties such as density, viscosity, and in some cases, surface tension.

- When these variables are calculated for two-phase flow, certain mixing rules and definitions unique to this application are encountered.

5- Definition of Variable Used in Two Phase Flow

- Liquid Holdup:
  - ✓ Liquid holdup is defined as the ratio of the volume of a pipe segment occupied by liquid to the volume of the pipe segment.

$$H_L = \frac{\text{Volume of liquid in a pipe segment}}{\text{Volume of pipe segment}}$$

  - ✓ Liquid holdup is a fraction which varies from zero for all gas flow to one for all liquid flow.
  - ✓ The most common method of measuring liquid holdup is to isolate a segment of the flow stream between quick-closing valves and to physical1y measure the liquid trapped.

5- Definition of Variable Used in Two Phase Flow

- Liquid Holdup:
  - ✓ Liquid holdup is a fraction which varies from zero for all gas flow to one for all liquid flow.
  - ✓ The remainder of the pipe segment is of course occupied by gas, which is referred to as gas holdup or gas void fraction.

$$H_g = 1 - H_L \qquad (14)$$

5- Definition of Variable Used in Two Phase Flow

- No Slip Liquid Holdup:
  - ✓ No-slip holdup, sometimes called input liquid content, is defined as the ratio of the ,volume of liquid in a pipe segment divided by the volume of the pipe segment which would exist if the gas and liquid travelled at the same velocity (no-slippage).
  - ✓ It can be calculated directly from the known gas and liquid flow rates from:

$$\lambda_L = \frac{q_L}{q_L + q_g} \qquad (15)$$

5- Definition of Variable Used in Two Phase Flow

- No Slip Liquid Holdup:
  - ✓ The no-slip gas holdup is defined as:

$$\lambda_g = 1 - \lambda_L = \frac{q_g}{q_L + q_g} \qquad (16)$$

  - ✓ It is obvious that the difference between the liquid holdup and the no-slip holdup is a measure of the degree of slippage between the gas and liquid phases.

$$\beta = H_L - \lambda_L \qquad (17)$$

5- Definition of Variable Used in Two Phase Flow

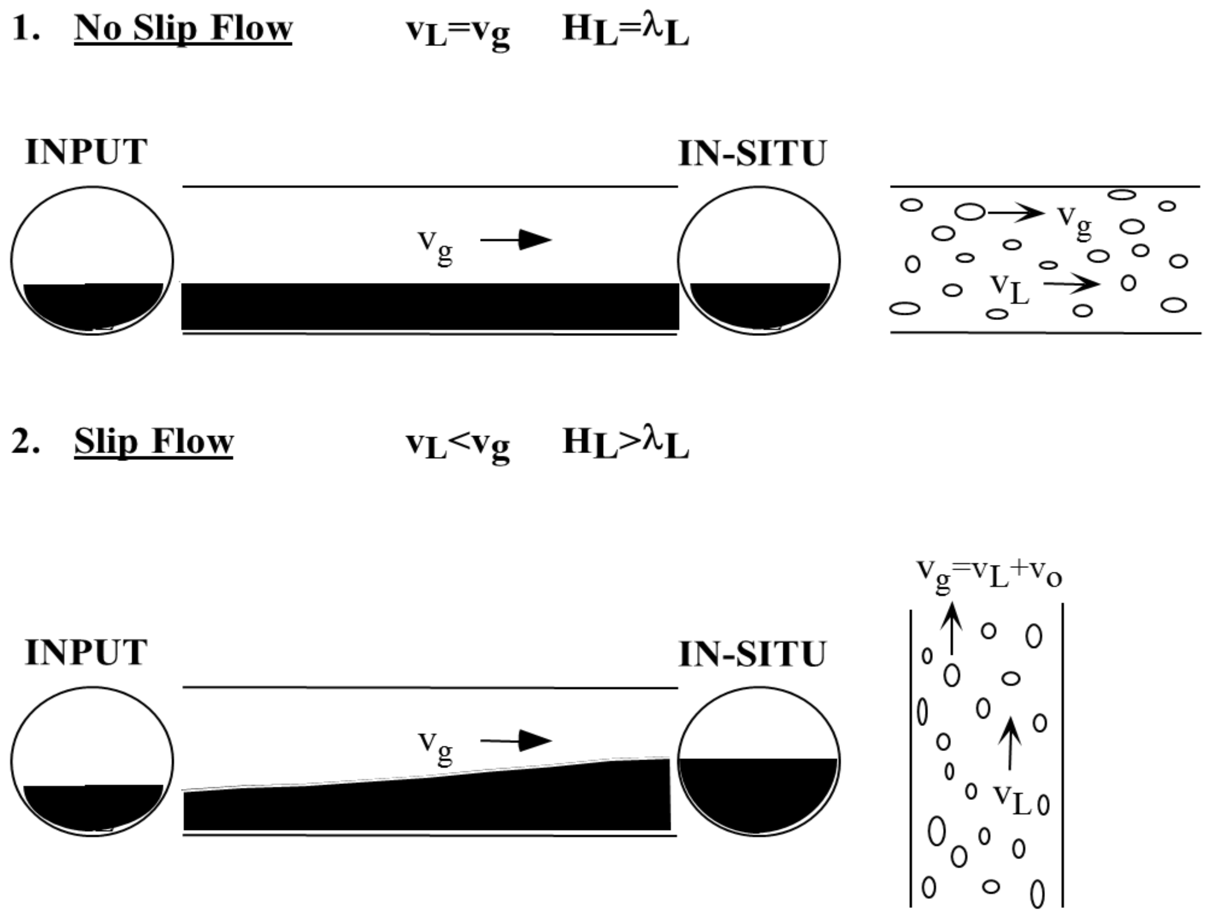


Figure 4: No slip and Slip flow

5- Definition of Variable Used in Two Phase Flow

- Density

  ✓ Gas density is a function of pressure and temperature which is calculated by:

$$\rho_g = \frac{P\,M}{zRT} = \frac{2.7\,P\gamma_g}{zT} \qquad (18)$$

❖ ρ_g is gas density = lb_m/ft^3

❖ P is pressure = psia

❖ T is temperature = ^oR

❖ Z is compressibility factor

❖ γ_g is gas gravity

5- Definition of Variable Used in Two Phase Flow

- Density
  - ✓ In calculation of oil density effect of dissolved gas on it's mass and volume should be encountered.

$$\rho_o = \frac{350.4\gamma_o + 0.0764\gamma_g R_s}{5.615 B_o} \qquad (19)$$

- ❖ ρ_o is oil density = lb_m/ft^3
- ❖ γ_o is oil specific gravity
- ❖ γ_g is gas specific gravity
- ❖ R_s is dissolved gas scf/stbd
- ❖ B_o is oil formation volume factor

5- Definition of Variable Used in Two Phase Flow

- Density

  - ✓ In-situ water density depends on the solid dissolved in the water, the temperature and any gas which may be dissolved in the water.

  - ✓ The total liquid density may be calculated from the oil and water densities and flow rates if no slippage between the oil and water is assumed.

$$f_o = \frac{q_o}{q_o + q_w} = \frac{q_o}{q_L}, \quad f_w = 1 - f_o = \frac{q_w}{q_o + q_w} = \frac{q_w}{q_L} \qquad (20)$$

$$\rho_l = \rho_o f_o + \rho_w f_w \qquad (21)$$

5- Definition of Variable Used in Two Phase Flow

- Density
  - ✓ Calculation of the two-phase density requires knowledge of the liquid holdup. Three equations for two phase density are used by various investigators in two-phase flow.

$$\rho_s = \rho_l H_l + \rho_g H_g \qquad (22)$$

$$\rho_n = \rho_l \lambda_l + \rho_g \lambda_g \qquad (23)$$

$$\rho_k = \rho_l \frac{\lambda_l^{\,2}}{H_l} + \rho_g \frac{\lambda_g^{\,2}}{H_g} \qquad (24)$$

Example 2

- Oil and natural gas flow in a 3-in-ID horizontal pipe. Use following data and calculate density of mixture.

R_p = 440 scf/stb

R_s = 190 scf/stb

API = 32

γ_g = 0.7

Z = 0.۵

B_o = 1.05

T = 190 °F

P = 1980 psia

Q_o = 5000 stb/d

Example 2

- Solution:

$$\gamma_o = \frac{141.5}{131.5 + API} = 0.865$$

$$\rho_o = \frac{350.4\gamma_o + 0.0764\gamma_g R_s}{5.615 B_o} = 53.13$$

$$\rho_g = \frac{PM}{ZRT} = 11.5$$

$$B_g = 0.0283\frac{ZT}{P} = 0.0046$$

$$Q_L = Q_{sc}B_o \times 5.615/86400 = 0.341\ \mathrm{ft^3/s}$$

$$Q_g = Q_o\,(R_p - R_s)\,B_g/86400 = 0.067\ \mathrm{ft^3/s}$$

$$\lambda_L = \frac{Q_L}{Q_g + Q_L} = 0.835 \qquad \lambda_g = 0.165$$

$$\rho_n = \rho_L\lambda_L + \rho_g\lambda_g = 46.26$$

5- Definition of Variable Used in Two Phase Flow

- Density

  - ✓ Eq. 22 is used by most investigators to determine the pressure gradient due to elevation change.

  - ✓ Some correlations are based on the assumption of no-slippage and therefore use Eq. 23 for two-phase density.

  - ✓ Eq, 24 is used by some investigators to define the density used in the friction loss term and Reynolds number.

5- Definition of Variable Used in Two Phase Flow

- Velocity

  - ✓ Many two-phase flow correlations are based on a variable called superficial velocity. The superficial velocity of a fluid phase is defined as the velocity which that phase would exhibit if it flowed through the total cross section of the pipe alone. Superficial and actual velocity for gas and liquid is shown in Eq.25 through 28.

$$V_{sg} = \frac{q_g}{A} \qquad (25) \qquad superficial \;\; gas \;\; velocity$$

$$V_g = \frac{q_g}{AH_g} \qquad (26) \qquad actual \;\; gas \;\; velocity$$

$$V_{sl} = \frac{q_l}{A} \qquad (25) \qquad superficial \;\; liquid \;\; velocity$$

$$V_l = \frac{q_l}{AH_l} \qquad (26) \qquad actual \;\; liquid \;\; velocity$$

5- Definition of Variable Used in Two Phase Flow

- Velocity
  - ✓ Two-phase flow velocity :

$$V_m = V_{sl} + V_{sg} \qquad (27)$$

  - ✓ The difference between actual gas and liquid velocity is called slip velocity.

$$V_s = V_g - V_l = \frac{V_{sg}}{H_g} - \frac{V_{sl}}{H_l} \qquad (28)$$

  - ✓ An alternate equation for no-slip liquid holdup is:

$$\lambda_l = \frac{V_{sl}}{V_m} \qquad (29)$$

5- Definition of Variable Used in Two Phase Flow

- Viscosity
  - ✓ The viscosity of the flowing fluid is used in determining a Reynolds number as well as other dimensionless numbers used as correlating parameters.
  - ✓ The concept of a two-phase viscosity is rather nebulous and is defined differently by various investigators.
  - ✓ The viscosity of various gases may be estimated from empirical correlations described in section 2.
  - ✓ Empirical correlations for both gas free and gas saturated crude oils are given in section 2.

5- Definition of Variable Used in Two Phase Flow

- Viscosity
  - ✓ The viscosity of an oil water mixture is usually calculated using the water-oil ratio as a weighting factor which is shown in equation 29.

$$\mu_l = \mu_o f_o + \mu_w f_w \qquad (29)$$

  - ✓ The following equations have been used to calculate a two-phase viscosity.

$$\mu_n = \mu_l \lambda_l + \mu_g \lambda_g \qquad (30)$$

$$\mu_s = \mu_l^{H_l} \times \mu_g^{H_g} \qquad (31)$$

5- Definition of Variable Used in Two Phase Flow

- Mass Flux, G (kg/m^{2}s).

$$G_L = \frac{W_L}{A_P} = Liquid\ \ mass\ \ flux$$

$$G_G = \frac{W_G}{A_P} = Gas\ mass\ \ flux$$

$$G = \frac{W_G + W_L}{A_P} = G_G + G_L \quad Total\ \ mass\ \ flux$$

- Drift Velocity, V_D (m/s):. The drift velocity of a phase is the velocity of the phase relative to a surface moving at the mixture velocity (center of volume).

$$V_{DL} = V_L - V_M \quad and \quad V_{DG} = V_G - V_M$$

5- Definition of Variable Used in Two Phase Flow

- Drift Flux, J (m/s): The drift flux represents the flow rate of a phase, per unit area, through a surface moving at the center of volume velocity.

$$J_L = H_L\left(V_L - V_M\right) \quad and \quad J_G = H_G\left(V_G - V_M\right)$$

- Diffusion Velocities, V_{ML} and V_{MG} (m/s): The diffusion velocity is the velocity of a phase relative to a surface moving at the center of mass velocity, as given by:

$$v_{ML} = v_L - \frac{G}{\rho_M} \quad and \quad v_{MG} = v_G - \frac{G}{\rho_M}$$

5- Definition of Variable Used in Two Phase Flow

- Surface Tension

  - ✓ The interfacial tension between natural gas and crude oil depends on oil gravity, temperature and dissolved gas, among other variables.

  - ✓ When the liquid phase contains both water and oil, the same weighting factors as used for calculating density and viscosity are used. That is :

$$\sigma_L = \sigma_o f_o + \sigma_w f_w \qquad (32)$$

Example 3

- Oil and natural gas flow in a 5.1-cm-ID horizontal pipe. The in-situ flow rates of the oil and natural gas are 0.025 m^3/min and 0.25 m^3/min, respectively. The corresponding liquid holdup is 0.35. Determine:
  - ✓ The gas and liquid superficial velocities, mixture velocity, and no-slip liquid holdup.
  - ✓ The actual velocities of the two phases.
  - ✓ The slip velocity between the gas-phase and the liquid-phase.
  - ✓ The drift velocities and drift fluxes of the two phases. Show that $J_G = -J_L$

Example 3

❑ Solution

$$A_p = \frac{\pi}{4}(0.051)^2 = 0.002043\, m^2$$

$$1.\quad v_{sL} = \frac{q_L}{A_p} = 0.2\ \frac{m}{s} \qquad v_{sG} = 2.04\ \frac{m}{s} \qquad v_m = 2.24\frac{m}{s} \quad and \quad \lambda_L = \frac{v_{sL}}{v_M} = 0.09$$

$$2. v_L = \frac{v_{sL}}{H_L} = 0.57\frac{m}{s} \quad and \quad v_G = \frac{v_{sG}}{1 - H_L} = 3.14\frac{m}{s}$$

$$3.\ v_{SLIP} = v_G - v_L = 2.57\frac{m}{s}$$

$$4. v_{DL} = v_L - v_M = -1.67\frac{m}{s} \qquad and \qquad v_{DG} = v_G - v_M = 0.9\frac{m}{s}$$

$$5. J_L = H_L v_{DL} = -0.58\frac{m}{s}$$

$$J_G = (1 - H_L)v_{DG} = +0.58\frac{m}{s} \qquad \to J_G + J_L = 0$$

6- Modification of the Pressure Gradient Equation for two phase flow

- The pressure gradient equation which is applicable to any fluid flowing in a pipe inclined at an angle 0 from horizontal is given in Eq. 13.
- The equation is usually adapted for two-phase flow by assuming that the gas-liquid mixture can be considered homogeneous over a finite volume of the pipe.
- For two-phase flow the elevation change component becomes:

$$\left(\frac{dp}{dl}\right)_{ele} = \frac{g}{g_c}\rho_s \sin\theta \qquad (33)$$

6- Modification of the Pressure Gradient Equation for two phase flow

- where ρ_s is the density of the gas-liquid mixture in the pipe element.

- Considering a pipe element which contains liquid and gas, the density of the mixture can be calculated from Eq.34.

$$\rho_s = \rho_l H_l + \rho_g H_g \qquad (34)$$

- The friction loss component is:

$$\left(\frac{dp}{dl}\right)_f = \frac{f_{tp}\rho_f v_m^{\ 2}}{2 g_c d} \qquad (35)$$

- f_{tp} and ρ_f are defined differently by different investigators.

6- Modification of the Pressure Gradient Equation for two phase flow

- Two-Phase Friction

  - ✓ Previously it was shown that the term $(dp/dl)_f$ represents the pipes pressure losses due to friction when gas and liquid flow simultaneously in pipes.

  - ✓ This term is not analytically predictable except for the case of laminar single-phase flow.

  - ✓ Therefore it must be determined by experimental means or by analogies to single phase flow.

6- Modification of the Pressure Gradient Equation for two phase flow

- Two-Phase Friction

  ✓ The method which has received by far the most attention is the one resulting in two phase friction factors.

  ✓ Among the most common definitions are the following:

$$\left(\frac{dp}{dl}\right)_{fl} = \frac{f_l \rho_l {v_{sl}}^2}{2g_c\, d} \qquad (36)$$

$$\left(\frac{dp}{dl}\right)_{fg} = \frac{f_g \rho_g {v_{sg}}^2}{2g_c\, d} \qquad (37)$$

$$\left(\frac{dp}{dl}\right)_{fT} = \frac{f_{tp} \rho_f {v_m}^2}{2g_c\, d} \qquad (38)$$

6- Modification of the Pressure Gradient Equation for two phase flow

- Two-Phase Friction

  - ✓ ln general, the two phase friction factor methods differ only in the way the friction factor is determined and to a large extent on the flow pattern.
  - ✓ For example, in the mist flow pattern Equation ۳۷, based on gas is normally used, whereas in the bubble flow regime Equation ۳۶ based on liquid is frequently used.
  - ✓ The definition of ρ_f can differ widely depending on the investigator as discussed previously.

6- Modification of the Pressure Gradient Equation for two phase flow

- Two-Phase Friction

  - ✓ Most correlations attempt to correlate friction factors with some form of a Reynolds number.

  - ✓ Recall that the single phase Reynolds number is defined as :

$$N_{\mathrm{Re}} = 1488 \frac{\rho v d}{\mu} \qquad (39)$$

    - ✓ ρ= density lb/ft^3
    - ✓ v= velocity ft/sec
    - ✓ d= pipe diameter ft
    - ✓ μ= viscosity cp

6- Modification of the Pressure Gradient Equation for two phase flow

Acceleration Component

- The acceleration component is completely ignored by some investigators and ignored in some flow regimes by others. When it is considered, various assumptions are made regarding the relative magnitudes of parameters involved to arrive at some simplified procedure to determine the pressure drop due to kinetic energy change.

- From the discussion of the various components contributing to the total pressure gradient it follows that the principal considerations for developing pressure gradient equations are developing methods for predicting liquid holdup and two-phase friction factor.

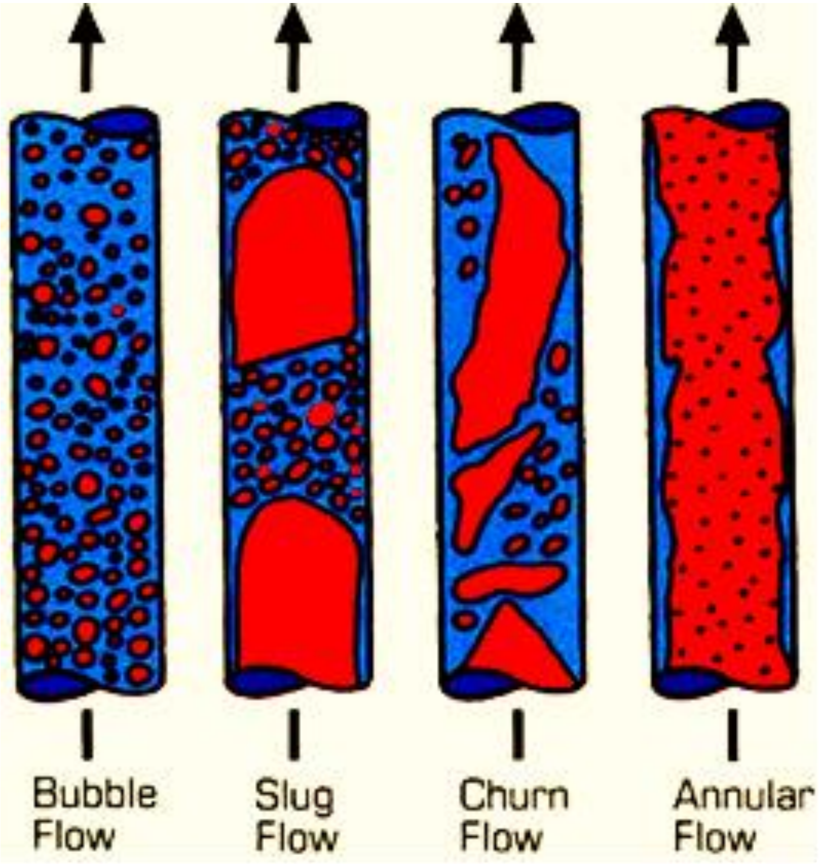
7- Flow Patterns

- Predicting the flow pattern that occurs at a given location in a well is extremely important.
- The empirical correlation or mechanistic model used to predict flow behavior varies with flow pattern.
- Brill and Beggs summarized numerous investigations that have described flow patterns in wells and that made attempts to predict when they occur.
- These predictions may be inadequate for high-pressure, high-temperature wells, or for wells producing oil and water or crude oils with foaming tendencies

7-1 Vertical Flow in Well

- For upward multiphase flow of gas and liquid, most investigators now recognize the existence of four flow patterns: **bubble flow, slug flow, churn flow, and annular flow.**
- These flow patterns, shown schematically in Fig.4, are described next.
- Slug and churn flow are sometimes combined into a flow pattern called intermittent flow.
- It is common to introduce a transition between slug flow and annular flow that incorporates chum flow. Some investigators have named annular flow as mist or annular-mist flow.

7-1 Vertical Flow in Well



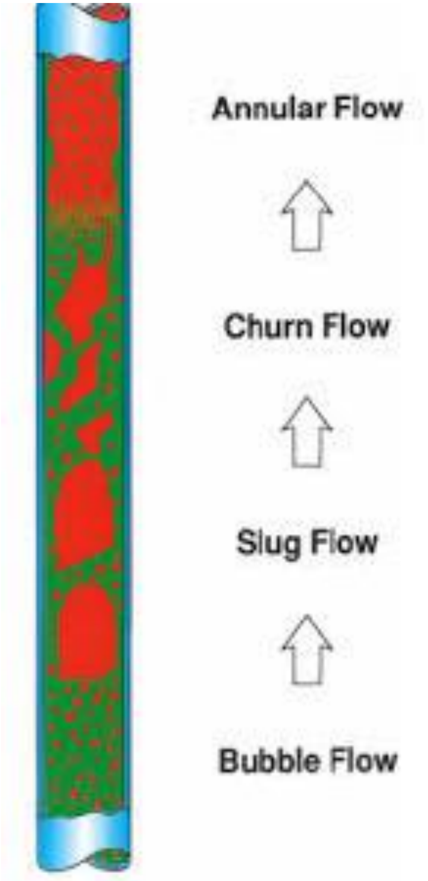


Figure 5: Flow patterns in upward vertical flow in wells.

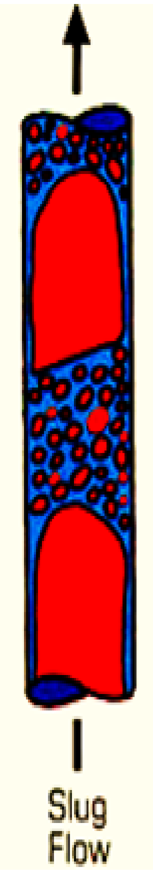
7-1 Vertical Flow in Well

❑ Bubble Flow

- ✓ Bubble flow is characterized by a uniformly distributed gas phase and discrete bubbles in a continuous liquid phase.
- ✓ Based on the presence or absence or slippage between the two phases, bubble flow is further classified into bubbly and dispersed-bubble flows.
- ✓ In bubbly flow, relatively fewer and larger bubbles move faster than the liquid phase because or slippage.
- ✓ In dispersed-bubble flow, numerous tiny bubbles are transported by the liquid phase, causing no relative motion between the two phases.

7-1 Vertical Flow in Well

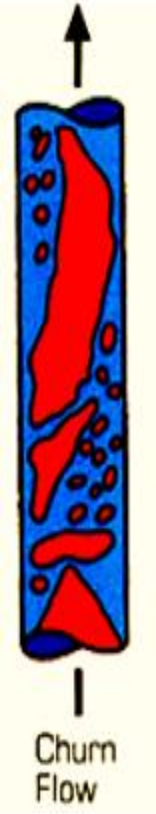
❑ Slug flow



- ✓ Slug flow is characterized by a series or slug units.
- ✓ Each unit is composed or a gas pocket called a Taylor bubble, a plug of liquid called a slug, and a film or liquid around the Taylor bubble flowing downward relative to the Taylor bubble.
- ✓ The Taylor bubble is an axially symmetrical, bullet-shaped gas pocket that occupies almost the entire cross-sectional area or the pipe.
- ✓ The liquid slug, carrying distributed gas bubbles, bridges the pipe and separates two consecutive Taylor bubbles.

7-1 Vertical Flow in Well

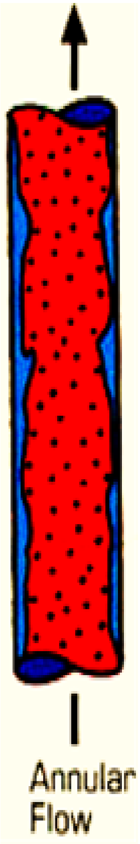
- Churn flow



- ✓ Churn flow is a flow of gas and liquid in which the shape of both the Taylor bubbles and the liquid slugs are distorted, Neither phase appears to be continuous.
- ✓ The continuity of the liquid in the slug is repeatedly destroyed by a high local gas concentration.
- ✓ An oscillatory or alternating direction of motion in the liquid phase is typical of churn flow.

7-1 Vertical Flow in Well

- Annular flow



- ✓ Annular flow is characterized by the axial continuity of the gas phase in a central core with the liquid flowing upward, both as a thin film along the pipe wall and as dispersed droplets in the core.
- ✓ At high gas flow rates more liquid becomes dispersed in the core, leaving a very thin liquid film flowing along the wall.
- ✓ The interfacial shear stress acting at the core/film interface and the amount of entrained liquid in the core are important parameters in annular flow.

7-2 Vertical Flow in Annulus

- In the petroleum industry, flow in wells normally occurs in a tubing string.
- However, many oil wells with high production rates produce through the casing/tubing annulus.
- This trend is dictated by economics, multiple completions, and regulated production rates.
- Other casing-flow applications are found in wells under various types of artificial lift.

7-2 Vertical Flow in Annulus

- In the past, annuli have been treated based on the hydraulic diameter concept.
- The hydraulic diameter is four times the flow area divided by the wetted perimeter.
- For annulus configuration:

$$d_h = d_{ci} - d_{to}$$

- An annulus is characterized by the existence of two circular pipes, where the flow occurs through the area bounded by the outer pipe inner wall and the inner pipe outer wall.
- Two geometrical parameters identify these configurations: the annulus pipe-diameter ratio, $K = d_{to}/d_{ci}$, and the degree of eccentricity.

7-2 Vertical Flow in Annulus

- The degree of eccentricity accounts for the displacement of the inner pipe center from the outer pipe center and is expressed by :

$$e = \frac{2\ D_{BC}}{\left(d_{ci} - d_{to}\right)}$$

- where d_t=outer diameter of the inner pipe (tubing), d_c=inner diameter of the outer pipe (casing). and D_{BC}=distance between the pipe centers. Annuli can have eccentricity values varying from zero to one.

- Fig. 6 shows cross sections of annuli with the same pipe-diameter-ratio value, K, and for eccentricities of 0.0, 0.5, and 1.0.

7-2 Vertical Flow in Annulus

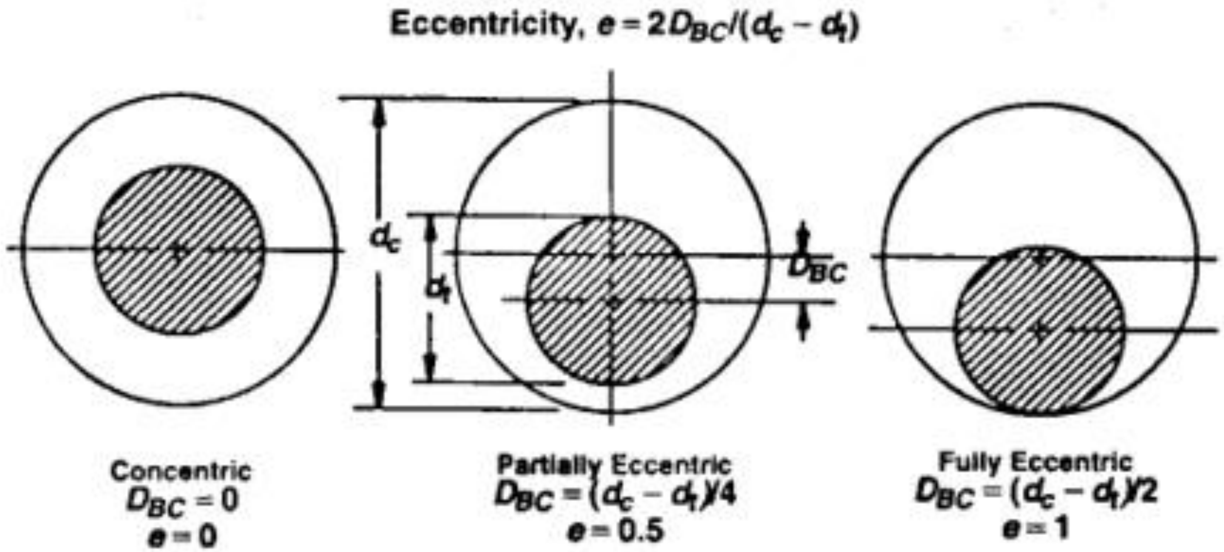


Figure 6: Annuli configuration

7-2 Vertical Flow in Annulus

- Friction factor for single phase flow:
  - Turbulent flow:
    - Empirical correlation:

$$f = C\, N_{Re}^{\;n}$$

      - C and n are determined empirically.
    - Semi-empirical correlation:
      - $f_{NC}=f(N_{Re}, F_C/F_{NC})$
      - F_C and F_{NC} are so called friction geometry parameters for circular and non-circular configurations.

7-2 Vertical Flow in Annulus

- Friction factor for single phase flow:
  - Newtonian laminar flow:
    - Friction factor in circular pipe:

$$f_p^{/} = \frac{F_p}{N_{Re}} = \frac{16}{N_{Re}}$$

$$F_p = Friction \quad geometry \quad parameter$$

$$F_p = 16 \quad for \quad pipe \quad flow$$

    - For a concentric annulus:

$$f_{CA}^{/} = \frac{F_{CA}}{N_{Re}} = \frac{16}{N_{Re}} \frac{\left(1-K^2\right)}{\left[\dfrac{1-K^4}{1-K^2} - \dfrac{1-K^2}{ln\left(\dfrac{1}{K}\right)}\right]}$$

7-2 Vertical Flow in Annulus

- Friction factor for single phase flow:
  - Newtonian laminar flow:
    - Friction factor for eccentric annulus:

$$f'_{EA} = \frac{F_{EA}}{N_{Re}} = \frac{1}{N_{Re}} \frac{4(1-K)^2(1-K^2)}{\phi \sinh^4\eta}$$

$$\phi = (\cosh\eta_i - \cosh\eta_o)^2 \left(\frac{1}{\eta_i - \eta_o} - 2\sum_{i=1}^{\infty} \frac{2n}{e^{2n\eta_i} - e^{2n\eta_o}} \right)$$

$$+ \frac{1}{4}\left(\frac{1}{\sinh^4\eta_o} - \frac{1}{\sinh^4\eta_i} \right)$$

$$note: \cosh\eta_i = \frac{K(1+e^2)+(1-e^2)}{2Ke} \quad , \cosh\eta_o = \frac{K(1-e^2)+(1+e^2)}{2Ke}$$

7-2 Vertical Flow in Annulus

- Friction factor for single phase flow:
  - Newtonian turbulent flow:
    - Friction factor for concentric annulus:

$$\frac{1}{\left[f_{CA}^{/}\left(\frac{F_P}{F_{CA}}\right)^{0.45\left[-\left(N_{Re}-3000\right)/10^6\right]}\right]^{0.5}}=4log\left(N_{Re}\left\{f_{CA}^{/}\left(\frac{F_P}{F_{CA}}\right)^{0.45\left[-\left(N_{Re}-3000\right)/10^6\right]}\right\}^{0.5}\right)-4$$

    - Friction factor for eccentric annulus:

$$\frac{1}{\left[f_{EA}^{/}\left(\frac{F_P}{F_{CA}}\right)^{0.45\left[-\left(N_{Re}-3000\right)/10^6\right]}\right]^{0.5}}=4log\left(N_{Re}\left\{f_{EA}^{/}\left(\frac{F_P}{F_{CA}}\right)^{0.45\left[-\left(N_{Re}-3000\right)/10^6\right]}\right\}^{0.5}\right)-4$$

7-2 Vertical Flow in Annulus

- The experimental data collected by Caetano et al. reveal that, although the same flow patterns described for wellbores occur in annuli, their characteristics can be substantially different.

- Thus, it is essential to define the flow patterns in these configurations. Fig.5 shows flow patterns fully eccentric annuli.

7-2 Vertical Flow in Annulus

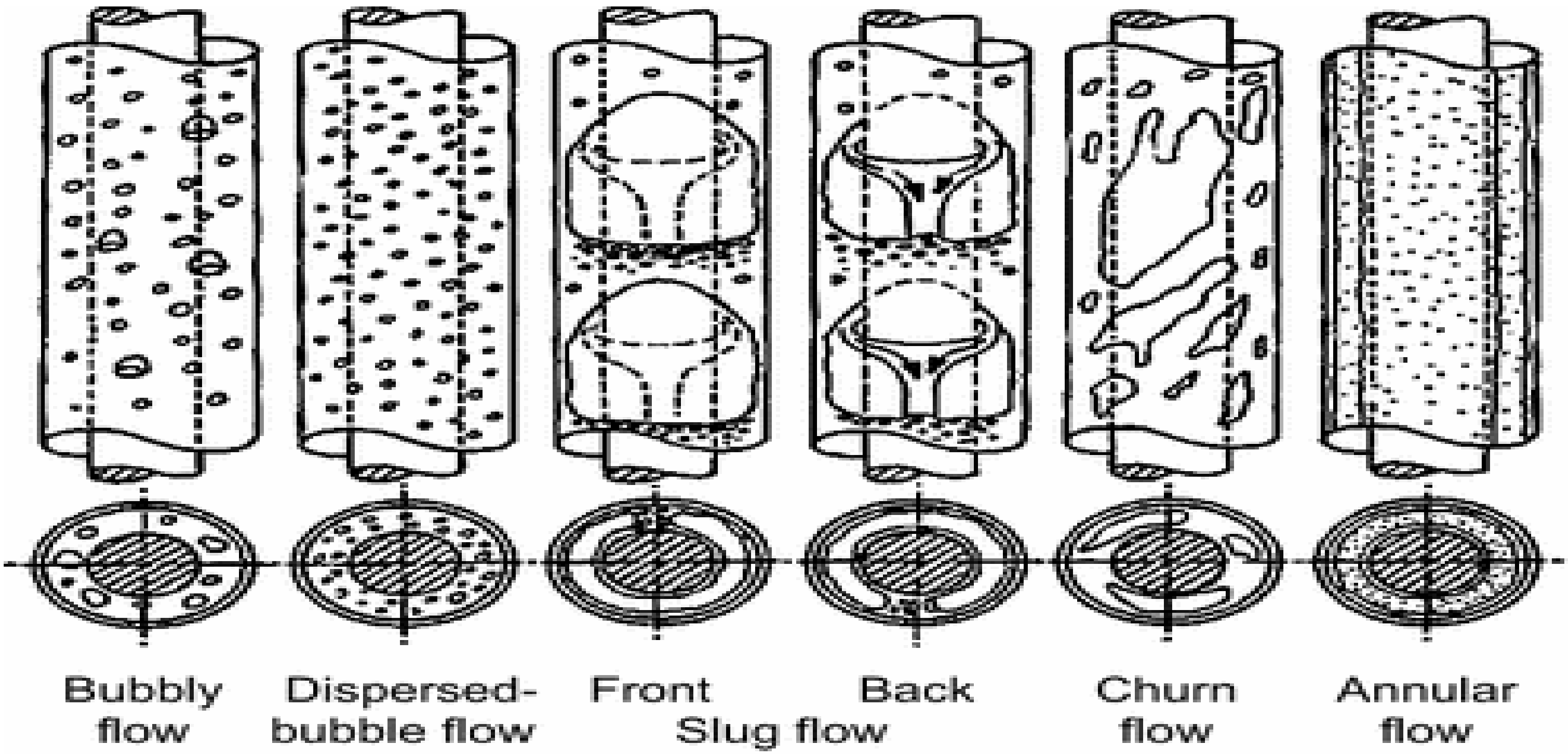


Figure 6: Flow pattern in Annulus

7-2 Vertical Flow in Annulus

❑ Bubble Flow

- ✓ The gas phase is dispersed into small discrete bubbles in a continuous liquid phase, forming an approximately homogeneous flow through the annulus cross-sectional area.
- ✓ The discrete bubbles occur in two different shapes. namely spherical bubbles and cap bubbles. The spherical bubbles are very small, on the order of 3 to 5 mm diameter, compared with the annulus-cap bubbles, which are relatively larger but still always smaller than half of the configuration hydraulic diameter.

7-2 Vertical Flow in Annulus

- Bubble Flow

  - ✓ The upward movement of the small spherical bubbles follows a zig-zag path, whereas the cap bubbles follow a straight path with a faster rise velocity. In a fully eccentric annulus, there is a tendency for the small bubbles and the cap bubbles to migrate into the widest gap of the annulus cross-sectional area. This causes a higher local void fraction relative to the cross-sectional average void fraction.

  - ✓ At high liquid velocities, the mixture appears to flow at the same velocity with no slippage between the phases, regardless of the annulus geometry.

7-2 Vertical Flow in Annulus

- Slug Flow

  - This now is characterized by large cap bubbles of gas moving upward, followed by liquid slugs that bridge the entire cross-sectional area and contain small spherically distributed gas bubbles. The large gas bubbles, which occupy almost the entire cross-sectional area of the annulus, are similar to the ones occurring in pipe now and also are termed Taylor bubbles.

  - The Taylor bubbles do not occupy the total cross-sectional area because they have a preferential channel through which most of the liquid ahead of the bubble flows backward.

7-2 Vertical Flow in Annulus

- Slug Flow

  - ✓ This preferential channel exists from the top to the bottom of the bubble and from the tubing wall to the casing wall. Because of the presence of this channel, no symmetry is observed for the Taylor bubble with respect to either vertical or horizontal planes.

  - ✓ The liquid phase flows backward in the form of films, around the Taylor bubble, and through the preferential channel, wetting both the tubing and the casing walls. This tends to create a high turbulent region behind the Taylor bubble. Contrary to the concentric annulus case, for a fully eccentric annulus the preferential liquid channel always is located where the pipe walls are in contact.

7-2 Vertical Flow in Annulus

- Churn Flow

  - ✓ The characteristics of churn flow are similar to those of pipe flow.

  - ✓ There is no change in flow characteristics observed with the annulus configuration.

7-2 Vertical Flow in Annulus

- Annular Flow

  - ✓ The gas is a continuous phase flowing in the core of the annulus cross-sectional area. The liquid flows upward, partially as wavy films around the tubing and casing walls and partially in the form of tiny spherical droplets entrained in the gas core.

  - ✓ The outer film that wets the casing wall is always thicker than the inner film flowing on the tubing wall. Liquid accumulation near the pipe-wall contact point is an additional characteristic of annular flow in a fully eccentric annulus.

  - ✓ This accumulation results from the merging of the casing and tubing liquid films, which probably happens as a result of the low local gas velocities.

7-2 Vertical Flow in Annulus

- Annular Flow
  - ✓ Comparison among flow patterns occurring in upward vertical flow in a pipe and in an annulus reveals that the existence of an inner pipe in the annulus changes the characteristics of slug flow and annular flow.
  - ✓ The Taylor bubbles in an annulus are not symmetric, having a preferential liquid flow channel through which most of the liquid phase is shed backward.
  - ✓ Two films exist in the annular-flow pattern, one flowing around the tubing wall and one around the casing wall.
  - ✓ These flow modifications seem to be a function of the pipe diameter ratio and the eccentricity of the annulus.

7-3 Horizontal and Near Horizontal Flow

- The existing flow patterns in these configurations can be classified as:
  - ✓ Stratified flow (Stratified-Smooth and Stratified-Wavy)
  - ✓ Intermittent flow (Slug flow and Elongated-Bubble (Plug) flow)
  - ✓ Annular flow
  - ✓ Dispersed-Bubble flow.
- Refer to the Fig. 6 these flow patterns are shown.

7-3 Horizontal and Near Horizontal Flow

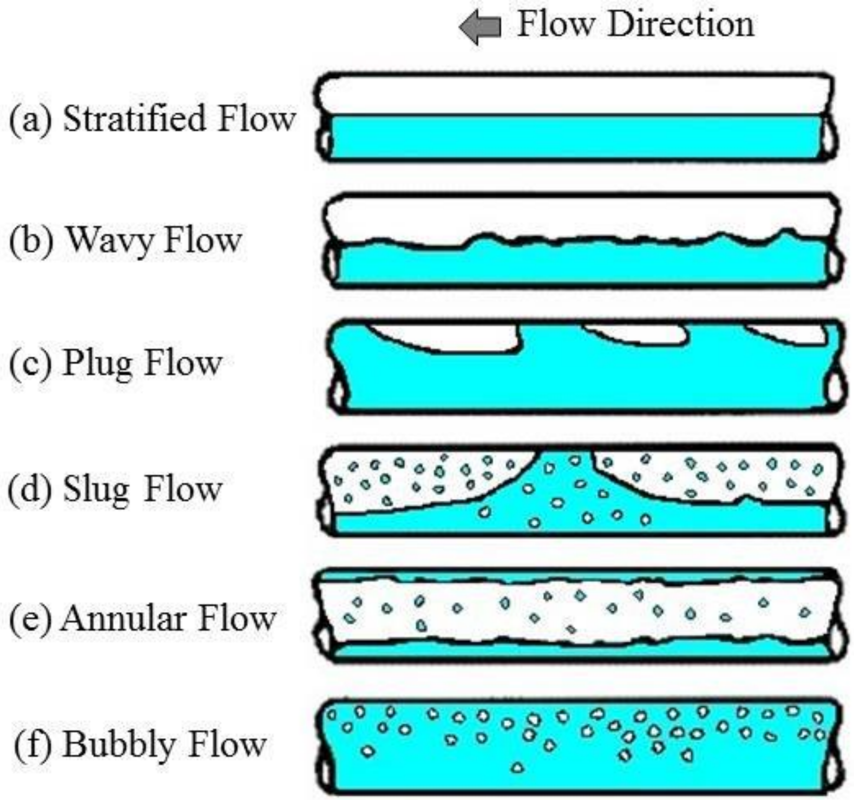


Figure 7: Flow patterns in horizontal and near-horizontal pipes

7-3 Horizontal and Near Horizontal Flow

- Stratified Flow (ST).
  - ✓ This flow pattern occurs at relatively low gas and liquid flow rates.
  - ✓ The two phases are separated by gravity, where the liquid-phase flows at the bottom of the pipe and the gas phase on the top.
  - ✓ The Stratified-flow pattern is subdivided into Stratified-Smooth (SS), where the gas-liquid interface is smooth, and Stratified-Wavy (SW), occurring at relatively higher gas rates, at which stable waves form at the interface.

7-3 Horizontal and Near Horizontal Flow

- Intermittent Flow (I).

  - ✓ Intermittent flow is characterized by alternate flow of liquid and gas.
  - ✓ Plugs or slugs of liquid, which fill the entire pipe cross-sectional area, are separated by gas pockets, which contain a stratified liquid layer flowing along the bottom of the pipe.
  - ✓ The mechanism of the flow is that of a fast moving liquid slug overriding the slow-moving liquid film ahead of it. The liquid in the slug body may be aerated by small bubbles, which are concentrated toward the front of the slug and at the top of the pipe. The Intermittent-flow pattern is divided into Slug (SL) and Elongated-Bubble (EB) patterns.

7-3 Horizontal and Near Horizontal Flow

- Intermittent Flow (I).
  - ✓ The flow behavior of Slug and Elongated-Bubble patterns is the same with respect to the flow mechanism.
  - ✓ The Elongated-Bubble pattern is considered the limiting case of Slug flow, when the liquid slug is free of entrained bubbles.
  - ✓ This occurs at relatively lower gas rates when the flow is calmer.
  - ✓ At higher gas-flow rates, where the flow at the front of the slug is in the form of an eddy with entrained bubbles (caused by picking up of the slow moving film), the flow is designated as Slug flow.

7-3 Horizontal and Near Horizontal Flow

- Annular Flow (A).
  - ✓ Annular flow occurs at very high gas-flow rates. The gas phase flows in a core of high velocity, which may contain entrained liquid droplets.
  - ✓ The liquid flows as a thin film around the pipe wall. The interface is highly wavy, resulting in a high interfacial shear stress. The film at the bottom is usually thicker than that at the top, depending upon the relative magnitudes of the gas- and liquid-flow rates.
  - ✓ At the lowest gas-flow rates, most of the liquid flows at the bottom of the pipe, while aerated unstable waves are swept around the pipe periphery and wet the upper pipe wall occasionally.

7-3 Horizontal and Near Horizontal Flow

- Annular Flow (A).
  - ✓ This flow occurs on the transition boundary between Stratified-Wavy, Slug and Annular flow.
  - ✓ It is not Stratified-Wavy because liquid is swept around and wets the upper pipe wall with a thin film. It is also not Slug flow because no liquid bridging of the pipe cross section is formed.
  - ✓ As a result, the frothy waves are not accelerated by the gas-phase but rather move slower.
  - ✓ Also, it is not fully developed Annular flow, which requires a stable film around the pipe periphery.

7-3 Horizontal and Near Horizontal Flow

- Annular Flow (A).
  - ✓ This flow pattern is designated sometimes as a "Proto Slug" flow. On the basis of the definitions and mechanisms of Slug and Annular flows, this regime is termed as Wavy-Annular (WA) and classified as a subgroup of Annular flow.
  - ✓ The difference between Slug flow and Wavy-Annular flow is more distinguishable in upward inclined flow.
  - ✓ During Slug flow, back flow of the liquid film between slugs is observed, whereas in Wavy Annular flow, the liquid moves forward uphill with frothy waves superimposed on the film.
  - ✓ These waves move much slower than the gas-phase.

7-3 Horizontal and Near Horizontal Flow

- Dispersed-Bubble Flow (DB)

  - ✓ At very high liquid-flow rates, the liquid-phase is the continuous-phase, in which the gas-phase is dispersed as discrete bubbles. The transition to this flow pattern is defined either by the condition where bubbles are first suspended in the liquid or when gas pockets, which touch the top of the pipe, are destroyed.

  - ✓ When this happens, most of the bubbles are located near the upper pipe wall. At higher liquid rates, the gas bubbles are dispersed more uniformly in the entire cross-sectional area of the pipe.

  - ✓ Under Dispersed-Bubble-flow condition, as a result of high liquid-flow rates, the two phases move at the same velocity, and the flow is considered homogeneous no-slip.

7-3-1 What Governs Flow Patterns?

- The two-phase flow pattern in pipes is governed by a mechanical force balance acting on each phase.
- There are several forces that act on each phase in different magnitudes and directions:
  - ✓ **Inertia** or momentum force ($\rho v^2 L^2$)
  - ✓ **Gravity** force ($\Delta\rho g L^2$)
  - ✓ **Viscous** force ($\mu v L$)
  - ✓ **Surface** tension forces (σ)
- For simplicity, only two forces, inertia and gravity, are considered in this discussion of what governs the flow pattern in pipes.

7-3-1 What Governs Flow Patterns?

- Inertia is the axial force along the pipe that pushes the fluids in the direction of flow; thus, it is a function of fluid velocity.

- In contrast, **gravity** is a force that acts downward and thus has a component in the opposite direction to the inertial force if the pipe is inclined upward.

- When gravity and inertia forces are relatively similar in magnitude, a combination of mixing and separation of phases occurs, resulting in intermittent flow.

- Froude number is commonly used to describe physical phenomena that involve both inertia and gravity forces.

7-3-1-1 Flow Pattern Matrix

- Fig.8 is a relative inertia-to-gravity matrix that shows how the two forces act together to govern two-phase flow patterns.
- Quadrant II (QII) is the relatively high inertia and low-to-intermediate gravity condition, in which the flow patterns are mainly dispersed.
- Quadrant III (QIII) combines relatively low inertia and high gravity, resulting in mainly segregated flows.
- Quadrant IV (QIV) is relatively high gravity and high inertia (i.e., the two forces are comparable to each other), which result in a combination of dispersed and segregated flows or intermittent flow.

7-3-1-1 Flow Pattern Matrix

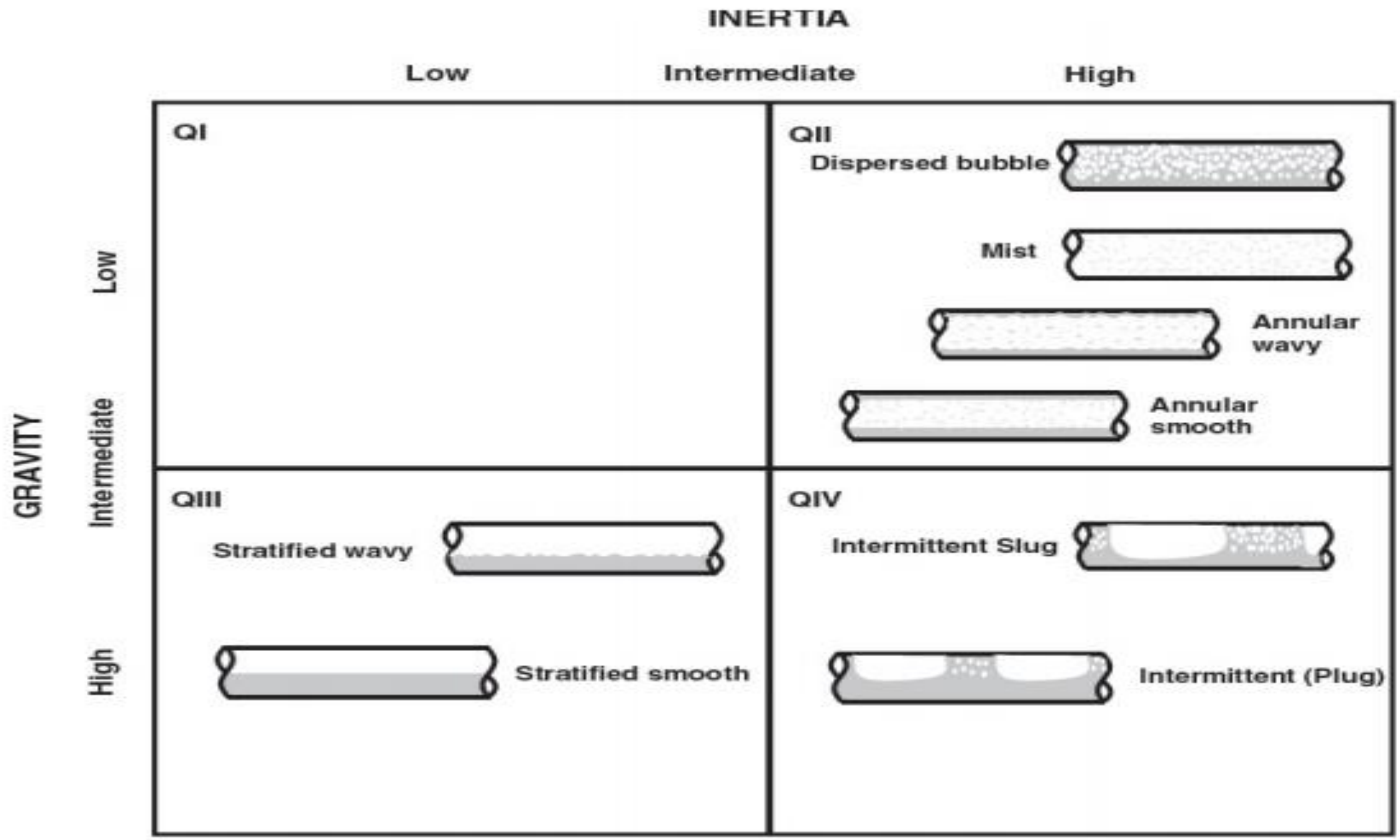


Figure 8: Flow patterns relative to inertia/gravity matrix

7-4 Flow Pattern Occurrence

- The following is a description of a typical sequence of flow patterns in an oil wellbore.
- Starting with a flowing bottom-hole pressure above the bubble-point pressure, only a liquid phase exists at the bottom.
- As the liquid flows upward, it experiences pressure decline resulting in the liberation of some of the gas dissolved in the liquid phase.
- The liberated gas appears as small bubbles in the continuous liquid phase, which characterizes the bubble-flow pattern.

7-4 Flow Pattern Occurrence

- This creates more and larger bubbles that start coalescing with each other.
- The coalescence creates larger Taylor bubbles separated by the continuous liquid phase. The slug-flow pattern thus occurs.
- This creates a chaotic two-phase flow, defined earlier as churn flow.
- The churn flow continues to exist until the gas flow rate is sufficiently high to push the liquid against the pipe wall. This characterizes the existence of annular flow.

7-4 Flow Pattern Occurrence

- After identifying the different flow patterns, tools are needed to predict their occurrence and flow behavior.
- This requires a thorough understanding of the mechanisms of each flow pattern.
- Ansari et al. used the best models for flow-pattern prediction and for flow pattern behavior. They are discussed later.

7-5 Flow Pattern Prediction

- Methods to predict the occurrence of the various flow patterns in wells have fallen into two categories.

- Empirical flow-pattern maps were drawn that could be used to predict the transitions.

- The second method to predict flow patterns considers the basic mechanisms that are important in causing a flow-pattern change. This approach is not restricted to a narrow range of flow parameters and has proved to be highly successful.

7-5 Flow Pattern Prediction

- The earlier approach for predicting flow patterns has been the empirical approach.
- Determination of the flow patterns was carried out mainly by visual observations.
- In most cases, the coordinates were chosen arbitrarily, without a physical basis.
- Thus, such empirical maps are expected to be reliable only in the range of conditions similar to those under which the data were acquired, and extension to other flow conditions is uncertain.

7-5 Flow Pattern Prediction

- Many different coordinate systems have been proposed for constructing flow-pattern maps, most of which are dimensional, such as the mass flow rates, the momentum fluxes, or the superficial velocities, as used by Mandhane et al. (1974).
- The Mandhane et al. flow-pattern map for horizontal flow, shown in Fig.7, is unique, being based on a large data bank [the American Gas Association (AGA)-American Petroleum Institute (API) Data Bank].

7-5 Flow Pattern Prediction

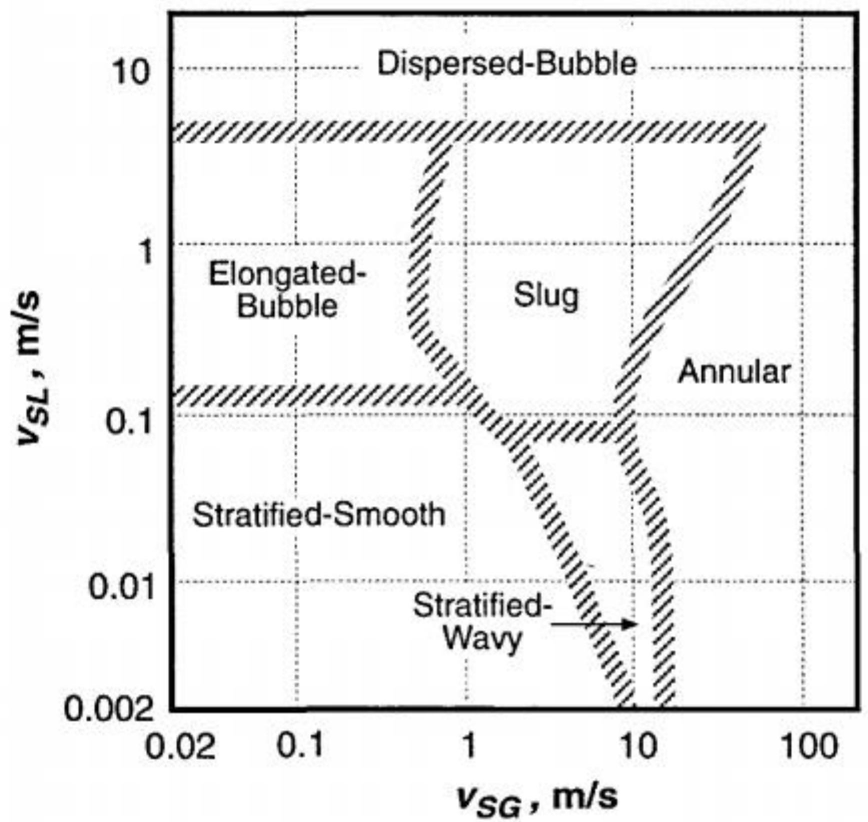


Figure 9: Flow-pattern map for horizontal pipes (after Mandhane et al., 1974)

7-5 Flow Pattern Prediction

- Various investigators have tried to extend the validity of their flow-pattern maps by choosing dimensionless coordinates or correction factors for fluid physical properties.

- An example of a flow-pattern map with correction factors, X and Y, for fluid properties is the Govier and Aziz (1972) map, shown in Fig. 8, developed for vertical flow.

7-5 Flow Pattern Prediction

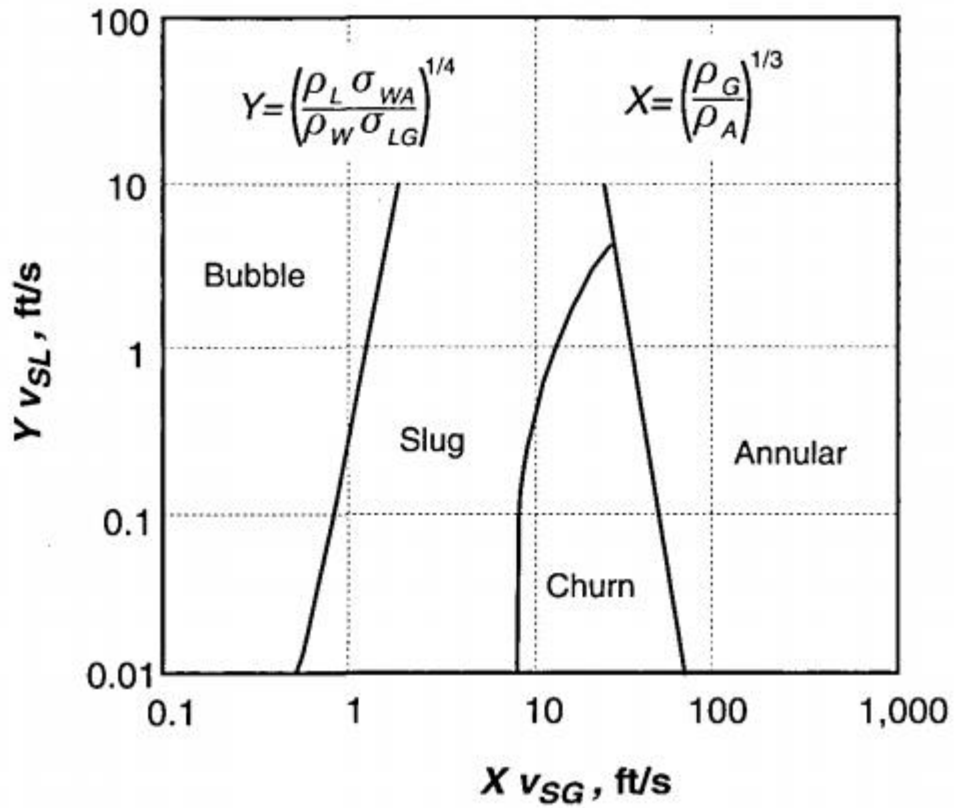


Fig. 10: Flow-pattern map for vertical pipes (after Govier and Aziz, 1972)

7-5 Flow Pattern Prediction

- Dimensionless coordinates have been proposed, for example, by Griffith and Wallis (1961) and Spedding and Nguyen (1980).

- Griffith and Wallis showed that the transition from slug to annular flow is governed by the dimensionless groups $\frac{V_{sg}}{V_m}$ and $\frac{V_m^2}{gd}$

  presented their flow-pattern map, as shown in Fig. 11, using these groups as coordinates.

7-5 Flow Pattern Prediction

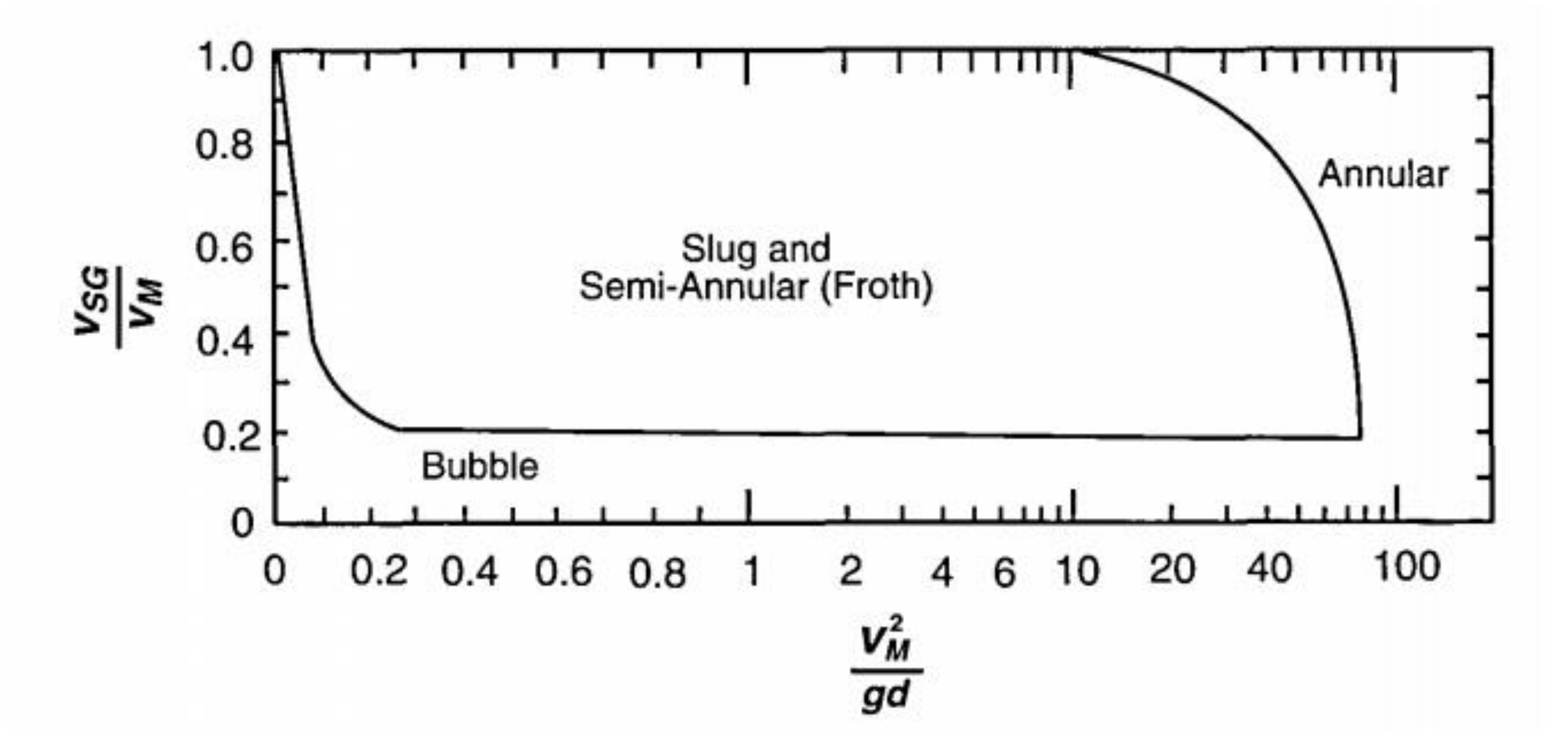


Fig.11: Flow map for vertical pipes (after Griffith and Wallis, 1961)

7-5 Flow Pattern Prediction

- During discussion of flow-pattern maps, one must mention the work published by Baker (1954) who has certainly been a pioneer in this area.

- His flow-pattern map, given in Fig.10, uses mixed dimensionless/dimensional coordinates, where G_L and G_G are the mass fluxes of the liquid and gas phases, respectively, and $A = [(\rho_G/0.075)(\rho_L/62.3)]^{1/2}$ and $\psi = 73/\sigma\ [(\mu_L)(\rho_L/62.3)^2]^{0.33}$ are correction factors for fluid properties in field units.

- Baker's flow-pattern map is probably the most durable one, as it is still in use in the petroleum industry.

7-5 Flow Pattern Prediction

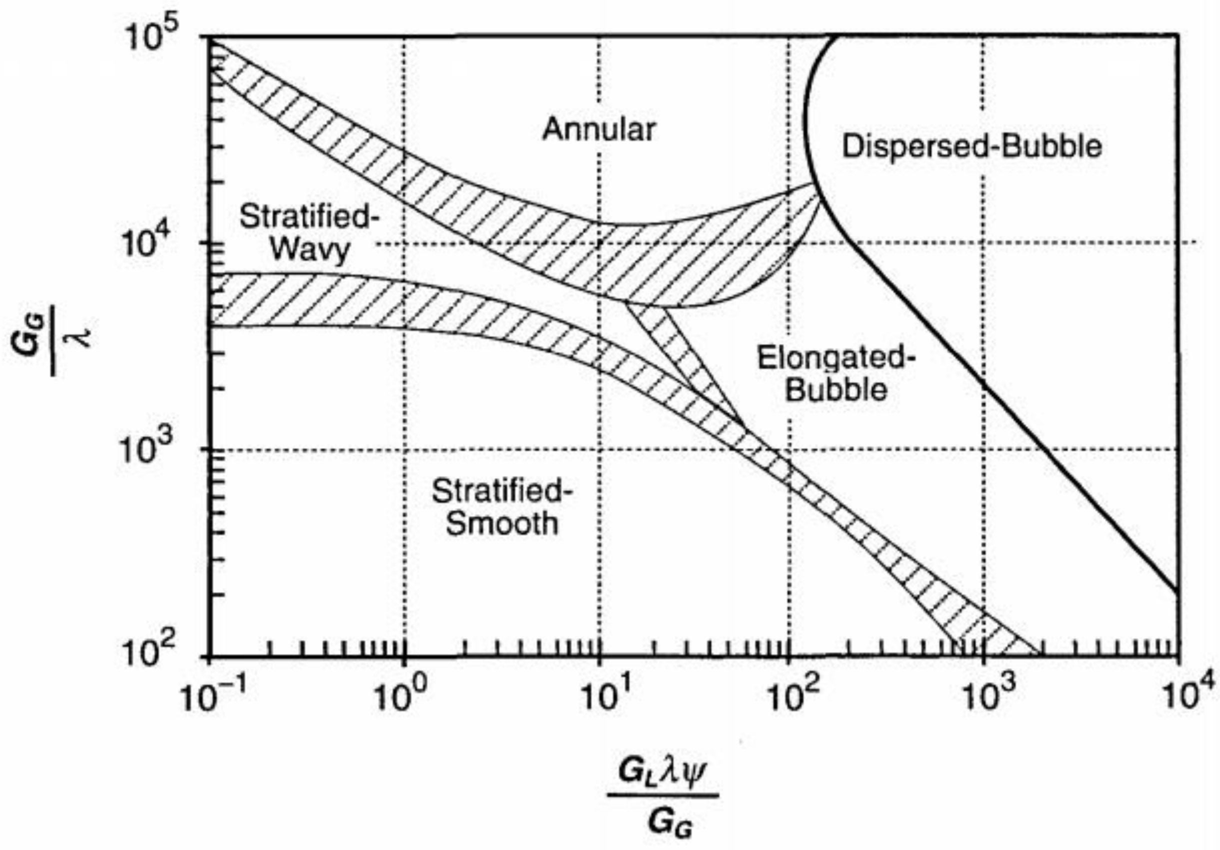


Fig. 12: Flow-pattern map for horizontal pipes (after Baker, 1954)

7-5 Flow Pattern Prediction

- To determine the flow pattern at specific flow conditions, calculate the in-situ no-slip liquid holdup and the mixture Froude number. Using the coordinates of (λ_L, N_{Fr}) in Fig.13 , identify the flow pattern.

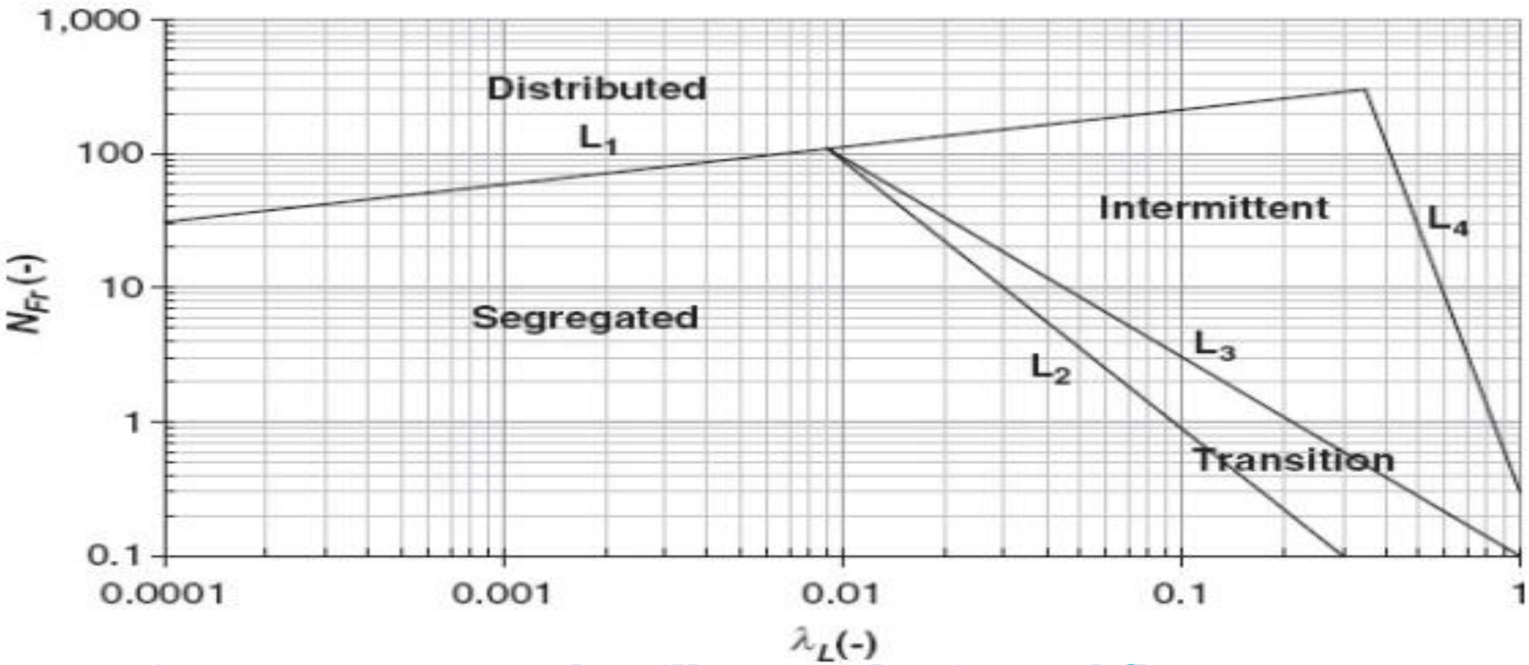


Figure 13:Beggs and Brill (1973) horizontal flow pattern map

Example 4

- A two-phase mixture of gas and crude oil flows in a horizontal 0.61-m (24- in.) ID pipeline. At a specific location, the in-situ flow parameters and fluid physical properties are known; these are given below. Determine the flow pattern at that location.

V_{sL} = 1.5 m/s

ρ_g = 115 kg/m^3

V_{sg} = 1 m/s

μ_L = 0.005 Pa.s (5 cp)

ρ_L = 800 kg/m^3

μ_g = 0.00002 Pa·s

σ = 0.03 N/m

Example 4

- Solution:

$$\lambda_L = \frac{V_{sL}}{V_{sL} + V_{sg}} = \frac{1.5}{2.5} = 0.6$$

$$N_{Fr} = \frac{V_M{}^2}{gD} = \frac{2.5^2}{9.8 \times 0.61} = 1.04$$

$$Using\ \ figure\ 13 \rightarrow intermittent\ \ flow\ region \rightarrow The\ \ flowpattern\ \ is\ \ plug\ \ or\ \ slug\ \ flow.$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model
  - ✓ The Taitel and Dukler (1976) model is applicable for steady-state, fully developed, Newtonian flow in horizontal and slightly inclined pipelines.
  - ✓ The starting point of the model is equilibrium stratified flow.
  - ✓ Assuming stratified flow to occur, the flow variables, including the liquid level in the pipe, are determined.
  - ✓ A stability analysis is then performed to determine whether or not the flow configuration is stable.
  - ✓ If the flow is stable, stratified flow occurs. If the flow is not stable, a change to non-stratified flow occurs, and the resulting flow pattern is determined.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model
  - ✓ Equilibrium-stratified-flow configuration is shown in Fig. 21.
  - ✓ The pipeline is inclined at θ inclination angle from the horizontal, and the gas and liquid average velocities are v_G and v_L, respectively.
  - ✓ The area for flow and the wetted perimeter of the gas and the liquid phases are A_G and S_G and A_L and S_L respectively. The interface length is S_I, and the liquid level (under equilibrium conditions) is h_L.
  - ✓ The objective of this part of the model is to determine the equilibrium liquid level in the pipe, h_L, for a given set of flow conditions: the gas- and liquid-flow rates, pipe diameter and inclination angle, and the fluid physical properties of the phases.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model

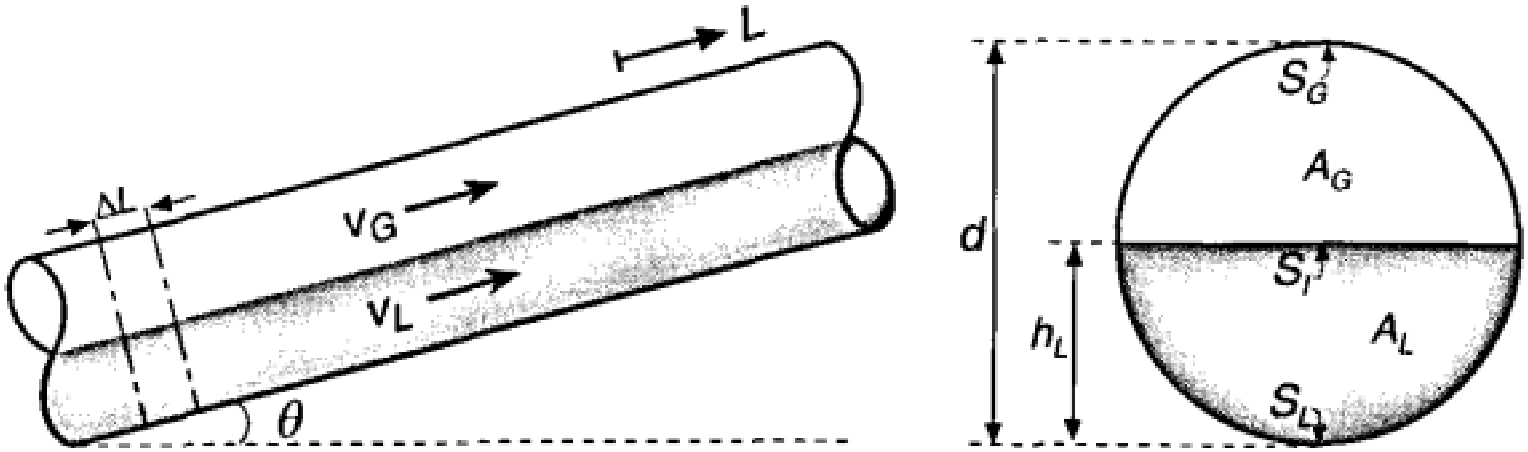


Figure ۱۴: Equilibrium stratified flow

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model

  - ✓ This is carried out by applying momentum balances on the gas and the liquid phases in a differential control volume with an axial length of ΔL, as shown in Fig. 22.

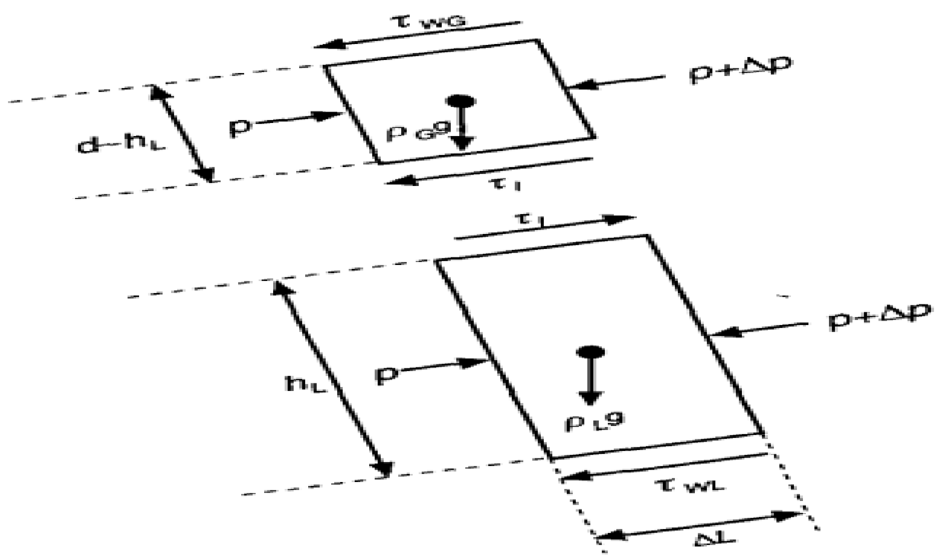


Figure ۱۵: Equilibrium stratified flow gas and liquid momentum balances

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model

  - ✓ The momentum (force) balances for the liquid and gas phases are given, respectively, by:

$$-A_L \left.\frac{dp}{dL}\right)_L - \tau_{WL} S_L + \tau_L S_L - \rho_L A_L g \sin\theta = 0 \qquad (88)$$

and

$$-A_G \left.\frac{dp}{dL}\right)_G - \tau_{WG} S_G - \tau_L S_L - \rho_G A_G g \sin\theta = 0$$

  - ✓ Eliminating the pressure gradient, the combined-momentum equation for the two phases is obtained, as:

$$\tau_{WG} \frac{S_G}{A_G} - \tau_{WL} \frac{S_L}{A_L} + \tau_L S_L \left(\frac{1}{A_L} + \frac{1}{A_G} \right) - (\rho_L - \rho_G) g \sin\theta = 0 \qquad (89)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model

  - ✓ The combined-momentum equation is an implicit equation for h_L, the liquid level in the pipe.

  - ✓ It combines all the forces that act on the liquid and gas phases, which in turn determine the location of the liquid level in the pipe. In order to solve the equation for h_L, it is necessary to determine the different geometrical and force variables in the equation.

  - ✓ The calculation of the forces in the equation is carried out with the single-phase-flow method based on the hydraulic diameter concept. The respective hydraulic diameters of the liquid and gas phases are given by: $d_L = \dfrac{4A_L}{S_L}$ and $d_G = \dfrac{4A_G}{S_G + S_L}$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model

  - ✓ The Reynolds numbers and the friction factors (for a smooth pipe) of each of the phases are:

$$f_L = C_L \left(Re_L\right)^{-n} = C_L \left(\frac{d_L v_L \rho_L}{\mu_L}\right)^{-n} \qquad (90)$$

$$f_G = C_G \left(Re_G\right)^{-n} = C_G \left(\frac{d_G v_G \rho_G}{\mu_G}\right)^{-n}$$

  - ✓ where $C_L = C_G = 16$ and m = n = 1 for laminar flow, and $C_L = C_G = 0.046$ and m = n = 0.2 for turbulent flow.

  - ✓ The wall shear stresses corresponding to each phase are:

$$\tau_{WL} = f_L \frac{\rho_L v_L^2}{2} \quad \text{and} \quad \tau_{WG} = f_G \frac{\rho_G v_G^2}{2} \qquad (90)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model

  - ✓ The interfacial shear stress is given, by definition, as:

$$\tau_I = f_I \frac{\rho_G \left(v_G - v_L\right)^2}{2} \qquad (11)$$

  - ✓ In this model, it is assumed that $f_I=f_G$. In addition, the interface velocity is neglected. With these approximations, the interfacial shear stress is equal to the gas-phase wall shear stress, namely, $\tau_I=\tau_{WG}$.

  - ✓ All the variables can be written in nondimensional form by choosing suitable scaleup parameters. The diameter, d, is used for the length variables, d^2 for the area, and v_{SL} and v_{SG} for the liquid and gas velocities, respectively.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model

  - ✓ The dimensionless variables are designated by a tilde (~), as given :

$$\tilde{S}_L = \frac{S_L}{d}\ ,\ \tilde{h}_L = \frac{h_L}{d}\ ,\ \tilde{A}_L = \frac{A_L}{d^2}\ ,\tilde{v}_L = \frac{v_L}{v_{SL}}\ \ and\ \tilde{v}_G = \frac{v_G}{v_{SG}}$$

  - ✓ Substituting $\tau_I = \tau_{WG}$ and rearranging:

$$\frac{\tau_{WL}}{\tau_{WG}}\frac{S_L}{A_L} - \left(\frac{S_L}{A_L} + \frac{S_G}{A_G} + \frac{S_L}{A_G}\right) + \left[\frac{(\rho_L - \rho_G)\, g \sin\theta}{\tau_{WG}}\right] \qquad (92)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model

  - ✓ Substituting the dimensionless parameters:

$$X^2 = \left[\left(\tilde{v}_L \tilde{d}_L \right)^{-n} \tilde{v}_L^{\,2} \frac{\tilde{S}_L}{\tilde{A}_L} \right] - \left[\left(\tilde{v}_G \tilde{d}_G \right)^{-m} \tilde{v}_G^{\,2} \left(\frac{\tilde{S}_G}{\tilde{A}_G} + \frac{\tilde{S}_L}{\tilde{A}_L} + \frac{\tilde{S}_L}{\tilde{A}_G} \right) \right] + 4Y = 0 \quad (93)$$

  - ✓ Two dimensionless groups emerge from the analysis, namely, X, the Lockhart and Martinelli parameter and Y, an inclination-angle parameter, given by:

$$X^2 = \frac{\dfrac{4C_L}{d} \left(\dfrac{\rho_L v_{SL} d}{\mu_L} \right)^{-n} \dfrac{\rho_L v_{SL}^{\,2}}{2}}{\dfrac{4C_G}{d} \left(\dfrac{\rho_G v_{SG} d}{\mu_G} \right)^{-m} \dfrac{\rho_G v_{SG}^{\,2}}{2}} = \frac{\left. -\dfrac{dp}{dL} \right)_{SL}}{\left. -\dfrac{dp}{dL} \right)_{SG}}$$

$$Y = \frac{\left(\rho_L - \rho_G \right) g \sin\theta}{\dfrac{4C_G}{d} \left(\dfrac{\rho_G v_{SG} d}{\mu_G} \right)^{-m} \dfrac{\rho_G v_{SG}^{\,2}}{2}} = \frac{\left(\rho_L - \rho_G \right) g \sin\theta}{\left. -\dfrac{dp}{dL} \right)_{SG}}$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model

  - ✓ All the dimensionless (tilde) variables are unique functions of the dimensionless liquid level $\tilde{h}_L = h_L / d$, Thus, it is proved that:

$$\tilde{h}_L = \tilde{h}_L \left(X , Y \right)$$

  - ✓ A generalized plot of h_L as a function of X and is given in Fig.23.
  - ✓ The solid lines represent the case where both phases are in turbulent flow (C_L = C_G = 0.046 and m = n=0.2), while the dashed lines are for the case of turbulent liquid and laminar gas flow (C_L = 0.046, C_G=16, n=0.2, m = 1).

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model

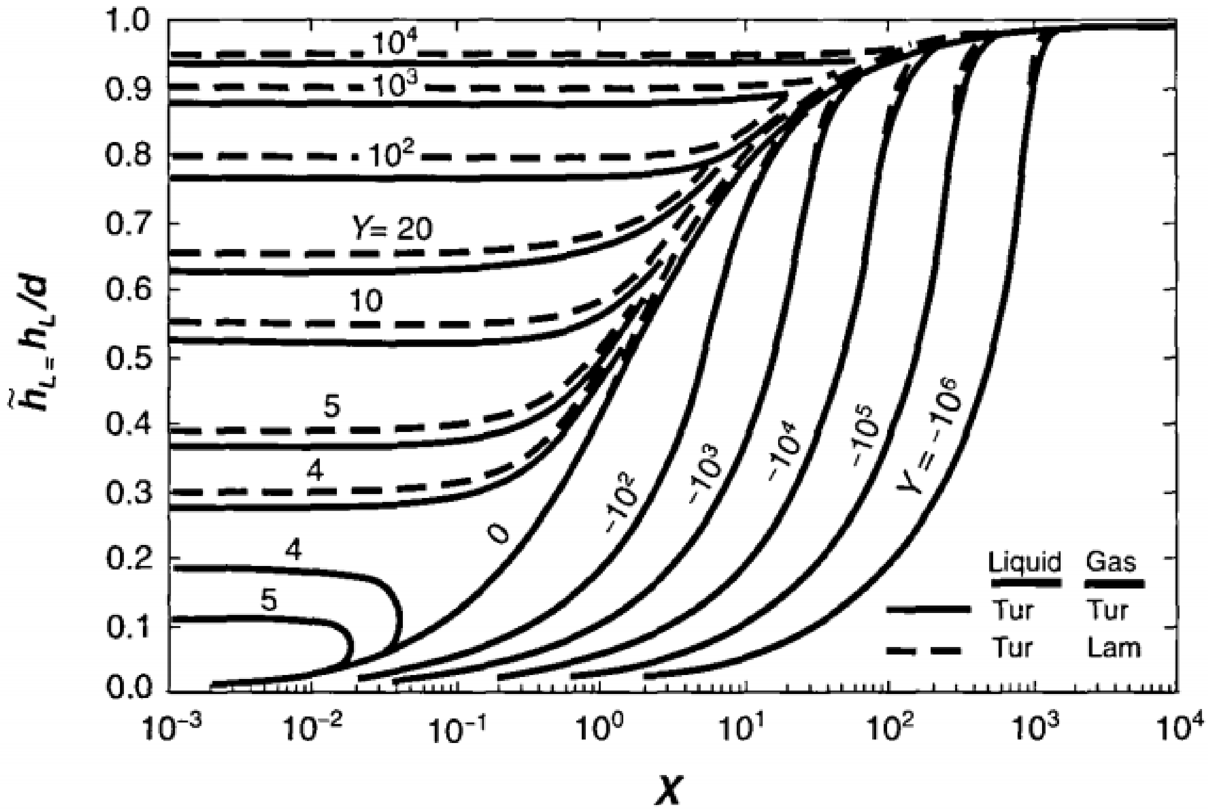


Figure ۱۶: Equilibrium liquid level in stratified flow

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model

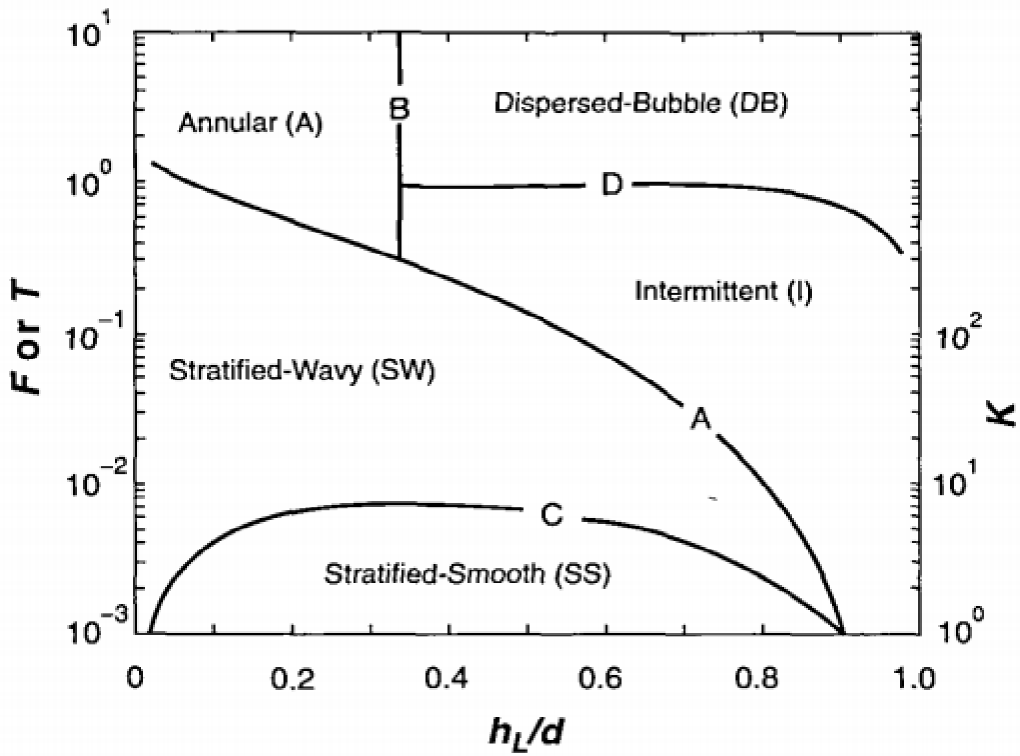


Figure ١٧: Generalized flow-pattern map for horizontal and slightly inclined flow (Taitel and Dukler, 1976)

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model

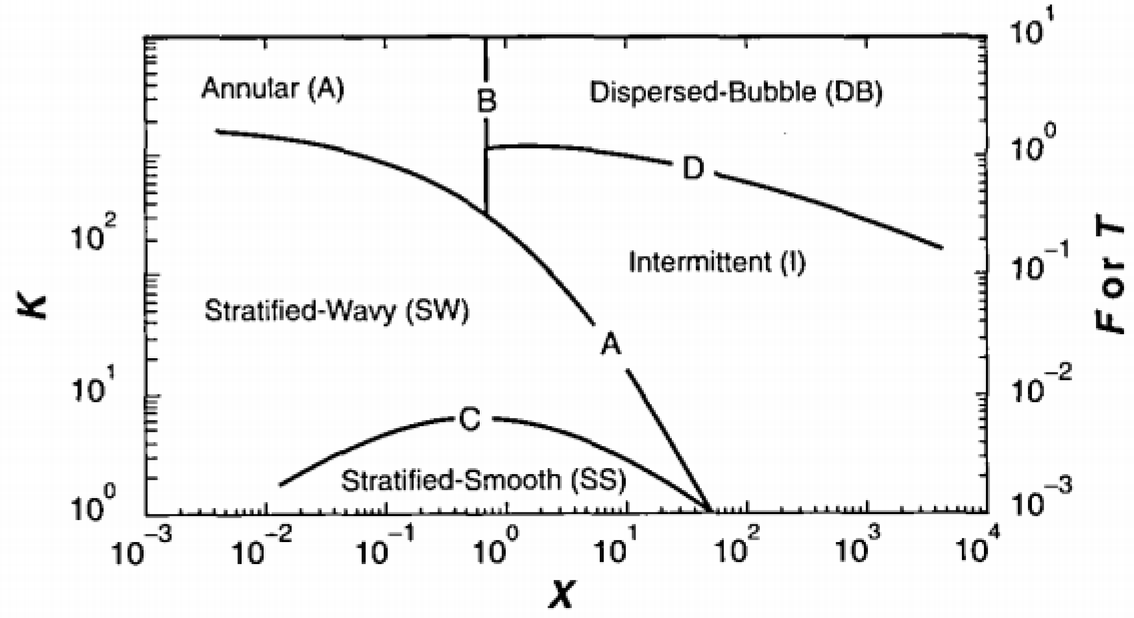


Figure ۱۸: Generalized flow-pattern map for horizontal flow (Taitel and Dukler, 1976)

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model
  - ✓ Stratified to Non-stratified Transition (Transition A):
    - ✓ The stratified-to-non-stratified transition criterion is:

$$F^2\left[\frac{1}{\left(1-\tilde{h}_L\right)^2}\frac{\tilde{v}_G^{\,2}\,\tilde{S}_I}{\tilde{A}_G}\right]\geq 1 \qquad (94)$$

$$F=\sqrt{\frac{\rho_G}{\left(\rho_L-\rho_G\right)}}\frac{v_{SG}}{\sqrt{dg\cos\theta}}$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model

  - ✓ Intermittent or Dispersed-Bubble to Annular Transition (Transition B): The criterion for the transition from intermittent or dispersed-bubble to annular flow is described schematically in Fig. 26.

  - ✓ When either the gas- or the liquid-flow rates are increased, the stratified-flow structure becomes unstable, and transition from stratified to non-stratified flow occurs.

  - ✓ Under unstable flow conditions, the following may occur: at low gas- and high-liquid flow rates, the liquid level in the pipe is high and the growing waves have sufficient liquid supply from the film, eventually blocking the cross-sectional area of the pipe. This blockage forms a stable liquid slug, and slug flow develops.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model

  - ✓ Intermittent or Dispersed-Bubble to Annular Transition (Transition B):

    - ✓ This is shown schematically in Fig. 26 (Part a), whereby, schematically, the crest of the wave reaches the top pipe wall before the trough of the wave reaches the bottom of the pipe.

    - ✓ However, at low-liquid and high-gas flow rates, the liquid level in the pipe is low. For this case, the wave at the interface do not have sufficient liquid supply from the film, and they are swept up and around the pipe by the high gas velocity.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model
  - ✓ Intermittent or Dispersed-Bubble to Annular Transition (Transition B):
    - ✓ Under these conditions, a liquid-film annulus is created rather than a slug, resulting in a transition to annular flow. This is described in Fig. 26 (Part b), where the trough of the wave reaches the bottom of the pipe before the crest of the wave touches the top wall of the pipe.
    - ✓ Thus, it is suggested that this transition depends uniquely on the liquid level in the pipe. Intuitively, a value of $h_L = 0.5$ was proposed for this transition. As shown schematically in Fig. ۲۶(Part c), for this case, the crest of the wave reaches the top pipe wall at the same time that the trough of the wave reaches the bottom of the pipe.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model

  - ✓ Stratified-Smooth to Stratified-Wavy Transition (Transition C) Criterion:

$$K \geq \frac{2}{\sqrt{\tilde{v}_L\, \tilde{v}_G}\sqrt{s}} \qquad s = 0.01$$

$$K^2 = \frac{\rho_G v_{SG}^2 \rho_L v_{SL}}{(\rho_L - \rho_G)\mu_L g \cos\theta} = \left[\frac{\rho_G v_{SG}^2}{(\rho_L - \rho_G)\mu_L g \cos\theta}\right]\left(\frac{\rho_L v_{SL} d}{\mu_L}\right) = F^2\, Re_{SL}$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model

  ✓ Intermittent to Dispersed-Bubble Transition (Transition D) Criterion:

$$T^2 \geq \left[\frac{8\tilde{A}_G}{\tilde{S}_L \tilde{v}_L^2 \left(\tilde{v}_L \tilde{d}_L \right)^{-n}} \right]$$

$$T = \left[\frac{\left(-\frac{dp}{dL} \right)_{SL}}{\left(\rho_L - \rho_G \right) g \cos\theta} \right]^{0.5}$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Taitel and Dukler (1976) Model

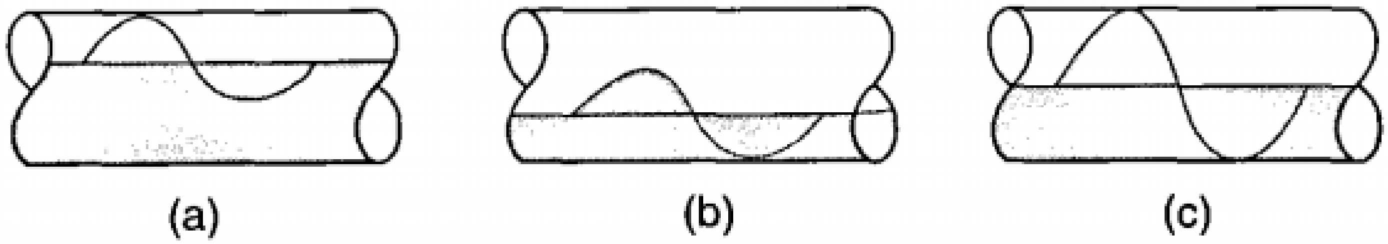


Figure ۱۹: Schematic of transition between intermittent or dispersed-bubble flow to annular flow

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Ansari et al. (1994) presented a comprehensive mechanistic model for vertical wells that consists of flow pattern prediction models and separate hydrodynamic models to predict liquid holdup and pressure gradient for each flow pattern, improved the Taitel et al. model by proposing additional annular flow transition criteria. Furthermore, Barnea et al. also modified the Taitel et al. model to include the effect of inclination angle on upward flow behavior. A typical flow pattern map generated from their models for vertical flow is shown in Fig. 14.

7-5-1 Mechanistic Modeling of Flow Pattern Map

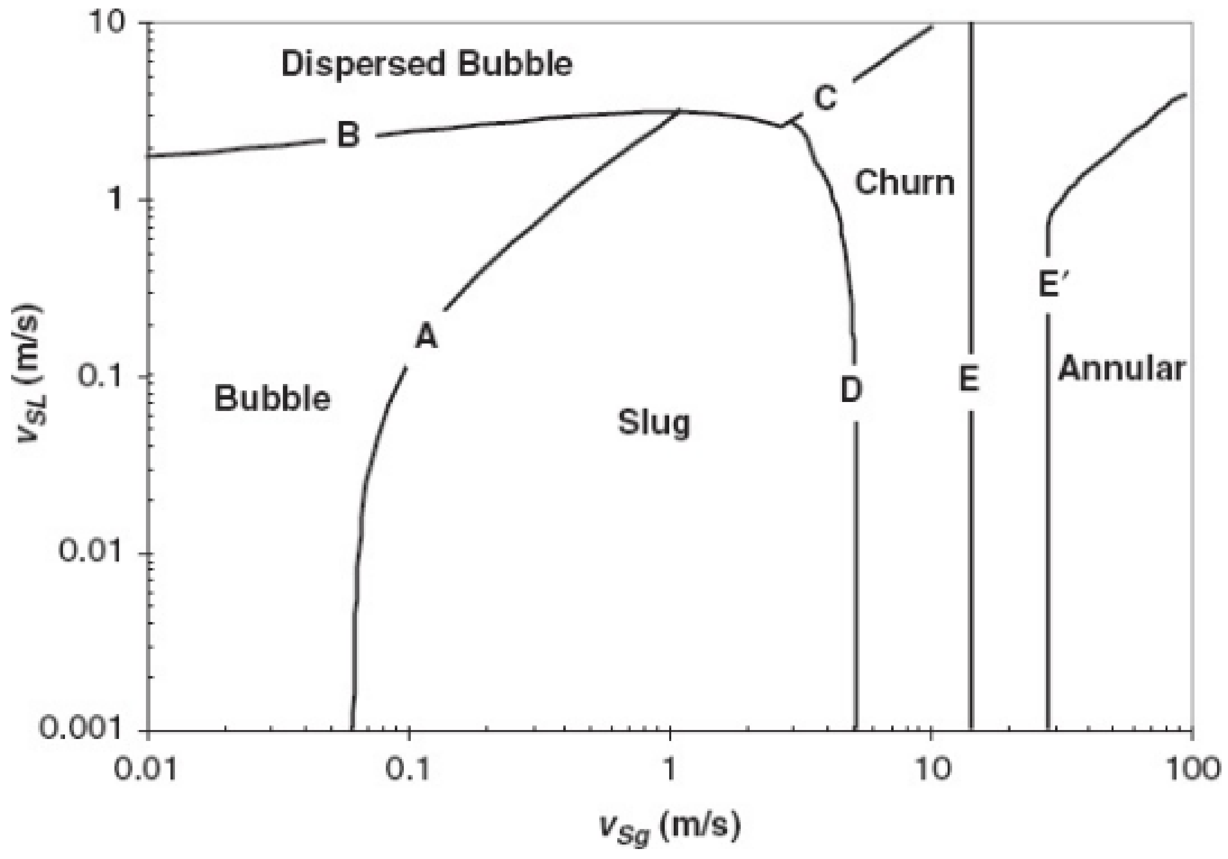


Figure ۲۰: Taitel et al. (1980) vertical flow pattern map

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Existence of Bubble Flow.

  - ✓ Bubble flow does not always exist in vertical pipes when large Taylor bubbles coexist.

  - ✓ It was observed that when the pipe diameter is smaller than a specific minimum diameter, small bubbles disappear because the Taylor bubble velocity is lower than the small-bubble rise velocity, resulting in small bubbles merging into the Taylor bubble and vanishing.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Existence of Bubble Flow.

  - ✓ Therefore, the critical minimum pipe diameter for bubble flow to exist can be determined by equating the velocity of small bubbles to the velocity of a Taylor bubble.

  - ✓ Small bubbles rise in a liquid column at a velocity given by Harmathy (1960) as:

$$V_{0\infty} = \left[\frac{5.48 g \sigma_L \left(\rho_L - \rho_g \right)}{\rho_L^{\ 2}} \right]^{0.25} \qquad (40)$$

  - ✓ Taylor bubble rise velocity in a liquid column is given by:

$$V_{TB} = 0.35\sqrt{gd} \qquad (41)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Existence of Bubble Flow.

  - ✓ Equating Eqs. 40 and 41 yields the minimum diameter for bubble flow to exist as:

$$d_{min} = 19.01\left[\frac{\left(\rho_L - \rho_g\right)\sigma_L}{\rho_L^{\,2} g}\right]^{0.5} \qquad (42)$$

  - ✓ For inclined upward flow (i.e., deviated wells), Barnea et al. (1985) proposed an additional inclination angle criterion for the existence of bubble flow:

$$\frac{cos\theta_{min}}{sin^2\theta_{min}} = \frac{3}{4}cos\left(45\right)\left[\frac{0.98V_{0\infty}^{\;2}}{gd}\right] \qquad (43)$$

Example 5

- A two-phase mixture of gas and crude oil flows in a horizontal pipeline. At a specific location, fluid physical properties are known; these are given below. Determine the minimum diameter for bubble flow to exist.

$\rho_g = 115\ kg/m^3$

$\rho_L = 800\ kg/m^3$

$\sigma = 0.03\ N/m$

Example 5

- Solution:

$$d_{min} = 19.01\left[\frac{(\rho_L - \rho_g)\sigma_L}{{\rho_L}^2 g}\right]^{0.5}$$

$$d_{min} = 19.01\left[\frac{(800 - 115)0.03}{800^2 \times 9.81}\right]^{0.5}$$

$$d_{\mathrm{min}} = 0.034\, m$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Bubble/Slug Transition [A]

  - ✓ The transition from bubble to slug flow is governed by two mechanisms: bubble collisions and the resulting bubble coalescence.

  - ✓ If bubbles stay away from each other, there are few collisions, little coalescence, and no transition to slug flow.

  - ✓ Thus, bubble collision is closely related to gas void fraction in the pipe (i.e., the distance between each bubble).

  - ✓ Experiments by Radovicich and Moissis (1962) have shown that bubble collision frequency increases sharply between void fractions of 0.25–0.3.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Bubble/Slug Transition [A]

  - ✓ Under this assumption, a cubic lattice of bubble-packing geometry gives a gas void fraction of 0.27, as shown in Fig. 15. Therefore, Taitel et al. (1980) adopted a 0.25 critical gas void fraction as the criterion for transition to slug flow. Thus, if the gas void fraction in a vertical or deviated pipe exceeds 0.25, a transition from bubble flow to slug flow occurs.

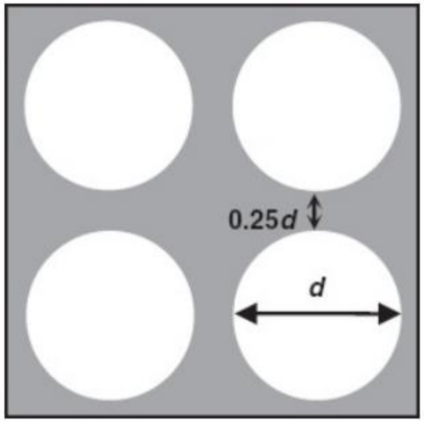


Figure ۲1:Bubble/slug transition gas void fraction bubble-packing geometry

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Bubble/Slug Transition [A]

  - ✓ To calculate gas void fraction we have:

$$V_s = \frac{V_{sg}}{\alpha} - \frac{V_{sL}}{1-\alpha} \qquad (44)$$

  - ✓ Where α is void fraction or 1-H_L . By solving gas void fraction :

$$V_s \alpha^2 - \left(V_m + V_s\right)\alpha + V_{sg} = 0 \qquad (45)$$

  - ✓ The slip velocity in Eq. 45 is given by ($V_s = V_{0\infty}$). Therefore, when the value of α obtained from Eq.45 is less than 0.25 and the operational condition is below transition [B] (see Fig. 14), bubble flow exists.

7-5-1 Mechanistic Modeling of Flow Pattern Map

❑ Bubble/Slug Transition [A]

- ✓ For the case of a deviated well, the Taitel et al. transition is modified by taking into account the component of the Harmathy bubble rise velocity along the pipe axis. Thus,

$$V_s = \left[\frac{5.48\sigma_L \left(\rho_L - \rho_g \right)}{\rho_L{}^2} \right]^{0.25} \qquad (46)$$

- ✓ To predict the entire transition [A] boundary in Fig.14, Eq.14 can be solved for v_{Sg} as a function of v_{SL} by substituting $\alpha = 0.25$ and the slip velocity into Eq. 14. Thus,

$$V_{sg} = 0.333 V_{sL} + 0.25 \left[\frac{5.48 g \sigma_L \left(\rho_L - \rho_g \right)}{\rho_L{}^2} \right]^{0.25} sin\theta \qquad (47)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Bubble/Dispersed Bubble Transition [B]

  - ✓ The bubble/dispersed bubble transition is a balance between the turbulence force in the continuous liquid phase and the bubble surface-tension force (Fig. 14), which will determine the bubble diameter under a given flow condition.

  - ✓ Conversely, if the bubble surface tension force is greater than the liquid turbulence force, bubbles will retain their shape and size, and bubble flow will prevail.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Bubble/Dispersed Bubble Transition [B]

  - ✓ Hinze (a995) proposed a bubble diameter expression at a given flow condition as follow:

$$d_b = \frac{k\left(\frac{\sigma_L}{\rho_L}\right)^{0.6}}{\left[\left(\frac{dp}{dL}\right)\left(\frac{V_m}{\rho_m}\right)\right]^{0.4}} \qquad (48)$$

$$k = 0.725 + 4.15\sqrt{\frac{V_{sg}}{V_m}}$$

$$\frac{dp}{dL} = Homogeneous \ \ frictional \ \ pressure \ \ gradient$$

$$with\,a\,mixture \ \ friction \ \ factor \left(\begin{array}{l} f_m = C\left(\frac{\rho_L V_m d}{\mu_L}\right)^{-n} \\ laminar: C = 64, n = 1 \\ Turbulent: C = 0.18 \end{array} \right)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Bubble/Dispersed Bubble Transition [B]

  - ✓ The critical agglomeration bubble diameter is defined as the bubble size above which bubbles will lose their solid sphere behavior, agglomerate, and eventually coalesce.

  - ✓ Brodkey (1967) proposed an empirical expression to predict critical bubble diameter, which was later modified by Barnea et al. (1982b) and given as:

$$d_c = 2\left[\frac{0.4\sigma_L}{\left(\rho_L - \rho_g\right)g}\right]^{0.5} \qquad (49)$$

  - ✓ When $d_b < d_c$, dispersed bubble flow exists; otherwise, the flow is bubble.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Slug/Dispersed Bubble Transition [C].

  ✓ When the gas void fraction, α, is greater than 0.25, dispersed bubble flow may occur if the turbulence forces in the liquid phase are large enough to break down Taylor bubbles into finely dispersed bubbles.

  ✓ Therefore, transition [B] is valid even if $\alpha > 0.25$. However, for dispersed bubble flow to occur when $\alpha > 0.25$, the no-slip gas void fraction must not exceed the maximum allowable rhombohedral bubble packing of 0.76.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Slug/Annular Transition [E and E′].
  - ✓ For the transition to annular flow, two transition boundaries have to be defined: [E] and [E′], shown in Fig. 5.5. For annular flow to occur, both transitions must be satisfied.
  - ✓ Transition [E] is the Taitel et al. (1980) annular transition, which defines the condition required to transport the largest liquid droplets upward in the gas core, preventing liquid fallback and accumulation.
  - ✓ Transition [E′] is the Barnea (1987) annular transition, which defines the liquid-film characteristics and flow condition in the transition to annular flow.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Slug/Annular Transition [E and E′].
  - ✓ One characteristic is the maximum liquid-film thickness to prevent the film bridging the pipe cross section and forming slug flow.
  - ✓ The other film behavior defined by the Barnea (1987) annular transition is the flow condition required to create a stable liquid film that is, to prevent a downward flow of the film, liquid accumulation, and slug flow formation.
  - ✓ Therefore, annular flow in vertical pipes will occur when all the following conditions are satisfied:

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Slug/Annular Transition [E and E′].

1. Gas flow rate is sufficiently high to transport the largest liquid droplets in the gas core upward; this is the liquid droplet fallback transition (Taitel et al.1980).

2. Liquid holdup in the pipe is low enough to prevent liquid film bridging the pipe cross section; this is the film-bridging transition (Barnea 1987).

3. Liquid-film thickness is small enough to be stable and flow upward; this is the film-instability transition (Barnea 1987).

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Slug/Annular Transition [E and E′].

  - ✓ The constant free-falling velocity of a liquid droplet in a gas field, often called "terminal velocity," is a result of a force balance between the downward gravity force and the upward buoyancy and drag forces ($F_g = F_B + F_D$), and expressed by turner as follows:

$$V_t = 3.1\left[\frac{\sigma_L g\, sin\theta\left(\rho_L - \rho_g\right)}{\rho_g{}^2}\right]^{0.25} \qquad (50)$$

  - ✓ when v_{Sg} exceeds v_t, given by Eq. 50, annular flow occurs.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Liquid-Film Bridging

  - ✓ Liquid-film bridging is a transition mechanism related to wave amplitude growth on the liquid/film interface, which occurs at relatively high liquid flow rates, as shown in the curved upper part of transition.

  - ✓ When the liquid film is thick enough to supply sufficient liquid to form a large wave, the wave will bridge the pipe cross-sectional area, blocking the passage of gas and forming a liquid slug, as shown in Fig. 16.

  - ✓ This bridging mechanism is governed by the minimum liquid holdup required to form a liquid slug, $H_{LLS,min} = 0.48$.

7-5-1 Mechanistic Modeling of Flow Pattern Map

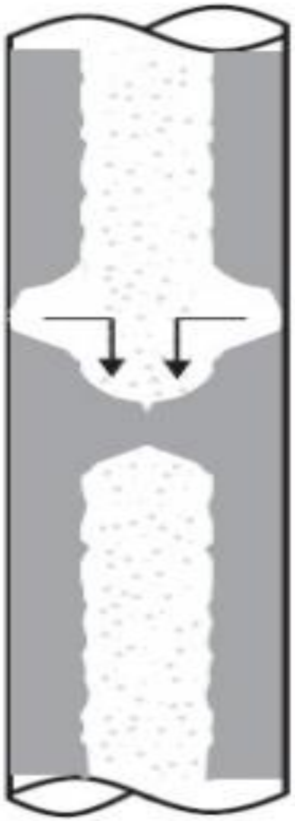


Figure 22:Film-bridging mechanism in annular flow transition

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Liquid-Film Bridging
  - ✓ This value corresponds to a maximum cubic lattice gas bubble packing of 0.52 in the slug body ($H_{LLS,min} = 1.0 - 0.52 = 0.48$).
  - ✓ Ansari et al. (1994) proposed a minimum liquid holdup value of 0.24 to form a liquid slug, which corresponds to the rhombohedral maximum gas bubble packing in a slug body of 0.76, below which it is impossible to form the liquid bridge necessary to form a slug.
  - ✓ Note that the minimum liquid holdup in the pipe required to block the pipe cross-sectional area is approximately 0.5.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Liquid-Film Bridging

  - ✓ However, since a slug can be formed with liquid holdup as low as 0.24 as proposed by Ansari et al. (1994), the maximum allowable liquid holdup to maintain annular flow in the pipe can be determined as follows:

$$\frac{A_L}{A_P H_{LLS,min}} = \frac{A_L}{A_P (0.24)} = \frac{H_L}{0.24} \geq 0.5 \qquad (51)$$

  - ✓ Eq. 51 gives the criterion for transition from annular flow to slug flow as:

$$H_L \geq 0.12 \qquad (52)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Liquid-Film Bridging

  - ✓ The liquid holdup in Eq. 52 consists of the liquid holdup in the film and the entrained liquid droplets in the gas core, resulting in the final form of the slug/annular transition criterion as follows:

$$\left[H_{Lf} + \lambda_{Lc}\left(1 - H_{Lf}\right)\right] \geq 0.12 \qquad (53)$$

  - ✓ Eq. 53 states that if the liquid holdup in the film and core is equal to or greater than 0.12, film bridging will occur and slug flow will prevail.

  - ✓ Note that Barnea (1987) proposed a critical liquid holdup transition value of 0.24, above which film bridging occurs.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Liquid-Film Bridging

  - ✓ The film liquid holdup in Eq. 53 is calculated by implicitly solving the dimensionless combined momentum equation for annular flow.

  - ✓ Two dimensionless parameters merge from the dimensional transformation of the combined momentum equation: the modified Lockhart and Martinelli (1949) parameters (X_M) and (Y_M). Both parameters are independent of the film liquid holdup and are functions of superficial parameters.

  - ✓ The dimensionless combined momentum equation is given as:

$$X^2 \frac{(1-f_E)}{H_{Lf}^{\;3}}\left(\frac{f_f}{f_{sL}}\right) - \frac{I}{H_{Lf}\,(1-H_{Lf})^{2.5}} + Y = 0 \qquad (54)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Liquid-Film Bridging
  - ✓ Eq. 54 is an implicit function of H_{Lf} and requires calculating f_E, I,f_f, f_{SL},X^2,and Y.
  - ✓ The entrainment fraction term, f_E, in Eq. 54 is defined as the fraction of liquid entrained in the gas core and can be predicted by several empirical closure relationships reported in the literature.
  - ✓ A recent experimental study by Magrini et al. (2010) showed that the Wallis (1969) correlation performed the best among other correlations when validated with their vertical experimental air/water data.

7-5-1 Mechanistic Modeling of Flow Pattern Map

Liquid-Film Bridging

- The Wallis correlation is given as:

$$f_E = 1 - exp\left\{-0.125\left[\left(10^4 \frac{V_{sg}\mu_g}{\sigma_L}\right)\left(\frac{\rho_g}{\rho_L}\right)^{0.5} - 1.5\right]\right\} \qquad (55)$$

- The ratio of the interfacial friction factor to the core superficial friction factor (I= f_i/f_{sc}) depends on the entrainments.

- For f_E>0.9 the dimensionless parameter I is:

$$I=1+300\delta_L \qquad (56)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Liquid-Film Bridging
  - ✓ For $f_E < 0.9$, the Whalley and Hewitt (1978) correlation is used and given as:

$$I = \left[1 + 24\left(\frac{\rho_L}{\rho_g}\right)^{\frac{1}{3}} \delta_L\right] \qquad (57)$$

  - ✓ The dimensionless film thickness in Eqs. 56 and 57 is related to the film liquid holdup by geometrical relationships and is obtained as follows.

$$H_{Lf} = 1 - \left(1 - 2\delta_L\right)^2 \qquad (58)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Liquid-Film Bridging

  - ✓ For inclined flow in deviated wells, the interfacial dimensionless parameter (I) is averaged on the basis of inclination angle between horizontal and vertical upward flows as follows. $$I_{\theta} = I_H cos^2\theta + I_V sin^2\theta \qquad (59)$$

  - ✓ where I_V is given by either Eq. 57 and I_H is the Henstock and Hanratty (1976) horizontal dimensionless friction parameter, given as:

$$I_H = 1 + 850\left[\frac{(0.42 N_{\mathrm{Re}f}^{\ 1.25} + 2.8\times10^{-4} N_{Re\,f}^{\ 2.25})^{0.4}}{N_{Re\,sg}^{\ 0.9}}\left(\frac{\mu_L}{\mu_g}\right)\left(\frac{\rho_L}{\rho_g}\right)^{0.5}\right] \qquad (60)$$

$$N_{\mathrm{Re}f} = \frac{\rho_L V_{sL} d\,(1 - f_E)}{\mu_L}$$

$$N_{\mathrm{Re}\,sg} = \frac{\rho_g V_{sg} d}{\mu_g}$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

Liquid-Film Bridging

- ✓ The Lockhart and Martinelli parameter (X) in Eq. 54 is the ratio of superficial liquid to superficial core frictional pressure gradients, given as:

$$X^2 = \frac{-\left(\frac{dp}{dL}\right)_{SL}}{-\left(\frac{dp}{dL}\right)_{Sc}} \qquad (61)$$

$$\left(\frac{dp}{dL}\right)_{SL} = -\frac{C_L\left(\rho_L V_{SL}\frac{d}{\mu_L}\right)^{-n}\left(\rho_L V_{SL}{}^2\right)}{2d}$$

$$\left(\frac{dp}{dL}\right)_{Sc} = -\frac{C_c\left(\rho_c V_{Sc}\frac{d}{\mu_c}\right)^{-n}\left(\rho_c V_{Sc}{}^2\right)}{2d}$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Liquid-Film Bridging

  - ✓ Where C_L = 64, n = 1 for laminar flow; and C_L= 0.184, n = 0.2 for turbulent flow, C_c = 0.184 and n = 0.2 because the core is always in turbulent flow.

  - ✓ The other core parameters are given as:

$$V_{sc} = V_{sg} + V_{SL} f_E$$

$$\lambda_{Lc} = \frac{V_{SL} f_E}{V_{Sg} + V_{SL} f_E}$$

$$\rho_c = \rho_g (1 - \lambda_{Lc}) + \rho_L \lambda_{Lc}$$

$$\mu_c = \mu_g (1 - \lambda_{Lc}) + \mu_c \lambda_{Lc}$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Liquid-Film Bridging

  ✓ The modified inclination angle parameter (Y_M) in Eq. 54 is given in terms of superficial core frictional pressure gradient as follows.

$$Y = \frac{(\rho_L - \rho_c)\, g \sin\theta}{-\left(\frac{dp}{dL}\right)_{sc}} \qquad (62)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Liquid-Film Bridging

✓ The following procedure is used to calculate operational film liquid holdup:

1. Assume a value of film liquid holdup, H_{Lf}.
2. Calculate entrainment fraction (f_E)by using the Wallis Eq. 55.
3. Calculate the ratio of the interfacial shear to the core superficial shear (I) based on f_E.
4. Calculate the film Reynolds number, then calculate the film (f_f) and superficial liquid (f_{SL}) friction factors by using the Blasius-type smooth-pipe equation
5. Calculate the gas core properties of v_{Sc}, λX_{Lc}, ρ_c, and μ_c

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Liquid-Film Bridging

6. Calculate the superficial liquid and core frictional pressure gradients.

7. Calculate the modified Lockhart and Martinelli parameters (X) and (Y).

8. Solve Eq. 5.29 by iterating on film liquid holdup HLF until convergence is reached.

9. Calculate the gas core liquid holdup, λ_{Lc},

10. Calculate the total liquid holdup as $H_L = H_{Lf} + \lambda_{Lc}\,(1 - H_{Lf})$. If the total liquid holdup is greater than 0.12, slug flow exists; otherwise, check the liquid-film instability criteria.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Liquid-Film Instability.

  - ✓ Similar to the transition mechanism for liquid-film bridging, the second annular transition criterion proposed by Barnea (1987) is also related to gas core blockage by liquid, but the blockage results from film instability. Instability of the liquid film (i.e., a downward flow followed by a complete collapse) and core bridging occur when the upward interfacial shear at the core/film interface is insufficient to overcome the film downward gravitational and liquid/wall shear forces, as shown in Fig.18.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Liquid-Film Instability.

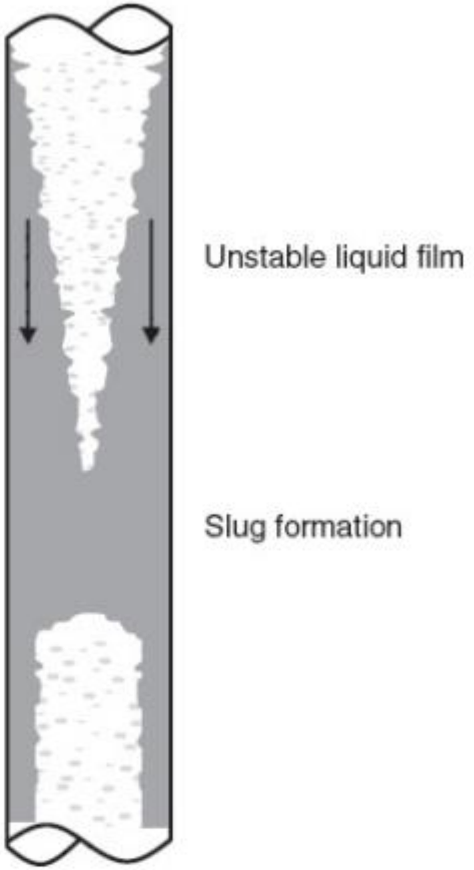


Figure 23:Film instability mechanism

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Liquid-Film Instability.
  - ✓ Annular flow liquid-film instability is governed by the combined momentum equation, which is solved for the minimum film liquid holdup at which interfacial shear at the core/film interface balances the downward film/wall shear and gravity forces acting on the film.
  - ✓ Therefore, if the actual operational film thickness determined from Eq. 54 is less than the minimum film thickness determined from Eq. 65, the liquid film is stable. In dimensionless form, the combined momentum equation for minimum film holdup can be expressed in term of modified Lockhart and Martinelli (1949) parameters (X_M) and (Y_M) as follows.

$$Y - \frac{\left(2 - 1.5 H_{Lf\,min}\right)\left(1 - f_E\right)^2}{H^3_{Lf\,min}\left(1 - 1.5 H_{Lf\,min}\right)}\left(\frac{f_f}{f_{SL}}\right) X^2 = 0 \qquad (65)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- To facilitate the flow pattern prediction, a flow chart is presented in Fig. 19 that summarizes the calculation procedure to determine the flow pattern.
- The letters in brackets are the flow pattern transition boundaries shown in Fig. 14.

7-5-1 Mechanistic Modeling of Flow Pattern Map

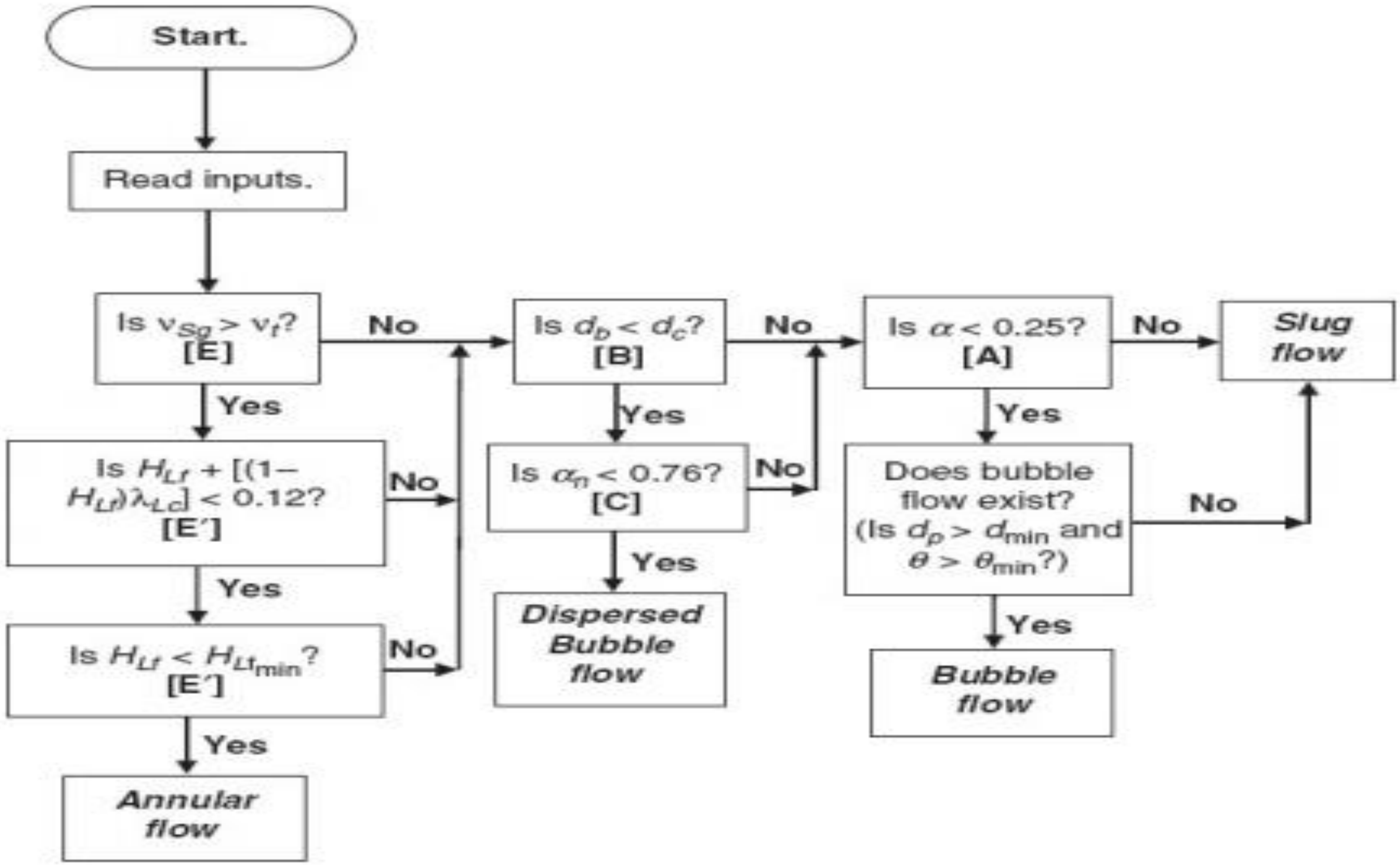


Figure 24:Vertical flow pattern prediction flow chart

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Holdup Calculations for Annular Flow

  - ✓ Fig. 20 gives a schematic description of annular flow in a concentric annulus. The model is based on equilibrium fully developed flow.

  - ✓ The phases are assumed to be incompressible. The two liquid films are assumed to have uniform thickness, bur with different values.

  - ✓ The gas and liquid droplets flowing in the annulus core are assumed to flow as a homogeneous mixture with the same velocity. The conservation of linear momentum for the outer (casing) liquid film yields :

$$\left(\frac{dp}{dL}\right)_{cf} + \tau_c \frac{S_c}{A_{cf}} - \tau_{ci} \frac{S_{ci}}{A_{cf}} + \rho_L g = 0 \qquad (66)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Holdup Calculations for Annular Flow

  - ✓ In this equation, (dp/dL) is the total pressure gradient for the casing liquid film. τ_c, and τ_{Ci} are the shear stresses at the casing wall and liquid film core-mixture interface, respectively. S_c, and S_{ci} are the wetted liquid perimeters on the casing wall and liquid film core-mixture interface, respectively. A_{cf} is the total area of the casing liquid film.

  - ✓ Similarly, a linear momentum balance for the inner (tubing) film yields :

$$\left(\frac{dp}{dL}\right)_{tf} + \tau_t \frac{S_t}{A_{tf}} - \tau_{ti} \frac{S_{ti}}{A_{tf}} + \rho_L g = 0 \qquad (67)$$

  - ✓ where all the terms have the same meaning as given for similar terms in Eq. 66.

7-5-1 Mechanistic Modeling of Flow Pattern Map

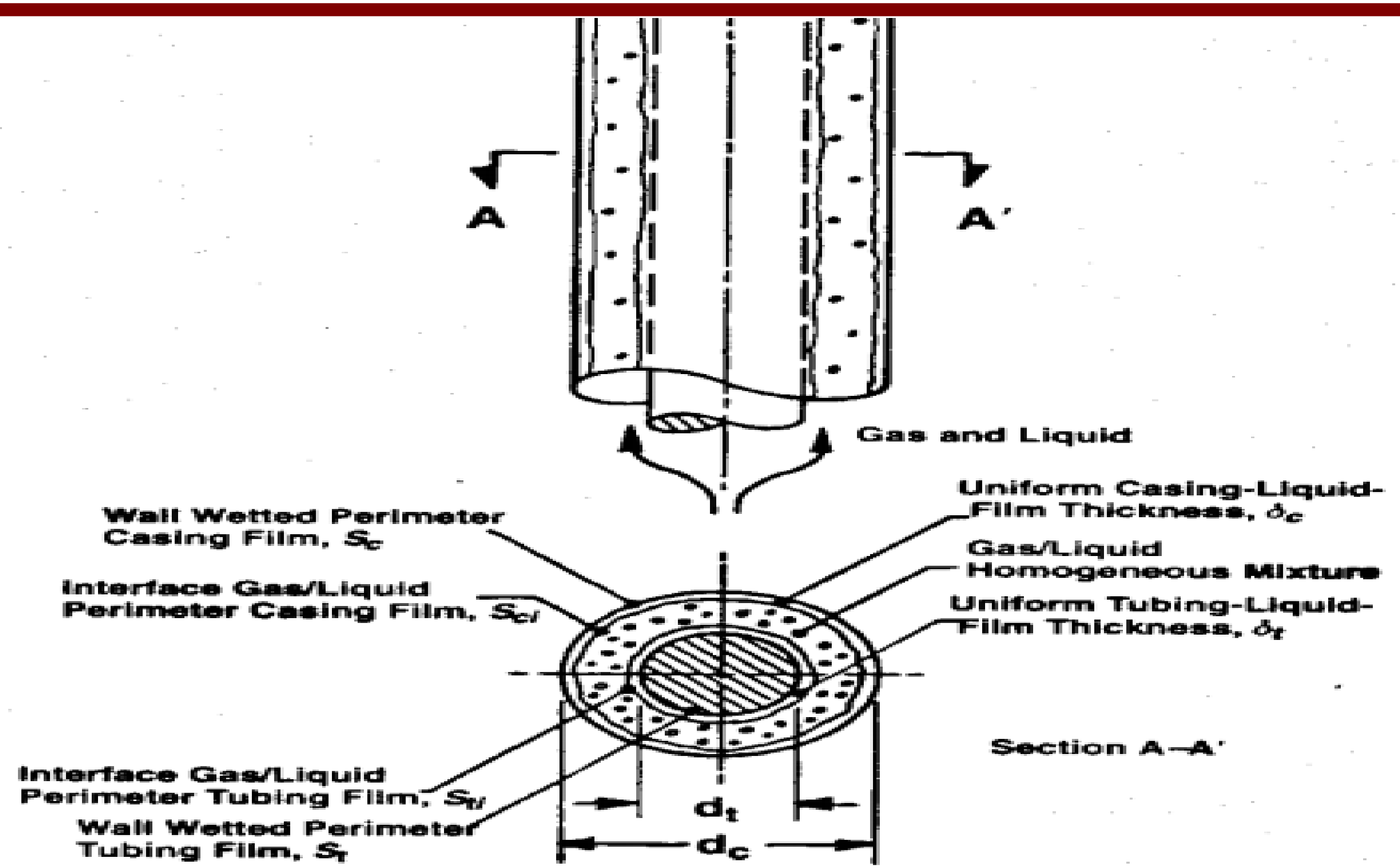


Figure 25: Annular flow in concentric tube

7-5-1 Mechanistic Modeling of Flow Pattern Map

Holdup Calculations for Annular Flow

- A linear momentum balance for the gas/droplets mixture flowing in the core of the annulus yields:

$$\left(\frac{dp}{dL}\right)_C + \tau_{Ci}\frac{S_{Ci}}{A_C} - \tau_{ti}\frac{S_{ti}}{A_C} + \rho_C g = 0 \qquad (68)$$

- where $(dp/dL)_c$ is the total pressure gradient for the mixture in the core, ρ_c is the density of the mixture in the core, and A_c is the area of the core occupied by the mixture.

- The gas/liquid interfaces are considered stable; hence, this equilibrium condition exists.

$$\left(\frac{dp}{dL}\right)_{cf} = \left(\frac{dp}{dL}\right)_{tf} = \left(\frac{dp}{dL}\right)_C \qquad (69)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Holdup Calculations for Annular Flow

  - ✓ With this equilibrium condition and the linear momentum equations given by Eqs. 67 through 69, two combined-momentum equations can be written.

$$-\tau_c \frac{S_c}{A_{cf}} + \tau_{Ci} \frac{S_{Ci}}{A_{cf}} + \tau_{Ci} \frac{S_{Ci}}{A_C} + \tau_{ti} \frac{S_{ti}}{A_C} - (\rho_L - \rho_C) g = 0 \qquad (70)$$

  - ✓ And :

$$-\tau_t \frac{St}{A_{ft}} + \tau_{ti} \frac{S_{ti}}{A_{tf}} + \tau_{Ci} \frac{S_{Ci}}{A_C} + \tau_{ti} \frac{S_{ti}}{A_C} - (\rho_L - \rho_C) g = 0 \qquad (71)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Holdup Calculations for Annular Flow

  - ✓ On the basis of the annulus geometry given in Fig. 20 and because the films are considered uniform in thickness, geometrical relationships for the relevant dimensions can be written.

  - ✓ The casing-liquid-film area, the tubing-liquid-film area, and the core area for the mixture are, respectively,

$$A_{cf} = \pi\delta_c\left(d_c - \delta_c\right) \qquad (72).$$

$$A_{tf} = \pi\delta_t\left(d_t + \delta_t\right) \qquad (73)$$

$$A_C = \frac{\pi}{4}\left[d_c^{\,2} - d_t^{\,2} - 4\delta_c\left(d_c - \delta_c\right) - 4\delta_t\left(d_t + \delta_t\right)\right] \qquad (74)$$

  - ✓ where δ_c and δ_t= casing-liquid-film thickness and tubing-liquid-film thickness, respectively.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Holdup Calculations for Annular Flow

  - ✓ The parameters associated with the casing-liquid film and tubing--liquid film, for both the wall and interface sides, are

$$S_c = \pi d_c \qquad (74)$$

$$S_{Ci} = \pi\left(d_c - 2\delta_c\right) \qquad (75)$$

$$S_t = \pi d_t \qquad (76)$$

$$S_{ti} = \pi\left(d_t + 2\delta_t\right) \qquad (77)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

❑ Holdup Calculations for Annular Flow

✓ By applying the hydraulic-diameter concept to the casing-liquid film, tubing-liquid film, and core and with Eqs. 72 through 77, the hydraulic diameters associated with each region can be written as :

$$d_{cfh} = 4\delta_c \left(1 - \frac{\delta_c}{d_c}\right) \qquad (78)$$

$$d_{cfh} = 4\delta_c \left(1 - \frac{\delta_c}{d_c}\right) \qquad (79)$$

$$d_{Ch} = \frac{\left[d_c{}^2 - d_t^2 - 4\delta_c\left(d_c - \delta_c\right) - 4\delta_t\left(\delta_t + d_t\right)\right]}{\left(d_c - 2\delta_c\right) + \left(2\delta_t + d_t\right)} \qquad (80)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

❑ Holdup Calculations for Annular Flow

- ✓ The wall shear stress is:

$$\tau = f^{\prime}\rho\frac{v^2}{2} \qquad (81)$$

- ✓ where τ is the respective wall shear stress, ρ is the density for the phase which wets the wall, v is the in-situ average phase velocity, and f is the associated Fanning friction factor, which is evaluated by a Blasius-type expression given by : $f^{\prime} = C N_{\mathrm{Re}}^{n} \qquad (82)$

- ✓ where n = -1 for laminar flow and n = - 0.25 for turbulent flow. The coefficient, C, is evaluated by taking into account the annulus configuration geometry.

- ✓ The Reynolds number in Eq.82 uses either d_{cfh} or d_{tfh}, depending on the wall shear stress being evaluated.

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Holdup Calculations for Annular Flow

  - ✓ Neglecting the liquid film velocities in comparison with the core velocity. the interfacial shear stress is approximated by

$$\tau_i = f_i^{/} \rho_c \frac{v_c^2}{2} \qquad (81)$$

  - ✓ where τ_i= interfacial shear stress, $Vc_=$ in-situ mixture velocity in the core. ρ_c= mixture density in the core, and f_i = Fanning friction factor associated with the interface. The mixture density and velocity were defined as

$$\rho_C = \rho_L H_{LC} + \rho_g \left(1 - H_{LC}\right) \qquad (82)$$

$$v_c = v_{sc} \frac{A}{A_C} \qquad (83)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Holdup Calculations for Annular Flow

  - ✓ The resulting casing-film-interface and tubing-film-interface friction factors are:

$$f_i^{/} = f_{SC}^{/}\left(1+300\frac{\delta_c}{d_c}\right) \qquad (83)$$

$$f_i^{/} = f_{SC}^{/}\left(1+300\frac{\delta_t}{d_t}\right) \qquad (84)$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

❑ Holdup Calculations for Annular Flow

✓ Finally liquid holdup for the casing-liquid film and tubing-liquid film is:

$$H_{Lcf} = \frac{4\delta_c\left(1-\frac{\delta_c}{d_c}\right)}{d_c\left(1-K^2\right)} \qquad (83) \quad \textit{casing liquid film holdup}$$

$$H_{Lct} = \frac{4K\,\delta_c\left(1+\frac{\delta_t}{d_t}\right)}{\left(1-K^2\right)\quad d_c} \qquad (85) \quad \textit{tubing liquid film holdup}$$

$$H_{LC} = \frac{v_{SL}F_E}{v_{SL}F_E + v_{sg}} \qquad (86) \quad \textit{core in situ holdup}$$

$$H_{L,T} = \frac{v_{SL}F_E}{v_{SL}F_E + v_{sg}\left(1-K^2\right)} \times \left[1-K^2-\frac{4\delta_c}{d_c}\left(1-\frac{\delta_c}{d_c}\right)-\frac{4\delta_t K}{d_c}\left(1+\frac{\delta_t}{d_t}\right)\right]$$

$$+ \quad \frac{4}{d_c\left(1-K^2\right)}\left[\delta_c\left(1-\frac{\delta_c}{d_c}\right)+\delta_t K\left(1+\frac{\delta_t}{d_t}\right)\right] \qquad (87) \quad \textit{Total liquid Holdup in Annulus}$$

7-5-1 Mechanistic Modeling of Flow Pattern Map

- Holdup Calculations for Annular Flow
  - ✓ K is the annulus pipe diameter ratio $K=d_t/d_c$
  - ✓ Also liquid film thickness ratio (δ_t/δ_c) can be found by figure 21 using eccentricity factor.

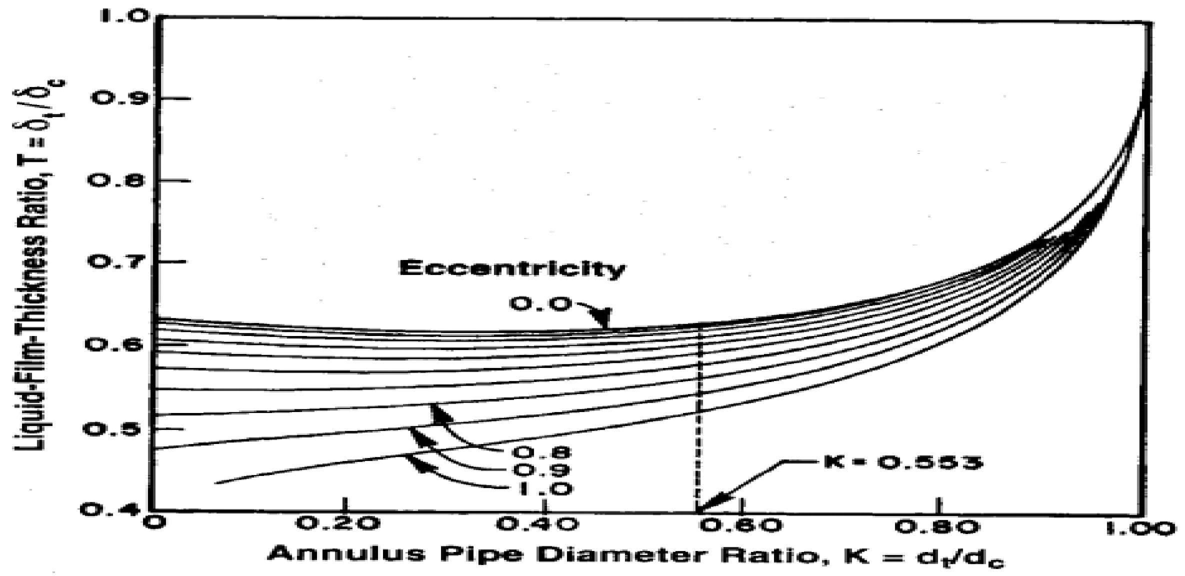


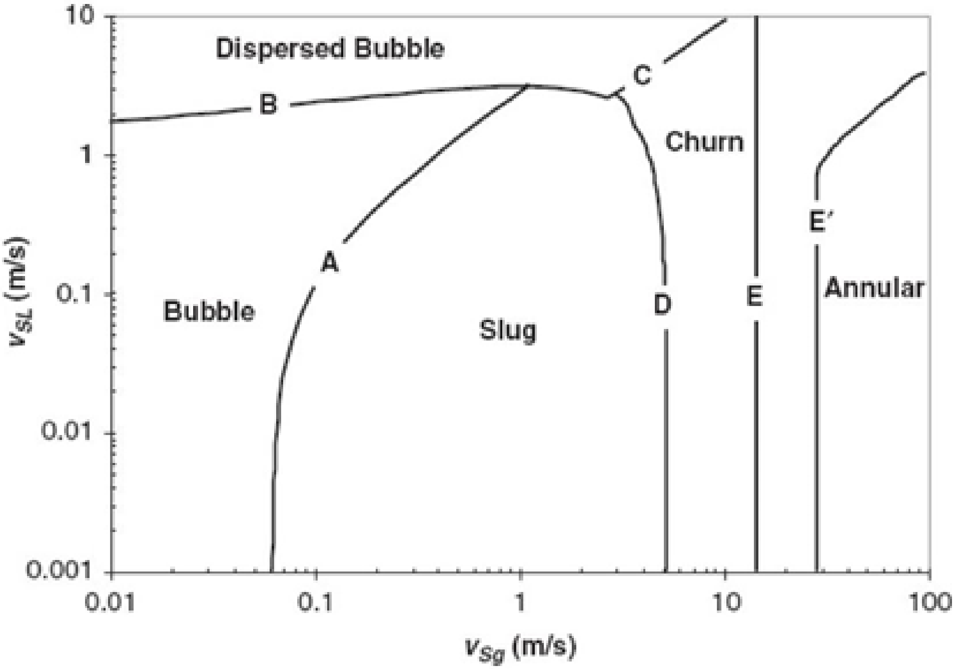
Figure 26:liquid film thickness ratio in annuli

Exercise 1

- In a two-phase flow pipeline, if the actual velocity of the liquid phase is 5 ft/s and the actual velocity of the gas phase is 40 ft/s. Calculate the degree of slippage and slip ratio. (Assuming superficial liquid velocity is 3.9 ft/s)

Exercise 2

- The Taitel et al. (1980) flow pattern map developed for vertical flow as given in Fig below. A vertical oil well is producing oil with 0.062 m (2.441 in) inner-diameter tubing. Answer the following question:



Exercise 2

1. What is the flow pattern for q_L=100 m^3/d and q_g=220 m^3/d ?
2. Determine the liquid holdup.
3. Over time, the tubing diameter at the given specific location is reduce to one third the original ID as a result of asphaltene deposition. Assuming the oil and gas flow rate remain constant, Determine the flow pattern.
4. Over time, the superficial gas velocity at the given specific location increases by 40% with the flow rate remaining constant. Determine the superficial liquid velocity. What is the flow pattern for this condition?

Exercise 3

- 4000 m^3/d of water are flowing in a 9-km-long smooth pipeline. The pipeline ID and inclination angle are 152 mm and +7°, respectively. The pipeline outlet pressure is 2.07 MPa (300 psi). Determine the pipeline inlet pressure if the water viscosity and density are 0.001 Pa.s and 1000 kg/m^3, respectively.


حتی در وضعیت بد هم بگوئید
الْحَمْدُ لِلَّه
شاید وضعتان میتوانست از این هم بدتر باشد.