A Study of Multiphase Flow Behavior in Vertical Wells

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Summary. This paper presents a physical model for predicting flow pattern, void fraction, and pressure drop during multiphase flow in vertical wells. The hydrodynamic conditions giving rise to various flow patterns are first analyzed. The method for predicting void fraction and pressure drop is then developed. In the development of the equations for pressure gradient, the contribution of the static head, frictional loss, and kinetic energy loss are examined. Laboratory data from various sources show excellent agreement with the model.

Introduction

A number of correlations are available for predicting pressure drop during multiphase flow. Because most of these correlations are entirely empirical, they are of doubtful reliability. The calculation procedures involved are also rather complicated. Therefore, a better approach is to attempt to model the flow system and then to test the model against actual data. Proper modeling of multiphase flow requires an understanding of the physical system.

When cocurrent flows of multiple phases occur, the phases take up a variety of configurations, known as flow patterns. The particular flow pattern depends on the conditions of pressure, flow, and channel geometry. In the design of oil wells and pipelines, knowledge of the flow pattern or successive flow patterns that would exist in the equipment is essential for choosing a hydrodynamic theory appropriate for that pattern.

The name given to a flow pattern is somewhat subjective. Hence, a multitude of terms have been used to describe the various possible phase distributions. In this paper, we will be concerned only with those patterns that are clearly distinguishable and generally recognized. The major flow patterns encountered in vertical cocurrent flow of gas and liquid are listed in standard textbooks and in the classic works of Orkiszewski,[1] Aziz *et al.*,[2] and Chierici *et al.*[3] The four flow patterns—bubbly, slug, churn, and annular—are shown schematically in Fig. 1.

At low gas flow rates, the gas phase tends to rise through the continuous liquid medium as small, discrete bubbles, giving rise to the name bubbly flow. As the gas flow rate increases, the smaller bubbles begin to coalesce and form larger bubbles. At sufficiently high gas flow rates, the agglomerated bubbles become large enough to occupy almost the entire pipe cross section. These large bubbles, known as "Taylor bubbles," separate the liquid slugs between them. The liquid slugs, which usually contain smaller entrained gas bubbles, provide the name of the flow regime. At still higher flow rates, the shear stress between the Taylor bubble and the liquid film increases, finally causing a breakdown of the liquid film and the bubbles. The resultant churning motion of the fluids gives rise to the name of this flow pattern. The final flow pattern, annular flow, occurs at extremely high gas flow rates, which cause the entire gas phase to flow through the central portion of the tube. Some liquid is entrained in the gas core as droplets, while the rest of the liquid flows up the wall through the annulus formed by the tube wall and the gas core.

In an oil well, different flow patterns usually exist at different depths. For example, near bottom hole we may have only one phase. As the fluid moves upward, its pressure gradually decreases. At the point where the pressure becomes less than the bubblepoint pressure, gas will start coming out of solution and the flow pattern will be bubbly. As pressure decreases further, more gas may come out of solution and we may see the whole range of flow patterns shown in Fig. 2.

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Here we discuss the hydrodynamic conditions that give rise to the various flow-pattern transitions. The method for estimating pressure drop in each flow regime is then developed. In developing the equations for pressure gradient, we note that for vertical flow of gas/liquid mixtures, 90 to 99% of the total pressure drop is usually caused by the static head. Accurate estimation of the in-situ gas void fraction is therefore of great importance.

Flow Pattern Transition

The often chaotic nature of multiphase flow makes it difficult to describe and to classify flow patterns and hence to ascribe criteria for flow-pattern transitions correctly. In addition, although flow patterns are strongly influenced by such parameters as phase velocities and densities, other less important variables—such as the method of forming the two-phase flow, the extent of departure from local hydrodynamic equilibrium, the presence of trace contaminants, and various fluid properties—can influence a particular flow pattern. Despite these deficiencies, a number of methods have been proposed to predict flow pattern during gas/liquid two-phase flow. Some of these methods could be extended to liquid/liquid systems with less accuracy.

One method of representing various flow-regime transitions is in the form of flow-pattern maps. Superficial phase velocities or generalized parameters containing these velocities are usually plotted to delineate the boundaries of different flow regimes. Obviously, the effect of secondary variables cannot be represented in a two-dimensional map. Any attempt to generalize the map requires the choice of parameters that would adequately represent various flow-pattern transitions. Because differing hydrodynamic conditions and balance of forces govern different transitions, a truly generalized map is almost impossible. Still, some maps are reasonably accurate. Among these, the map proposed by Govier *et al.* [4] has found wide use in the petroleum industry. The flow-pattern map of Hewitt and Roberts[5] has also been widely accepted in academia and the power-generating industry.

An alternative, more flexible approach is to examine each transition individually and to develop criteria valid for that specific transition. Because this approach allows physical modeling of individual flow patterns, it is more reliable than the use of a map.

Bubbly/Slug-Flow Transition. Transition from the condition of small bubbles dispersed throughout the flow cross section (bubbly flow) to that when the bubble becomes large enough to fill the entire cross section (slug flow) requires agglomeration or coalescence. Bubbles, other than very small ones, generally follow a zigzag path when rising through a liquid. This results in collision between bubbles, with consequent bubble agglomeration and formation of larger bubbles, which increases with increasing gas flow rate. Radovich and Moissis[6] examined the behavior of bubbles theoretically by considering a cubic lattice in which the individual bubbles fluctuate. They showed that at a void fraction of 0.3, the collision frequency

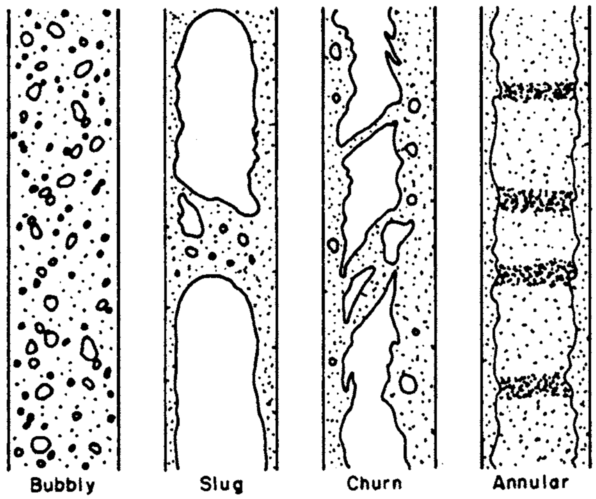

Fig. 1—Flow patterns in vertical cocurrent two-phase flow.[9]

becomes so high that a transition to slug flow is to be expected. Griffith and Snyder[7] experimentally verified that this transition occurs at a void fraction of 0.25 to 0.3. Hasan and Kabir[8] also found the transition to take place at a void fraction of about 0.25, even in an annular (casing/tubing) geometry.

Thus, $f_g=0.25$ may be taken as the criterion for transition between bubbly and slug flow. This criterion still needs to be expressed in terms of measured variables, such as superficial phase velocities. Taitel *et al.*[9] equate the slip between the phases with the terminal rise velocity, v_∞, of a single bubble. On the other hand, Hasan *et al.*[8] used the relationship between void fraction and superficial gas velocity in bubbly flow (to be developed later) in the following manner:

$$v_g=v_{gs}/f_g=1.2v_M+v_\infty \quad \text{(1)}$$

or

$$v_{gs}=(1.2v_{Ls}f_g+v_\infty f_g)/(1-1.2f_g).$$

Using $f_g=0.25$, we obtain

$$v_{gs}=0.429v_{Ls}+0.357v_\infty. \quad \text{(2)}$$

An expression for the terminal rise velocity of bubbles in a vertical system may be derived by balancing the buoyant and the drag forces on the bubble. Eq. 3, derived by Harmathy,[10] has been supported by data from many independent sources:

$$v_\infty=1.53[g\sigma(\rho_L-\rho_g)/\rho_L{}^2]^{0.25}. \quad \text{(3)}$$

Using the Harmathy correlation for v_∞, we can rewrite Eq. 2 as

$$v_{gs}=0.429v_{Ls}+0.546[g\sigma(\rho_L-\rho_g)/\rho_L{}^2]^{0.25}. \quad \text{(4)}$$

Taitel *et al.*[9] and Taitel and Dukler[11] arrived at a similar equation with slightly lower values for the constants in Eq. 4. For a stagnant water column at atmospheric pressure, Eq. 4 predicts that slug flow will occur whenever superficial gas velocity exceeds 0.29 ft/sec [0.088 m/s].

In deriving Eq. 2, we assumed that the void-fraction relationship for bubbly flow would be applicable up to the point of transition to slug flow. This is not strictly true because any transition is likely to be gradual. Indeed, if Eq. 1 is replaced by an expression appropriate for slug flow, the transition criterion given by Eq. 2 will have to be modified by replacing the bubble-rise velocity, v_∞, by the Taylor bubble-rise velocity, $v_{\infty T}$. The difference between $v_{\infty T}$ and v_∞ is not large, however, and Eq. 2 adequately represents the transition between bubbly and slug flow.

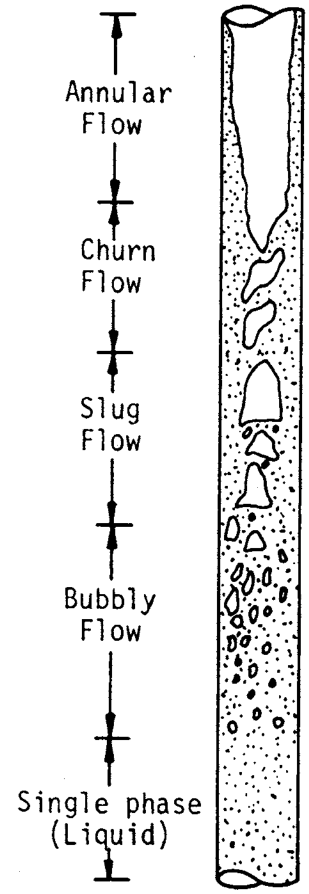

Fig. 2—Flow patterns in a vertical oil well.[2]

An interesting aspect of the transition between bubbly and slug flow, the influence of pipe diameter, was pointed out by Taitel *et al.* The terminal rise velocity of small bubbles (Eq. 3) is dependent on fluid properties but is independent of tube diameter. The rise velocity of a Taylor bubble,[12] however,

$$v_{\infty T}=0.35\sqrt{gd(\rho_L-\rho_g)/\rho_L} \sim 0.35\sqrt{gd}, \quad \text{(5)}$$

is dependent on the pipe diameter. When $v_{\infty T}>v_\infty$, the nose of the Taylor bubble sweeps the smaller bubbles ahead of it. However, when $v_{\infty T}<v_\infty$, which is possible in smaller pipes, the rising smaller bubbles approach the back of the Taylor bubble, coalesce with it, increase its size, and ultimately cause a transition to slug flow. Under such conditions, bubbly flow cannot exist, except at very small gas flow rates that do not allow formation of any Taylor bubbles. For air/water systems at standard conditions, the small bubble-rise velocity would be equal to Taylor bubble-rise velocity when the pipe diameter is about 1.8 in. [4.6 cm]. For smaller tubes, therefore, one may not observe any bubbly flow at all and flow-pattern data from such tubes may not be scaled up to larger tubes. This is evident in Hewitt and Roberts'[5] map (data gathered with 0.4- to 1.2-in. [1- to 3-cm] -diameter tubes), where no bubbly flow exists at low liquid flow rates.

Figs. 3 and 4 show the bubbly/slug-transition data from a number of sources as reported by Weisman and Kang.[13] Fig. 3 shows that the data gathered with pipes larger than 1.8 in. [4.6 cm] agree very well with Eq. 4. The only exception is the data of Spedding and Nguyen,[14] who did not consider the bubbly/slug transition seriously. The data of Weisman and Kang, although gathered from a 1-in. [2.5-cm] pipe, were included in Fig. 3 because for freon-113, which was used to gather these data, the terminal rise velocity is about 0.48 ft/sec [0.15 m/s], considerably lower than the Taylor bubble-rise velocity of about 0.57 ft/sec [0.17 m/s] for a 1-in. [2.5-cm] pipe. Thus, Eq. 4 is expected to apply to these data. The

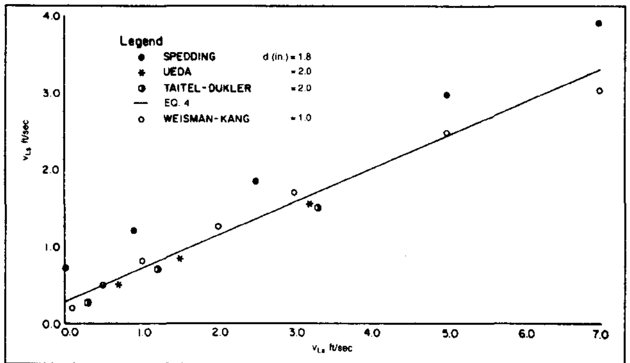

Fig. 3—Bubbly/slug-flow-transition data from large-diameter pipes.

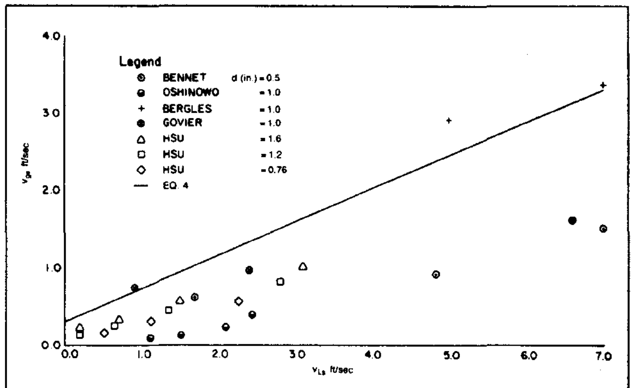

Fig. 4—Bubbly/slug-flow-transition data from small-diameter (<1.8-in.) pipes.

lower value of the terminal rise velocity means that the transition line for the Freon-113 system would be parallel to the line shown in Fig. 3, with a lower value (0.17 ft/sec [0.05 m/s]) for the intercept. The agreement of the Weisman-Kang data with our theory is therefore quite good.

The data plotted in Fig. 4, gathered from smaller-diameter pipes, clearly show that transition to bubbly flow occurs at much lower superficial gas velocities than Eq. 4 suggests. This is especially true of the data of Bennet *et al.*,[15] gathered in 0.4-in. [1-cm] tubes, and the data of Hsu and Dudokovic,[16] gathered in 0.76-in. [1.9-cm] tubes. This transition to slug flow at lower v_{gs} than suggested by Eq. 2 agrees with the previous discussion of the difficulty of maintaining bubbly flow in small-diameter tubes, even at very low gas flow rates. Bubbly flow evidently does exist even for very-small-diameter tubes, albeit only at low superficial gas velocities. The data presented in Fig. 4, therefore, appear to indicate that when $v_{\infty T} < v_{\infty}$, the tendency for bubbles to agglomerate and the possibility of transition to slug flow are increased, but not guaranteed, as Taitel and Dukler[11] implied. Many other parameters—such as the method of forming two-phase flow, vibration in the system, and the mixture velocity (shear force)—probably influence the transition to slug flow for smaller pipes.

Transition to Dispersed Bubbly Flow. Eq. 2 applies only to transition from bubbly to slug flow at low or moderate flow rates. At high flow rates, the turbulence tends to break up the larger agglomerated bubbles, thus inhibiting transition to slug flow. In such cases, bubbly flow persists even when the void fraction exceeds 0.25. This type of bubbly flow resulting from the breakdown and dispersion of larger bubbles in the liquid is known as the dispersed bubbly flow.

Taitel *et al.*[9] presented an analysis for the onset of dispersed bubbly flow based on the maximum bubble diameter possible under highly turbulent conditions as given by Hinze.[17] If the turbulence produced is not high enough, the bubble diameter could still be large enough to coalesce and transition to slug flow will occur at $f_g = 0.25$. If, however, the flow is highly turbulent so that the bubble sizes are smaller than a certain critical value, the agglomeration is suppressed and bubbly flow can exist beyond $f_g = 0.25$. Using the expression for critical bubble diameter given by Brodkey[18] and equating it to the maximum bubble diameter possible under turbulent flow condition, Taitel *et al.* arrived at the following expression for mixture velocity:

$$v_M^{1.12} = 4.68(d)^{0.48}[g(\rho_L - \rho_g)/\sigma]^{0.5}(\sigma/\rho_L)^{0.6}(\rho_M/\mu_L)^{0.08}. \quad \text{(6)}$$

Thus, if the mixture velocity is greater than that given by Eq. 6, bubbly flow will persist even when $f_g > 0.25$. However, Taitel *et al.* show that even for small gas bubbles, the gas void fraction can, at most, be 0.52. At higher void fraction, transition to slug (or churn) flow must take place.

Prediction of Eq. 6 for air/water systems at 77°F [25°C] in a 2-in. [5-cm] pipe ($v_M = 11.8$ ft/sec [3.6 m/s], equivalent to 4,000 B/D [636 m^3/d]) is comparable to the air/water data of Weisman and Kang[13] ($v_{Ls} = 10$ ft/sec) [3 m/s]). The empirical relationship proposed by Weisman and Kang is also quite similar, except that it predicts the superficial liquid velocity above which dispersed bubbly flow would persist. Eq. 6 appears to be more appropriate because the shear stress acting to break up the bubbles would be a result of the total fluid velocity. The dispersed bubbly-flow region of the Hewitt-Roberts map also agrees substantially with the predictions of Eq. 6.

Slug/Churn-Flow Transition. In slug flow, Taylor-type bubbles, formed by the agglomeration of smaller bubbles, occupy most of the pipe cross-sectional area. The Taylor bubbles are axially separated by a liquid slug in which small bubbles are dispersed. The liquid confined by the Taylor bubble and the tube wall flows around the bubble as a falling film. The interaction between this falling film and the rising Taylor bubble increases with the increasing flow rate. The upper limit of the slug flow occurs when the interaction becomes high enough to break up the long bubbles, causing the transition to churn flow.

A semitheoretical method of predicting this limitation interaction (known as flooding) is given by Porteous,[19] who relates the Taylor bubble-rise velocity, $v_{\infty T}$, given by Eq. 6 to the total mixture velocity, v_M:

$$v_M = 0.3 v_{\infty T}\sqrt{\rho_L/\rho_g} = 0.103\sqrt{gd(\rho_L - \rho_g)/\rho_g}. \quad \text{(7)}$$

At higher pressures (>10 atm [1013 kPa]) when ρ_g is large, Porteous's method predicts very low values of v_M (often lower than that needed to sustain slug flow) for transition to churn flow. Therefore, we recommend the use of Hewitt and Roberts' map to predict this transition at pressures higher than 10 atm [1013 kPa]. The slug/churn-transition curve of the map may be approximated by

$$\rho_g v_{gs}^2 = 0.00673(\rho_L v_{Ls}^2)^{1.7}, \text{ if } \rho_L v_{Ls}^2 < 50 \quad \text{(8)}$$

and

$$\rho_g v_{gs}^2 = 17.1 \log_{10}(\rho_L v_{Ls}^2) - 23.2, \text{ if } 50 < \rho_L v_{Ls}^2 < 3{,}300. \quad \text{(9)}$$

Note that if $\rho_L v_{Ls}^2 > 3{,}300$ lbm/ft-sec^2 [4911 kg/m·s^2], the existing flow pattern is either dispersed bubbly or annular.

Taitel and Dukler[11] proposed that when $v_M/\sqrt{gd} > 50$, the slug/churn-flow transition occurs at a gas volume fraction exceeding 0.86. Taitel *et al.*[9] developed a correlation for this transition by looking at churn flow as an entrance effect and a preliminary stage to the fully developed slug flow. This approach needs verification. The churn-flow pattern is easily confused, and very little agreement exists regarding the point of transition between slug and churn flow.[13]

Transition to Annular Flow. At high gas flow rates, transition from churn or slug flow to annular flow takes place. The liquid flows upward along the tube wall, while the gas flows through the center of the tube. The liquid film has a wavy interface, and the waves could break down and be carried away as entrained droplets.

The transition between churn (or slug) and annular flow has been studied by Wallis[20] and by Jones and Zuber,[21] who analyzed the minimum gas flow rate required to reverse the direction of flow of a falling liquid film. A different physical model was adapted by Taitel *et al.*, who examined the drag force necessary to keep the entrained liquid droplets in suspension. Reasoning along the line of Turner *et al.*,[22] they suggested that when the gas velocity is not sufficient to keep the liquid droplets in suspension, the droplets will fall back, accumulate, form a bridge, and finally establish churn or slug flow. The minimum velocity required to keep the droplets in suspension may be determined from a balance of the drag forces on these droplets and the gravitational forces acting on them:

$$(\tfrac{1}{2})C_d(d_d^2/4)\rho_g v_g^2=(d_d^3/6)g(\rho_L-\rho_g) \quad \text{(10a)}$$

or

$$v_g=(2/\sqrt{3})[g(\rho_L-\rho_g)d_d/\rho_g C_d]^{0.5}. \quad \text{(10b)}$$

For the droplet diameter, d_d, Taitel *et al.* used the correlation suggested by Hinze[17] for maximum stable drop size:

$$d_d=(N_{\mathrm{Wec}})\sigma/\rho_g v_g^2. \quad \text{(11)}$$

Substitution of Eq. 11 into Eq. 10 gives

$$v_g=(4N_{\mathrm{Wec}}/3C_d)^{0.25}[\sigma g(\rho_L-\rho_g)/\rho_g^2]^{0.25}. \quad \text{(12)}$$

Following Turner *et al.*'s suggestion, Taitel *et al.* used a value of 30 for the critical Weber number, N_{Wec}, and a value of 0.44 for the drag coefficient, C_d. In any case, because N_{Wec} and C_d are raised to the one-fourth power, v_g is not very sensitive to the values of these parameters. Eq. 12 is simplified further if we note that in annular flow the gas void fraction approaches unity when $v_{gs} \sim v_g$. Thus,

$$v_{gs}=3.1[\sigma g(\rho_L-\rho_g)/\rho_g^2]^{0.25}. \quad \text{(13)}$$

Taitel *et al.* present data from six different sources that agree remarkably well with the predictions of Eq. 13. We point out that Eq. 13, as well as Hewitt and Roberts' map and Wallis'[20] method, suggests that the transition to annular flow is not affected by the liquid flow rate. For an air/water system at standard conditions, Eq. 13 predicts a gas velocity of about 50 ft/sec [15 m/s].

Pressure Gradient

The total pressure gradient, dp/dz, during multiphase flow (as in single-phase flow) is the sum of the gravitational [static head, $(dp/dz)_H$], frictional [$(dp/dz)_F$], and acceleration [kinetic head, $(dp/dz)_A$] components:

$$(dp/dz)=(dp/dz)_H+(dp/dz)_F+(dp/dz)_A \quad \text{(14)}$$

$$=(-1/g_c)[(\rho_M g)+(2f_M v_M^2\rho_M/d)+(\rho_M v_M dv_M/dz)].$$

For oil production, the accelerational term is often very small and is usually neglected. But the contribution of this term across any section of the pipe, as suggested by the third term of Eq. 14, is $\rho_M v_M(v_{M2}-v_{M1})$ and can be easily evaluated. Note that the product $\rho_M v_M$ equals the total mass flux (mass flow rate per unit area), through the string and remains constant, although the mixture density and velocity individually may vary.

For vertical flow, except during annular flow, the static head is the major contributor to the total head loss, and in some cases (low gas fraction and low flow rates), it may account for more than 95% of the total gradient. Because the gas void fraction, f_g, is so important in the calculation of the static head of the fluid column, accurate estimation of the void fraction is of paramount importance in vertical-multiphase-flow analysis. The determination of the frictional head losses also requires an estimate of the mixture density and hence the gas void fraction in the pipe.

The gas void fraction depends on the in-situ velocity of the gas phase relative to the mixture. Because of the terminal rise velocity of the gas bubbles arising from the density difference between the phases, the in-situ gas velocity, v_g, is greater than the mixture velocity, v_M. In addition, the bubbles tend to flow through the central portion of the channel where the local mixture velocity is higher than the average velocity. These two factors, the terminal rise velocity and the local mixture velocity in the vicinity of most of the bubbles, depend on the flow pattern and are discussed individually for each flow regime.

Bubbly Flow. ***Void Fraction in a Tubing.*** If we designate the central mixture velocity to be C_0 times the average mixture velocity, v_M, then

$$v_g=C_0 v_M+v_\infty. \quad \text{(15)}$$

If the flow is ideal bubbly, which is possible with pure liquids and very low gas flow rates, the bubbles do not affect each other's motion and Eq. 15 is not strictly valid. In such cases, the drift flux model developed by Ishii,[23] Zuber and Findlay,[24] and Wallis[20] should be used. Indeed, Eq. 15 is a special form of the drift flux model that is valid when the bubbles are affected by each other's wake and the tube wall. For most practical systems, especially for oil wells, fluids are rarely pure and Eq. 15 is quite appropriate.

Because the in-situ velocity of the gas phase is equal to the superficial gas velocity divided by the gas void fraction, Eq. 15 may be used to arrive at the following expression for the gas void fraction:

$$v_{gs}/f_g=C_0 v_M+v_\infty \quad \text{(16a)}$$

or

$$f_g=v_{gs}/(C_0 v_M+v_\infty). \quad \text{(16b)}$$

For turbulent flow, the mixture velocity at the axis of the pipe is 1.2 times the average mixture velocity. If the gas bubbles are assumed to flow mostly through the central portion of the pipe, as has been shown to be the case for vertical flow, then the value of C_0 is 1.2. A number of correlations are available for the terminal rise velocity of small bubbles. With the recommended Harmathy correlation for v_∞ and with $C_0=1.2$, we arrive at Eq. 17 for the in-situ gas void fraction during vertical bubbly flow:

$$f_g=v_{gs}/\{1.2v_M+1.50[g(\rho_L-\rho_g)\sigma/\rho_L^2]^{1/4}\}. \quad \text{(17)}$$

For a laminar-velocity profile, when the maximum velocity at the axis is twice the average velocity, and when we assume that the bubbles are confined at the central portion of the tube (this assumption is not very reasonable any more because the velocity profile is no longer flat over a significant portion of the channel), then $C_0=2.0$. The value of C_0 probably lies between 1.2 and 2.0 and is likely to be closer to 1.2 because for most practical cases, the Reynolds number based on bubble velocity is much greater than 2,100. The classic work of Zuber and Findlay[24] established a value of $C_0=1.2$ for an air/water system in a 2-in. [5-cm] pipe. Later data of Zuber *et al.*[25] for an air/water system in a 5.5-in. [14-cm] pipe indicate a higher value of 1.6 for C_0. Recent works with bubbles rising up stagnant liquid columns also indicate a higher value for C_0. For example, Mashelkar,[26] Zahrdnik and Kastanek,[27] and Haug[28] estimated C_0 to be 2.0. The data of Hasan *et al.*[8] from an air/water system in a 5-in. [12.7-cm] pipe also suggest a value of about 1.96.

Eq. 17 may be rearranged in the following manner:

$$v_{gs}/f_g=C_0 v_M+v_\infty, \quad \text{(18)}$$

which suggests a linear relationship between the in-situ gas velocity, v_g (or v_{gs}/f_g), and the mixture velocity, v_M. Data gathered for

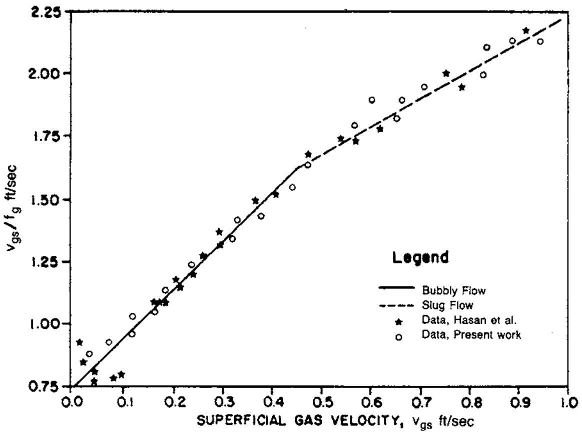

Fig. 5—Ratio of superficial gas velocity to void fraction as a function of gas velocity—5-in. ID circular channel.

this study in a 5-in. [12.7-cm] pipe with air and water and by Hasan *et al.* are shown in Fig. 5, which validates Eq. 18. Our present data indicate a slightly lower value of C_0 (~ 1.6) and a higher value of v_∞ (~ 0.8), more in line with Eq. 17, than that reported earlier.[8]

These disagreements over the exact values of the constants in Eq. 16 reflect the influence of such secondary variables as channel diameter and system pressure,[29] the gas-injection method, the distance from gas-injection point, and various fluid properties. Until the effect of these secondary variables are properly established, it is best to use Eq. 17 (i.e., with $C_0=1.2$, the most widely used value, and the Harmathy equation for v_∞) to estimate gas void fraction in vertical bubbly flow.

Void Fraction in a Tubing/Casing Annulus. The analysis presented for the derivation of Eq. 16 should also apply when the multiphase flow occurs in an annular geometry, such as in the casing of an oil well. Hasan *et al.* showed that Eq. 16 can indeed be used to represent void fraction for an annular system. They argue, however, that the presence of a tubing would change the velocity and bubble-concentration profile from those present in a circular channel. This would influence C_0 and may affect v_∞. The data of Hasan *et al.* indicated that terminal rise velocity, v_∞, does not change significantly because of the presence of any tubing. However, they found the parameter C_0 to increase linearly with the tubing-to-casing-diameter ratio, d_t/d_c, at a rate of $0.371(d_t/d_c)$. Using avalue of 1.2 for C_0 for circular channels, we propose that for an annular geometry

$$C_0=1.20+0.371(d_t/d_c). \quad \text{(19)}$$

Fig. 6 shows some typical data of Ref. 8 and our recent data gathered with an air/water system in a 5-in. [12.7-cm] casing with three different tubing sizes. The validity of Eq. 15 is apparent.

For estimating the total pressure gradient, Eq. 14 can be used with the mixture density calculated from void fraction estimated with Eq. 17. The frictional component can be computed by treating the multiphase mixture as a homogeneous fluid. Thus,

$$(-\mathrm{d}p/\mathrm{d}z)=(2f_M v_M{}^2\rho_M/g_c d)+(g/g_c)\rho_M+[\rho_M v_M(\mathrm{d}v_M/\mathrm{d}z)], \quad \text{(20)}$$

where

$$\rho_M=\rho_g f_g+\rho_L(1-f_g) \quad \text{(21)}$$

and

$$f_g=v_{gs}/\{(1.2+0.371 d_t/d_c)v_M+1.5[g(\rho_L-\rho_g)\sigma/\rho_L{}^2]^{1/4}\}. \quad \text{(17)}$$

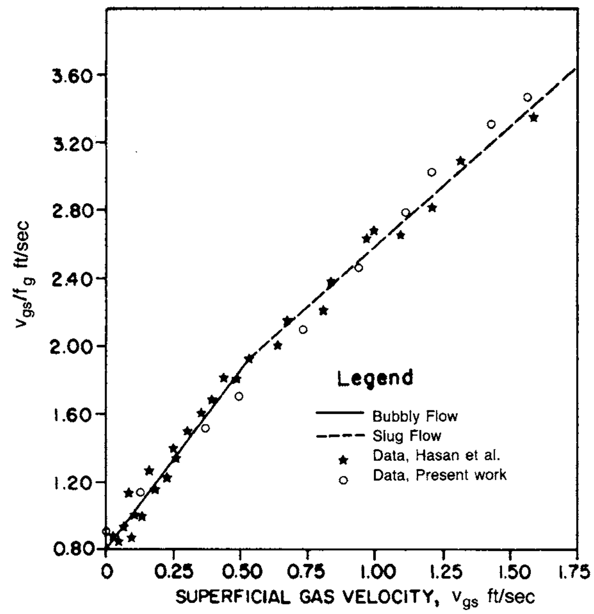

Fig. 6—Ratio of superficial gas velocity to void fraction as a function of gas velocity—1.5-in. tubing, 5-in. casing.

To estimate the friction factor for the mixture, f_M, one may use a Blasius-type correlation valid for smooth pipes because most oil wells become smooth as a result of wax deposition. If the actual pipe roughness is known, however, one may estimate f_M from any standard charts. In any case, the contribution of the frictional component to the total pressure gradient is very small (typically $<10\%$), and refinement for its evaluation is only of academic interest. The accelerational component, represented by the third term of Eq. 14, usually contributes even less than the frictional term, except in vigorous boiling situations such as boilers. The term may be neglected for all practical purposes.

Slug Flow. ***Void Fraction in a Tubing.*** The analysis for slug flow is very similar to that for bubbly flow. Indeed, Eq. 16 applies for the void fraction for slug flow as well, with different constants. Thus,

$$f_g=v_{gs}/(C_1 v_M+v_{\infty T}). \quad \text{(22)}$$

For the value of the flow parameter C_1, we can make an analysis similar to the one we did for bubbly flow. Because the flow is almost surely turbulent and because the bubbles ride the flat portion of the velocity profile, we expect C_1 to be 1.2. This was found to be the case by Nicklin *et al.*[12] and by Hasan *et al.*[8] and is the accepted value for the parameter.

The Taylor bubble-rise velocity, $v_{\infty T}$, in slug flow is given by

$$v_{\infty T}=C_2[gd(\rho_L-\rho_g)/\rho_L]^{0.5}. \quad \text{(6)}$$

A number of researchers[30-34] have presented experimental data and theoretical analyses for the parameter C_2 in Eq. 6. These analyses and data indicate that C_2 is influenced by the forces of inertia, viscosity, and surface tension. The extensive data of White and Beardmore,[33] along with those of Dumitrescu[32] and Wallis,[35] have been represented by the following single equation[20]:

$$C_2=0.345\left[1-\exp\left(\frac{-0.01N_f}{0.345}\right)\right]\left[1-\exp\left(\frac{3.37N_{\mathrm{Eo}}}{m}\right)\right], \quad \text{(23)}$$

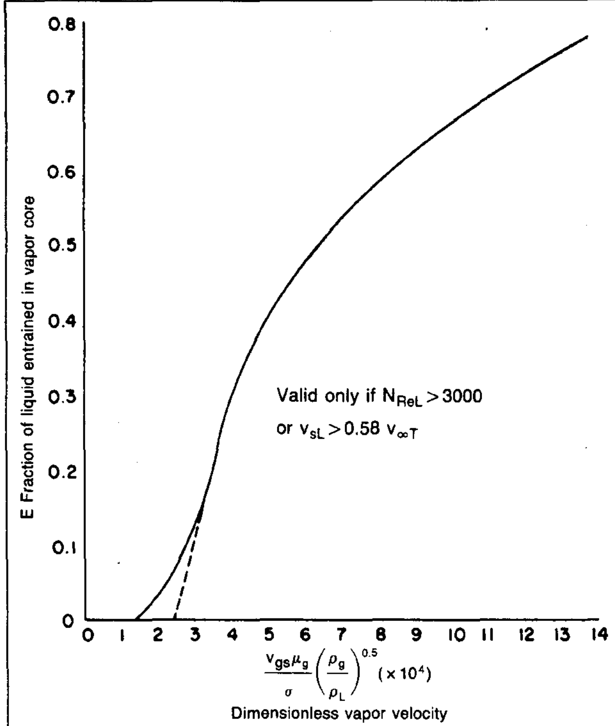

Fig. 7—Entrainment as a function of gas velocity.[37]

where

$$N_f=[d^3g(\rho_L-\rho_g)\rho_L]^{0.5}/\mu_L, \quad \ldots\ldots (24)$$

$$N_{Eo}=gd^2(\rho_L-\rho_g)/\sigma, \quad \ldots\ldots (25)$$

and

$$m=10, \text{ for } N_f>250, \quad \ldots\ldots (26a)$$

$$m=69N_f^{0.35}, \text{ for } 18<N_f<250, \quad \ldots\ldots (26b)$$

and

$$m=25, \text{ for } N_f<18. \quad \ldots\ldots (26c)$$

For large values of N_f (>300) and N_{Eo} (>100), Eq. 23 reduces to

$$C_2=0.345. \quad \ldots\ldots (27)$$

For air/water flow through a 2-in. [5-cm] pipe at standard conditions, $N_f=35{,}000$ and $N_{Eo}=322$. Even for a highly viscous crude with 100-cp [100-mPa·s] viscosity under similar conditions (with $\sigma=30$ dynes/cm [30 mN/m]), $N_f=350$ and $N_{Eo}=817$. Thus, for most practical purposes, $C_2=0.345$ and Eq. 22 for void fraction during slug flow becomes

$$f_g=v_{gs}/[1.2v_M+0.345\sqrt{gd(\rho_L-\rho_g)/\rho_L}\,]. \quad \ldots\ldots (28)$$

Eqs. 22 and 28 may be rearranged in the form of Eq. 18, which allows estimation of the parameter C_1 and the Taylor bubble-rise velocity, $v_{\infty T}$, from a plot of v_{gs}/f_g vs. v_M. Such a plot of our data from a 5-in. [12.7-cm] pipe is shown , along with our earlier data,[8] in Fig. 5. Our estimate of 1.18 for C_1 is in excellent agreement with Eq. 28. However, our data give a slighly lower estimate of 0.30 for C_2 than Eq. 27, but Eq. 27 agrees well with the theoretical value of 0.328 predicted by Davis and Taylor.[31] However, $C_2=0.345$ has found much wider acceptance and therefore is recommended.

Void Fraction in a Tubing/Casing Annulus. As in the case of bubbly flow, void fraction during slug flow in an annular geometry may be represented by Eq. 22.[8] In flowing up an annulus, Taylor bubbles loose their symmetry about the channel axis. This means that both C_1 and $v_{\infty T}$ would be affected by the casing and the tubing dimensions. Hasan *et al.* found both C_1 and C_2 to be linearly dependent on the tubing-to-casing-diameter ratio and recommended the use of equivalent diameter d_e $(=d_t-d_c)$ in place of the diameter d in Eq. 5 for $v_{\infty T}$. Their proposed correlations for C_1 and $v_{\infty T}$ are

$$C_1=1.182+0.90(d_t/d_c) \quad \ldots\ldots (29)$$

and

$$v_{\infty T}=[0.30+0.22(d_t/d_c)][\sqrt{g(d_t/d_c)(\rho_L-\rho_g)/\rho_L}\,]. \quad \ldots\ldots (30)$$

We point out that Griffith[34] also observed similar variation of C_1 and $v_{\infty T}$. His data, however, indicate a weaker dependence of C_1 on d_t/d_c. He also suggests use of casing diameter instead of the equivalent diameter for estimating $v_{\infty T}$. Our recent data, shown in Fig. 6, agree quite well with Eqs. 29 and 30.

As in bubbly flow, the total pressure gradient can be obtained by adding the three components, making use of void fraction predicted by Eq. 28. The estimation of the frictional component presents some difficulty because some of the liquid flows downward in a film against the Taylor bubble, while most of the liquid flows upward in the liquid slugs. Wallis[20] suggested that the wall shear stress around the vapor bubble be ignored. This is equivalent to saying that the liquid actually flows a distance of $z(1-f_g)$ instead of z. With this assumption, the total pressure gradient becomes

$$(-dp/dz)_a=(2f_Mv_M{}^2\rho_L/g_cd)(1-f_g)+(g/g_c)\rho_M+(-dp/dzA), \quad \ldots\ldots (31)$$

where ρ_M is calculated from $\rho_gf_g+\rho_L(1-f_g)$ and f_g is estimated from Eq. 30. Note that Wallis and Govier and Aziz[30] suggest using ρ_L, along with v_M, in calculating the frictional component. The product $\rho_L(1-f_g)$ is very nearly equal to ρ_M for low-pressure systems, indicating the similarity in evaluating the frictional term in slug and bubbly flow. The contribution of the frictional component is no longer negligible but is still small (typically 10% of the total gradient). Acceleration, however, is still small and may be neglected or evaluated with the approach suggested for bubbly flow.

Wallis indicated that Eq. 28 slightly underestimates the void fraction, especially when long bubbles are present. His method for accounting for the presence of long bubbles, however, leads to too low an estimate of the pressure gradient. In view of the complicated calculations involved and the questionable improvement in accuracy, the procedure is not recommended. We believe that Eq. 31, along with the void fraction from Eq. 28, is adequate.

Churn Flow. The churn- or froth-flow pattern is rather difficult to analyze and has not been investigated extensively. The works of Harmathy,[10] Peebles and Garber,[36] and Zuber and Findlay[24] suggest that the analysis procedure presented for bubbly and slug flow may be applied to the churn-flow pattern as well. After reviewing these works, Govier and Aziz recommended that the equations developed for slug flow—Eq. 28 for predicting void fraction and Eq. 31 for predicting pressure drop—be used for the churn-flow regime as well. However, the chaotic nature of the flow would tend to make the mixture velocity and the gas-concentration profiles flat. These profiles would suggest a value of about 1.16 for the flow parameter C_0, rather than 1.2, which was used for the slug flow regime.

Annular Flow. In annular flow, the gas flows through the central core of the pipe while the liquid flows along the tube wall as a film. Therefore, the system may be looked upon as the single-phase flow of gas through a tube of slightly reduced diameter because of the liquid film. If we assume ideal annular flow (i.e., no liquid is being carried as droplets in the gas phase and the gas/liquid interface is smooth), then the problem reduces to that of estimating pressure

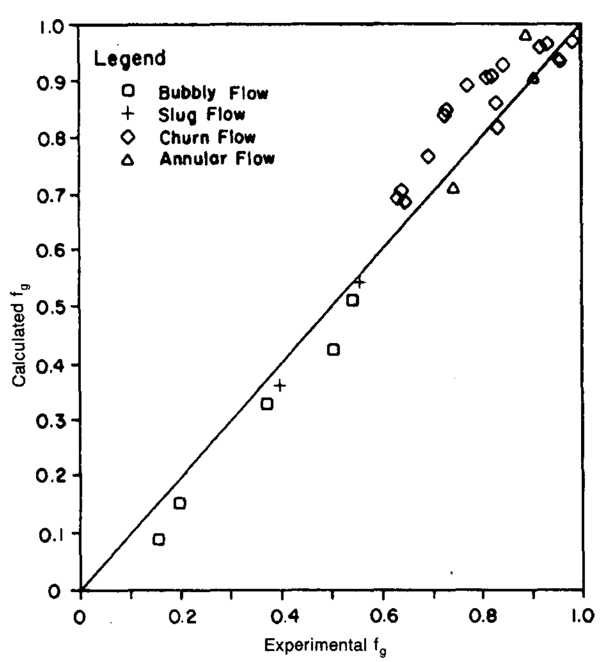

Fig. 8—Comparison of the theory with the void-fraction data of Beggs.

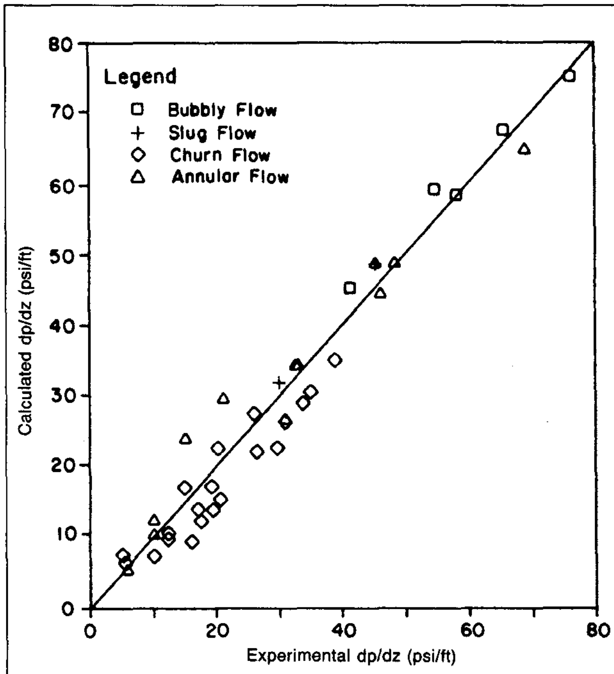

Fig. 9—Comparison of the theory with the pressure-gradient data of Beggs and Sevigny.

drop in single-phase gas flow. In that case, we do not even need to estimate void fraction because we are looking at truly single-phase flow. The liquid film thickness is typically less than 5% of the tube diameter, thus introducing little error even if it is neglected in calculating tube diameter.

Unfortunately, however, annular flow is rarely ideal. Usually, a substantial fraction of the liquid is carried as droplets in the gas stream. The existence of the liquid droplets causes two problems: the fluid density in the core is now different from that of gas alone, and the mixture is now a truly two-phase mixture with the possibility of the liquid phase moving at a slower velocity than the gas phase. In addition, the gas/liquid interface is usually wavy. Thus, determining the appropriate friction factor becomes very difficult.

In general, we may assume that the liquid-droplet velocity is equal to that of the gas in the core. With this assumption in mind, we can write Eq. 32 for the total pressure gradient during annular flow:

$$-\mathrm{d}p/\mathrm{d}z=(1/g_c)[g\rho_c+(2f_c\rho_c v_g{}^2/d)+(\rho_c v_g \mathrm{d}v_g/\mathrm{d}z)]. \quad \ldots (32)$$

The problem then reduces to estimating the density of the fluid in the core, ρ_c, and the friction factor, f_c, for gas flowing through a rough pipe. Note that in Eq. 32 the accelerational contribution contains the differential $\mathrm{d}v_g/\mathrm{d}z$. Using the gas law, we can write Eq. 32 in terms of $\mathrm{d}p/\mathrm{d}z$:

$$-\frac{\mathrm{d}p}{\mathrm{d}z}=\frac{1}{g_c}\frac{[g\rho_c+(2f_c\rho_c v_g{}^2/d)]}{[1-(\rho_c v_g{}^2/pg_c)]}. \quad \ldots (33)$$

The estimation of void fraction needed to calculate the in-situ gas velocity is discussed later.

Entrainment Estimation. To determine the density of the fluid flowing through the core, it is necessary to estimate the fraction of total liquid, E, that is entrained in the gas core. Steen and Wallis[37] suggested that when the liquid film is fully turbulent $[d\rho_M v_M(1-x)\rho_L>3{,}000]$, the fraction E of the input liquid entrained in the vapor core is a unique function of the critical vapor velocity, $(v_{gs})_c$, defined as follows:

$$(v_{gs})_c=v_{gs}\mu_g(\rho_g/\rho_L)^{0.5}/\sigma \quad \ldots (34)$$

$$=\rho_M v_M x\mu_g/(\sigma\sqrt{\rho_L\rho_g}).$$

The Steen and Wallis correlation for E with critical vapor velocity is shown in Fig. 7. The curve in Fig. 7 may be represented by the following equations:

$$E=0.0055[(v_{gs})_c\times10^4]^{2.86}, \text{ if } (v_{gs})_c\times10^4<4 \quad \ldots (35)$$

and

$$E=0.857\log_{10}[(v_{gs})_c\times10^4]-0.20, \text{ if } (v_{gs})_c\times10^4>4. \quad \ldots (36)$$

Of course, entrainment cannot be more than 1. Therefore, if Eq. 36 predicts an entrainment value greater than 1, it is set equal to 1. Once entrainment is estimated, the gas-core density, ρ_c, can be determined easily, considering the gas core as the system with a diameter approximately equal to that of the pipe. In that case, superficial gas velocity remains equal to v_{gs}, but the superficial liquid velocity becomes Ev_{Ls} because only the entrained fraction of the liquid is in the system. Hence, for the core fluid (not the entire pipe),

$$f_{gc}=v_{gs}/(v_{gs}+Ev_{Ls}) \quad \ldots (37a)$$

and

$$\rho_c=f_{gc}\rho_g+(1-f_{gc})\rho_L$$

$$=(v_{gs}\rho_g+Ev_{Ls}\rho_L)/(v_{gs}+Ev_{gs}). \quad \ldots (37b)$$

Collier[29] suggests an alternative method based on the work of Hutchinson and Whalley,[38] which views the process of liquid entrainment as one in which liquid is being torn out as droplets into the gas stream, while liquid droplets from the gas stream are also being deposited onto the liquid film. The method needs verification.

Film Friction Factor. A number of correlations are available for predicting the liquid film roughness or the film friction factor, f_c. Probably the best among these is that proposed by Wallis[20]:

$$f_c=f_{fg}(1+300\delta/d), \quad \ldots (38)$$

which reduces to

$$f_c=f_{fg}[1+75(1-f_g)]. \quad \ldots (39)$$

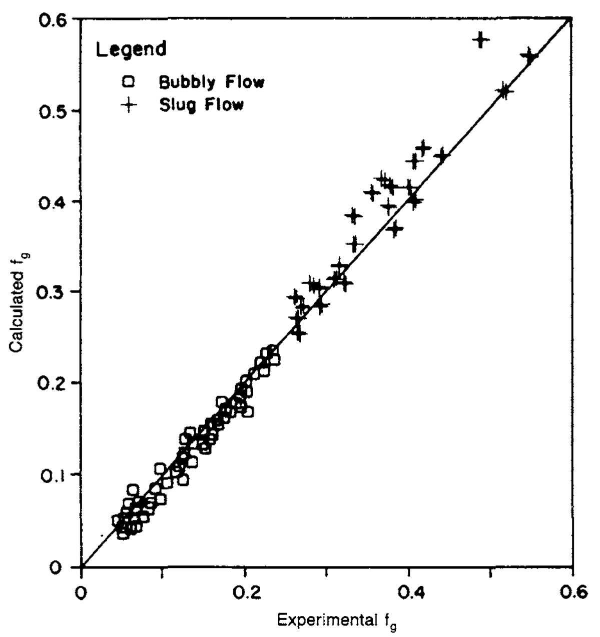

Fig. 10—Comparison of the theory with the void-fraction data of Lau.

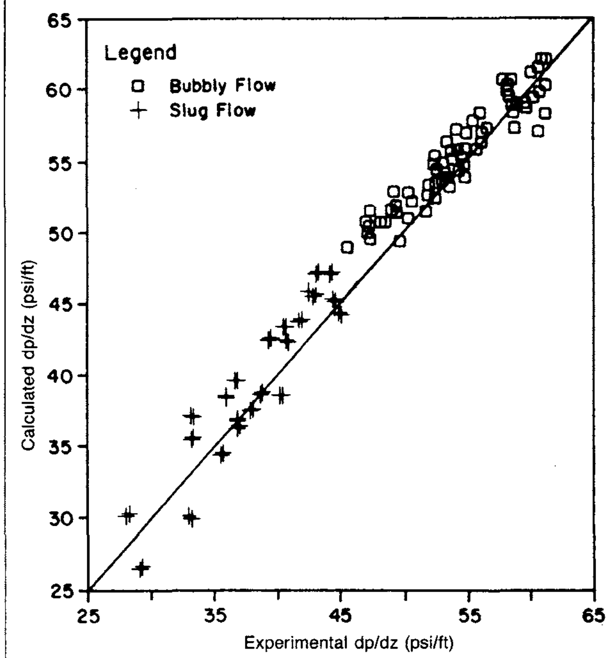

Fig. 11—Comparison of the theory with the pressure-gradient data of Lau.

An estimate of the liquid film thickness, δ, or the gas void fraction, f_g, is needed because the film friction factor is dependent on this quantity. This may be done with a suitable empirical correlation, such as the Lockhart-Martinelli,[39] the Hughmark,[40] or the Guzhov *et al.*[41] correlation. Because of its accuracy,[42] we recommend the Lockhart-Martinelli correlation, which expresses void fraction in terms of the Lockhart-Martinelli parameter, X. The graphical relationship can be expressed by the following simple equation[20]:

$$f_g=(1+X^{0.8})^{-0.378}. \qquad (40)$$

The Lockhart-Martinelli parameter, X, is defined in terms of the ratio of the frictional pressure gradient for the liquid, flowing alone in the channel, to that for the gas phase flow:

$$X=\sqrt{(\mathrm{d}p/\mathrm{d}z_L)_F/(\mathrm{d}p/\mathrm{d}z_g)_F}. \qquad (41)$$

For turbulent flow of both gas and liquid phases, X can be written in terms of the gas mass fraction, x, and fluid properties as follows:

$$X=[(1-x)/x]^{0.9}\sqrt{(\rho_g/\rho_L)}\,(\mu_L/\mu_g)^{0.1}. \qquad (42)$$

The gas void fraction in case of annular flow means the channel volume not occupied by the liquid film. Hence, the gas-mass-fraction calculation should include the entrained liquid droplets.

Comparison With Laboratory Data

The void-fraction and pressure-gradient predictions of the theory presented in this paper for vertical upflow of two-phase fluids are compared with published data from diverse sources. In Fig. 8, the predictions of the theory are plotted vs. the void-fraction data of Beggs[43] for vertical systems. Beggs' data were gathered with air/water in 1.5- and 1-in. [3.8- and 2.5-cm] pipes at 4- to 7-atm [405- to 709-kPa] pressures and 50 to 100°F [10 to 38°C] temperatures. The agreement between the data and predictions is excellent for all flow regimes except churn flow. Fig. 8 shows that the theory appears to overestimate void fraction during churn flow slightly, suggesting a somewhat higher value for C_1 than is actually used in Eq. 28. However, we also point out that the highly fluctuating nature of churn flow makes accurate data gathering difficult.

Fig. 9 compares the pressure gradient data of Beggs[43] and Sevigny[44] with the predictions of our theory. The agreement is excellent for all flow regimes. Our theory predicts the pressure-gradient data of Beggs with a standard deviation of 0.025 psi/ft [0.566 kPa/m] and an average error of −2.0%. Beggs' correlation, containing seven optimized parameters, predicts his own pressure-gradient data with +2.0% average error and a standard deviation of 0.016 psi/ft [0.362 kPa/m]. We estimate the void-fraction data of Beggs with an average percentage error of +0.427% and a standard deviation of 0.063. The average percentage error of Beggs' correlation in predicting his void-fraction data is 0.882% and the standard deviation is 0.0225. Our predictions for Sevigny's pressure-gradient data are slightly worse; the average error is −2.8% and the standard deviation is 0.0326 psi/ft [0.737 kPa/m].

Fig. 10 is a similar plot of Lau's[45] data gathered for an air/water system in a 1.90-in. [4.83-cm] pipe at pressures below 7 atm [709 kPa] and at room temperatures. All the data fall in the bubbly- and slug-flow regimes. The theory appears to underestimate the void fraction in bubbly-flow regime slightly. However, the pressure-gradient data, shown in Fig. 11, indicate no systematic deviation from theory. On the average, our theory underestimated void fraction by 4.6% with a standard deviation of 0.02. The average error in predicting the pressure-gradient data is +2.35% and the standard deviation is 0.0135 psi/ft [0.3054 kPa/m].

Conclusions

This study presents a model for predicting flow behavior of two-phase gas/oil mixtures in vertical oil wells. The major advantage of the proposed method is that it is based on the physical behavior of the flow and therefore is more reliable than available correlations under diverse production conditions. Data from various sources verify the accuracy of the model.

Specifically, the model is capable of predicting flow regime, void fraction, and pressure drop at any point in the flow string. Development of the model is summarized below and the proposed equations are presented in the Appendix.

1. The bubbly/slug transition criteria, void fraction, and pressure drop in bubbly- and slug-flow regimes are developed from our experimental work. Data from other sources are also used to support the proposed theory.
2. An appropriate model is proposed to handle two-phase bubbly and slug flow in an annular channel, such as in a tubing/casing annulus. Our laboratory data are used to develop this model.

3. Transition criteria for the existence of annular-flow and pressure-drop prediction are based on sound hydrodynamic theory. However, the same cannot be stated for the churn flow. Because of lack of extensive work, slug/churn transition criteria remain a weak link in the model.

Nomenclature

C_d = drag coefficient
C_0 = flow parameter in bubbly flow, dimensionless
C_1 = flow parameter in slug flow, dimensionless
C_2 = constant given by Eq. 2, dimensionless
d = pipe diameter, ft [m]
d_c = casing diameter, ft [m]
d_d = droplet (liquid) diameter, ft [m]
dp/dz = pressure gradient: $(dp/dz)_F$, $(dp/dz)_A$, and $(dp/dz)_H$ are three components [frictional, accelerational, and potential (static head)] of total pressure gradient, lbf/ft^2-ft [Pa/m]
d_t = tubing diameter, ft [m]
E = entrainment
f = Fanning friction factor, dimensionless
f_g = in-situ volume fraction of lighter phase (void fraction), dimensionless
g = acceleration due to gravity, ft/sec^2 [m/s^2]
g_c = conversion factor, 32.2 lbm-ft/lbf-sec^2 [1]
m = parameter in Eq. 23
N_{Eo} = Eötvös number based on pipe diameter, $gd^2(\rho_L-\rho_g)/\sigma$
N_f = inverse viscosity number $[d^3g(\rho_L-\rho_g)\rho_L]^{0.5}/\mu_L$
N_{Fr} = Froude number, v^2/gd
N_{Re} = Reynolds number, $dv\rho/\mu = dG/\mu$
N_{Wec} = critical Weber number
p = pressure, lbf/ft^2 [Pa]
v = in-situ velocity of any given phase, ft/sec [m/s]
v_M = superficial velocity of mixture, total volumetric flow rate/total cross-sectional area, ft/sec [m/s]
v_s = superficial velocity of any given phase, phase flow rate/total cross-sectional area, ft/sec [m/s]
v_∞ = terminal bubble-rise velocity in bubbly flow, ft/sec [m/s]
$v_{\infty T}$ = terminal rise velocity of a single Taylor bubble (in slug flow), ft/sec [m/s]
x = mass fraction (quality) of gas phase (or lighter phase), dimensionless
X = Lockhart-Martinelli parameter, $[(dp/dz)_{FL}/(dp/dz)_{Fg}]^{0.5}$; $\sqrt{\rho_g/\rho_L}\ [(1-x)/x]^{0.9}(\mu_L/\mu_g)^{0.1}$ for turbulent flow, dimensionless
z = axial distance in pipe, ft [m]
δ = film thickness, ft [m]
μ = viscosity, lbm/ft-sec [kg/m·s]
ρ = density, lbm/ft^3 [kg/m^3]
σ = surface tension, lbm/sec^2 [N/m]

Subscripts

A = accelerational
c = core fluid (in case of annular flow)
e = equivalent
f = fluid (single or multiphase)
F = frictional
g = gas
gcs = superficial critical gas
gs = superficial gas
H = static head
L = liquid
Ls = superficial liquid
m = mixture (multiphase)
o = oil

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Appendix—Summary of the Model

Bubbly.

Transition Criteria: $v_{gs} < (0.429 v_{Ls} + 0.357 v_\infty)$

or $f_g < 0.52$ and $v_M{}^{1.12} > 4.68(d)^{0.48}[g(\rho_L - \rho_g)/\sigma]^{0.5} \times (\sigma/\rho_L)^{0.6}(\rho_M/\mu_L)^{0.08}$.

Void Fraction: $f_g = v_{gs}/(C_0 v_M + v_\infty)$, $C_0 = 1.2 + 0.371(d_t/d_c)$,

$\rho_M = (1-f_g)\rho_L + f_g\rho_g$.

Pressure Drop: $(dp/dz)_F = 2 f_M v_M{}^2 \rho_M / g_c d$,

f_M from $(d v_M \rho_L/\mu_L)$.

Slug.

Transition Criteria: $v_{gs} > (0.429 v_{Ls} + 0.357 v_\infty)$

and $v_{gs}{}^2\rho_g < [17.1 \log_{10}(\rho_L v_{Ls}{}^2) - 23.2]$,

if $v_{Ls}{}^2\rho_L > 50$.

$v_{gs}{}^2\rho_g < 0.00673(v_{Ls}{}^2\rho_L)^{1.7}$, if $v_{LS}{}^2\rho_L < 50$.

Void Fraction: $f_g = v_{gs}/(C_1 v_M + v_{\infty T})$, $C_1 = 1.18 + 0.90(d_t/d_c)$,

$\rho_M = (1-f_g)\rho_L + f_g\rho_g$.

Pressure Drop: $(dp/dz)_F = 2 f_M v_M{}^2 \rho_L (1-f_g)/g_c d$,

f_M from $(d v_M \rho_L/\mu_L)$.

Churn.

Transition Criteria: $v_{gs} 3.1[\sigma g(\rho_L - \rho_g)/\rho_g{}^2]^{0.25}$

and $v_{gs}{}^2\rho_g > [17.1 \log_{10}(\rho_L v_{Ls}{}^2) - 23.2]$,

if $v_{Ls}{}^2\rho_L > 50$.

$v_{gs}{}^2\rho_g > 0.00673(v_{Ls}{}^2\rho_L)^{1.7}$ if $v_{LS}{}^2\rho_L < 50$.

Void Fraction: $f_g = v_{gs}/(C_1 v_M + v_{\infty T})$, $C_1 = 1.15 + 0.90(d_t/d_c)$,

$\rho_M = (1-f_g)\rho_L + f_g\rho_g$.

Pressure Drop: $(dp/dz)_F = 2 f_M v_M{}^2 \rho_L (1-f_g)/g_c d$,

f_M from $(d v_M \rho_L/\mu_L)$.

Annular.

Transition: $v_{gs} > 3.1[\sigma g(\rho_L - \rho_g)/\rho_g{}^2]^{0.25}$.

Void Fraction: $f_g = (1 + X^{0.8})^{-0.378}$,

$\rho_c = (v_{gs}\rho_g + E v_{Ls}\rho_1)/(v_{gs} + E v_{gs})$.

Pressure Drop: $f_c = 0.079[1 + 75(1-f_g)]/N_{\mathrm{Reg}})^{0.25}$,

$X = (\rho_g/\rho_L)^{0.5}[(1-x)/x]^{0.9}(\mu_L/\mu_g)^{0.1}$.

SI Metric Conversion Factors

ft × 3.048*	E−01	=	m
in. × 2.54*	E+00	=	cm

*Conversion factor is exact.

SPEPE

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Erratum

A.R. Hasan and C.S. Kabir note the following errors in their papers.

"A Study of Multiphase Flow Behavior in Vertical Wells" (May 1988 *SPEPE*, Pages 263-72): (1) in Eq. 6, ρ_m should be ρ_L; (2) in Eq. 30, the term under the square root should be $g(d_c - d_t)[(\rho_L - \rho_g)/-\rho_L]$; (3) the left side of Eq. 31 should be $(-dp/dz)$ and the right side should read $(-dp/dz)_A$; (4) in the Appendix, Line 3 of bubbly transition criteria, ρ_M should be ρ_L; and (5) in Figs. 9 and 11, the units for the x and y axes should be psf/ft.

"Predicting Multiphase Flow Behavior in a Deviated Well" (Nov. 1988 *SPEPE*, Pages 474-82): (1) in Eq. 5, ρ_m should be ρ_L; (2) the subheadings **Bubbly Flow** should be **Bubbly Flow.** ***Void Fraction in a Tubing,*** **Bubbly Flow in a Tubing** should be ***Void Fraction in a Tubing/Casing Annulus,*** **Slug Flow** should be **Slug Flow.** ***Void Fraction in a Tubing,*** and **Slug Flow in a Tubing** should be ***Void Fraction in a Tubing/Casing Annulus;*** (3) in Table 2, Line 2 under bubbly transition criteria, ρ_m should be ρ_L and in slug rise velocity, d should be d_c; and (4) in Figs. 7, 9, and 10, the units for the x and y axes should be psf/ft.