

PRESSURE DROP IN WELLS PRODUCING OIL AND GAS

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ABSTRACT

A simple, mechanistically based scheme for the calculation of the pressure drop in wells producing oil and gas in the single-phase liquid, bubble and slug flow patterns is described and checked with independent field data. The scheme is based on an identification of the flow pattern through a modification of the flow pattern map of Govier, Radford and Dunn and the application of the mechanical energy balance in a form appropriate for the flow pattern as suggested by Govier and Aziz. Predictions for some 48 wells are compared with field data and with the predictions of Orkiszewski, Duns and Ros, and Hagedorn and Brown. The proposed method gives results at least as good as any of the others, is more soundly based on the mechanism of flow and is independent of the data with which it is confirmed.

A computer program for the method and a typical printout are given.



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INTRODUCTION

MANY METHODS have been proposed for the estimation of the pressure drop in wells which produce a mixture of oil and gas. These are reviewed in detail by Govier and Aziz⁽¹⁾. Most of the methods are strictly empirical, but for certain of the flow patterns which may be encountered methods based on the mechanism of flow may now be developed. This is especially true for the bubble and the slug flow patterns and, as will be discussed in a following paper, the annular-mist flow pattern encountered with gas-condensate wells.

Orkiszewski⁽²⁾ has developed a pressure drop prediction scheme based on an identification of the flow pattern and the application of selected mechanistic and empirical methods to the individual flow patterns. His original appraisal included, among others, the Poettmann and Carpenter⁽³⁾ and related correlations. The most accurate methods, the Duns and Ros⁽⁴⁾ and the Hagedorn and Brown⁽⁵⁾, were then tested along with his own scheme against 148 well conditions. Orkiszewski's comparison showed that his method gave improved accuracy over that of Duns and Ros and Hagedorn and Brown. Espanol *et al.*⁽⁶⁾ confirmed this with data from 44 different wells.

Orkiszewski's method is rather complex and is not entirely consistent with present understandings of the flow mechanism in the bubble and the slug flow patterns. Although the scheme covers all flow patterns, it has not been fully tested for the froth or the annular-mist patterns.

The objective here is the development of a sound mechanistically-based prediction method for the flow patterns commonly encountered in oil wells — those where the oil is the continuous phase, i.e., the single phase, the bubble and the slug flow patterns. The work is an extension of that discussed by Govier and Aziz⁽¹⁾ and by Aziz, Fortems and Settari⁽⁷⁾. Another paper dealing with the remaining flow patterns is being prepared.

THE MECHANICAL ENERGY EQUATION

All methods for the prediction of the relationship between the pressure gradient, the flow rates, the fluid properties and the geometry of the flow duct involve one form or another of the mechanical energy equation. For a small elevation change, Δz , the equation may be written (ref. 2)

$$\Delta P = \Delta P_{HH} + \Delta P_{KE} + \Delta P_f \dots \dots \dots (1)$$

where

$$\Delta P = \text{pressure drop over the small elevation change, } \Delta z$$

$$\Delta P_{HH} = \text{hydrostatic head component of } \Delta P$$

$$= \frac{g}{g_c} \rho \Delta z \dots \dots \dots (2)$$

$$\rho = \text{density of fluid or } in \text{ situ mixture}$$

$$\Delta P_{KE} = \text{pressure drop due to kinetic energy effects}$$

$$= \frac{\Delta V^2}{2g_c} \rho \dots \dots \dots (3)$$

- α = velocity profile correction term
 V = average velocity of fluid or mixture
 ΔP_f = friction component of ΔP
 $= \frac{2fV^2\rho\Delta z}{g_c D}$ (4)
 f = Fanning friction factor
 D = inside diameter of pipe.

Where ΔP_f is due to the shear of a single phase fluid, the friction factor, f , is known to be a function of the Reynolds number, $Re = DV\rho/\mu$ and the relative roughness, k/D . In regions of laminar flow the relative roughness has no effect and

$$f = \frac{16\mu}{DV_o} \quad (5)$$

For turbulent flow, among the many available relationships that of Colebrook⁽³⁾ is probably the most suitable:

$$\frac{1}{\sqrt{f}} = 4 \log \frac{D}{2k} + 3.48 - 4 \log \left(1 + 9.35 \frac{D}{2k Re \sqrt{f}} \right) \quad (6)$$

Equations 1 to 6, with some appropriate integration technique, may be applied directly to the flow of any single-phase fluid in a well of finite depth, z . The equations are also applicable to the flow of gas-liquid mixtures in the bubble and slug flow patterns, provided the terms in them are properly interpreted. The first step in this process is the identification of the flow pattern.

FLOW PATTERN IDENTIFICATION

In wells which produce both oil and gas the gas may be entirely in solution in the oil at the bottom of the well or free gas may be present. In any event as the mixture flows upward to lower pressure regions below

the original bubble-point pressure of the oil, gas comes out of solution and any free gas expands. The flow pattern may initially be that of a single-phase liquid, but typically both the bubble and the slug flow patterns are encountered in the upper reaches of the well. With large quantities of free gas, the froth flow pattern may be reached. The situation is illustrated schematically in Figure 1.

Orkiszewski⁽²⁾ bases his flow pattern identification on the flow pattern map of Griffith and Wallis⁽⁹⁾ for the bubble and slug flow patterns and on the map of Duns and Ros⁽⁴⁾ for the other flow patterns. Govier and Aziz discuss the various available methods for flow pattern prediction and conclude that a simple map based on the work of Govier, Radford and Dunn⁽¹⁰⁾ is the most suitable. This map is reproduced as Figure 2. The bubble to slug and the slug to froth flow pattern transition lines are described by the following equations:

For the bubble to slug transition

$$YV_{SL} = 0.01 [(1.96 X V_{SG})^{5.81}] \quad (7)$$

where

- V_{SL}, V_{SG} = superficial velocity of the liquid and the gas phase respectively
 $Y = (\rho_L \sigma_{WA} / \rho_W \sigma)^{1/4}$
 $X = (\rho_G / \rho_A)^{1/8} Y$
 ρ_L and ρ_G = density of liquid and gas under flowing conditions, respectively
 ρ_A = density of air at standard atmospheric conditions (60°F, 14.65 psia)
 σ = interfacial tension of liquid-gas system at flowing conditions
 σ_{WA} = interfacial tension of water-air system at standard atmospheric conditions

For the slug to froth transition

$$YV_{SL} = 0.263 (XV_{SG} - 8.61) \text{ for } YV_{SL} \leq 4 \quad (8)$$

When $YV_{SL} > 4$ the transition is from slug to annular mist flow is at $XV_{SG} = 26.5$.

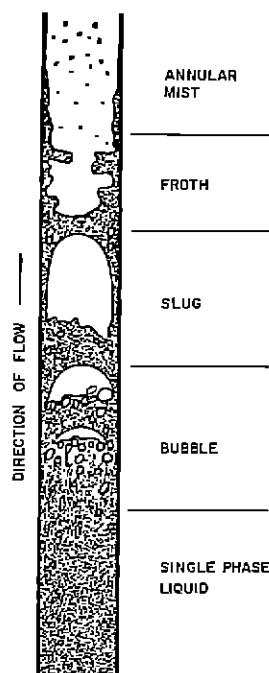


FIGURE 1 — Schematic diagram of flow pattern in vertical flow of gas-liquid mixtures.

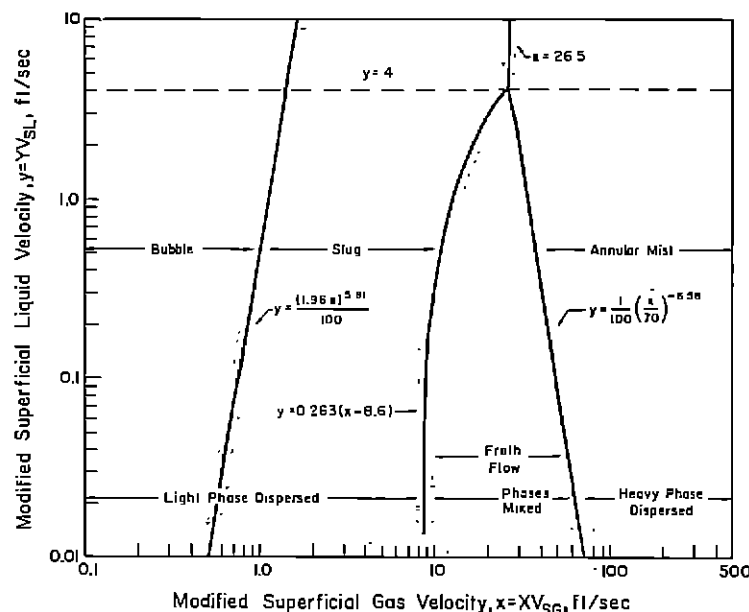


FIGURE 2 — Generalized flow pattern map for flow of gas-liquid mixtures.

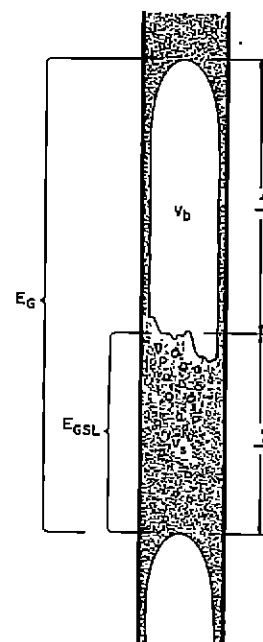


FIGURE 3 — Elements of the slug flow pattern.

Bubble Flow

The bubble flow pattern is characterized by small bubbles of gas dispersed in a continuous oil phase. The difference in the densities of the two phases causes the bubbles to travel at a velocity higher than the average velocity of the liquid or of the mixture as a whole.

The hydrostatic head component of the total pressure drop may be calculated from Equation 2, provided the *in situ* density of the gas liquid mixture is known:

$$\Delta P_{HH} = \frac{g}{g_c} \rho_M \Delta z \quad (9)$$

The *in situ* mixture density is given by

$$\rho_M = E_G \rho_G + (1 - E_G) \rho_L \quad (10)$$

The *in situ* volume fraction of the gas phase, E_G , may be estimated from continuity considerations and a knowledge of the rise velocity of the bubbles⁽¹¹⁾

$$E_G = \frac{V_{SG}}{V_{bf}} \quad (11)$$

where

V_{bf} = the rise velocity of the bubbles in the flowing stream

Zuber and Findlay⁽¹¹⁾ and others have shown that the rise velocity of bubbles in a flowing stream may be calculated from

$$V_{bf} = C_0 V_M + V_{bs} \quad (12)$$

where

$C_0 \approx 1.2$ for turbulent flow

$$V_M = V_{SG} + V_{SL} \quad (13)$$

V_{bs} = the rise velocity of the bubbles in a stagnant liquid

The rise velocity of bubbles in a stagnant liquid, V_{bs} , may be determined from Equation 10 and ΔP_{HH} from *at.*⁽¹²⁾, applicable to a swarm of bubbles in the so-called churn turbulent bubbly regime, i.e., at high Reynolds numbers where the bubbles lose their separate identity:

$$V_{bs} = 1.41 \left[\frac{c g (\rho_L - \rho_G)}{\rho_L^2} \right]^{1/4} \quad (14)$$

With E_G determined from Equations 11, 12 and 14, ρ_M may be determined from Equation 10 and ΔP_{HH} from Equation 9.

The frictional component of the pressure drop may be calculated from Equation 15 by recognizing that the liquid phase is in contact with the pipe wall,

$$\Delta P_f = \frac{2 f_L \bar{v}_M V_M^2}{g_c D} \Delta z \quad (15)$$

where f_L is calculated from Equation 5 or Equation 6 depending on whether the Reynolds number, evaluated as $Re = DV_M \rho_L / \mu_L$, is less than or exceeds 2100. The appropriate value of k/D is used with Equation 6.

Kinetic energy effects are very small over the flow rates where the bubble flow pattern is encountered so the term ΔP_{KE} of Equation 1 may be neglected. The final ΔP is therefore the sum of ΔP_{HH} and ΔP_f .

Slug Flow

As the gas rate increases with increasing amount of gas coming out of solution, the smaller bubbles collide, coalesce and form larger, cap-shaped bubbles as shown in Figure 1. The cap-shaped bubbles are usually referred to as Taylor bubbles, after Taylor⁽¹³⁾, who studied their motion. The liquid between such large bubbles is referred to as the slug. The slug contains small bubbles, but most of the gas phase is contained in the large bubbles.

The hydrostatic head component of the pressure drop may again be predicted by Equations 9 and 10, with E_G estimated through Equations 11 and 12 but with V_{bs} determined from the equation of Neal⁽¹⁴⁾ for the rise velocity of Taylor bubbles with the proportionality factor C determined as proposed by Wallis⁽¹⁵⁾, i.e.,

$$V_{bs} = C \left[\frac{g D (\rho_L - \rho_G)}{\rho_L} \right]^{1/2} \quad (16)$$

where

$$C = 0.345 \left[1 - \exp \left(- \frac{0.01N}{0.345} \right) \right] \left[1 - \exp \left(\frac{3.37 - E_0}{m} \right) \right]$$

$$N = \frac{[D^3 g (\rho_L - \rho_G) \rho_L]^{1/2}}{\mu_L} \quad (\text{liquid viscosity number})$$

$$m = \begin{cases} 10 & \text{for } N > 250 \\ 69N^{-0.35} & \text{for } 18 < N < 250 \\ 25 & \text{for } N < 18 \end{cases}$$

$$E_0 = \frac{g D^2 (\rho_L - \rho_G)}{\sigma} \quad (\text{Eotvos number})$$

Equations 11, 12 and 16 enable us to predict E_G , assuming all of the gas is in the larger bubbles moving at the velocity, V_{bf} . This provides a reasonable estimate of the average *in situ* volume fraction of gas over the small change in elevation, Δz , provided that Δz is great enough to include several large bubbles and their intervening slugs. The hydrostatic head component may be estimated from this value of E_G and Equations 9 and 10.

Alternatively the hydrostatic component may be estimated from a consideration of the actual distribution of the gas phase in the mixture. Govier and Aziz⁽¹⁾ discuss the relationship between E_G , the volumes and dimensions of the Taylor bubbles and the slugs, and the volume fraction of gas in the liquid slug, E_{GSL} . Based on continuity considerations, work of Griffith and Wallis⁽⁹⁾, and data of Akagawa and Sakaguchi⁽¹⁶⁾, Govier and Aziz⁽¹⁾ show that

$$E_G = \frac{v_b + E_{GSL} v_s}{A(L_s + L_b)} \quad (17)$$

where

$$v_b = \text{the volume of the Taylor bubble} \quad (18)$$

$$= A(0.913 L_b - 0.526 D) \quad (18)$$

$$v_s = \text{the volume of the slug} = A L_s \quad (19)$$

$$E_{GSL} = E_G^{1.8} \quad (20)$$

$$L_s \geq 8 D \text{ or } L_s \leq 10 D \quad (21)$$

These relations enable a prediction to be made of E_{GSL} , L_s and L_b from the known values of E_G and D . The value of L_s so determined is only the minimum value to be expected, but because greater L_s values lead to approximately proportionately greater L_b values, this restriction is not serious. For practical purposes, L_s may be taken as $10D$. Equations 18 and 20, although based on air-water data, are expected to give a reasonable approximation for oil-gas systems.

Figure 3 illustrates the situation in slug flow, and from it and the preceding relations we may express

the pressure drop due to hydrostatic head over the elevation change, Δz , as

$$\Delta P_{\text{HT}} = \frac{g}{g_c} (\rho_G L_b + \rho_L L_s) \frac{\Delta z}{L_b + L_s} \quad (22)$$

where

$$\rho_s = E_{\text{GSL}} \rho_G + (1 - E_{\text{GSL}}) \rho_L \quad (23)$$

The component of the pressure drop due to friction may be estimated by recognizing that the bulk of the frictional effect is due to the liquid slug (see Govier and Aziz⁽¹¹⁾) and that this may be estimated by single-phase methods.

Either of two approaches may be used. The fraction of the total length affected may be taken as $(1 - E_G)$, with the slugs considered to be free of gas bubbles, yielding

$$\Delta P_f = \frac{2 f_L V_M^2 \rho_L (1 - E_G)}{g_c D} \Delta z \quad (24)$$

A method, preferable at least in principle, is to take account of the estimated fraction $L_s / (L_b + L_s)$ and the estimate of the volume fraction of gas in the slug, E_{GSL} , thus

$$\Delta P_f = \frac{2 f_L V_M^2 \rho_s}{g_c D} \left(\frac{L_s}{L_b + L_s} \right) \Delta z \quad (24a)$$

where, in either case, f_L is evaluated from Equation 6 at $Re = DV_M \rho_L / \mu_L$ and the appropriate k/D .

THE CALCULATION METHOD

A program incorporating the concepts described has been developed. The principle features of the program are discussed in the following.

Starting from the well head where the pressure, P_H , and temperature, T_H , are known and the depth $z_H = 0$,

1. assume $\Delta z = 1/10$ of the total depth,
2. assume $\Delta P = 100$ psi,
3. P and T are determined at the mid-point of the first depth increment, $z = z_H + 0.5 \Delta z$, $P = P_H + 0.5 \Delta P$,

$$T = T_H + \frac{(T_B - T_H)z}{z_B}$$

4. From the subroutine PROP the fluid properties and flow rates are determined at P and T as follows:

- RS — Solution gas-oil ratio from the correlation of Standing⁽¹⁷⁾. When RS is greater than the producing gas-oil ratio, $RS = \text{GOR}$ and the pressure gradient is calculated for single-phase flow;
- Bo — Formation volume factor from the correlation of Katz⁽¹⁸⁾;
- Z — Compressibility factor from the Standing-Katz⁽¹⁹⁾ chart at pseudo-reduced pressure and temperature, p_p , and T_p , estimated from gas gravity⁽¹⁸⁾. In the subroutine COMPR the value of Z is determined at the reduced conditions by interpolation;
- ρ_L, ρ_G — the liquid and gas densities are determined as the ratio of mass and volumetric flow rates.

The volumetric flow rates, in ft^3/sec , are

$$Q_L = 6.49 \times 10^{-5} Q_o (B_o + \text{WOR})$$

$$Q_G = 3.27 \times 10^{-7} Z Q_o (R - RS) \frac{T + 460}{P}$$

where

- Q_o = Oil production rate, STbbl/day
- WOR = Water-oil ratio, STbbl/STbbl
- R = Producing gas-oil ratio, SCF/STbbl

The mass flow rates, in lb/sec , are

$$M_L = 0. [4.05 \times 10^{-3} (SG_o + SG_w \times \text{WOR})$$

$$+ 8.85 \times 10^{-7} \times SG_o RS]$$

where

- SG = specific gravity
- μ_o = Viscosity of the oil from the correlation of Chew and Connally⁽²⁰⁾
- μ_G = Viscosity of the gas from the correlation of Lee *et al.*⁽²¹⁾

5. The flow pattern is determined in the sub-routine REGM1 according to the flow pattern map shown in Figure 2.
6. In the subroutine DENS1, the *in situ* density and the volume fraction of gas are calculated from the equations appropriate for the flow pattern (see discussion under the separate flow patterns).
7. In the subroutine FRIC1 the frictional contribution, ΔP_f , to the total pressure drop is calculated. The friction factor is evaluated from Equation 6 through subroutine COLE at the appropriate Reynolds number and relative roughness.
8. The hydrostatic head term is evaluated from Equation 9 in bubble flow and from Equation 22 in slug flow. The total pressure gradient is then the sum of the frictional and hydrostatic head components.
9. The calculated value of ΔP is compared with the assumed value. If $\Delta P < 100$ psi and if the difference between the assumed and calculated values is less than ± 0.10 psi, the calculation proceeds to the next step. If $\Delta P > 100$ psi the depth increment is divided by 2 and steps 1 to 8 are repeated. If the assumed and calculated values of ΔP are not within 0.10, the assumed value is replaced by the calculated ΔP and steps 1 to 8 are repeated.
10. Subroutine OUTPUT is called and the values at the depth increment are printed (see sample output in the Appendix)
11. Steps 1 to 10 are repeated for the new conditions of the next depth increment until $\Sigma \Delta z$ equals the total depth.

The program listing is given in the Appendix.

Comparison of Predictions with Field Data and With Other Methods

A search of the literature has revealed relatively complete field data on the pressure gradients in some 48 wells producing at gas-oil ratios within the range of this study. The relevant field data are identified and summarized in Table 1. The free gas index has been calculated and is given for the top, mid-depth and bottom-hole conditions.

Of the 48 wells studied, 38 are those reported by Espanol⁽²⁾, one is the same as that used in a sample calculation by Orkiszewski⁽²²⁾, one is taken from Poettmann and Carpenter⁽²³⁾, and seven were obtained from the files of the Energy Resources Conservation Board. For each well the table gives the API gravity, the oil production rate, the gas-oil ratio, the depth of the well, the tubing diameter, and the measured well-head and bottom-hole pressures. The data are listed in order of increasing top hole gas-liquid ratio, as roughly measured by a "free gas index", FGI, defined as

$$\text{FGI} = R - KP \text{ (scf/bbl)}$$

where

- R = producing gas-oil ratio, scf/bbl
- K = $1(\text{API}/100) - 0.05$
- P = pressure, psia

The approximate percentage of well depth for each flow pattern, as determined by this method, i.e., single-phase liquid, bubble and slug, is shown. The bottom hole pressure and the hydrostatic head and frictional components of the total pressure drop as predicted by the proposed method are given, and the per cent errors in pressure drop and in bottom hole pressure are recorded. For comparative purposes, the per cent errors in the predicted bottom hole pressure by the methods of Orkiszewski, of Hagedorn and Brown and of Duns and Ros are also given. The calculated results from the Hagedorn and Brown and the Duns and Ros methods were taken from Espanol⁽⁶⁾. It is clear from this comparison that no single method may be claimed to be more accurate than all other methods in all cases.

The data for wells No. 1, 11 and 16 are clearly suspect. For well No. 1, both the Orkiszewski and the present method show a large error between the pre-

dicted and measured values of the bottom hole pressure, in spite of the fact that the flow is essentially single phase and the pressure drop is almost entirely due to the hydrostatic head. Well No. 11 shows substantial errors for all four methods, and well No. 16 for all but the Duns and Ros method, hardly to be expected when more than two-thirds of the depth is occupied by single-phase liquid, and again, essentially all of the pressure drop is due to the hydrostatic head. In addition, the results of all the prediction methods, except that of Hagedorn and Brown, for well No. 48, where again essentially all of the pressure drop is due to the hydrostatic head, raise a question about the validity of the data.

It may be observed in Table 1 that the hydrostatic component of the total pressure drop is orders of magnitude larger than the frictional component in all but four cases. Only at the high oil flow rates of wells No. 24, 27, 45 and 47 is the frictional pressure drop at all

TABLE 1 — SUMMARY OF DATA AND COMPARISON OF RESULTS

Well No.	Ident.*	API Gravity	Oil Rate bbl/day	WOR bbl/bbl	GOR Scf/bbl	Well Depth ft.	Tubing Dia. ft.	Measured Pressure, psi		Free Gas Index Mfd			Approximate per cent Depth for Flow Pattern		
								Bottom	Top	Bottom	Depth	Top	S.Ph.	B	SL
1	CB 66	38.4	136		143	5046	.203	1637	391	- 416	- 202	12	95	5	0
2	E 12	40	218		171	12037	.198	4368	440	-1358	- 670	17	100	0	0
3	E 29	44	415		516	12449	.198	5140	1085	-1490	- 698	93	92	8	0
4	E 38	44	118		485	12449	.198	4965	960	-1450	- 669	111	90	10	0
5	CB 68	38.4	114		250	5012	.203	1631	271	- 295	- 68	159	65	30	5
6	E 32	44	135		450	12445	.198	4875	675	-1450	- 631	187	85	15	0
7	E 25	44	132		518	12446	.198	4887	850	-1388	- 600	187	85	15	0
8	E 23	44	131		523	12439	.198	4722	850	-1320	- 564	192	88	12	0
9	CB 33	36.6	233		441	4839	.166	2056	742	- 209	- 1	207	40	40	20
10	E 31	44	116		493	12454	.198	4967	650	-1444	- 602	240	80	20	0
11	E 40	40	407		443	7300	.198	1945	495	- 238	+ 16	270	68	12	20
12	E 16	44	498		478	10553	.198	3695	450	- 960	- 328	303	70	10	20
13	E 33	44	164	0.26	333	12453	.198	4339	48	-1360	- 523	314	69	11	20
14	CB 44	41.4	74		811	9101	.166	4042	1355	- 660	- 171	318	73	27	0
15	E 36	44	71	0.98	389	12458	.198	4675	20	-1430	- 525	381	69	19	12
16	E 8	40	150		486	10201	.198	2122	275	- 260	+ 65	390	63	20	17
17	E 27	44	130	0.23	572	12453	.198	4801	425	-1300	- 447	406	68	20	12
18	E 10	40	93		443	4304	.198	1094	50	+ 60	+ 243	426	0	45	55
19	E 30	44	135		756	12449	.198	4683	800	-1070	- 313	444	65	25	10
20	E 39	40	378		582	7300	.198	2232	375	- 200	+ 125	451	38	17	45
21	E 24	44	103		485	12457	.198	3656	70	- 940	- 241	458	60	17	23
22	E 37	44	136		485	12456	.198	3850	50	-1020	- 277	466	53	20	27
23	CB 67	38.4	138		581	5054	.203	1633	317	+ 36	+ 255	475	0	55	45
24	O 22	18.7	1850		575	3890	.249	1500	670	+ 370	+ 426	483	0	0	100
25	E 4	46	282		699	12024	.198	2985	350	- 525	+ 15	556	60	12	28
26	E 21	44	111		873	9667	.198	3044	750	- 314	+ 133	580	48	37	15
27	E 2	43	1656		873	6225	.167	2720	750	- 161	+ 213	588	60	5	35
28	E 18	44	89		897	9664	.198	2986	625	- 268	+ 192	653	43	40	17
29	E 34	44	81	1.38	675	12438	.198	4595	50	-1120	- 232	656	50	25	25
30	E 35	44	141		1091	12451	.198	5137	1100	- 910	- 124	662	48	35	17
31	E 28	44	129		899	12443	.198	4440	500	- 833	- 65	704	40	33	27
32	CB 22	41.4	235		1200	8862	.203	3825	1290	- 193	+ 268	729	0	53	47
33	E 19	44	103		1014	9667	.198	2965	725	- 142	+ 295	731	33	45	22
34	E 26	44	141		1000	12441	.198	4508	645	- 760	- 6	748	38	32	30
35	E 14	44	336		889	8156	.198	1907	360	+ 145	+ 447	749	13	20	67
36	E 1	37	490		1100	6000	.167	3006	1093	+ 138	+ 444	750	40	20	40
37	E 9	40	205		987	10328	.198	2742	450	+ 27	+ 428	830	23	30	47
38	E 20	43	82		1112	9673	.198	3022	625	- 36	+ 420	875	32	48	20
39	E 13	36	101	0.04	1069	7214	.198	2662	480	+ 244	+ 582	920	0	45	55
40	E 42	43	51		1171	8454	.198	2925	575	+ 60	+ 506	952	15	65	20
41	E 43	43	61		1433	8456	.198	3382	950	+ 148	+ 610	1072	10	78	12
42	E 17	44	59		1617	8472	.198	3339	900	+ 315	+ 790	1266	0	70	30
43	E 44	43	57		1675	8468	.198	3260	835	+ 440	+ 899	1358	0	75	25
44	E 11	40	53		1460	4308	.198	1338	220	+ 990	+ 1186	1383	0	0	100
45	CB 55	47.3	1104		2645	11429	.166	4734	2465	+ 640	+ 1121	1602	0	0	100
46	P 1	44.4	60		2250	10961	.203	3870	1264	+ 725	+ 1238	1752	0	65	35
47	CB 56	47.3	1296		2796	11429	.166	4577	2465	+ 860	+ 1306	1753	0	0	100
48	E 22	44	44		9975	9665	.198	3000	1750	+ 8805	+ 9048	9292	0	0	100

*Code: CB — Energy Resources Conservation Board; E — Espanol⁽⁶⁾; O — Orkiszewski⁽¹⁾; P — Poettmann and Carpenter⁽⁴⁾

significant. In calculating the total pressure drop in the slug flow region, both approaches, i.e. Equations 22 and 24a, and Equations 9, 10 and 24, were tested to determine ΔP_{RH} and ΔP_r . The results obtained for ΔP_{RH} were essentially the same. The proposed method (Eqs. 22 and 24a), using a different proportion of the tubing subject to friction, gives different predictions for the total pressure drop only in cases where ΔP_r is significant.

On the whole, the results of calculating ΔP_r from Equation 24a were superior to those based on Equation 24. The values of ΔP_{RH} and ΔP_r shown in Table 1 were obtained from Equations 22 and 24a.

Single-phase flow predominates with a modest amount of bubbles but no slug where the top hole FGI is about +200 or less and where the mid-depth FGI is in the range of -600 to -200; this is the case for wells No. 1 to 8 and 10. For these tests the proposed and the Orkiszewski method give essentially the same results; the method of Hagedorn and Brown is slightly superior and that of Duns and Ros is slightly in-

ferior. It is significant to note that in almost all cases the errors are negative.

At FGI values at the top in the range of about 200 to 1000 (wells No. 9, 11 to 38, 40 and 41), with two or three exceptions, the flow pattern is a combination of single-phase, bubble and slug flow; the proportion of single-phase flow decreases and that of the bubble and slug increases as the top hole FGI increases. In this region the proposed method is modestly superior to the methods of Orkiszewski and of Duns and Ros, which are slightly better than that of Hagedorn and Brown.

With top hole FGI values exceeding about 1000 (wells No. 39, and 42 to 48), single-phase flow is not predicted, and the flow pattern tends increasingly from a mixture of bubble and slug to slug. In this region the proposed method is slightly superior to the others.

Summarizing the results of Table 1 and neglecting the suspect data referred to earlier, the average of the absolute errors in the pressure drop calculated

Calculated by Proposed Method bottom hole pressure, psia	ΔP_{RH} psi	ΔP_r psi	Error in Calc'd ΔP , % Proposed Method	Error in Calculated Bottom Hole Pressure, Per Cent				No.
				Proposed Method	Orkiszewski	Hagedorn- Brown	Duns-Ros	
2123	1732	1	+39.0	+29.7	+29.7	—	—	1
4456	4013	3	+ 2.3	+ 2.0	+ 2.0	+ 2.1	+ 1.7	2
4823	3730	10	- 7.8	- 6.2	- 6.3	- 5.9	- 7.0	3
4721	3762	1	- 6.1	- 4.9	- 4.9	- 4.7	- 7.8	4
1902	1639	1	+20.0	+16.6	+16.2	—	—	5
4447	3776	1	-10.2	- 8.8	- 8.8	- 8.6	-10.9	6
4572	3725	1	- 7.8	- 6.4	- 6.4	- 6.3	- 8.7	7
4566	3718	1	- 4.0	- 3.3	- 3.3	- 3.2	- 8.4	8
2248	1508	4	+14.6	+ 9.3	+ 7.4	—	—	9
4376	3733	1	-13.7	-11.9	-11.9	-11.7	-14.1	10
2723	2224	7	+53.7	+40.0	+36.7	+23.6	+37.8	11
3557	3097	14	- 4.3	- 3.7	- 7.6	-11.0	- 5.0	12
3856	3809	6	-11.3	-11.1	- 9.5	-51.3	-14.3	13
3974	2619	1	- 2.5	- 1.7	- 1.7	—	—	14
4270	4255	3	- 8.7	- 8.7	- 7.2	-73.3	-13.2	15
3295	3024	2	+63.5	+55.3	+53.4	+33.0	+ 4.1	16
4248	3830	2	-12.6	-11.5	-12.1	-37.3	-14.4	17
1139	1093	3	+ 4.3	+ 4.1	+13.7	-59.3	- 2.4	18
4291	3499	2	-10.1	- 8.4	- 8.5	- 8.4	-10.2	19
2364	1985	9	+ 7.1	+ 5.9	- 4.8	-24.6	- 0.2	20
3490	3428	3	- 4.6	- 4.5	-10.7	-30.0	-10.0	21
3327	3281	6	-13.8	-13.6	-24.9	-41.6	-17.9	22
1711	1403	2	+ 5.9	+ 4.8	+ 2.6	—	—	23
1722	993	58	+26.7	+14.8	+ 1.9	—	—	24
3566	3213	8	+22.0	+19.5	+14.9	- 9.3	+15.2	25
3436	2968	1	+17.1	+12.9	+12.5	+13.3	+10.7	26
2685	1729	206	- 1.8	- 1.3	- 3.6	- 1.0	+ 1.0	27
3292	2680	1	+13.0	+10.2	+ 9.6	+10.4	+ 8.0	28
4123	4078	6	-10.4	-10.3	- 7.2	-84.2	-16.5	29
4382	3292	2	-18.7	-14.7	-14.8	-15.2	-15.4	30
3769	3280	2	-17.0	-15.1	-15.8	-31.6	-19.0	31
3607	2327	4	- 8.6	- 5.7	- 6.8	—	—	32
3331	2618	1	+16.3	+12.3	+11.7	+12.7	+11.0	33
3877	3242	3	-16.3	-14.0	-14.6	-29.9	-17.2	34
2270	1903	12	+23.5	+19.0	+ 6.8	-27.5	+46.0	35
2816	1698	25	- 9.9	- 6.3	- 9.5	- 7.8	- 6.4	36
3123	2673	6	+16.6	+13.9	+13.1	-23.7	+11.1	37
3239	2622	1	+ 9.1	+ 7.2	+ 6.5	+ 2.4	+ 4.3	38
2456	1988	2	- 9.4	- 7.7	- 5.2	-38.4	-12.4	39
2871	2310	0	- 2.3	- 1.8	- 2.6	- 3.6	- 6.6	40
3208	2271	0	- 7.2	- 5.1	- 5.1	- 6.6	- 6.5	41
3101	2220	1	- 9.8	- 7.1	- 7.6	- 8.7	- 8.4	42
3057	2237	1	- 8.4	- 6.2	- 6.4	- 8.6	- 8.4	43
1189	969	2	-13.4	-11.1	-17.9	- 3.4	-19.6	44
5186	2397	324	+19.9	+ 9.5	- 0.6	—	—	45
3946	2706	1	+ 2.9	+ 2.0	—	—	—	46
5300	2370	465	+34.3	+15.8	+16.8	—	—	47
3631	1878	2	+50.5	+21.0	+23.3	+ 5.0	+23.1	48

by the proposed method is 11.5 per cent. The average absolute errors in the calculated bottom hole pressures are 8.9, 8.9, 20.5 and 11.1 per cent, respectively, for the proposed method and the methods of Orkiszewski, Hagedorn and Brown, and Duns and Ros. The proposed method is not demonstrated to be superior to the method of Orkiszewski, but the fact that it is more soundly based on the mechanism of flow, while giving results of equal accuracy to the method of Orkiszewski, is significant.

CONCLUSIONS

The proposed method, based on mechanistic considerations, permits ready identification of the flow pattern, and the calculation of the *in situ* volume fraction of the gas phase and the pressure gradient. The predicted pressure drop compares favourably with measured values in 44 of the 48 wells for which adequate data are reported. The absolute error is about the same as for the Orkiszewski method, which has been shown previously⁽²⁾ to be superior to the Hagedorn and Brown and the Duns and Ros method.

The computer program developed permits rapid evaluation at discrete depth intervals of the flow pattern and all other factors influencing the pressure gradient. The complete pressure profile is determined.

Uncertainties in some of the field data and a lack of complete and reliable data covering the full range of flow rates have hindered the development of a fully reliable, mechanistically based computation method. Additional reliable data are needed to permit further checking and possible refinement of the proposed method.

ACKNOWLEDGMENTS

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NOMENCLATURE

A	= cross-sectional area of pipe, ft ²
B _m	= formation volume factor, bbl/STbbl
C	= proportionality factor (Eq. 16)
C _n	= distribution coefficient (Eq. 12)
D	= inside diameter of tubing, ft.
E _G	= cross-sectional average <i>in situ</i> volume fraction of the gas phase
E _{GSL}	= volume fraction of gas in the liquid slug
E _o	= Eotvos number (defined following Eq. 16)
f	= Fanning friction factor (Eqs. 5 and 6)
FGI	= free-gas index, SCF/bbl
g	= acceleration of gravity, ft/sec ²
g _a	= dimension conversion factor = 32.2 lb _m /ft/lb _{sec} ²
k/D	= relative roughness of pipe
L _b	= length of the Taylor bubble, ft (Eq. 17)
L _s	= length of the liquid slug, ft (Eq. 21)
M	= mass flow rate, lb _m /sec.
N	= Wallis liquid viscosity number (defined following Eq. 16)
P	= pressure, psia
ΔP	= total pressure drop over small elevation change, Δz, psi (Eq. 1)
ΔP _f	= friction component of ΔP, psi (Eq. 1)
ΔP _{hst}	= hydrostatic head component of ΔP, psi (Eq. 1)
ΔP _{KE}	= kinetic energy component of ΔP, psi (Eq. 1)
Q	= volumetric flow rate, ft ³ /sec.
R	= producing gas oil ratio, SCF/bbl
Re	= Reynolds number = DVρ/μ
RS	= solution gas oil ratio, ft ³ /bbl
SG	= specific gravity
T	= temperature (°F or °R)
V	= average velocity, ft/sec.
V _{bl}	= rise velocity of bubbles in a flowing stream, ft/sec. (Eq. 12)
V _{st}	= rise velocity of bubbles in a stagnant liquid, ft/sec. (Eq. 14 or 16)
V _M	= average velocity of the mixture, ft/sec. (Eq. 13)


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CUC 6600 FIN V3.0-P213 OPT=0 72/04/
SUBROUTINE COMPRA (P,T,GAS,GRVGS,R,OL,CG,DSST,DNSL,DMSG,
I AMT, VISCAL, VISCGL, RS, CEPT, SLP, WGR, GRM)
C CALCULATION OF OIL AND GAS PROPERTIES
C DESCRIPTION OF VARIABLES
P - PRESSURE, PSIA
T - TEMPERATURE, DEGREE F
RS - SOLUTION OF GAS, SCF/STB
OL - LIQUID FLOW RATE, CUB/SEC
OG - GAS FLOW RATE, CUB/SEC
BO - FORMATION VOLUME FACTOR OF OIL, RB/STB
ZCOEF - COMPRESSIBILITY OF GAS
DNSL - DENSITY OF LIQUID, LBM/CUFT
DMSG - DENSITY OF GAS, LBM/CUFT
VISCAL - VISCOSITY OF OIL, L/DUIN, CP
VISCGL - VISCOSITY OF GAS, CP
LEE, GONZALEZ, EAKIN (1966) CORRELATION FOR VISCGL
GM = GAS MOLE FRACTION
API0=API-5/CEVDS = 131.5
TRANK = 1 + 450
RS=GRVGS*(P/10.13)**((0.0125-API0)/(10.13*(0.00991**T)))*1.205
IF (RS .GT. 8) GO TO 26
CALL COMPRA (P, T, GRVGS, ZCOEF,
OG,OL,27-1.0E-7*ZCOEF*OGST*(R-451)*(TRANK/P)
)MSG=RS*(1.0E-7*ZCOEF*OGVGS*(R-451))
DMSG=OG*OG
GM = GRVGS*26.966
C = (17.77 - 0.0063*GM)*(TRANK**1.5)/(122.4 + 12.9*GM - TRANK)
R = 2.57 + 191.57/TRANK + 0.0095*GM
T = 1.11 + 0.066
VISCAL = (C*P**2)*DMSG/62.43**T**1.12*0.001
GO TO 26
45 = 2
C SINGLE PHASE LIQUID FLOW AT THIS PRESSURE
CONTINUE
P=RS*SGR((GRVGS/GRVGS) + 1.2541
)OG=1.922 + 0.00147*P**1.175
OL = 6.9E-5*OG**T*(80-OD)
AML = OOST*(0.00005*(GRVGS*CF*WGR) + 0.85E-07*GRVGS*RS)
DNSL=AML/OL
AML=AML*AMG
VISCGL = CEPT*(100./T)**SLP
R = 0.2 + 0.00110*(10.0001*RS)
B = 0.63 + 0.57*(10.0007*RS)
VISCAL=VISCGL*B
RETURN
END

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CUC 6600 FIN V3.0-P213 OPT=0 72/04/
5 1.016, 1.014, .243, .394, .451, .514, .582, .636, .688, .729,
7 .765, .796, .841, .876, .904, .924, .941, .963, .940, .995, 1.009,
7 1.019, .607, .644, .679, .715, .752, .789, .826, .864, .901, .939,
7 .875, .865, .958, .934, .933, .967, .981, .997, 1.012, 1.024 /
DATA (175*(N+1), N=1,201), M=1.61 /
1 .45, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.4, 2.6, 2.8, 3. /
DATA (175*(N+1), N=1,201), M=1.61 /
1 1.058, 1.048, 1.041, 1.04, 1.036, 1.023, 1.029, 1.033, 1.014,
2 1.021, 1.016, 1.013, 1.01, 1.009, 1.007, 1.004, 1.004, 1.001,
3 1.0, .999, .99, .98, .97, .96, .95, .94, .93, .92, .91, .9,
4 1.1, 1.1, 1.1, 1.1, 1.1, 1.1, 1.1, 1.1, 1.1, 1.1, 1.1, 1.1,
5 .949, .949, .953, .954, .968, .975, .98, .983, .987, .992, .998,
6 .998, 1.0, 1.002, 1.006, .785, .819, .85, .871, .885, .898, .911,
7 .921, .93, .945, .958, .967, .973, .978, .98, .982, .987, 1.001,
8 1.004, .588, .609, .773, .774, .776, .812, .817, .854, .876, .889,
9 .902, .924, .941, .953, .961, .97, .981, .989, .994, 1.001,
1 1.007, .931, .959, .979, .991, .994, .995, .996, .997, .998, .999,
2 .999, .931, .945, .955, .964, .978, .987, .994, 1.002, 1.008 /
DATA (175*(N+1), N=1,201), M=1.61 /
3 .951, .520, .640, .707, .753, .789, .817, .837, .859, .876,
4 .901, .925, .941, .952, .962, .977, .987, .994, 1.002, 1.009,
5 .966, .480, .605, .682, .732, .773, .801, .829, .856, .881, .895,
6 .92, .536, .693, .900, .975, .986, .988, 1.022, 1.011, .926, .926,
7 .571, .658, .71, .757, .791, .817, .841, .859, .889, .915, .936,
8 .946, .957, .974, .985, .995, 1.003, 1.01, .951, .946, .911, .936,
9 .891, .742, .778, .805, .832, .851, .884, .913, .930, .941, .954,
1 .972, .984, .995, 1.003, 1.011, .97, .971, .972, .982, .986, .912,
2 .755, .784, .815, .837, .871, .9, .924, .934, .95, .97, .981,
3 .995, 1.001, 1.012, .971, .951, .954, .926, .887, .872,
4 .766, .799, .824, .861, .893, .916, .931, .947, .968, .971, .995,

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CUC 6600 FIN V3.0-P213 OPT=0 72/04/
SUBROUTINE COMPRA (P,T,GAS,GRVGS,R,OL,CG,DSST,DNSL,DMSG,
I AMT, VISCAL, VISCGL, RS, CEPT, SLP, WGR, GRM)
C CALCULATION OF OIL AND GAS PROPERTIES
C DESCRIPTION OF VARIABLES
P - PRESSURE, PSIA
T - TEMPERATURE, DEGREE F
RS - SOLUTION OF GAS, SCF/STB
OL - LIQUID FLOW RATE, CUB/SEC
OG - GAS FLOW RATE, CUB/SEC
BO - FORMATION VOLUME FACTOR OF OIL, RB/STB
ZCOEF - COMPRESSIBILITY OF GAS
DNSL - DENSITY OF LIQUID, LBM/CUFT
DMSG - DENSITY OF GAS, LBM/CUFT
VISCAL - VISCOSITY OF OIL, L/DUIN, CP
VISCGL - VISCOSITY OF GAS, CP
LEE, GONZALEZ, EAKIN (1966) CORRELATION FOR VISCGL
GM = GAS MOLE FRACTION
API0=API-5/CEVDS = 131.5
TRANK = 1 + 450
RS=GRVGS*(P/10.13)**((0.0125-API0)/(10.13*(0.00991**T)))*1.205
IF (RS .GT. 8) GO TO 26
CALL COMPRA (P, T, GRVGS, ZCOEF,
OG,OL,27-1.0E-7*ZCOEF*OGST*(R-451)*(TRANK/P)
)MSG=RS*(1.0E-7*ZCOEF*OGVGS*(R-451))
DMSG=OG*OG
GM = GRVGS*26.966
C = (17.77 - 0.0063*GM)*(TRANK**1.5)/(122.4 + 12.9*GM - TRANK)
R = 2.57 + 191.57/TRANK + 0.0095*GM
T = 1.11 + 0.066
VISCAL = (C*P**2)*DMSG/62.43**T**1.12*0.001
GO TO 26
45 = 2
C SINGLE PHASE LIQUID FLOW AT THIS PRESSURE
CONTINUE
P=RS*SGR((GRVGS/GRVGS) + 1.2541
)OG=1.922 + 0.00147*P**1.175
OL = 6.9E-5*OG**T*(80-OD)
AML = OOST*(0.00005*(GRVGS*CF*WGR) + 0.85E-07*GRVGS*RS)
DNSL=AML/OL
AML=AML*AMG
VISCGL = CEPT*(100./T)**SLP
R = 0.2 + 0.00110*(10.0001*RS)
B = 0.63 + 0.57*(10.0007*RS)
VISCAL=VISCGL*B
RETURN
END

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CUC 6600 FIN V3.0-P213 OPT=0 72/04/
SUBROUTINE REMLIO (OL,OG,DNSL,DMSG,DMSG1,
COMMON / ROUNDS
)ALSS, ALSE, ALEM, A, IDEM
C FLOW REGIME IDENTIFICATION ACCORDING TO FIG.8-9 IN BOYLM & WIL
A=0.785398*P**2
SGMA=0.162
VSL=OL/A
VSG=OG/A
T=(DNSL*SGMA/(42.4*SGMA))**0.25
A=T*(DMSG/D.008F)**0.33333
T=T*VSL
A=T*VSG
ALRS=0.51*100./T**0.177
IF (ALRS) 1,1,2
1 IDENT=1
GO TO 100
2 IF (T-1) 3,3,4
4 IF (T-26.5) 7,7,10
3 ALSF=T/0.263 *H
4 ALSF=T/0.263 *H
IF (ALSF) 2,7,8
7 IDENT=7
GO TO 100
5 ALFM=T*(100./T**T*(1-152)
IF (ALFM) 9,9,10
9 IDENT=9
GO TO 100
10 IDENT=10
130 RETURN
END

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CUC 6600 FIN V3.0-P213 OPT=0 72/04/
SUBROUTINE DENSLIO (OL,OG,DNSL,DMSG,VISCAL,VISCGL,
)BL,SL,DNSL,DMSG
COMMON / ROUNDS
)ALSS, ALSF, ALFM, F, IDENT
C CALCULATION OF IN-SITU DENSITY
C VB = BURBLE VELOCITY IN STAGNANT LIQUID
C VHF = BURBLE VELOCITY IN FLOWING LIQUID
C
SIGMA = 0.162
G = 0.2
A=0.785398*P**2
VSG = OG/A
VM = (OL + OG)/A
GO TO (1,2,3,4) IDENT
C BURBLE FLOW
1 CONTINUE
VB=ALSF*((SIGMA*H*(DNSL-DMSG))/(DNSL*0.51))**0.251
VBF = 1.25*VSG*VB
EG = VSG/VBF
EL = 1-EG
DMSFL=DNSL-EG*DMSG
GO TO 100
C SLUG FLOW
2 XNF=ALSF*((SIGMA*H*(DNSL-DMSG)*DNSL)/VISCAL
XNEO=G*(DNSL-DMSG)*10**7/SIGMA
IF (XNF = 1.5*10**7) GO TO 21
20 PNUM = 25
GO TO 25
21 IF (XNF = 25) 22,22,23
22 PNUM = 62/(XNF**0.35)
GO TO 24
23 PNUM = 10
24 AL = 0.0184/H**0.5
IF (AL,1,1-10) GO TO 210
C1 = 1-EXP(AL)
GO TO 210
200 C1 = 1
210 AZ = (3.31-XNEO)/2*PM
IF (22,1,1-10) GO TO 220

```

```

TIME DENS1 TRACE CDC 6600 FTH V3.0-P213 OPT=0 72/04/
C2 = 1. - EXP(A21)
GO TO 230
230 C2 = 1.
COEF = 0.345 * C2
VB = COEF * SQRT(G * D * (DMSL - DMSG) / DMSL)
VBF = 1.244 * VB
EG = VBF / VBF
EGSL = EG * 1.8
BL = (DMSL * (EGSL - EG) - 0.5261) / (EG - 0.913)
SL = 10 * D
DMSG = EGSL * DMSG + (1.8 - EGSL) * DMSL
DMSL = (DMSG * BL - DMSG * SL) / (BL - SL)
IF (IDENT - 3) 70, 71, 70
70 DMS = DMSL
GO TO 100
71 CONTINUE
C MIST FLOW
4 EG = 1. / (1. - QL / QG)
EL = 1. - EG
DMSMT = EL * DMSL - EG * DMSG
IF (IDENT - 1) 80, BL, 80
80 DMS = DMSMT
GO TO 100
C TRANSITION FLOW IS CALCULATED AS WEIGHTED AVERAGE VALUE OF SLUG AND
C MIST FLOW VALUES (DMSL AND DMSMT)
R1 DMS = (XLFM - X) * DMSL / (XLFM - XLSF) + (X - XLSF) * DMSMT / (XLFM - XLSF)
100 RETURN
END

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```

TIME FRIC TRACE CDC 6600 FTH V3.0-P213 OPT=0 72/04/
SUBROUTINE FRIC(1D, CL, DG, DMSL, DMSG, VISCL, VISC, SIGMA, XMSID, VH,
1 REL, REG, F, DMS, DMSG, SL, BL, TF)
COMMON / ROUNDS /
1 XLBS, XLSF, XLFM, X, IDENT
C CALCULATION OF THE FRICTION TERM
A = 0.78539 * D ** 2
SIGMA = 0.142
VH = (CL - 0.01) / A
GO TO (1, 2, 3, 4, 1), IDENT
C BUBBLE FLOW
1 REL = 1.488 * DMSL * D * VH / VISCL
CALL COLE(1REL, XMSID, F)
TF = (2.0 * F * VH * DMSL) / (32.2 * D)
GO TO 100
C SLUG FLOW
2 CONTINUE
REL = 1.488 * DMSL * D * VH / VISCL
CALL COLE(1REL, XMSID, F)
IFSI = (2.0 * F * VH * DMSG * SL) / (32.2 * D * (R1 - SL))
10 IF (IDENT - 1) 10, 4, 10
IF = TFSL
GO TO 100
C MIST FLOW
4 VSG = QG / A
REG = 1.488 * DMSG * D * VSG / VISC
C CALCULATE RELATIVE ROUGHNESS FOR MIST FLOW XD
XN = 4.52E-07 * ((VSG * VISCL / SIGMA) ** 2) * DMSG / DMSL
IF (XN = 0.105) 21, 21, 22
21 XD = 3.4 * SIGMA / (DMSG * D * VSG ** 2)
GO TO 23
22 XD = 17.4 * SIGMA * (XN ** 0.102) / (DMSG * D * VSG ** 2)
23 IF (XD - 0.001) 24, 25, 25
24 XD = 0.001
25 IF (XD - 0.51) 26, 26, 27
27 XD = 0.5
26 CALL COLE(1REG, XD, F)
TFMT = 2.1 * DMSG * VSG * VSG / (32.2 * D)
IF (IDENT - 1) 30, 31, 30
30 IF = TFMT
GO TO 100
C TRANSITION FLOW IS CALCULATED AS A WEIGHTED AVERAGE OF VALUES FOR
C SLUG AND MIST FLOW (TFSL AND TFMT)
31 TF = (XLFM - X) * TFSL / (XLFM - XLSF) + (X - XLSF) * TFMT / (XLFM - XLSF)
100 RETURN
END

```

```

TIME COLE TRACE CDC 6600 FTH V3.0-P213 OPT=0 72/04/
SUBROUTINE COLE(IP, XD, F)
C CALCULATION OF FRICTION FACTOR FROM COLEBROOK EQUATION USING
C NEWTON - RAPHSON ITERATION TECHNIQUE
C
DEFL / XD
C1 = 6.67 * DE / R
C2 = 1.73716 * ALOG(DE) - 2.28
FNI = (1.1 - 7.716 * ALOG(DE) - 2.281 * C2)
3 FUNC = C2 - 1.73716 * ALOG(1.1 - C1 / SQRT(F)) - 1. / SQRT(F)
DER = 6.666 * C1 / (F ** 1.5 - C1 * F) + 0.5 / (F ** 1.5)
FNF = FUNC / DER
IF (ABS(FNI - F) / FNI - 0.005) 1, 1, 1, 2
2 F = FNI
GO TO 3
1 F = FNI
RETURN
END

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TIME OUTPUT TRACE CDC 6600 FTH V3.0-P213 OPT=0 72/04/
SUBROUTINE OUTPUT (TF, DELZ, DMS, IDENT, EG, VBF, REL, REG, COEF,
1 DELP, ITER, ZST, DMSG, DMSL, VISC, VISCL, QG, QL, F, PST, NOWELL,
1 LINE)
DATA (BL, ISLUG, IFROTH, MIST/3HRL, 4HSLUG, 5HFROTH, 4HNMST/
DATA (SNGL /3HSL/
LINE = LINE + 1
IF (LINE - 67, 30) GO TO 55
GO TO 77
55 WRITE (6, 66) NOWELL
LINE = 0
CONTINUE
77 FRIC = TF * DELZ / 144.
HYDR = DMS * DELZ / 144.
GO TO (1, 2, 3, 4, 6), IDENT
1 IF (PST = 188)
REY = REL
GO TO 22
2 IF (PST = 15LUG)
REY = REL
GO TO 22
3 IF (PST = 188TH)
REY = REL
GO TO 11
4 IF (PST = MIST)
REY = REG
PKIN = (1. - COEF) * (1 - DELP)
GO TO 44
6 IF (PST = 15NGL)
REY = REL
EG = 0.
GO TO 33
11 CONTINUE
WRITE (6, 5555) (ITER, ZST, DMSG, DMSL, VISC, VISCL, QG, QL,
1REY, F, IF (PST = EG, DELZ, FRIC, HYDR, PKIN, PST)
GO TO 5
22 CONTINUE
WRITE (6, 6666) (ITER, ZST, DMSG, DMSL, VISC, VISCL, QG, QL,
1REY, F, IF (PST = EG, VBF, DELZ, FRIC, HYDR, PST)
GO TO 5
44 CONTINUE

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TIME OUTPUT TRACE CDC 6600 FTH V3.0-P213 OPT=0 72/04/
WRITE (6, 7777) (ITER, ZST, DMSG, DMSL, VISC, VISCL, QG, QL,
1REY, F, IF (PST = EG, DELZ, FRIC, HYDR, PKIN, PST)
GO TO 5
33 CONTINUE
WRITE (6, 8888) (ITER, ZST, DMSL, VISCL, QG, QL, REY, F, IF (PST =
1EG, DELZ, FRIC, HYDR, PST)
CONTINUE
5555 FORMAT (1H, 1), F8.1, F5.1, F6.1, F6.3, F7.3, IPE10.2, E10.2,
1OPE8.0, F5.3, 2X, A5, F5.3, A5, F7.0, IPE9.2, E10.2, OPE8.1)
6666 FORMAT (1H, 1), F8.1, F5.1, F6.1, F6.3, F7.3, IPE10.2, E10.2,
1OPE8.0, F5.3, 2X, A5, F5.3, F6.3, F7.0, IPE9.2, E10.2, OPE8.1)
7777 FORMAT (1H, 1), F8.1, F5.1, F6.1, F6.3, F7.3, IPE10.2, E10.2,
1OPE8.0, F5.3, 2X, A5, F5.3, A5, F7.0, IPE9.2, 2E10.2, OPE8.1)
8888 FORMAT (1H, 1), F8.1, 5X, F6.1, 7X, F6.3, 2X, F3.1, 5X, IPE10.2,
1OPE8.0, F5.3, 2X, A5, F5.1, A5, F7.0, IPE9.2, E10.2, 10X, OPE8.1)
66 FORMAT (1H), 10X, *WELL NO. *, 12, * CONTINUED*, //)
RETURN
END

```

DESCRIPTION OF INPUT CARDS AND SAMPLE DATA

Card	Format
1. Well number 27	12
2. Intercept and slope of temperature and dead oil viscosity 2, 1.72	2F10.3
3. Specific gravity of oil, gas and water 0.806 0.78 1.	3F10.5
4. Inside diameter of tubing (ft) relative roughness of tubing, surface tension of oil (lb./sec ²) 0.198 0.0008 0.066	3F10.5
5. Depth (ft) and temperature (°F) on the bottom and at the well head 12453. 192. 0. 100.	4F10.5
6. Production rate of oil (STBbl/day), gas-oil ratio (SCF/STBbl), water-oil ratio (STBbl/STBbl) 130. 572. 0.231	3F10.5
7. Pressure (psia) on the bottom and top of the well 4801. 425.	2F10.5

Sample Output

WELL NO	DEPTH FT	PRESSURE BOTTOM PSIA	TEMPERATURE ROT TOP DEGREE F	PRODUCTION RATE OIL STB/D	GAS MSCF/D	SPEC. GRAVITY OIL	GAS	DIAMETER OF WELL FT	WFL HOUGH F/D	SURFACE TENSION LBM/SEC	GAS-OIL RATIO SCF/STB	
27	12453.	4801.	192.	100.	130.	74.4	.806	.780	.198	.0008	.0060	572.

I	DEPTH FT	DENSITY GAS OIL LBM/CUFT	VISCOSITY GAS OIL CENTIPOISE	FLOW RATE GAS OIL CUFT/SEC	REYN. NO.	FRICTION FACT.	FLOW PATT.	VOL. FRAC. GAS	BURBLE VELOC. FT/SEC	DELTA FT	P R E S S U R E FRICTION PSI	S U R F A C E HYDRAULIC PSI	D R O P KINEMATIC PSI	PRESS. PSIA		
4	311.3	1.9	50.7	.011	1.441	2.01E-02	1.10E-02	10192.	.008	SLUG	.316	2.067	311.	1.06E-01	7.37E+01	498.8
3	822.6	2.3	50.4	.012	1.361	1.60E-02	1.11E-02	9609.	.008	SLUG	.272	1.904	311.	1.51E-01	7.79E+01	576.9
2	934.0	2.6	50.1	.012	1.250	1.25E-02	1.13E-02	9205.	.008	SLUG	.232	1.784	311.	1.26E-01	8.16E+01	658.6
2	1245.3	3.0	49.7	.012	1.145	1.02E-02	1.14E-02	8440.	.008	SLUG	.197	1.685	311.	1.06E-01	8.47E+01	743.4
2	1556.6	3.5	49.4	.012	1.056	8.10E-03	1.15E-02	8742.	.008	REL	.166	1.585	311.	9.53E-02	8.03E+01	833.7
2	1867.9	3.9	49.0	.012	.970	6.37E-03	1.17E-02	8721.	.008	REL	.136	1.522	311.	8.29E-02	8.26E+01	926.4
2	2179.3	4.4	48.6	.013	.893	4.94E-03	1.19E-02	8741.	.008	REL	.109	1.470	311.	7.37E-02	8.46E+01	1021.1
2	2490.6	4.9	48.2	.013	.823	3.76E-03	1.20E-02	8633.	.008	REL	.086	1.424	311.	6.55E-02	8.61E+01	1117.3
2	2801.9	5.4	47.7	.014	.761	2.77E-03	1.22E-02	8490.	.008	REL	.065	1.395	311.	5.94E-02	8.73E+01	1214.6
2	3113.2	5.9	47.3	.014	.705	1.95E-03	1.24E-02	8205.	.008	REL	.046	1.367	311.	5.46E-02	8.82E+01	1312.8
2	3424.6	6.4	46.9	.015	.655	1.25E-03	1.26E-02	8470.	.008	REL	.030	1.345	311.	5.07E-02	8.88E+01	1411.6
2	3735.9	6.9	46.5	.015	.609	6.42E-04	1.28E-02	9778.	.008	REL	.016	1.378	311.	4.75E-02	8.92E+01	1510.9
2	4047.2	7.4	46.1	.016	.569	1.19E-04	1.29E-02	10123.	.008	REL	.003	1.313	311.	4.49E-02	8.94E+01	1610.3
1	4358.5	45.9			.551	0.0	1.30E-02	10367.	.009	SFL	0.0		311.	4.42E-02	8.92E+01	1705.6
2	4669.9	45.9			.541	0.0	1.30E-02	10571.	.009	SFL	0.0		311.	4.41E-02	8.92E+01	1808.9
2	4981.2	45.8			.530	0.0	1.30E-02	10775.	.009	SFL	0.0		311.	4.39E-02	8.91E+01	1908.0
2	5292.5	45.8			.521	0.0	1.30E-02	10980.	.008	SFL	0.0		311.	4.37E-02	8.90E+01	2007.1
2	5603.8	45.7			.511	0.0	1.31E-02	11185.	.009	SFL	0.0		311.	4.36E-02	8.89E+01	2106.0
2	5915.2	45.7			.502	0.0	1.31E-02	11390.	.008	SFL	0.0		311.	4.35E-02	8.88E+01	2204.9
2	6226.5	45.7			.493	0.0	1.31E-02	11596.	.008	SFL	0.0		311.	4.33E-02	8.87E+01	2303.6
2	6537.8	45.6			.484	0.0	1.31E-02	11803.	.008	SFL	0.0		311.	4.32E-02	8.86E+01	2402.2
2	6849.1	45.6			.476	0.0	1.31E-02	12005.	.009	SFL	0.0		311.	4.31E-02	8.85E+01	2500.8
2	7160.5	45.5			.468	0.0	1.31E-02	12217.	.008	SFL	0.0		311.	4.29E-02	8.84E+01	2599.2
2	7471.8	45.5			.460	0.0	1.31E-02	12424.	.008	SFL	0.0		311.	4.28E-02	8.83E+01	2697.6
2	7783.1	45.4			.452	0.0	1.32E-02	12632.	.008	SFL	0.0		311.	4.27E-02	8.82E+01	2795.4
1	8094.4	45.4			.445	0.0	1.32E-02	12840.	.008	SFL	0.0		311.	4.26E-02	8.81E+01	2894.0
1	8405.8	45.3			.438	0.0	1.32E-02	13049.	.008	SFL	0.0		311.	4.25E-02	8.80E+01	2992.0
1	8717.1	45.3			.431	0.0	1.32E-02	13257.	.008	SFL	0.0		311.	4.24E-02	8.79E+01	3089.9
1	9028.4	45.2			.424	0.0	1.32E-02	13467.	.007	SFL	0.0		311.	4.23E-02	8.78E+01	3187.8
1	9339.7	45.2			.418	0.0	1.32E-02	13676.	.007	SFL	0.0		311.	4.22E-02	8.77E+01	3285.5
1	9651.1	45.1			.412	0.0	1.32E-02	13886.	.007	SFL	0.0		311.	4.21E-02	8.76E+01	3383.2
1	9962.4	45.1			.405	0.0	1.32E-02	14097.	.007	SFL	0.0		311.	4.20E-02	8.75E+01	3480.7
1	10273.7	45.1			.399	0.0	1.33E-02	14307.	.007	SFL	0.0		311.	4.19E-02	8.74E+01	3578.2
1	10585.0	45.0			.394	0.0	1.33E-02	14518.	.007	SFL	0.0		311.	4.18E-02	8.73E+01	3675.5
1	10896.4	45.0			.388	0.0	1.33E-02	14729.	.007	SFL	0.0		311.	4.17E-02	8.72E+01	3772.8
1	11207.7	44.9			.383	0.0	1.33E-02	14941.	.007	SFL	0.0		311.	4.16E-02	8.71E+01	3869.9
1	11519.0	44.9			.377	0.0	1.33E-02	15153.	.007	SFL	0.0		311.	4.15E-02	8.70E+01	3967.0
1	11830.3	44.8			.372	0.0	1.33E-02	15365.	.007	SFL	0.0		311.	4.14E-02	8.69E+01	4063.9
1	12141.7	44.8			.367	0.0	1.33E-02	15578.	.007	SFL	0.0		311.	4.14E-02	8.68E+01	4160.8
1	12453.0	44.7			.362	0.0	1.34E-02	15791.	.007	SFL	0.0		311.	4.13E-02	8.67E+01	4257.6

NOTE REYNOLDS NUMBER REFERS TO GAS, REG. IN MIST FLOW AND TO LIQUID, WEL. ELSEWHERE

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