A Study of Two-Phase Flow in Inclined Pipes

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Introduction

The prediction of pressure drop and liquid holdup occurring during two-phase gas-liquid flow in pipes is of particular interest to the petroleum, chemical, and nuclear industries. In the nuclear industry, two-phase flow occurs in reactor cooling equipment, and liquid holdup greatly affects heat transfer. Two-phase flow occurs frequently in chemical processing, and the design of processing equipment and piping systems requires knowledge of pressure drop, liquid holdup, and often flow pattern. In the petroleum industry, two-phase flow occurs in pipelines and in oil and gas wells. More than one-half the natural gas gathered in the U. S. flows in two-phase flowlines. Most gas-producing wells produce some liquid and most oil wells produce some gas. As the natural reservoir energy is depleted, many wells are equipped with artificial-lift systems such as gas lift. To design these systems, a method of predicting two-phase-flow pressure gradients is required.

Although extensive research in two-phase flow has been conducted during the last 25 years, most of this research has concentrated on either horizontal or vertical flow. Several good correlations exist for predicting pressure drop and liquid holdup in either horizontal or vertical flow, but these correlations have not been successful when applied to inclined flow. Many gathering lines and long-distance pipelines pass through areas of hilly terrain. This presents no problem in single-phase flow because the potential energy lost going uphill is regained in the downhill section. This is not the case for two-phase flow, because the liquid holdup, and thus the mixture density, are usually much lower in downhill flow. For this reason, pressure recovery in the downhill sections is usually neglected in the design of two-phase pipelines.

The number of directional or inclined wells is increasing as the search for petroleum moves into previously unexplored areas. In offshore drilling, several directional wells are usually drilled from one platform for economic reasons. Deviations of 35° to 45° from the vertical are common. In the permafrost areas of Alaska and Canada, the cost of drilling-rig foundations and the difficulty of transportation require that several wells be directionally drilled from one location. Existing vertical-flow correlations frequently fail to predict pressure gradients in these wells within acceptable limits.

Gathering lines from offshore wells usually are laid along the sea floor that slopes up to the shore. The elevation pressure gradient in a pipeline with a very small upward inclination from horizontal can be much greater than the frictional pressure gradient. Therefore, in order to predict pressure drop, the liquid holdup must be accurately predicted. The ability to predict liquid holdup also is essential for designing field processing equipment, such as gas-liquid separators. When the flow rates of wells pro-

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Gas-liquid flow in inclined pipes was investigated to determine the effect of pipe inclination angle on liquid holdup and pressure loss. Correlations for liquid holdup and friction factor were developed for predicting pressure gradients for two-phase flow in pipes at all angles for many flow conditions.

ducing into gathering lines are changed, the amount of liquid holdup in the lines may change and result in overloading of processing equipment.

The two-phase flow problem is complicated by such phenomena as slippage between phases, change of flow pattern, and mass transfer between phases. The gas-liquid interface may be smooth or wavy and energy may be transferred between phases. These factors result in a much greater pressure loss than can be explained by the reduced area available to flow for each phase. When angle of flow is added to such variables as fluid properties, flow rates, and pipe diameter, the problem is indeed formidable.

Very little work has been published on the effect of inclination on two-phase flow in pipes. Pressure gradients in directional wells are usually calculated using a vertical flow correlation such as that of Hagedorn and Brown[7] or of Orkiszewski.[12] This is satisfactory if the well is inclined only slightly from vertical. Pressure gradients in pipelines may be calculated using a horizontal-flow correlation such as that of Dukler *et al.*[3] or Eaton.[4] The pressure drop caused by elevation change may be calculated if liquid holdup can be determined.

In 1958, Flanigan[5] proposed a method to calculate two-phase pressure drop in pipelines in hilly terrain. The method was developed using field data obtained from a 16-in. pipeline and the data of Baker.[1] Flanigan used the Panhandle equation for gas flow with an efficiency factor to calculate the frictional pressure drop. The elevation pressure drop was calculated using an elevation factor, the liquid density, and a summation of the uphill elevation changes. Flanagan concluded that the inclination angle of the hills had no effect on the elevation factor and that there was no pressure recovery on the downhill side of the hill. A correlation was given for elevation factor as a function of superficial gas velocity. This correlation includes, to some extent, the effect of pressure recovery on the downhill side, since the over-all pressure drop was used in developing the correlation. A correlation for efficiency factor as a function of superficial gas velocity and gas-liquid ratio was given also.

In 1962, Sevigny[14] studied the flow of air and water mixtures through a 0.8245-in.-diameter pipe inclined at angles of plus and minus 90°, 60°, 30°, 15°, 10°, and 5° from horizontal, and at 0°. Holdup was not measured, and the pressure drop not accounted for by elevation change was included in a friction-loss term. The no-slip or input density was used to calculate elevation pressure drop, in a method similar to that used by Poettmann and Carpenter[13] in their study of vertical flow. Sevigny presented a correlation for two-phase friction factor as a function of input liquid content, gas Reynolds number, and liquid Reynolds number. The accuracy of this correlation is very questionable. For some conditions, the method gives a pressure recovery of more than 100 percent in downhill flow, which is clearly impossible.

In 1967, Guzhov *et al.*[6] presented results of a study of inclined two-phase flow. Their paper included much of the information that was reported earlier by Mamayev[9] in 1965 and Odishariya[11] in 1966. Their data were obtained using 2-in. pipe at angles between +9° and −9° from horizontal. Liquid holdup was found to be a function of Froude number and input gas content. An equation was given for holdup in uphill plug flow that was independent of angle, but the equations for downhill flow included an inclination effect. A correlation was given for the ratio of two-phase friction factor to single-phase friction factor as a function of gas-input content, with Froude number as a parameter. Their friction factor correlation should be used with caution since the friction factor ratio becomes unbounded as the flow approaches all gas.

A study of slug flow in inclined pipes was reported by Singh and Griffith[15] in 1970. They measured pressure drop and liquid holdup in pipe with diameters of 0.626, 0.822, 1.063, 1.368, and 1.600 in., at inclination angles of plus and minus 10° and 5° from horizontal, and at 0°. An expression was derived for bubble-rise velocity from which liquid holdup could be calculated. Liquid holdup was found to be independent of inclination angle, which would result in 100-percent calculated pressure recovery in the downhill section of an inclined pipe. Frictional pressure drop was calculated using the Fanning friction factor, the liquid density, and the mixture velocity. Acceleration pressure drop was neglected.

After the literature survey was completed, it was concluded that no correlation exists for predicting the liquid holdup and pressure drop that occur during two-phase flow in pipes at all angles. The purpose of this study, then, was to find a correlation of this type that would be particularly applicable in designing pipelines for hilly terrain and tubing strings for inclined wells.

Theory

The equation used to calculate pressure gradient when gas or liquid, or both, flow in a pipe is

$$\frac{-dp}{dZ} = \frac{\dfrac{g}{g_c}\sin\theta\,[\rho_L H_L + \rho_g(1-H_L)] + \dfrac{f_{tp} G_m v_m}{2 g_c d}}{1 - \dfrac{[\rho_L H_L + \rho_g(1-H_L)]\,v_m\,v_{sg}}{g_c p}} \quad . \quad \ldots \quad (1)$$

The development of this equation is found in the Appendix. Eq. 1 reduces to the equation for single-phase liquid or single-phase gas flow as $H_L \rightarrow 1$ or $H_L \rightarrow 0$, respectively. Also, as the angle of the pipe, θ, becomes zero, +90°, or −90°, Eq. 1 becomes applicable to horizontal or vertical flow.

Eq. 1 contains two unknowns: H_L, which must be determined to calculate the in-situ density, and f_{tp}, which is used to calculate friction losses. The purpose of this study was to develop correlations for predicting H_L and f_{tp} from fluid and system properties that are known. To accomplish this, an experimental apparatus was designed and built so that flow rates, pressure gradient, inclination angle, and liquid holdup could be measured.

Experimental Study

The data from which the correlations were developed were taken in transparent acrylic pipes 90 ft long. Sufficient length was provided between the gas-liquid mixing point and the test section to eliminate entrance effects before any measurements were made.

The parameters studied and their range of variation were (1) gas flow rate (0 to 300 Mscf/D); (2) liquid flow rate (0 to 30 gal/min); (3) average system pressure (35 to 95 psia); (4) pipe diameter (1 and 1.5 in.); (5) liquid holdup (0 to 0.870); (6) pressure gradient (0 to 0.800 psi/ft); (7) inclination angle ($-90°$ to $+90°$); and (8) flow pattern.

Fluids used were air and water. For each pipe size, liquid and gas rates were varied so that all flow patterns were observed when the pipe was horizontal. After a particular set of flow rates was set, the angle of the pipe was varied through the range of angles so that the effect of angle on holdup and pressure drop could be observed. Liquid holdup and pressure drop were measured at angles of plus and minus 90°, 85°, 75°, 55°, 35°, 20°, 15°, 10°, and 5° from horizontal, and at 0°. In all, 584 two-phase flow tests were run. Data for all the tests are in Ref. 2.

Liquid System

A schematic diagram of the test system is shown in Fig. 1. The liquid was stored in a 100-cu-ft tank equipped with a heat exchanger. The gas was passed through this heat exchanger to equalize the gas and liquid temperatures. The temperature of the two-phase mixture was measured at each end of the test section. Liquid flow was provided by a Pacific centrifugal pump with an output capacity of 60 gal/min at 95 psig. The liquid flow rate was controlled by a choke valve upstream of the meters. After passing through the flow system and separator, the liquid was returned to the tank and recirculated.

The liquid-metering system consisted of a Foxboro magnetic flowmeter and two rotameters. The magnetic flowmeter was equipped with a Dynalog recorder and had two ranges, 0 to 30 and 0 to 100 gal/min. For low flow rates, Brooks rotameters with ranges of 0 to 1 and 0 to 5 gal/min were used. The meters were calibrated frequently, using a calibration tank.

Gas System

Compressed air was provided by a Clark Model HO-6-4C four-stage, reciprocating compressor equipped with a tank having a volume of 20 cu ft. The output

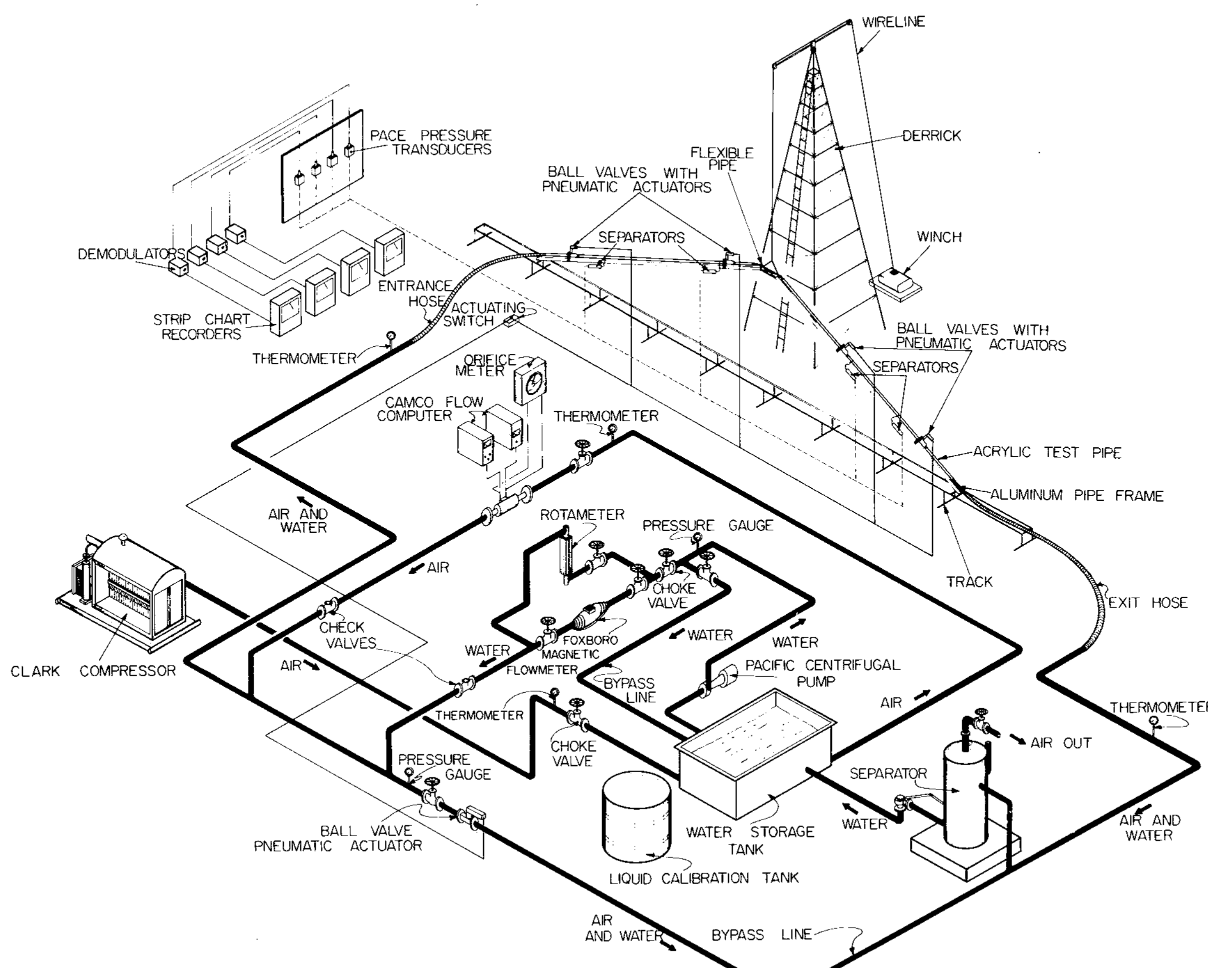

Fig. 1—Schematic diagram of test system.

capacity of the compressor was 150 Mscf/D at 3,500 psig. For the high-gas-rate tests, the volume tank was charged to a high pressure and the air was passed through a regulator to provide the necessary flow capacity. The gas flow rate was controlled by a choke valve upstream of the heat exchanger. After passing through the flow system and separator, the air was vented to the atmosphere.

An orifice meter and a Camco flow computer were used to measure the gas rate. Orifice plates were easily changed and the optimum size was used for each flow rate. The meters were calibrated with a water manometer and dead-weight tester.

Liquid-Holdup Measurement

Liquid holdup was measured with pneumatically actuated, quick-closing ball valves. This same type of system was used by Eaton[4] and by Singh and Griffith.[15] Two valves were used in the uphill section and two in the downhill section. A similar valve was placed in the bypass line. The valves were full-opening ball valves, with inside diameters equal to the inside diameter of the pipe, so that the flow stream was not disturbed by passing through the open valves.

The valves were opened and closed by Ramcon rotary pneumatic actuators. Air pressure was admitted to the actuator chamber through a solenoid valve and the five valves could be actuated simultaneously by a single switch. Thus the holdup could be measured without disturbing the gas and liquid flow rate settings. After the valves were closed, the amount of liquid trapped between the valves was measured to determine holdup.

Pressure-Drop Measurement

Four pressure taps (see Fig. 1) were located in the test section, two on the uphill side and two on the downhill side. Pressure was transmitted through plastic tubing to pressure transducers located in an instrument house. Small separators were provided at each pressure tap to keep liquid from entering the pressure lines. The pressure taps were numbered P_1, P_2, P_3, and P_4. Absolute pressure was measured at P_1, and differential pressures were measured at P_1-P_2, P_2-P_3, P_3-P_4, and P_1-P_4.

Pressure drop was measured with variable reluctance pressure transducers. The output of these transducers was fed to demodulators for amplification and then to strip-chart recorders. Diaphragms in the transducers could be changed to provide a differential pressure range of 0 to 1 psi and 0 to 50 psi. The pressure-measuring system was designed so that the transducers could be calibrated with a manometer before each test.

Transducers rather than manometers were used to measure pressure drops. Because of the fluctuating nature of two-phase flow, a recording of the pressure drop was made so that an average value could be determined.

Test Section

The test section frame consisted of two 45-ft sections of aluminum pipe hinged at the center. Two 45-ft sections of acrylic pipe, through which the two-phase mixture flowed, were mounted on this frame and were joined together by a section of flexible pipe of the same ID. The aluminum frame was mounted on a track so that when the center of the frame was hoisted by an electrically powered winch to change the angle of inclination, each end of the frame was free to slide on the track. A wire line was strung over a 50-ft derrick and the flow system could be raised to any angle between horizontal and $+90°$ or $-90°$.

Testing Procedure

The testing procedure, designed to emphasize the effect of inclination change on holdup and pressure drop, consisted of the following steps:

1. The gas rate was set with a choke valve and was monitored with the Camco flow computer and the differential pressure recorder.
2. The liquid rate was set with a choke valve and bypass valve.
3. After steady-state conditions were reached, the pressure recording system was activated and the various pressures and pressure drops were recorded.
4. After pressures were recorded for a sufficient length of time and all flow rates and temperatures were recorded, the holdup valves were actuated and the holdup was recorded for both the uphill and downhill sections. This step was repeated at least 10 times and the holdup readings were averaged. By using the bypass systems, the flow rate settings were not disturbed by this step.
5. The angle of the pipe was changed and Steps 1 through 4 were repeated for each angle tested.
6. The gas rate was changed and Steps 2 through 5 were repeated.
7. The liquid rate was changed and Steps 1 through 6 were repeated.

Each test took approximately 45 minutes, depending on the time required to reach steady state. Tests were first conducted with 1-in.-ID pipe. The pipe was then changed to 1.5-in. pipe, and tests were repeated at the same mass flux rates. A total of 584 average liquid-holdup and pressure-drop measurements were obtained.

Development of Correlations

The development of Eq. 1 revealed that correlations must be devised for two parameters in order to calculate pressure gradients in two-phase, inclined flow. These parameters are liquid holdup, H_L, and two-phase friction factor, f_{tp}.

Liquid-Holdup Correlation

In plotting liquid holdup vs inclination angle for constant flow rates, it was discovered that holdup has a definite dependency on angle, as can be seen in Fig. 2. The reversal of these curves at approximately 50°, plus and minus, from horizontal was unexpected. This phenomenon may be explained by considering the effects of gravity and viscosity on the liquid phase. As the angle of the pipe is increased from horizontal, gravity forces acting on the liquid cause a decrease in liquid velocity, thus increasing slippage and liquid holdup. As the angle is increased further, the liquid eventually bridges the entire pipe, reducing slippage between phases and thus decreasing holdup. It should be noted that segregated flow was not observed at any angle greater than + 3° from horizontal. In downhill flow, in which the flow pattern is almost always segregated, an increase in angle in the negative direction results in increased liquid velocity and decreased holdup. As the angle is further increased in the negative direction, the flow pattern changes to semiannular and, finally, to annular. More of the liquid is then in contact with the pipe surface, and viscous drag causes a decrease in liquid velocity and an increase in liquid holdup.

These curves revealed also that the degree of holdup change with angle was different for different flow rates. Many attempts were made to find a functional relationship between holdup and flow rates, pipe size, angle, and other variables. It was finally decided to normalize the holdup by dividing the holdup at any angle by the holdup at 0°. Thus,

$$\frac{H_L(\theta)}{H_L(0)} = \psi , \quad \ldots \ldots \ldots \ldots \quad (2)$$

where

$H_L(\theta)$ = holdup at angle θ,
$H_L(0)$ = horizontal holdup,
ψ = inclination correction factor.

A plot of ψ vs θ for several flow conditions is shown in Fig. 3.

To correlate horizontal holdup, a multiple, linear, stepwise regression analysis was performed, with holdup as the dependent variable and with the following independent variables: liquid velocity number, N_{Lv}; gas velocity number, N_{gv}; diameter number, N_D; Reynolds number, N_{Re}; Froude number, N_{FR}; pressure ratio, p/p_a; gas-liquid ratio, q_g/q_L; and input liquid content, λ.

This analysis revealed that the most significant independent variables were Froude number and input liquid content. These same two variables were used by Guzhov *et al.*[6] and by Nezhilskii and Khodanovich.[10] Various forms of the Reynolds number, including those of Dukler *et al.*,[3] Hagedorn and Brown,[7] and Hughmark,[8] were tried, but none proved to be significant. This may possibly be explained by the fact that both liquid and gas phases were in turbulent flow in all tests. Therefore viscous effects were negligible. The use of only two fluids — air and water — could also tend to mask viscous effects. An equation was obtained of the form,

$$H_L(0) = A\ \lambda^{\alpha}\ N_{\mathrm{FR}}^{\beta} , \quad \ldots \ldots \quad (3)$$

which predicted the experimental data farily well except at low values of holdup. This equation was independent of flow pattern, but it was obvious that it would not apply to all flow conditions.

At this point, efforts were directed to correlating the inclination correction factor, ψ. A study of the graphs of ψ vs θ revealed that the curves were all of the same general shape and reached a maximum or minimum at approximately the same angle, plus and minus 50° from horizontal. A least-squares curve fit was applied to each curve, using a polynominal in the sine of a multiple of angle; that is,

$$\psi = \sum_{i=0}^{n} B_i \sin^i (\gamma\theta) \quad \ldots \ldots \quad (4)$$

It was found that ψ could be predicted for all flow conditions by an equation of the form

$$\psi = 1 + B_1 \sin(1.8\theta) + B_3 \sin^3(1.8\theta) ,$$

or

$$\psi = 1 + C(\sin\phi - \tfrac{1}{3}\sin^3\phi) , \quad \ldots \quad (5)$$

where $\phi = 1.8\theta$. For uphill flow,

$$C_+ = (\psi_{\max} - 1)\ 1.5 , \quad \ldots \ldots \quad (6)$$

and for downhill flow,

$$C_- = (1 - \psi_{\min})\ 1.5 , \quad \ldots \ldots \quad (7)$$

where $\psi_{\max}$ and $\psi_{\min}$ are the maximum and minimum values of ψ for a particular set of flow conditions.

To develop a method for predicting values of C, it was necessary to divide the tests into three regimes on the basis of the flow pattern of the test when the pipe was in the horizontal position. If the horizontal flow pattern was mist, bubble, or froth, the change in holdup with inclination was insignificant.

The tests were divided into the three flow regimes described by Dukler *et al.*[3] These are segregated flow, intermittent flow, and distributed flow (see Fig. 4). Regression analysis revealed that the C values could be correlated with input liquid content (λ), Froude number (N_{FR}), and liquid velocity number (N_{LV}). Dif-

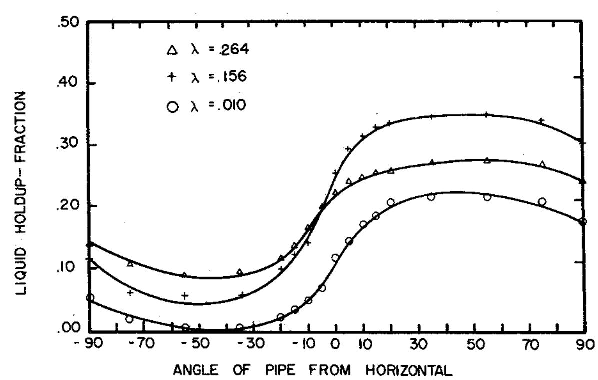

Fig. 2—Liquid holdup vs angle.

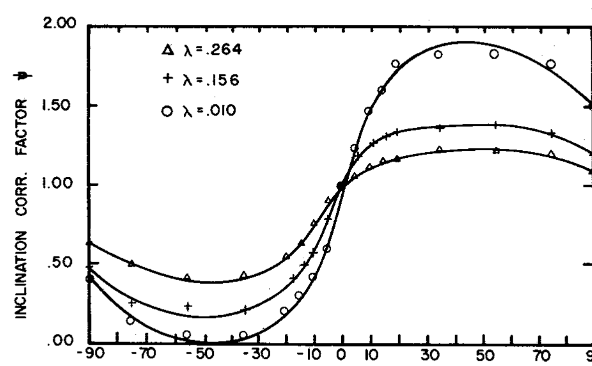

Fig. 3—Inclination correction factor vs angle.

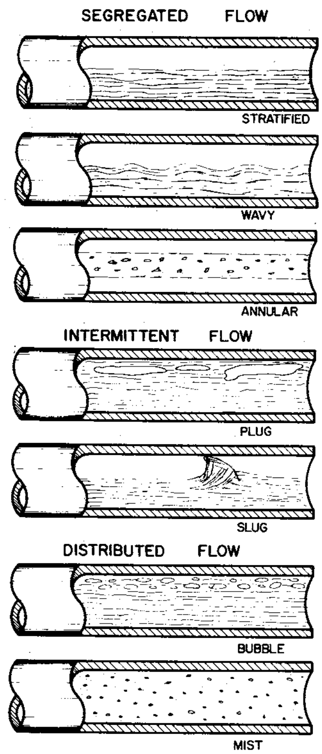

Fig. 4—Horizontal flow patterns.

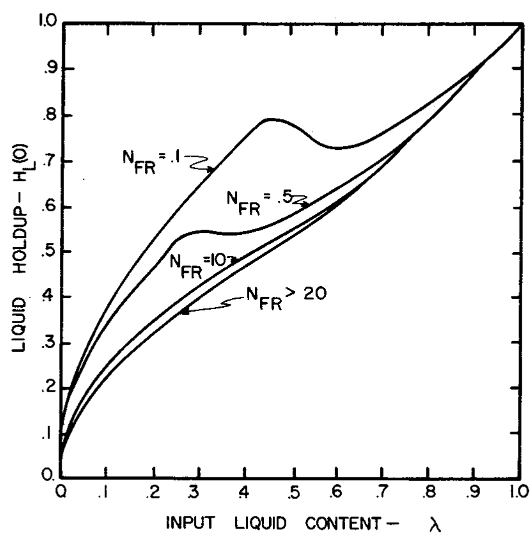

Fig. 5—Horizontal liquid holdup vs input liquid content.

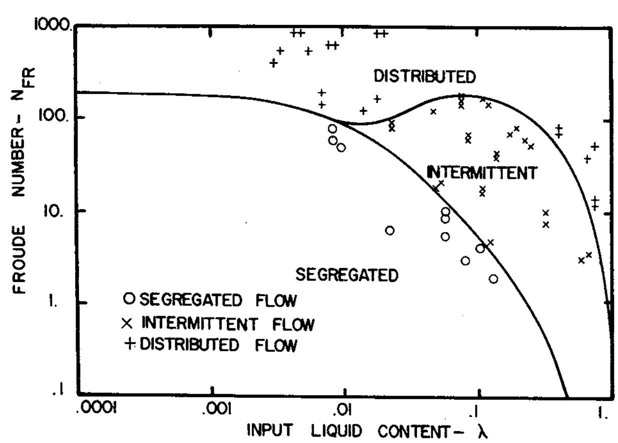

Fig. 6—Flow-pattern map — horizontal flow.

ferent equations were obtained for segregated, intermittent, and distributed flow regimes in uphill flow; but it was found that one equation could predict C for all downhill flow. This may be explained by the fact that the flow regime changes from intermittent to segregated as the flow stream breaks over the hill. The equations obtained were of the form

$$C = (1 - \lambda) \ln(D\, \lambda^{\delta}\, N_{FR}{}^{\varepsilon}\, N_{Lv}{}^{\zeta}) ,$$

where D, δ, ε, and ζ are different for the various flow conditions. As was stated earlier, for uphill distributed flow, holdup was not a function of angle, and therefore C is zero for this condition.

When it was found necessary to resort to flow regimes to correlate the inclination correction factor, the correlation for horizontal holdup was reexamined. A different equation of the form of Eq. 3 was obtained for each of the three flow regimes. This improved considerably the accuracy of the prediction for low holdup values. A plot of horizontal liquid holdup vs input liquid content with Froude number as a parameter is shown in Fig. 5. The sharp breaks in the curves for $N_{FR} = 0.1$ and $N_{FR} = 0.5$ indicate a change in flow pattern from segregated to intermittent. One of the problems caused by using different equations for different flow regimes is that a discontinuity appears in a curve of holdup vs input liquid content at low values of Froude number. Although the holdup does decrease greatly as the flow changes from segregated to intermittent, the change is not so rapid as is indicated by the equations. This situation could possibly be eliminated by defining a transition zone between flow regimes.

Since different holdup correlations are used for different horizontal flow regimes, it was necessary to develop a method to predict flow regime in horizontal two-phase flow. When the Froude numbers were plotted vs input liquid content on log-log paper the tests fell into groups in different areas. These areas are shown on the flow pattern map (Fig. 6). Because of its simplicity, this map may or may not strictly define the flow pattern into which a given flow condition will fall, but it does determine the equations to be used in predicting liquid holdup using the present correlation. Equations were fitted to the lines of Fig. 6 so that the flow pattern could be determined without referring to the graph. The flow pattern may be determined as follows.

1. If $N_{FR} < L_1$, the flow pattern is segregated.
2. If $N_{FR} > L_1$ and $> L_2$, the flow pattern is distributed.
3. If $L_1 < N_{FR} < L_2$, the flow pattern is intermittent.
4. L_1 and L_2 are given by

$$L_1 = \exp(-4.62 - 3.757X - 0.481X^2 - 0.0207X^3) \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and

$$L_2 = \exp(1.061 - 4.602X - 1.609X^2 - 0.179X^3 + 0.635 \times 10^{-3}X^5) ,$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where

$$X = \ln(\lambda)\,.$$

Equations have now been developed to predict liquid holdup in two-phase flow for all conditions. These equations are given in Table 1. After the values for $H_L(0)$ and C are determined, holdup at any angle is calculated from

$$H_L(\theta) = H_L(0)\,\{1 + C[\sin(1.8\theta) - \tfrac{1}{3}\sin^3(1.8\theta)]\}, \quad . \; . \; . \; . \; . \; . \quad (11)$$

with the restrictions that $H_L(0) \geq \lambda$ and $0 \leq H_L(\theta) \leq 1$.

Friction-Factor Correlation

Values for the two-phase friction factor, f_{tp}, were obtained by solving the pressure gradient equation, Eq. 1, for f_{tp}:

$$f_{tp} = \left[\frac{dp}{dZ}\left(1 - \frac{\rho_{tp} v_m v_{sg}}{g_c p}\right) - \frac{g}{g_c}\sin\theta\,\rho_{tp}\right]\frac{2 g_c d}{G_m v_m} \quad . \; . \; . \quad (12)$$

The two-phase friction factor was normalized by dividing it by a no-slip friction factor, f_{ns}, which would apply if the fluids were flowing at the same velocity. The no-slip friction factor is obtained from a Moody diagram or, for smooth pipe, from

$$f_{ns} = \left[2\log\left(\frac{N_{\mathrm{Re}ns}}{4.5223\log N_{\mathrm{Re}ns} - 3.8215}\right)\right]^{-2}, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (13)$$

where

$$N_{\mathrm{Re}ns} = \frac{[\rho_L\lambda + \rho_g(1-\lambda)]v_m d}{\mu_L\lambda + \mu_g(1-\lambda)},$$

or

$$N_{\mathrm{Re}ns} = \frac{G_m d}{\mu_L\lambda + \mu_g(1-\lambda)} \quad . \; . \; . \; . \; . \; . \quad (14)$$

This Reynolds number approaches the Reynolds number for liquid or gas as λ approaches 1 or zero, respectively.

The normalized friction factor was found to be a function of input liquid content, λ, and liquid holdup, $H_L(\theta)$. It is possible that the dependence on holdup could have been eliminated by including holdup in the Reynolds number, but after several forms of Reynolds number were tried with little success, this approach was abandoned. Regression analysis with normalized friction factor as the dependent variable and λ and $H_L(\theta)$ as independent variables indicated a relationship of the type

$$\frac{f_{tp}}{f_{ns}} = f\left\{\frac{\lambda}{[H_L(\theta)]^2}\right\}. \quad . \; . \; . \; . \; . \; . \; . \; . \quad (15)$$

The equation for friction factor is

$$\frac{f_{tp}}{f_{ns}} = e^S, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (16)$$

where

$$S = [\ln(y)]/\{-0.0523 + 3.182\ln(y) - 0.8725\,[\ln(y)]^2 + 0.01853\,[\ln(y)]^4\},$$

and

$$y = \frac{\lambda}{[H_L(\theta)]^2}\,.$$

Eq. 16 becomes unbounded at a point in the interval $1 < y < 1.2$; and for y in this interval, the function S is calculated from

$$S = \ln(2.2\,y - 1.2).$$

As the flow approaches all gas,

$$\lambda \to 0, \qquad S \to 0, \qquad \text{and} \qquad f_{tp} \to f_{ns} \to f_{sp}.$$

As the flow approaches all liquid,

$$\frac{\lambda}{H_L^2} \to 1, \qquad S \to 0, \qquad \text{and} \qquad f_{tp} \to f_{ns} \to f_{sp}.$$

A plot of the normalized friction factor vs input liquid content, with holdup as a parameter, is shown in Fig. 7.

The correlations for liquid holdup and friction factor were developed using dimensionless variables. Both the holdup and friction factor correlations degenerate to single-phase conditions as the flow approaches all liquid or all gas. As the correlations were developed from data obtained using only two fluids — water and air — and two pipe sizes — 1

TABLE 1—EQUATIONS FOR PREDICTING LIQUID HOLDUP

Horizontal Flow Pattern	Horizontal Holdup	C+	C−
Segregated	$H_L(0) = \frac{0.98\lambda^{0.4846}}{N_{FR}^{0.0868}}$ (Eq. 19)	$C+ = (1-\lambda)\ln\left[\frac{0.011\,N_{LV}^{3.539}}{\lambda^{3.768}\,N_{FR}^{1.614}}\right]$ (Eq. 22)	$C- = (1-\lambda)\ln\left[\frac{4.7\,N_{LV}^{0.1244}}{\lambda^{0.3692}\,N_{FR}^{0.5056}}\right]$ (Eq. 25)
Intermittent	$H_L(0) = \frac{0.845\lambda^{0.5351}}{N_{FR}^{0.0173}}$ (Eq. 20)	$C+ = (1-\lambda)\ln\left[\frac{2.96\lambda^{0.305}\,N_{FR}^{0.0978}}{N_{LV}^{0.4473}}\right]$ (Eq. 23)	Same as Segregated (Eq. 25)
Distributed	$H_L(0) = \frac{1.065\lambda^{0.5824}}{N_{FR}^{0.0609}}$ (Eq. 21)	$C+ = 0$ (Eq. 24)	Same as Segregated (Eq 25)

and 1.5 in. — extrapolation to larger pipes and different fluids will have to be tested; but it has definitely been established that pipe inclination has a considerable effect on liquid holdup in two-phase flow.

Discussion and Application of Results

The foregoing correlations for calculating liquid holdup and pressure gradients occurring in two-phase flow are applicable to many situations in the petroleum and chemical industries. Designing well completions and gas-lift systems in directionally drilled wells requires a knowledge of the effect of inclination on two-phase flow. For example, the present work indicates that the pressure gradient occurring in a well inclined at 15° to 20° from vertical can be greater than the pressure gradient in a vertical well.

One of the principal reasons for undertaking this study was to develop a method for predicting the pressure drop occurring in two-phase pipelines constructed in hilly terrain. The degree of pressure recovery in downhill two-phase flow has been a subject of conjecture for many years. Most previous methods, such as that of Flanigan,[5] have assumed that there is no pressure recovery in downhill two-phase flow. The study dealt with here indicates that this is true in some cases, but that under many flow conditions the pressure recovery is considerable and must be taken into account in designing pipelines.

Calculation Procedure

The pressure gradient equation presented here is relatively simple to apply and involves no trial-and-error solutions for isothermal flow. If temperature, and therefore z-factor, is a function of distance, a trial-and-error solution is necessary. Equations were developed for all unknowns so that referring to graphs is unnecessary, making the procedure easily programmable for the computer.

The pressure gradient equation may be used to calculate a pressure traverse in two ways. In the first method, a pressure decrement is chosen; and if temperature is a function of distance, a distance increment is estimated. The equation is solved for the distance increment corresponding to this pressure decrement.

$$\Delta Z = \Delta p \left[\frac{1 - \dfrac{\bar{\rho}_{tp}\bar{v}_m\bar{v}_{sg}}{g_c\bar{p}}}{\dfrac{g}{g_c}\sin\theta\bar{\rho}_{tp} + \dfrac{f_{tp}\bar{G}_m\bar{v}_m}{2\,g_c d}} \right] \quad . \quad . \quad (17)$$

When this method is used, all the barred variables in Eq. 17 must be evaluated at the average pressure and temperature. If the calculated distance increment is not close enough to the estimated value, a new value is assumed and the calculation is repeated until the two values agree. The accuracy of the calculation depends on the size of the pressure decrement.

The second method, which would be applicable for isothermal flow at a constant inclination angle, consists of solving the equation for pressure gradient at several pressure values, plotting the reciprocal of pressure gradient vs pressure, and integrating to obtain ΔZ; that is,

$$\Delta Z = \int_{p_1}^{p_2} \frac{dZ}{dp}\, dp \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The procedure to calculate the pressure gradient at a given pressure and temperature is outlined as follows.

1. Calculate ρ_L, ρ_g, v_{sL}, v_{sg}, v_m, G_m, λ, N_{FR}, $N_{Re_{ns}}$, and N_{Lv} at p and T.
2. Calculate L_1 and L_2 using Eqs. 9 and 10.
3. Determine the flow pattern, using the following criteria: (a) if $N_{FR} < L_1$, segregated; (b) if $N_{FR} > L_1$ and $> L_2$, distributed; and (c) if $L_1 < N_{FR} < L_2$, intermittent.
4. Calculate $H_L(0)$ using Eqs. 19 and 20 or Eq. 21, Table 1.
5. Calculate C using Eqs. 22, 23, 24, or 25, Table 1.
6. Calculate $\phi = 1.8\,\theta$ and $\psi = 1 + C(\sin\phi - \frac{1}{3}\sin^3\phi)$.
7. Calculate $H_L(\theta) = H_L(0)\,\psi$, and ρ_{tp}.
8. Calculate f_{tp}/f_{ns} using Eq. 16.
9. Calculate f_{ns} using Eq. 13, or obtain it from a Moody diagram.
10. Calculate $f_{tp} = (f_{tp}/f_{ns})\,f_{ns}$.
11. Calculate dp/dZ using Eq. 1.

This procedure is repeated with small pressure changes, and the pressure traverse is calculated using one of the methods described above.

Statistical Analysis of Results

The statistical analysis consisted of calculating the errors between observed and predicted values of liquid holdup and pressure gradient and the standard deviation of these errors. Errors were calculated on a percentage basis as

$$\text{Percent error} = \frac{\text{calculated value} - \text{observed value}}{\text{observed value}} \times 100\,,$$

so that a positive error indicates that the predicted value is too high. The average error was calculated by

$$\text{Average percent error} = \frac{\sum_{0}^{N} \text{percent error}}{N}$$

The standard deviation of the percentage errors

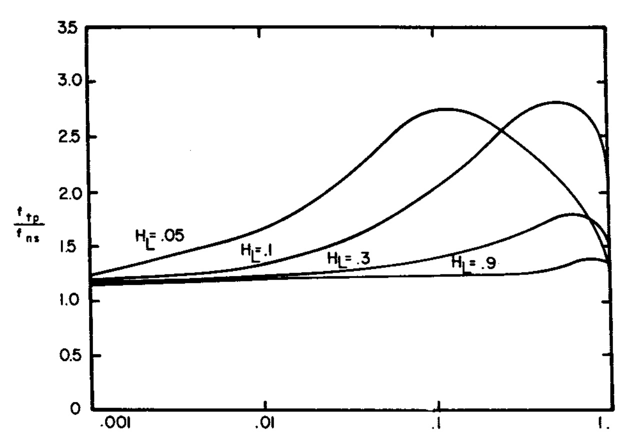

Fig. 7—Two-phase friction factor.

was calculated by

Standard deviation =

$$\left[\frac{N\sum_{0}^{N}(\%\text{ error})^2-\left(\sum_{0}^{N}\%\text{ error}\right)^2}{N^2}\right]^{0.5}.$$

Table 2 lists the average percentage errors and standard deviations for each inclination angle and for all tests.

Conclusions

The following conclusions are drawn from this study.

1. The inclination angle of a pipe in which two-phase flow is occurring definitely affects liquid holdup and pressure drop for most flow conditions.
2. In inclined two-phase flow, the liquid holdup reaches a maximum at an angle of approximately +50° and a minimum at approximately −50° from horizontal. The fact that holdup is approximately equal at angles of +90° and +20° explains why vertical holdup correlations can be used with some degree of success for horizontal flow.
3. Pressure recovery in the downhill section of a two-phase pipeline in hilly terrain can definitely exist and should be considered in pipeline design.
4. The accuracy of a liquid holdup correlation for horizontal two-phase flow can be improved by considering flow pattern.
5. Input liquid content, λ, and Froude number, N_{FR}, are very important correlating parameters in two-phase flow.
6. Friction loss in two-phase flow is greatly affected by liquid holdup.
7. A correlation has been developed that can predict liquid holdup and pressure gradients occurring in two-phase, air-water flow in 1- and 1.5-in. smooth, circular pipes at any angle of inclination.

Nomenclature

A_p = pipe area
C = coefficient
d = pipe diameter
f = friction factor
g = acceleration due to gravity
g_c = gravitational constant
G = mass flux rate = w/A_p
G_m = mixture mass flux rate = $G_L + G_g$
h = vertical elevation
H_L = liquid holdup fraction
L_1 = correlation boundary, Eq. 9
L_2 = correlation boundary, Eq. 10
M = molecular weight
N_D = diameter number = $d\left(\frac{\rho_L g}{\sigma}\right)^{0.5}$
N_{gv} = gas velocity number = $v_{sg}\left(\frac{\rho_L}{g\sigma}\right)^{0.25}$
N_{Lv} = liquid velocity number = $v_{sL}\left(\frac{\rho_L}{g\sigma}\right)^{0.25}$
N_{FR} = Froude number = v_m^2/gd
N_{Re} = Reynolds number = $\rho v d/\mu$
p = pressure
q = in-situ volumetric flow rate
R = gas constant
S = correlating factor, Eq. 16
T = temperature
v = velocity
v_m = mixture velocity = $(q_L + q_g)/A_p$
v_{sg} = superficial gas velocity − q_g/A_p
v_{sL} = superficial liquid velocity = q_L/A_p
w = mass flow rate

TABLE 2—SUMMARY OF STATISTICAL RESULTS

Inclination Angle (degrees)	Number of Tests	Liquid Holdup: Average Percent Error	Liquid Holdup: Standard Deviation	Pressure Gradient: Average Percent Error	Pressure Gradient: Standard Deviation
0	58	−0.87	5.51	+2.57	6.03
+ 5	33	−1.46	7.07	+1.57	6.52
+10	32	−0.34	5.70	+1.43	5.84
+15	32	−0.14	5.44	+1.30	6.27
+20	32	−0.95	5.72	+2.03	6.64
+35	30	+0.63	6.83	+0.97	6.92
+55	30	+0.78	5.63	+1.22	5.50
+75	31	+0.83	5.75	+1.77	7.59
+85	16	−0.12	5.11	+0.24	6.32
+90	27	+0.18	7.27	+1.79	5.96
All uphill tests	263	−0.09	6.19	+1.43	6.45
− 5	33	+0.13	7.45	+1.98	9.32
−10	32	−0.84	8.90	−1.51	17.73
−15	32	+1.16	8.09	+0.57	15.94
−20	32	+0.29	7.17	+1.47	10.30
−35	30	−4.17	10.59	+2.95	11.24
−55	30	−3.13	13.57	−1.83	11.85
−75	31	−0.75	9.13	+0.05	9.61
−85	16	−1.20	6.68	−0.39	3.11
−90	27	+5.47	11.23	+0.44	6.47
All downhill tests	263	−0.34	9.82	+0.47	11.89
All tests	584	−0.28	7.98	+1.11	9.30

w_f = irreducible friction losses
z_g = gas compressibility factor
Z = distance of axial flow
γ = specific gravity
Δ = difference
θ = angle from horizontal
λ = input liquid content = $q_L/(q_L + q_g)$
μ = viscosity
μ_m = mixture viscosity = $\mu_L\lambda + \mu_g(1-\lambda)$
ρ = density
ρ_{ns} = no-slip density = $\rho_L\lambda + \rho_g(1-\lambda)$
ρ_{tp} = two-phase mixture density = $\rho_L H_L + \rho_g(1-H_L)$
σ = liquid surface tension
ϕ = angle
ψ = inclination correction factor, Eq. 5

Subscripts

a = atmospheric
acc = acceleration (kinetic energy)
el = elevation (gravity, potential energy)
f = friction
g = gas
L = liquid
m = mixture
ns = no-slip (two-phase)
sp = single phase
tp = two phase

Arbitrary Constants

$A, B, D, \alpha, \beta, \gamma, \delta, \varepsilon, \zeta$

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References

1. Baker, Ovid: "Discussion on How Uphill and Downhill Flow Affect Pressure Drop in Two-Phase Pipelines through Hilly Country," *Oil and Gas J.* (Nov. 11, 1957) 150-152.
2. Beggs. H. D.: "An Experimental Study of Two-Phase Flow in Inclined Pipes," PhD dissertation, U. of Tulsa (1972).
3. Dukler, A. E., Baker, O., Cleveland, R. L., Hubbard, M. G. and Wicks, M., III: "Gas-Liquid Flow in Pipelines, Part 1, Research Results," Monograph NX-28, U. of Houston (May, 1969).
4. Eaton, B. A.: "The Prediction of Flow Patterns, Liquid Holdup and Pressure Losses Occurring During Continuous Two-Phase Flow in Horizontal Pipelines," PhD dissertation, The U. of Texas at Austin (1966).
5. Flanigan, O.: "Effect of Uphill Flow on Pressure Drop in Design of Two-Phase Gathering Systems," *Oil and Gas J.* (March 10, 1958) **56,** 132.
6. Guzhov, A. I., Mamayev, V. A. and Odishariya, G. E.: "A Study of Transportation in Gas Liquid Systems," paper IGU/C19-67 presented at 10th Intl. Gas Conference, Hamburg, Germany, June 6-10, 1967.
7. Hagedorn, A. R. and Brown, K. E.: "Experimental Study of Pressure Gradients Occurring During Continuous Two-Phase Flow in Small-Diameter Vertical Conduits," *J. Pet. Tech.* (April, 1965) 475-484.
8. Hughmark, G. A.: "Holdup in Gas-Liquid Flows," *Chem. Eng. Prog.* (April, 1962) **58,** 62.
9. Mamayev, V. A.: "Some Problems in the Hydrodynamics of Joint Transport of Gas and Liquid," *Int. Chem. Eng.* (April, 1965) **5,** 318-322.
10. Nezhilskii, B. M. and Khodanovich, I. E.: "Experimental Studies of True Gas Content in Descending and Ascending Sections of a Nonhorizontal Pipeline," *Nauch-Tekh. Sb. Ser. Gazovoe Delo* (1970) **11,** 21-25.
11. Odishariya, G. E.: "On Certain Problems in Gas Liquid Flow Through Pipelines," *Neft Khoz* (Sept., 1966) **9,** 54-59.
12. Orkiszewski, J.: "Predicting Two-Phase Pressure Drops in Vertical Pipes," *J. Pet. Tech.* (June, 1967) 829-838.
13. Poettmann, F. H. and Carpenter, P. G.: "The Multiphase Flow of Gas, Oil, and Water Through Vertical Flow Strings with Application to the Design of Gas-Lift Installations," *Drill. and Prod. Prac.*, API (1952) 257.
14. Sevigny, Rene: "An Investigation of Isothermal, Cocurrent, Two-Fluid Two-Phase Flow in an Inclined Tube," PhD dissertation, U. of Rochester, Rochester, N. Y. (1962).
15. Singh, G. and Griffith, P.: "Determination of the Pressure Drop Optimum Pipe Size for a Two-Phase Slug Flow in an Inclined Pipe," *J. Eng. Ind. Trans.*, ASME (Nov., 1970) **92,** 717-726.

APPENDIX

Development of Pressure-Gradient Equation

The basis for any fluid-flow calculation is an energy balance for the flowing fluid between two points. Assuming no external work is done on or by the fluid, a steady-state mechanical energy balance equation in differential form may be written for 1 lb_m of fluid as

$$\frac{dp}{\rho_{tp}} + \frac{g}{g_c}\,dh + \frac{v_m dv_m}{g_c} + d(w_f) = 0\,, \qquad \text{(A-1)}$$

where $d(w_f)$ represents the irreversible friction losses. For flow in a pipe up or down an incline,

$$dh = \sin\theta\, dZ\,, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(A-2)}$$

where dh is the vertical distance moved, θ is the angle of the pipe from horizontal, and dZ is the axial distance moved. Substitution of Eq. A-2 into Eq. A-1 gives

$$\frac{dp}{dZ} = -\left[\frac{g}{g_c}\rho_{tp}\sin\theta + \rho_{tp}\frac{v_m}{g_c}\frac{dv_m}{dZ} + \rho_{tp}\frac{d(w_f)}{dZ}\right] \quad . \quad . \quad . \quad \text{(A-3)}$$

This may be written as

$$-\frac{dp}{dZ} = \left(\frac{\partial p}{\partial Z}\right)_{el} + \left(\frac{\partial p}{\partial Z}\right)_{acc} + \left(\frac{\partial p}{\partial Z}\right)_{f}\,; \quad . \quad \text{(A-4)}$$

that is, the total pressure drop is the sum of the pressure drops due to potential energy change, kinetic energy change, and friction loss.

Frictional Pressure Gradient

By definition, the frictional pressure gradient is given by

$$\left(\frac{\partial p}{\partial Z}\right)_f = \frac{f_{tp}\,\rho_{ns}\,v_m^2}{2\,g_c d} = \frac{f_{tp}\,G_m v_m}{2\,g_c d} \quad . \quad . \quad . \quad \text{(A-5)}$$

In two-phase flow, the no-slip density may be different from the in-situ density because of slippage between the phases. The no-slip density, ρ_{ns}, is used in the friction-loss equation because the energy entering and leaving a differential element of the pipe by way of the flowing fluid is a function of the properties of the fluid entering and leaving the differential element, and not of the fluid in place.

Acceleration Pressure Gradient

To analyze the kinetic energy or acceleration term, which is negligible for most practical cases, several simplifying assumptions are made.

$$v_m = v_{sL} + v_{sg} = \frac{G_L}{\rho_L} + \frac{G_g}{\rho_g};$$

therefore,

$$\left(\frac{\partial p}{\partial Z}\right)_{\text{acc}} = \frac{\rho_{tp} v_m}{g_c} \frac{dv_m}{dZ}$$

$$= \frac{\rho_{tp} v_m}{g_c} \left[\frac{d}{dZ}\left(\frac{G_L}{\rho_L}\right) + \frac{d}{dZ}\left(\frac{G_g}{\rho_g}\right)\right].$$

While $\frac{d}{dZ}\frac{G_L}{\rho_L}$ is not zero, it may be assumed to be small compared with $\frac{d}{dZ}\frac{G_g}{\rho_g}$ because of the difference in the compressibilities of the liquid and the gas. Therefore,

$$\left(\frac{\partial p}{\partial Z}\right)_{\text{acc}} = \frac{\rho_{tp} v_m}{g_c} \frac{d}{dZ}\left(\frac{G_g}{\rho_g}\right)$$

$$= \frac{\rho_{tp} v_m}{g_c}\left[\frac{\rho_g \frac{d}{dZ}(G_g) - G_g \frac{d}{dZ}(\rho_g)}{\rho_g^2}\right]$$

$$= \frac{\rho_{tp} v_m}{g_c}\left[\frac{\frac{d}{dZ}(G_g)}{\rho_g} - \frac{G_g}{\rho_g^2}\frac{d}{dZ}(\rho_g)\right]. \quad \text{(A-6)}$$

It may be assumed also that the change in gas mass flux is much smaller than the change in gas density, or

$$\frac{\frac{d}{dZ}(G_g)}{\rho_g} << \frac{G_g}{\rho_g^2}\frac{d}{dZ}(\rho_g).$$

Incorporating this assumption into Eq. A-6 gives

$$\left(\frac{\partial p}{\partial Z}\right)_{\text{acc}} = -\frac{\rho_{tp} v_m}{g_c}\frac{G_g}{\rho_g^2}\frac{d}{dZ}(\rho_g) \quad . \quad . \quad . \quad \text{(A-7)}$$

From the engineering gas law,

$$\rho_g = \frac{pM}{z_g RT},$$

and

$$\frac{d}{dZ}(\rho_g) = \frac{d}{dZ}\left(\frac{pM}{z_g RT}\right)$$

$$= \frac{M}{z_g RT}\frac{dp}{dZ} + \frac{p}{z_g RT}\frac{d}{dZ}(M)$$

$$- \frac{pM}{z_g^2 RT}\frac{d}{dZ}(z_g) - \frac{pM}{z_g RT^2}\frac{dT}{dZ}. \quad \text{(A-8)}$$

Dividing Eq. A-8 by

$$\rho_g = \frac{pM}{z_g RT}$$

gives

$$\frac{d}{dZ}(\rho_g) = \rho_g\left(\frac{1}{p}\frac{dp}{dZ} + \frac{1}{M}\frac{dM}{dZ} - \frac{1}{z_g}\frac{dz_g}{dZ} - \frac{1}{T}\frac{dT}{dZ}\right) \quad . \quad . \quad \text{(A-9)}$$

In analyzing the relative magnitudes of the remaining terms in Eq. A-9, it may be assumed that

$$\frac{1}{M}\frac{dM}{dZ} - \frac{1}{z_g}\frac{dz_g}{dZ} - \frac{1}{T}\frac{dT}{dZ} << \frac{1}{p}\frac{dp}{dZ}.$$

Therefore,

$$\frac{d}{dZ}(\rho_g) = \frac{\rho_g}{p}\frac{dp}{dZ} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(A-10)}$$

Substitution of Eq. A-10 into Eq. A-7 gives

$$\left(\frac{\partial p}{\partial Z}\right)_{\text{acc}} = -\frac{\rho_{tp} v_m}{g_c}\frac{G_g}{\rho_g^2}\frac{\rho_g}{p}\frac{dp}{dZ},$$

or

$$\left(\frac{\partial p}{\partial Z}\right)_{\text{acc}} = -\frac{\rho_{tp} v_m v_{sg}}{g_c p}\frac{dp}{dZ} \quad . \quad . \quad . \quad . \quad . \quad \text{(A-11)}$$

Gravity Pressure Gradient

Calculation of the pressure gradient caused by elevation change,

$$\left(\frac{\partial p}{\partial Z}\right)_{\text{el}} = \frac{g}{g_c}\rho_{tp}\sin\theta, \quad . \quad . \quad . \quad . \quad . \quad \text{(A-12)}$$

requires a procedure to determine the in-situ density of the gas-liquid mixture, ρ_{tp}. For this purpose, a liquid holdup factor is defined as

$$H_L = \frac{\text{Volume of liquid in an element}}{\text{Volume of the element}}.$$

The in-situ density of the fluid mixture may now be represented by

$$\rho_{tp} = \rho_L H_L + \rho_g(1 - H_L) \quad . \quad . \quad . \quad \text{(A-13)}$$

Therefore, Eq. A-12 becomes

$$\left(\frac{\partial p}{\partial Z}\right)_{\text{el}} = \frac{g}{g_c}[\rho_L H_L + \rho_g(1 - H_L)]\sin\theta. \quad \text{(A-14)}$$

By substitution of Eqs. A-5, A-11, and A-14 into Eq. A-4, the total pressure gradient is

$$-\frac{dp}{dZ} = \frac{g}{g_c}[\rho_L H_L + \rho_g(1-H_L)]\sin\theta + \frac{f_{tp} G_m v_m}{2 g_c d}$$

$$- \frac{[\rho_L H_L + \rho_g(1-H_L)] v_m v_{sg}}{g_c p}\frac{dp}{dZ},$$

or

$$-\frac{dp}{dZ} = \frac{\frac{g}{g_c}\sin\theta[\rho_L H_L + \rho_g(1-H_L)] + \frac{f_{tp} G_m v_m}{2 g_c d}}{1 - \{[\rho_L H_L + \rho_g(1-H_L)] v_m v_{sg}\}/g_c p}$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(A-15)}$$

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