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Pressure Drop Correlations for Inclined Two-Phase Flow

Pressure drop behavior in two-phase inclined flow was studied. For bubble and slug flow, a no-slip friction factor calculated from the Moody diagram was found adequate for friction head loss calculations. In downhill stratified flow, the friction pressure gradient is calculated based on a momentum balance equation for either phase assuming a smooth gas-liquid interface. For this calculation, a prior knowledge of the liquid holdup is needed. For annular-mist flow, a friction factor correlation is presented that is a function of holdup ratio and no-slip Moody friction factor. Results agree well with the experimental data and correlations were further verified with Prudhoe Bay and North Sea data.

Introduction

Accurate prediction of pressure drop is extremely important when designing inclined two-phase flow systems such as hilly terrain pipelines or directional wells. Pipe inclination can appreciably affect flow patterns, slippage between phases and energy transfer between phases. No method exists for performing these calculations that is accurate for all flow conditions.

Historically, pressure gradients in inclined flow have often been calculated using vertical or horizontal two-phase flow correlations. This is often satisfactory if the pipe inclination is very near the inclination for which the correlation was developed. However, in many applications this may not be the case.

In 1958, Flanigan [6] proposed a method to calculate two-phase pressure drop in hilly terrain pipelines. This method involved the use of a correlation for liquid holdup in the uphill sections of a pipeline as a function of superficial gas velocity. The Flanigan liquid holdup correlation was developed using field data obtained from a 16-in. (40.64-cm) pipeline and the data of Baker [1]. The Flanigan method ignores pressure recovery in downhill sections, and the effect of pipe inclination angle on the liquid holdup was considered negligible. Flanigan used the Panhandle equation for gas flow with an efficiency factor dependent on the liquid volume present to calculate the pressure drop due to friction. Frequently, the Dukler, et al. [4] friction head loss correlation with the Flanigan hill correction is used for inclined two-phase flow pressure drop calculations.

In 1972, Beggs and Brill [2] presented a two-phase flow correlation that predicts liquid holdup and pressure drop as functions of inclination angle. They developed an empirical liquid holdup correlation that depended on a predicted flow pattern for horizontal flow, no-slip holdup, Froude number and an inclination angle correction factor. A correlation was also presented for predicting two-phase friction factor normalized with the no-slip friction factor calculated from the Moody diagram for smooth pipe. Modifications commonly used to improve results with the Beggs and Brill correlation are to use a rough pipe normalizing friction factor and to reduce uphill and downhill liquid holdup predictions according to the recommendation of Payne, et al. [13]

Spedding, et al [15] in 1982 reported pressure drop data in two-phase inclined flow in pipes for both uphill and downhill flow. The experiments were performed in a 4.55-cm-dia pipe for air-water flow at near atmospheric pressure.

Experimental Program

An experimental facility was designed and constructed to obtain the desired test data. A schematic diagram of the test facility is shown in Fig. 1. The test sections consisted of an inverted U-shaped 1.5-in. (3.8-cm) i.d. nominal steel pipe. The closed end of the U-shaped test sections could be raised or lowered to any angle from 0 to ±90 deg from horizontal. Each leg of the *U* was 56-ft (17 m) long with 22-ft (6.7-m) entrance lengths followed by 32-ft (9.8-m) long test sections on both uphill and downhill sides. Each test section could be isolated from the rest of the piping by pneumatic ball valves that could be opened or closed simultaneously when calibrating holdup sensors. Pressure taps 30.5 ft (9.3 m) apart were located in each test section to permit measuring absolute and differential pressures using Validyne transducers shown in Fig. 2. A 7-ft (2.1-m) long transparent lexan pipe section was located in each test section to permit flow pattern observations and mounting of capacitance-type holdup sensors.

The oil and gas phases were carefully metered before mixing, using turbine meters, orifice meters or rotameters, depending on the phase and the flow rates. The two-phase mixture flowed through the test sections and into a horizontal separator. The air was vented to the atmosphere and the liquid passed through a solid particle filter and into a storage tank.

Kerosene and lube oil were used as the liquid phases. The surface tension, density and viscosity of the kerosene at 60°F (15.5°C) were 26 dynes/cm (26×10^{-5} N/cm), 51 lb_m/ft^3 (817 $kg.m^{-3}$) and 2 cp (2×10^{-3} Pa.s), respectively. Corresponding values for the lube oil were 35 dynes/cm (35×10^{-5}N/cm), 53 lb_m/ft^3 (849 $kg.m^{-3}$) and 29 cp (29×10^{-3} Pa.s).

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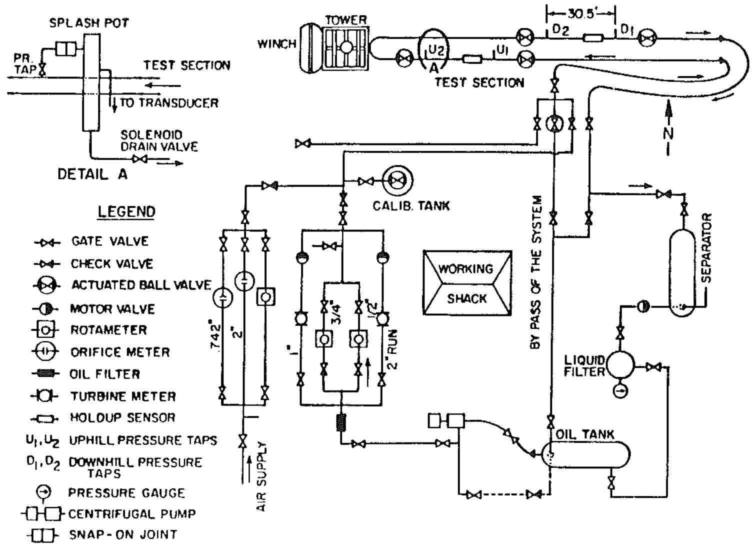

Fig. 1 Schematic diagram of inclined flow test facility

Temperatures between 18°F (−7.8°C) and 132°F (55.6°C) were encountered during the tests. The experimental program is discussed in detail by Mukherjee [9].

Pressure Drop Correlations

The general form of the steady-state mechanical energy balance equation for two-phase flow yields the following pressure balance:

$$\Delta p_t = \Delta p_h + \Delta p_a + \Delta p_f \tag{1}$$

where

$\Delta p_h = \gamma_m\, L \sin(\theta)$ except for stratified flow, and
$= \gamma_g\, L \sin(\theta)$ for stratified flow,

and $\quad \gamma_m = H_L \gamma_L + H_g \gamma_g$.

Also, $\quad \Delta p_a = 0$ in case of stratified flow, and

$$= \frac{\gamma_m v_m v_{sg}\, dp}{g\, \bar{p}} \text{ in other flow patterns.}$$

The part of total pressure drop designated as Δp_f represents the irreversible losses due to the shear of fluids at the pipe wall. For single-phase flow

$$\Delta p_f = \frac{f\, L\, v^2\, \gamma}{2\, g\, d} \tag{2}$$

If $\quad N_{\text{Re}} \le 2000$, then $f = 64/N_{\text{Re}}$. $\quad$ (3)

Otherwise, the Colebrook equation can be used

$$\frac{1}{\sqrt{f}} = 1.74 - 2\ \text{Log}_{10}\left(2\frac{\epsilon}{d} + \frac{18.7}{N_{\text{Re}}\sqrt{f}}\right) \tag{4}$$

where $\quad N_{\text{Re}} = \dfrac{d\, v\, \rho}{\mu}$. $\quad$ (5)

More simple expressions exist for special cases of smooth pipe or fully turbulent flow. For single-phase flow, all the flow and fluid properties are those of the flowing fluids. For two-phase flow, these properties are calculated for in-situ gas and liquid mixtures. Depending on the flow pattern, these mixture properties are calculated based on weighted average properties of individual phases with volumetric fractions as weighting factors. To achieve this, both flow patterns and phase holdups at these flow patterns have to be predicted. Brief discussions on prediction algorithms for flow pattern, holdup and friction head loss follow.

Nomenclature

A = area
d = pipe diameter
$\dfrac{dp}{dL}$ = spatial derivative of p, pressure gradient
D_E = equivalent diameter of cross section of pipe occupied by given phase
e = relative percent error, (calc. value − meas. value) × 100/meas. value
f = Moody friction factor
g = acceleration due to gravity
h_L = depth of liquid phase in pipe cross section along centerline
H = holdup or volume fraction
L = length of pipe
N_{gv} = gas velocity number, $v_{sg}\, (\rho_L/g\,\sigma_L)^{1/4}$
N_{Lv} = liquid velocity number, $v_{sL}\, (\rho_L/g\,\sigma_L)^{1/4}$
N_L = liquid viscosity number, $\mu_L\, [1/(\rho_L\, \sigma_L{}^3)]^{1/4}$
N_{Re} = Reynolds number, $d\, v\, \rho/\mu$
P = perimeter of pipe
p = pressure
r = pipe radius
T_i = interface shear stress
T_w = wall shear stress
v = velocity
W = width
γ = specific weight
δ = angle subtended by liquid-gas interface at center of pipe
Δ = difference
ϵ = absolute pipe roughness
λ = no-slip holdup or volume fraction
μ = viscosity
ρ = density
σ = surface tension, standard deviation
θ = pipe inclination angle from horizontal

Subscripts

a = acceleration
c = calculated
f = friction
g = gas
h = hydrostatic
i = interface
L = liquid
m = mixture
ns = no-slip
R = ratio
sg = superficial gas
sL = superficial liquid
t = total

Flow Pattern Correlation. Mukherjee and Brill [10] presented a complete study on flow pattern transitions in inclined two-phase flow. The flow patterns used in this work are adequately defined in the referred paper.

Two transitions were fitted in uphill flow. The bubble-slug transition was found to be linear and at 45 deg with the axis. The equation of these sets of straight lines was found to be

$$N_{LV} = 10^{**}[\text{Log}_{10}(N_{gv}) + 0.940 + 0.074 \sin\theta - 0.855 \sin^2\theta + 3.695\, N_L] \quad (6)$$

The slug-annular mist transition was found to be identical for all uphill and downhill angles. However, liquid viscosity was found to have a marked effect on this transition. Increased liquid viscosity accelerates the transition from slug to annular-mist flow. The equation of this transition curve was

$$N_{gv} = 10^{**}(1.401 - 2.694\, N_L + 0.521\, N_{Lv}{}^{0.329}) \quad (7)$$

and is independent of inclination angle.

In downhill and horizontal flow, the equation for the bubble-slug transition was found to be

$$N_{gv} = 10^{**}[0.431 + 1.132 \sin\theta - 3.003\, N_L - 1.138\, (\text{Log}_{10}N_{Lv})\sin\theta - 0.429\, (\text{Log}_{10}N_{Lv})^2 \sin\theta] \quad (8)$$

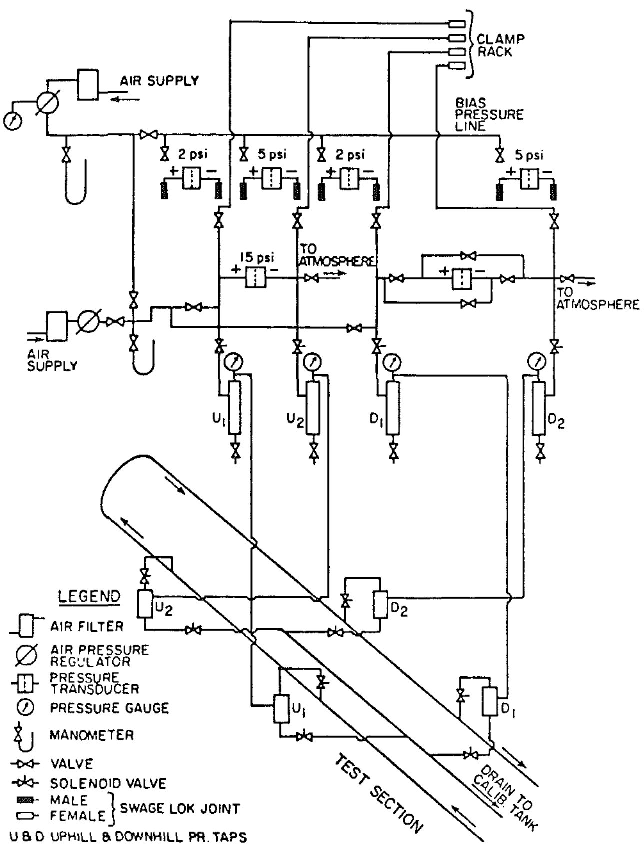

Fig. 2 Schematic diagram of the pressure measurement system

This transition generates a family of curves for different angles of inclination and liquid viscosities. In horizontal flow this transition becomes a function of viscosity only and is a vertical straight line.

The downhill stratified flow boundary is given by the equation

$$N_{Lv} = 10^{**}(0.321 - 0.017\, N_{gv} - 4.267 \sin\theta - 2.972\, N_L - 0.033\, (\text{Log}_{10}N_{gv})^2 - 3.925 \sin^2\theta) \quad (9)$$

Liquid Holdup Correlation. Three liquid holdup correlations were developed using regression analysis [11]. One of these was for horizontal and uphill flow. The other two were one each for downhill stratified flow and other downhill flow patterns. The general holdup equation is shown in equation (10). The regression coefficients are given in Table 1.

$$H_L = \text{EXP}\left\{ C_1 + C_2 \sin\theta + C_3 \sin^2\theta + C_4 N_L^2) \frac{N_{gv}{}^{C_5}}{N_{Lv}{}^{C_6}} \right\} \quad (10)$$

Friction Head Loss in Bubble and Slug Flow. For the data observed to be in the bubble and slug flow patterns, the total friction head loss was calculated from equation (1) as follows:

$$\Delta p_f = \Delta p_t - \Delta p_h - \Delta p_a \quad (11)$$

Since $$\Delta p_f = \frac{f\, L\, v_m^2 \gamma_m}{2\, g\, d} \quad (12)$$

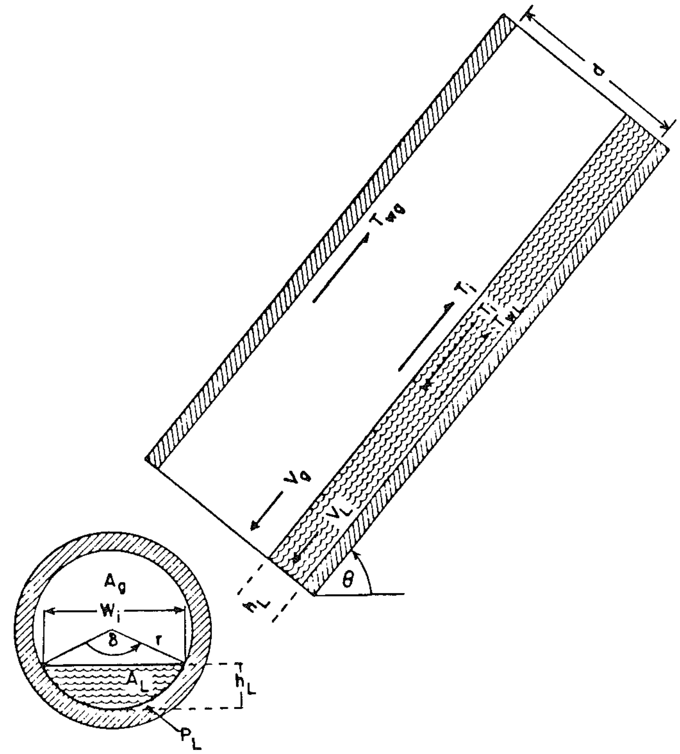

Fig. 3 Control volume and definition of variables for stratified flow model

Table 1 Coefficients for liquid holdup equation

Flow Direction	Flow Pattern	Values of Coefficients					
		C_1	C_2	C_3	C_4	C_5	C_6
Uphill and Horizontal Flow	All	−0.380113	0.129875	−0.119788	2.343227	0.475686	0.288657
Downhill Flow	Stratified	−1.330282	4.808139	4.171584	56.262268	0.079951	0.504887
	Other	−0.516644	0.789805	0.551627	15.519214	0.371771	0.393952

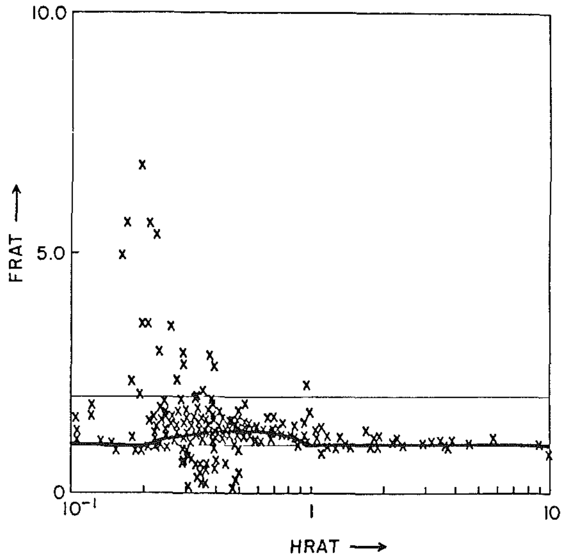

Fig. 4 Plot of friction factor ratio versus holdup ratio for annular flow at all angles of inclination

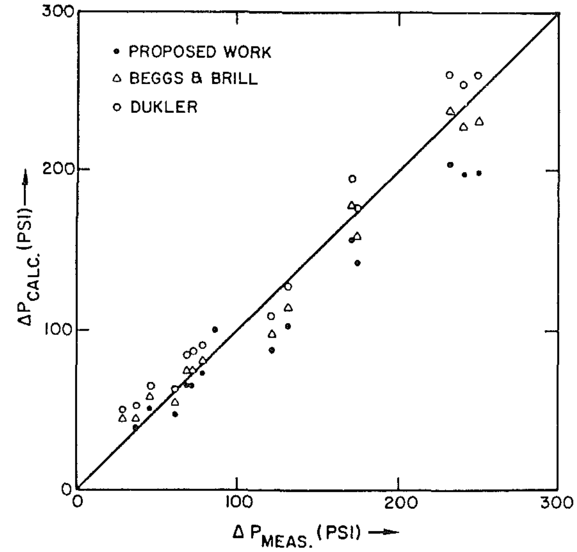

Fig. 5 Comparison of different pressure loss correlations based on Alaskan data reported by Brill, et al. [3]

then
$$f_c = \frac{\Delta p_f \cdot 2\, g\, d}{L\, v_m^2 \gamma_m} \tag{13}$$

where $v_m = v_{sL} + v_{sg}$

and $\gamma_m = H_L \gamma_L + H_g \gamma_g$.

From the total friction head loss the friction factor f_c was calculated from equation (13). When this friction factor was divided by a no-slip Moody friction factor for each data point, the values of the quotient were found to be very near unity. As a result, for bubble and slug flow the no-slip Moody friction factor is proposed for friction head loss calculation. The friction pressure drop equation is then

$$\Delta p_f = \frac{f_{ns} L v_m^2 \gamma_m}{2\, g\, d} \tag{14}$$

where f_{ns} = Moody friction factor based on Reynolds number, N_{Rens},

$$N_{\mathrm{Rens}} = \frac{d\, v_m \rho_{ns}}{\mu_{ns}}$$

where

$$\rho_{ns} = \lambda_L \rho_L + \lambda_g \rho_g$$

and $\mu_{ns} = \lambda_L \mu_L + \lambda_g \mu_g$.

Friction Head Loss in Stratified Flow. Assuming a smooth gas-liquid interface in stratified flow, momentum balance equations for steady-state flow may be written for each phase. For the gas phase,

$$A_g \frac{dp}{dL} = -(T_{wg} P_g + T_i W_i) - \gamma_g A_g \sin\theta \tag{15}$$

For the liquid phase,

$$(A - A_g)\frac{dp}{dL} = -[T_{wL}(\pi d - P_g) - T_i W_i] - \gamma_L A_L \sin\theta \tag{16}$$

where T_{wg}, T_{wL}, T_i are shear stresses at the pipe wall for the gas and liquid phases and at the interface, respectively, as shown in Fig. 3. From equations (15) and (16) it is evident that either equation can be used to calculate the pressure gradient in stratified flow. Equation (15) was chosen for the calculation of pressure gradient.

For large diameter pipes it is advisable to eliminate the effect of interface by adding equations (15) and (16) to obtain

$$-A\frac{dp}{dL} = (T_{wL}(\pi d - P_g) + T_{wg} P_g) + (\gamma_L A_L + \gamma_g A_g)\sin\theta \tag{17}$$

From simple geometrical considerations based on Fig. 3, it can be shown that δ, W_i, A_L, A_g, P_L and P_g are all related to h_L and d. The relations are given in the foregoing:

$$\delta = 2\cos^{-1}\left(1 - 2\frac{h_L}{d}\right) \tag{18}$$

$$W_i = 2\, d\sqrt{\frac{h_L}{d} - \left(\frac{h_L}{d}\right)^2} \tag{19}$$

$$H_L = \frac{A_L}{A} = \frac{1}{2\pi}(\delta - \sin\delta) \tag{20}$$

$$H_g = 1 - H_L \tag{21}$$

$$P = P_L + P_g \tag{22}$$

$$P_g = (1 - \delta/2\pi)P \tag{23}$$

$$D_{EL} = d\,\frac{(\delta - \sin\delta)}{\delta + 2\sin\dfrac{\delta}{2}} \tag{24}$$

$$D_{Eg} = d\,\frac{\{2\pi - (\delta - \sin\delta)\}}{2\pi - \delta + 2\sin\dfrac{\delta}{2}} \tag{25}$$

where δ is in radians.

Govier and Aziz [7] suggested that the wall shear stresses, T_{wL} and T_{wg}, can be evaluated approximately by assuming single-phase flow to occur in the cross section occupied by the phase. With these assumptions the following relations hold:

$$T_{wL} = \frac{f_L \gamma_L v_L^2}{2\, g} \tag{26}$$

$$T_{wg} = \frac{f_g \gamma_g v_g^2}{2\, g} \tag{27}$$

where f_L and f_g are Moody friction factors based on N_{ReL} and N_{Reg}, respectively, and

$$N_{ReL} = \frac{D_{EL} v_L \rho_L}{\mu_L}$$

and $N_{Reg} = \frac{D_{Eg} v_g \rho_g}{\mu_g}$

where

$$v_L = v_{sL}/H_L$$

and $v_g = v_{sg}/H_g$.

Calculation of total pressure gradient for stratified flow can be carried out employing any of equations (15)–(17) using the following steps:

1 Calculate H_L based on equation (10).
2 With this value of H_L, equation (20) is solved iteratively for δ. A value of 0.001 is a good first guess for δ. From equation (20), A_L is also calculated.
3 Knowing the value of δ, h_L/D can be calculated using equation (18) and D_{EL} and D_{Eg} are evaluated from equations (24) and (25), respectively.
4 Knowing δ and P, P_g and P_L are calculated from equations (23) and (22), respectively.
5 Knowing the values of D_{EL} and D_{Eg}, equations (26) and (27) can be solved for T_{wL} and T_{wg}.
6 With these parameters determined, any of equations (15)–(16) can be solved for dp/dL provided T_i is known or negligible. If T_i is not known and cannot be neglected, equation (17) can be used to calculate dp/dL.

Friction Head Loss in Annular Flow. In vertical uphill annular flow, one normally expects a coaxial and symmetric liquid annulus around a circular gas core. This particular geometry of annular flow can be mathematically modeled. However, in inclined and horizontal two-phase annular flow the liquid annulus is eccentric. The thickness of the liquid annulus is less at the top of the pipe than at the bottom. The shape and size of this annulus depends primarily on the pipe inclination angle, individual phase flow rates and direction of flow. As a result, only a global empirical model for the calculation of friction head loss is considered feasible at this stage.

Holdup ratios versus frction factor ratios were plotted for all annular flow data in Fig. 4, where

$$H_R = \frac{\lambda_L}{H_L}$$

and friction factor ratio

$$f_R = \frac{f_c}{f_{ns}}$$

where

$$f_c = \frac{\Delta p_f \, 2 \, g \, d}{L \, v_m^2 \gamma_{ns}}$$

and Δp_f was calculated from equation (1), neglecting Δp_a.

With the prevailing contention of no slippage between phases in annular flow, one would expect holdup ratio to be near unity in this case. However, a wide range of holdup ratios is observed in Fig. 4. This is partly due to the inclusion of numerous slug-annular mist transition flow data in the annular-mist region. Also, this is inevitable where the flow pattern transition bands are replaced by well-defined and sharp curvilinear transitions. Another major reason for the deviation of holdup ratio from unity is the error in the instrumental measurement of very low liquid holdup commonly observed in true annular flow at very high gas flow rates. In many of these data liquid holdup values near one percent or less are quite common. In this range of liquid holdup, errors in holdup of more than 100 percent may not significantly affect the calculated hydrostatic head loss based on slip properties. However, the error is reflected in the values of the holdup ratio since a very small quantity is divided by another small quantity and both are subject to some error. Distribution of points on both sides of unit holdup ratio in Fig. 4 unmistakably reflects the mixture of uphill and downhill flow data.

A smooth curve was drawn to fit the data in Fig. 4 and the values of f_R and H_R are noted and tabulated in Table 2.

For calculation of friction pressure drop in annular flow the following steps are followed:

1 H_L is obtained from the liquid holdup correlation presented in equation (10).
2 $H_R = \frac{\lambda_L}{H_L}$ is calculated.
3 A value of f_R is interpolated from Table 2 corresponding to H_R calculated in step 2.
4 No-slip friction factor, f_{ns}, is calculated with the Colebrook equation.
5 $f_c = f_R * f_{ns}$
6 $\Delta p_f = \frac{f_c \gamma_{ns} v_m^2 L}{2 \, g \, d}$

Table 2 Values of friction factor ratios versus holdup ratios in annular two-phase flow

f_R	H_R
1.00	0.01
0.98	0.20
1.20	0.30
1.25	0.40
1.30	0.50
1.25	0.70
1.00	1.00
1.00	10.00

Table 3 Statistical parameters for the pressure drop correlations applied to observed data

Oil	Angle (degrees)	No. of Data	Average % Error	Standard Deviation
Kerosene	5	55	7.79	24.59
	20	72	3.86	25.60
	30	63	1.24	16.05
	45	4	-12.93	1.77
	50	53	-5.00	15.29
	60	4	-9.81	3.25
	70	68	-2.60	14.52
	80	41	-4.38	12.89
	90	47	-1.42	15.92
	0	61	-3.33	16.20
	-5	39	28.60	119.16
	-20	52	37.39	172.23
	-30	43	14.92	53.59
	-45	3	-4.37	5.11
	-50	56	7.98	18.08
	-60	2	-1.04	7.89
	-70	51	0.91	19.71
	-80	42	3.37	20.96
	-90	43	1.91	21.41
Lube Oil	30	24	0.80	13.67
	90	40	-1.10	13.13
	0	37	7.24	15.86
	-30	16	41.74	117.57
	-90	25	2.46	25.27

Table 4 Statistical results for pressure drop calculations based on field data from the Prudhoe Bay Field and the North Sea

Source of Data / Parameter	Alaska Data			North Sea Data			
	Proposed	Beggs & Brill	Dukler	Proposed	Hagedorn & Brown	Beggs & Brill	Orkiszewski
No. of Data	14	14	14	130	130	130	130
$\bar{e}$	-9.50	3.65	11.86	-3.3	3.6	-10.5	-5.2
σ	14.67	12.38	14.23	15.3	9.7	18.2	30.5

This value of Δp_f is used in equation (1) to obtain the total pressure drop.

Evaluation of Pressure Drop Correlations

The pressure drop correlations were applied to the founding data and results are shown in Table 3. For very low observed values of pressure drop, absolute error is a more meaningful statistic than relative error. Nevertheless, only relative errors were used resulting in large average percent errors and standard deviations for some of the low angles in Table 3.

The proposed correlations, together with the Beggs and Brill [2] and the Dukler, et al. [5] correlations were applied to data obtained on 12 and 16-in-(30.48 and 40.64-cm-) dia flowlines in the Prudhoe Bay Field of Alaska reported by Brill, et al. [3]. Results are shown in Table 14 and individual results are plotted in Fig. 5. The proposed correlation, together with the Hagedorn and Brown [8], Beggs and Brill, and Orkiszewski [12] correlations were also applied to data for high volume wells from the North Sea as reported by Rossland [14]. The proposed correlation predicted the measured pressure drops fairly accurately, as shown by the small average relative percent errors presented in Table 4 for each of the two data sets.

The excellent performance of the proposed correlations on these large diameter data suggests that they can be used successfully beyond the range of the founding data. The unique feature of the correlations is their dependence on flow pattern for each angle of inclination.

Conclusion

Flow pattern dependent pressure drop correlations are presented for inclined two-phase flow. The accuracy of the correlations was validated using the field data from Prudhoe Bay flowlines and North Sea wells. This correlation is valid for both upward and downward two-phase flow.

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