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The Calculation of Pressure Gradients In High-Rate Flowing Wells

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ABSTRACT

Work on the calculation of vertical two-phase flow gradients by Cia. Shell de Venezuela has been based mainly on the "energy-loss" method proposed by Poettmann and Carpenter in 1952.

The "energy-loss-factor" correlation proposed by Poettmann and Carpenter was based on relatively low-rate flow data. This correlation proved inapplicable to high-rate flow conditions. In an attempt to establish a satisfactory correlation for high rates, a series of experiments was carried out at rates up to 5,000 B/D in Cia. Shell de Venezuela's La Paz field in Venezuela, using tubing strings fitted with electronic surface-recording pressure elements.

As a result of these experiments a correlation between energy-loss factor and mass flow rate was established which is believed to be applicable to a wide range of conduit sizes and crude types at high flow rates (e.g., above 900 B/D for 2⅞-in. OD tubing). It is anticipated that the resulting gradient calculations will have an accuracy of the order of ± 5 per cent.

At lower flow rates the energy-loss factor cannot be considered as constant for any mass rate of flow, but varies with the free gas in place and the mixture velocity. No satisfactory correlating parameter was obtained. As a practical compromise for low flow rates, a modification of the curve proposed by Poettmann and Carpenter was used. In practice this was found to give gradient accuracies of approximately ± 10 per cent down to flow rates as low as 300 B/D in 2⅞-in. tubing.

INTRODUCTION

Production operations in Cia. Shell de Venezuela's light- and medium-crude fields are principally concerned with high-rate flowing or gas-lift wells. Under these conditions the analysis of well performance, the selection of production strings and the design of gas-lift installations are vitally dependent on an accurate knowledge of the pressure gradients involved in vertical two-phase flow.

Initially, attempts were made to establish the gradients empirically as done by Gilbert,[1] but the results were not reliable due to scarcity of data over a full range of rates and gas-oil ratios. Several methods of calculation based on energy-balance considerations were attempted, but the computations were cumbersome and the results discouraging. In 1952 a paper was published by Poettmann and Carpenter[2] which proposed a new approach. Their method was also based on an energy-balance equation, but it was original in that no attempt was made to evaluate the various components making up the total energy losses. Instead, they proposed a form of analysis which assumed that all the significant energy losses for mutiphase flow could be correlated in a form similar to that of the Fanning equation for frictional losses in single-phase flow. They then derived an empirical relationship linking measurable field data with a factor which, when applied to the standard form of the Fanning equation, would enable the energy losses to be determined.

The basic method was applied in Venezuela to the problem of annular flow gradients in the La Paz and Mara fields[3]. This involved establishing a new energy-loss-factor correlation to cover high flow rates and, also, some adaptation of the method to permit mechanized calculation using punch-card machines. The final result was a set of gradient curves for La Paz and Mara conditions which proved to be surprisingly accurate.

With the encouraging results of the annular flow calculations, several attempts were made to obtain a corresponding set of curves for tubing flow. Here, unfortunately, little progress could be made. The original correlation of Poettmann and Carpenter was based on rather limited data derived from low-rate observations in 2⅜- and 2⅞-in. OD tubing. It did not cover the higher range of production rates, and extrapolation proved unsuccessful.

A new correlation covering high flow rates was required, but this proved to be extremely difficult to establish since tubing flow pressure measurements at high rates did not exist—due to the difficulty of running pressure bombs against high-velocity flow.

The necessity for reliable tubing flow data increased with the development of the new concessions in Lake Maracaibo, where high-rate tubing flow from depths of 10,500 ft became routine.

Thus, it was decided to set up a full-scale test to establish a reliable energy-loss factor for tubing flow conditions. A La Paz field light-oil producer with a potential of approximately 12,000 B/D on annular flow was chosen. To obtain full pressure gradients, a special tubing string was installed which was equipped with electronic surface-recording pressure measuring devices,

*Recently transferred by Shell to Indonesia.

[1]References given at end of paper.

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Discussion of this and all following technical papers is invited. Discussion in writing (three copies) may be sent to the office of the *Journal of Petroleum Technology*. Any discussion offered after Dec. 31, 1961, should be in the form of a new paper. No discussion should exceed 10 per cent of the manuscript being discussed.

located at intervals in the string. The production rate was varied by means of adjustable bottom-hole chokes, and by this means a set of gradients covering a wide range of flow rates was obtained for both 2⅞- and 3½-in. OD tubing.

This paper describes the analysis of the experimental results, the establishment of the energy-loss correlation and the accuracy of the resulting depth-pressure gradients.

THE GRADIENT EQUATION

From energy-balance considerations, Poettmann and Carpenter[2] derived the following relationship for the pressure gradient in vertical two-phase tubing flow.

$$144\frac{\Delta p}{\Delta h} = \bar{\rho} + \frac{fQ^2M^2}{7.413 \times \bar{\rho} \times D^5 \times 10^{10}} \quad . \quad . \quad (1)$$

where Δp = pressure difference (in psi) between any two points in the vertical conduit,

Δh = difference in height (in ft) between the two points,

$\bar{\rho}$ = integrated average density (in lb/cu ft) of the two-phase mixture between the two points,

Q = oil flow rate (in STB of oil/D),

M = total mass (in lb of the gas and oil associated with 1 bbl of tank oil),

D = inside diameter (in ft) of the pipe, and

f = dimensionless factor used as the correlating function for total energy loss due to irreversibilities of the fluid in flow.

Eq. 1 is the basic form employed in this paper.

To establish a correlation linking the energy-loss factor f with field data, Poettmann and Carpenter retained the analogy with single-phase flow and the Fanning equation by correlating f against the product of diameter, density and velocity, $D \rho v$, which corresponds to the numerator of the Reynolds number. The viscosity term, which forms the denominator of the Reynolds number, could not be evaluated for two-phase flow conditions and is ignored. This appeared to be a logical development, for as Poettmann and Carpenter point out: "The fact that viscosity is not one of the variables involved in the vertical multiphase flow is both fortunate and to be expected. This is because of the fact that the degree of turbulence is of such a magnitude that, of the total energy loss, W_f, that portion resulting from viscous shear is negligible". Various other workers in this field[4,5] have made the same observation.

In terms of field units, the $D \rho v$ term is equivalent to mass rate of flow divided by diameter or, in the terms already defined, QM/D.

THE LA PAZ EXPERIMENTS

FIELD INSTALLATION AND DATA

A packer was set in the casing at 6,250 ft just above the productive interval. The tubing string was latched into the packer and carried surface-recording Maihak pressure-transmitting elements spaced at approximately 1,000-ft intervals in the lower part of the string and 500-ft intervals in the upper part. The pressure elements were connected to the outside of the tubing on "limpet"-type gas-lift valve mandrels and, thus, offered no impedance to flow within the tubing. The well was produced on tubing flow against a constant tubinghead pressure, which was maintained by a back-pressure regulator. Different flow rates were obtained by means of retrievable bottom-hole chokes set in an Otis nipple immediately above the packer. This arrangement enabled the tubinghead pressure to be maintained at a constant value for all flow rates. Two separate installations were used, one employing 3½-in OD EU tubing, the other 2⅞-in.

The stability and repeatability of the pressure-recording elements under operating conditions were found to be excellent. Recalibration of the elements after their use in the 3½-in. tubing string indicated no change in the calibration constants—notwithstanding that, in the period after completion of the 3½-in. test and before recalibration, the instruments had been subjected to the buffeting turbulence of casing flow for some six months.

Since the recalibration was carried out in a different instrument shop, on a different tester, by different personnel and gave identical results to the first calibration, it is safe to assume that the stability of the instruments during the period of the actual test can be relied upon and that an accuracy of ± 1 per cent in the subsurface pressure readings can be expected.

The tubinghead pressure was read from a new manometer gauge, calibrated especially for the experiment and capable of ± 1-psi accuracy over the pressure range considered.

Special precautions were taken to ensure that the oil and gas gauging procedure was as accurate as possible. The GOR figures quoted are based on gas production from a single-stage 60-psig separator.

The well was always in gauge at the time that the pressure measurements were being taken.

The survey results from the test in 3½-in. and 2⅞-in. tubing are detailed in Tables 1 and 2, respectively, and examples of the gradient curves drawn from the results are shown in Fig. 1.

The maximum deviation of the observed points from the smooth curve was generally less than 2 per cent,

TABLE 1—DATA FROM 3½-in. OD TUBING FLOW EXPERIMENT

Prod. (B/D)	GOR (vol/vol)	THP (psia)	Maihak Pressures (in psia) at Depths Shown (ft) 488	993	1494	1998	4015	5018	5989
5082	129	165	290	400	500	615	1041	1250	1565
5051	129	167	290	404	498	615	1045	1238	1539
4931	128	167	290	403	493	609	1034	1228	1517
4780	124	165	275	380	472	605	1004	1206	1485
4321	129/130	164	269	365	447	558	941	1137	1401
4057	123	165	256	336	402	511	866	1079	1346
3799	122	165	250	330	—	493	850	1052	1313
3264	132	165	243	315	378	468	773	950	1168
3113	133	165	231	299	353	438	735	914	1144
2245	122	165	218	263	314	321(?)	593	744	945
1937	131	161/169	211	260	299	365	558	685	845
1346	122	165	192	227	252	299	454	558	719
956	128	165/167	195	225	240	295	445	505	640
176	135	159/169	185	215	227	265	373	473	569

TABLE 2—DATA FROM 2⅞-IN. OD TUBING FLOW EXPERIMENT

Prod. (B/D)	GOR (vol/vol)	THP (psia)	Maihak Pressures (in psia) at Depths Shown (ft) 516	1010	1998	3019	4007	5018	6001
3189	132	150	304	427	647	888	1125	1387	1618
3164	129	150	298	416	629	862	1093	1356	1580
3013	132	150	295	413	627	856	1083	1339	1563
2881	132	150	288	401	606	830	1046	1296	1517
2692	136	150	284	389	583	802	1010	1265	1471
2510	140	150	271	370	555	762	959	1186	1396
2025	135	147/159	250	328	473	654	822	1037	1219
1881	148	138/162	238	312	450	616	767	962	1139
1648	137	145/165	239	295	420	575	714	—	1056
1252	142	145/157	217	256	346	471	577	723	834
818	162	135/181	191/255	231/255	305	405	481	586	677

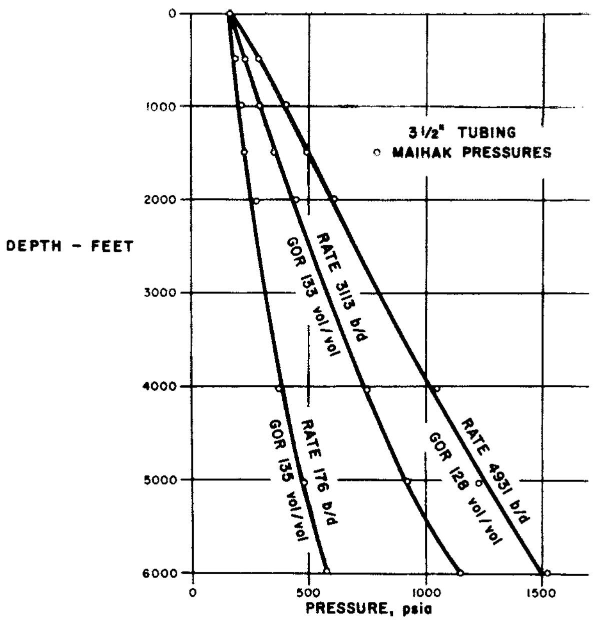

FIG. 1—EXAMPLES OF GRADIENT CURVES FOR THE LA PAZ WELL, 3½-IN. TUBING.

and the arithmetic mean deviation of all observed points from the smooth curves was 0.15 per cent.

LABORATORY DATA

From surface samples of oil and gas taken from the well immediately prior to the vertical flow experiments, a set of pressure-density curves was obtained for GOR's of 119.9, 131.5 and 141.1 vol/vol at temperatures of 140°, 160° and 180°F.

These curves formed the basis for the pressure-density calculations made in the analysis of the results.

A tabulation of the physical properties of the crude oil and gas from the well is given in Table 3.

ANALYSIS OF FIELD DATA

The first step in the analysis of the field data was to examine the individual depth-pressure traverses and attempt to fit them to the basic energy equation to determine the value of the energy-loss factor.

This was done by calculating depth-pressure traverses from the basic energy equation using the individual gauged production and an arbitrary spread of values for the energy-loss factor f. The theoretical curves were then superimposed on the actual curve recorded in the well, and the appropriate mean value of f selected together with limiting values of f to give $\pm$ 5 per cent accuracy over the measured pressure range.

The fit of the calculated curves to the observed data was generally excellent, particularly at the higher rates of flow.

The resulting values of f with $\pm$ 5 per cent upper and lower limits were then plotted against the QM/D parameter. The correlation is the excellent fit illustrated in Fig. 2. Both 2⅞ and 3½-in. OD tubing results apparently follow a single consistent trend.

ADDITIONAL SUPPORTING DATA

The correlation obtained in the plot of f vs QM/D was a far closer fit than had been anticipated. The closeness of this fit and the fact that the correlation showed no appreciable divergence with tubing size indicated that it might be generally applicable to two-phase flow conditions over a similar range of QM/D values in other sizes or types of conduit.

TABLE 3—PHYSICAL PROPERTIES OF CRUDE OIL AND GAS, LA PAZ TEST WELL

Specific Gravity of Tank Oil	0.8548
Gravity of Separator Gas	0.8170 (Air = 1.0)
Separator Gas-Oil Ratio	130 vol/vol
Gravity of Tank Gas	1.270 (Air = 1.0)
Tank Gas-Oil Ratio	5 vol/vol
Average Flow-String Temperature	180°F
Viscosity of Oil at Atmospheric Pressure:	
200°F	1.87 cp
160°F	2.58 cp

In an attempt to establish further supporting data, previous work on the subject of two-phase flow was re-examined.

ANNULAR FLOW

The analysis of the annular flow conditions in La Paz and Mara fields made in 1955[3] established a high-rate correlation for the energy-loss factor for the annulus between 2⅞-in. OD tubing and 7-in casing.

In this analysis it was not necessary to consider the problem of the equivalent diameter of the annular conduit. The diameter term in the energy equation, though unknown, was in fact constant because only one particular size of annulus was under consideration. The energy-loss correlation was drawn up with parameters of QM/D and f/D^5 where D, a composite term of true and equivalent diameters (discussed later), was unknown but constant and the term f/D^5 could be evaluated as a single unit from the field data.

The annular flow data, upon which the 1955 investigation was based, was recalculated using a standard form of equivalent hydraulic diameter to compare the results with the high-rate tubing flow correlation obtained in the La Paz experiments.

The equivalent diameter D_e is derived as follows.

$$D_e = 4 \times \frac{\text{Cross-sectional Area}}{\text{Wetted Perimeter}}$$

$$= D_c - D_t = 4 \times \frac{\pi/4\ (D_c^2 - D_t^2)}{\pi\ (D_c + D_t)}$$

where D_c = ID casing, and
D_t = OD tubing.

In Eq. 1, which is derived from a basic energy equation,[2] the term D^5 is in fact a compound of the square of the cross-sectional area and a hydraulic diameter.

In tubing flow, the diameter used for the calculation

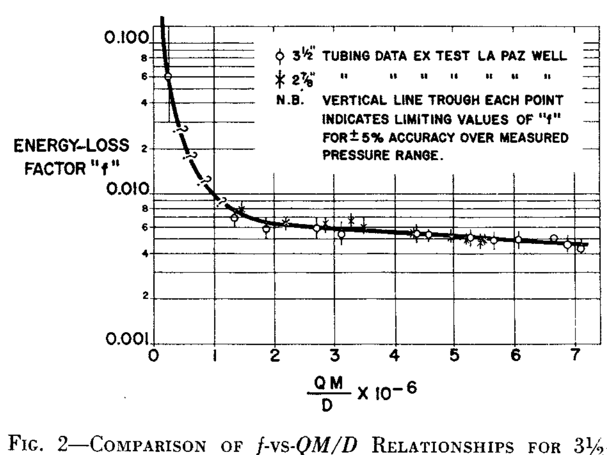

FIG. 2—COMPARISON OF f-VS-QM/D RELATIONSHIPS FOR 3½- AND 2⅞-IN. TUBING DATA, LA PAZ WELL.

of area is the same as the hydraulic diameter, thus raising no difficulties. However, for annular flow the term D^5 is equivalent to $(D_c^2 - D_t^2)\ (D_c - D_t)$.

Furthermore, in the correlating parameter QM/D, against which f is plotted for tubing flow, for annular flow this term becomes $QM/(D_c + D_t)$.

To compare the annular flow correlation of 1955 with that derived in the experiments in La Paz, the values of f/D^5 from the original data were multiplied by $(D_c^2 - D_t^2)^2\ (D_c - D_t)$ to give a value for f, and the correponding values of QM were divided by $(D_c + D_t)$ to give a value for QM/D_e.

The resulting correlation is shown in Fig. 3. As can be seen, there is very close agreement between the two sets of data.

This close support is important in that it should now be possible to calculate vertical flowing gradients for an annular conduit of any size using the form of the equivalent hydraulic diameter stated previously. In the case of a small-sized outer conduit, the presence of tubing collars and upsets may cause additional restrictions to flow. However, in areas where little or no empirical data on annular flow exists, this method is good enough as a first approach.

HORIZONTAL PIPELINE FLOW

Earlier work in Venezuela had derived a friction-factor correlation for two-phase pipeline flow in 4- and 6-in. lines,[6] which has been used in the Mara-Maracaibo area with consistently accurate results at high rates of flow.

The analysis used in this derivation was basically similar to the energy-balance equation used for vertical flow and assumed that the energy losses could be correlated in a form similiar to the Fanning equation as was used in the analysis of the La Paz data. Consequently, the friction-factor values so derived are directly comparable with the energy-loss factor of the current investigation.

These flow-line friction factors were plotted against QM/D and superimposed on the tubing flow correlation (see Fig. 3). The flow-line data support the tubing flow correlation closely at high rates but diverge at values of QM/D less than 2.5×10^6, the point which was considered in the original flow-line analysis to be the lower limit of truly turbulent flow in flow lines.

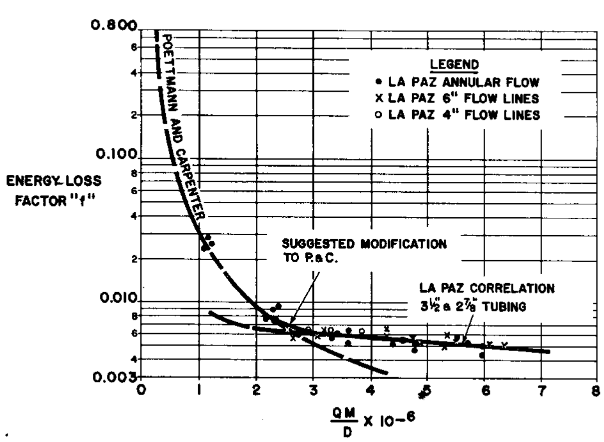

Fig. 3

TUBING FLOW DATA
(POETTMANN AND CARPENTER)

The friction-factor correlation derived originally by Poettmann and Carpenter is the best average curve drawn through a group of widely scattered points calculated from 49 surveys. These surveys were taken mainly at low flow rates, less than 300 B/D, in tubing sizes of 2⅜- and 2⅞-in. OD.

At the low-rate end, there were no local data available to confirm the correlation, other than the extreme low-rate end of the La Paz annular flow correlation which appears conformable.

Above values of $D \rho v = 30$ (i.e., $QM/D = 2.0 \times 10^6$ approximately), the Poettmann and Carpenter correlation diverges from the general trend of the high-rate correlation established from the La Paz tubing data, the annular flow data and the flow-line data. The divergence probably has no significance as, above this value of QM/D, the Poettmann and Carpenter correlation was not strongly supported by their basic data. The low-rate correlation of Poettmann and Carpenter can be readily adjusted to fit smoothly into the high-rate correlation derived from the other data.

THE SIGNIFICANCE OF THE ENERGY-LOSS CORRELATION

The excellent agreement of the tubing flow, annular flow and pipeline correlations above a value of $QM/D = 3.0 \times 10^6$ indicates that at high flow rates the correlation of f against QM/D is well justified for practical purposes and that, given the correct PVT data, it should be possible to make computations of pressure changes in both vertical and horizontal* flow with an accuracy of $\pm$ 5 per cent.

At low flow rates (i.e., at values of QM/D less than 2.0 to 3.0×10^6), the different sets of empirical data diverge considerably.

As a hypothesis, it has been assumed that the flattened portion of the f correlation in Fig. 3 (i.e., at values of QM/D greater than 2.0 to 3.0×10^6) represents the truly turbulent phase of flow in which little or no gas slippage occurs; while below this point, the sharply rising portion of the curve represents slippage.

Since the energy loss by slippage will depend to some degree on the amount of free gas present and on the over-all velocity of the mixture, it can be expected that, for the same mass rate of flow under different conditions of pressure, the energy loss due to slippage and hence the energy-loss factor will not be constant.

The wide spread of f values for the same value of QM/D in the Poettmann and Carpenter correlation and the different correlation obtained from the La Paz data are then explicable by the variation in GOR's and pressure. These variations will result in variations of the percentage free gas in place and the mixture velocity.

Attempts were made to correlate f with a variety of parameters incorporating variables of percentage free gas and mixture velocity. Very few data are available at low rates, and the attempts to establish a satisfactory correlation with more complex parameters proved unsuccessful. It was decided to accept Poettmann

*Horizontal flow in this sense means the flow through lines in which differences in end elevations are small enough to be ignored and in which no appreciable variations in elevation occur along the route.

and Carpenter's correlation, somewhat modified as shown in Fig. 3 to fit smoothly in the well defined high-rate correlation derived from the La Paz data, and to calculate assuming a constant energy-loss factor for a given mass rate of flow.

It was considered that such an approximation would give adequately accurate results for normal field conditions in view of the original results of Poettmann and Carpenter.

To establish an independent check on the accuracy of gradient calculations at low rates, a set of gradient curves were calculated for Lake Maracaibo conditions using the modified Poettmann and Carpenter correlation just proposed. These curves were then checked against a total of 78 flowing bottom-hole pressure surveys made in Lake Maracaibo wells at flow rates between 300 and 1,600 B/D in 3½-in. OD tubing and between 500 and 1,250 B/D in 2⅞-in. OD tubing. It appears that an accuracy of $\pm$ 10 per cent is achievable under these conditions, which is reasonably satisfactory for gas-lift design or flowing-well analysis.

The accuracy obtained by using what is admittedly an approximation to the true energy-loss factor can be explained by the following points.

1. At high flow rates ($QM/D > 2.0 \times 10^6$) under all normal pressure conditions, the value of $\Delta p/\Delta h$ is vitally dependent on the value of f since the term $\frac{(f)(Q^2)(M^2)}{(7.413)(10^{10})(\bar{\rho})(D^5)}$ (see Eq. 1) assumes significant proportions with respect to $\bar{\rho}$.

2. At medium or low production rates (i.e., QM/D values lower than 2.0×10^{-6}), it appears that at high pressure the first term ($\bar{\rho}$) in the gradient equation is dominant. Consequently, under such conditions, deviations in the value of f are of less significance in the final value of $\Delta p/\Delta h$.

3. At low flow rates and low pressures, the term $\frac{(f)(Q^2)(M^2)}{(7.413)(10^{10})(\bar{\rho})(D^5)}$ again assumes significance in the calculation of the gradient. However, it is under such conditions that "heading" also begins to manifest itself in the long flow string of the producing well. The over-all gradient is then in fact fluctuating, and a high degree of accuracy in its estimation is not possible.

It should be realized, moreover, that the use of a more complex factor which is not related solely to the mass flow rate, but to such variables as the percentage free gas in place, would result in an extremely complex calculation requiring the derivation of separate values of f for each pressure increment. With the present field methods of oil and gas measurements, it is doubtful whether a higher accuracy could be obtained even with a more complex solution.

REVERSED CURVATURE

A further point of significance in connection with the question of calculated pressure gradients is that of the reversal of curvature of the calculated pressure-depth curves at low pressure—i.e., the condition occasionally arises where the calculated pressure traverse in the low-pressure range is concave downwards instead of concave upwards.

In the past[2,3] this reversal had been considered as an anomaly due solely to the inadequacies of the method of calculation. In the course of the La Paz experiments, however, this behavior was observed in the recorded traverses at high rates of flow in both 3½- and 2⅞-in. tubing (Fig. 1). To establish a check independent of the Maihak readings, a special test was carried out in another La Paz well. During a workover, six closely spaced retrievable gas-lift mandrels were run in the upper part of the tubing string. Dummy valves were inserted in each mandrel and the tubing plugged-off below the bottom mandrel with a retrievable Otis-S-plug. The well was produced on casing flow to a well-head separator, thus ensuring a low casinghead pressure. The pressure at the level of each mandrel was obtained by pulling each dummy valve in turn and injecting gas at a very low rate down the tubing until the injection pressure stabilized.

The pressure at the open mandrel could thus be obtained from the observed tubinghead pressure, measured with a highly accurate gauge and corrected to allow for the weight of the gas column. The casinghead pressure was also read with a newly calibrated standard manometer. On completion of the test, the tubing plug was pulled and the pressure at the tubing shoe read with an Amerada bomb.

As shown in Fig. 4, the observed pressure-depth relationship was then compared with a calculated curve obtained from the energy-loss correlation shown in Fig. 3.

It can be seen that, although the calculated curve is not a perfect fit to the observed data, the deviation is small and the reversed curve is a better approximation to the actual conditions than that obtained by extrapolating the concave upwards section of the calculated curve.

It is concluded, therefore, that the calculated curves should be drawn with the reversed curvature inherent in the calculation and that no attempt should be made to extrapolate the lower part of the curve back over the low-pressure range.

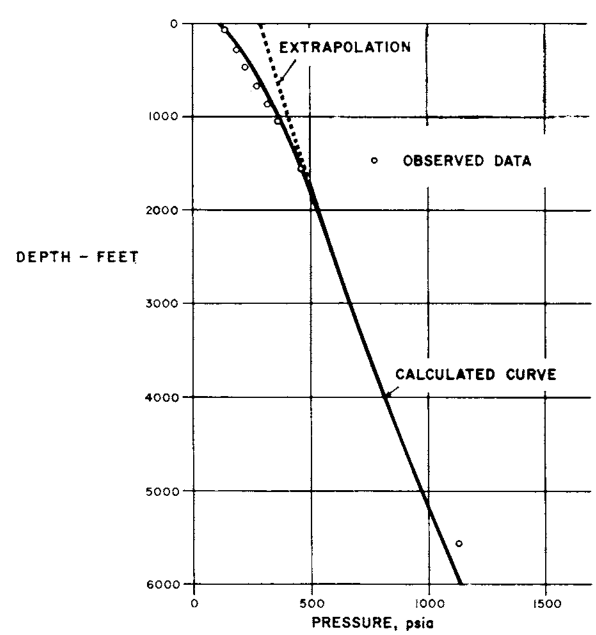

Fig. 4—Observed Pressure-Depth Relationship vs Calculated Curve Obtained from the Energy-Loss Correlation of Fig. 3. La Paz Well.

CHECK DATA

The check data available at low rates of flow have been discussed and analyzed in the section entitled "The Significance of the Energy-Loss Correlation". These 78 readings represent the total low-rate data available for Lake Maracaibo conditions.

Very few data are available at the higher flow rates, the difficulty being, of course, that special equipment is required to obtain readings in tubing against high vertical flow rates. Some measurements have been taken using either a long string or sinker bars above the bombs or an anchoring device supporting the bomb which latches in a tubing-collar recess. While there are insufficient readings for any satisfactory statistical analysis, it would appear that an average accuracy of the order of $\pm$ 5 per cent can be expected at these higher rates.

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