Liquid Holdup Correlations for Inclined Two-Phase Flow

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Summary

Liquid holdup behavior in two-phase inclined flow was studied in an inclined pipe-flow simulator. Two sets of empirical equations, one each for uphill and downhill flow, are presented. For downhill stratified flow, a third equation is presented. The liquid holdup equations are functions of dimensionless liquid and gas velocity numbers in addition to liquid viscosity number and angle of inclination. These four parameters uniquely define the flow-pattern transitions in inclined two-phase flow. Consequently, the holdup equations are also implicitly flow-pattern dependent.

Introduction

An accurate prediction of liquid holdup is required to compute the hydrostatic head loss in two-phase inclined flow. In this case, hydrostatic head may be the most important of the pressure gradient components. There are many liquid holdup correlations in the literature, but almost all are for horizontal or vertical uphill flow.

Eaton *et al.*[1] proposed a holdup correlation based on data for natural gas, water, crude oil, and distillate oil mixtures in 2- and 4-in. (5- and 10-cm) diameter horizontal pipes. This correlation is based on five dimensionless groups reflecting various physical properties, flow rates, system pressures, and pipe diameters. A study by Vohra *et al.*[2] showed that this correlation performed best on a collection of horizontal data taken by Eaton *et al.*[1] and Beggs and Brill.[3] Cunliffe[4] found that using the Eaton *et al.* correlation to predict the total liquid volume in a wet gas pipeline was quite successful for determining the incremental volume of liquid removed from rate increases.

Using dynamic similarity analysis, Dukler *et al.*[5] developed a holdup correlation for horizontal two-phase flow. This holdup correlation is implicit in liquid holdup, requiring an iterative calculation. Experience has shown that most wet gas-transmission applications will result in a no-slip liquid holdup calculation when the Dukler *et al.* correlation is used.

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The Beggs and Brill correlation was developed from data obtained in an air/water flow system with 1- and 1½-in. (2.5- and 3.8-cm) diameter pipes. They considered a range of inclination angles from 0 to $\pm 90°$. Use of the correlation requires first determining the holdup for horizontal flow according to predicted horizontal flow patterns. The horizontal holdup is then corrected for angle of inclination. Palmer[6] found that the Beggs and Brill liquid holdup was overpredicted for both uphill and downhill flow and suggested proper correction factors.

For uphill flow from 0 to 9°, Guzhov *et al.*[7] proposed a holdup correlation that is independent of inclination angle. Hughmark and Pressburg[8] developed a general holdup correlation for gas/liquid flow covering a wide range of physical properties and diameters. This correlation is based on data taken in 1-in. (2.54-cm) diameter pipe for vertical uphill flow of air, water, oils of different viscosities, and carefully selected data of other investigators.

In addition to these empirical liquid holdup correlations, at least two analytical holdup correlations deserve mention. Bonnecaze *et al.*[9] developed a slug flow model for inclined pipe based on a mass and force balance around a simplified slug unit. The pressure drop contributions caused by the liquid film and the gas bubble were neglected. Using this holdup correlation, they correlated pressure drop data obtained in 1½-in. (3.8-cm) diameter pipe inclined at various angles around $\pm 10°$ to find an expression for friction factor. The holdup equation and friction factor correlation were compared with field data taken in a 6-in. (15.24-cm) diameter, 10,000-ft (3048-m) long pipe with a maximum deviation of 5%.

With a very similar mechanistic approach, Singh and Griffith[10] proposed a simple model for two-phase slug flow in inclined pipes. Most of their model parameters were experimentally determined using five different diameters of copper pipe at 5, 10, and 15° inclinations with an air/water system. For stratified flow, the authors developed a holdup model based on Chezy's open-channel flow equation. This equation shows that the liquid holdup is independent of gas flow, which is contrary

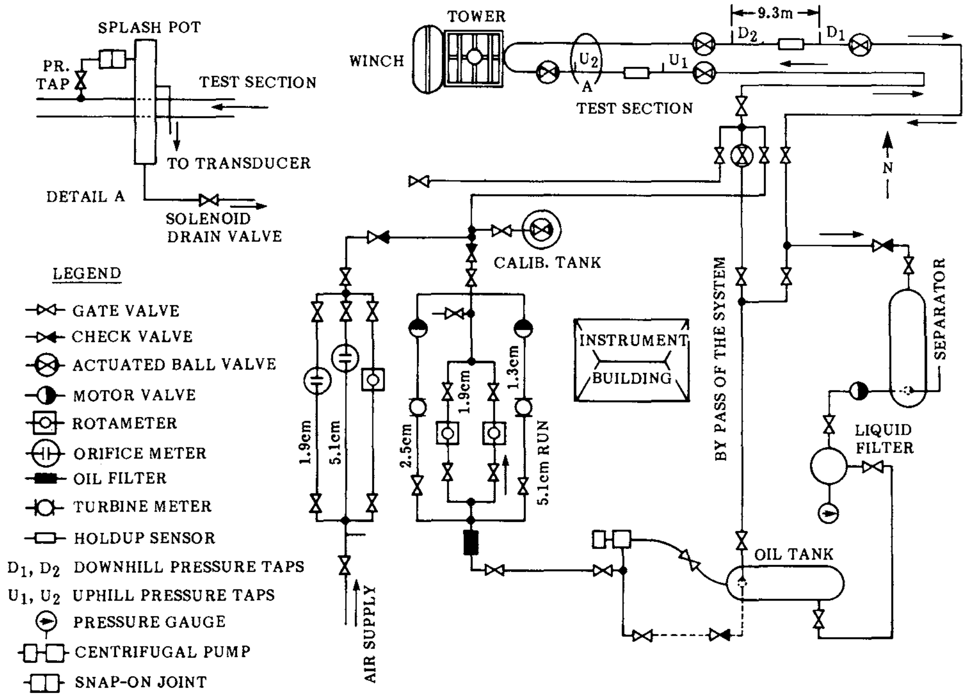

Fig. 1—Schematic of inclined flow simulator.

to accepted phenomena. They also suggested a method of calculating liquid holdup for annular flow that is iterative in nature. Their liquid holdup models reproduced their data with reasonable accuracy.

Experimental Program

An experimental facility was designed and constructed to obtain the desired test data. Fig. 1 is a schematic of the test facility. The test sections consisted of an inverted U-shape 1.5-in. (3.8-cm) ID nominal steel pipe. The closed end of the U-shape test sections could be raised or lowered to any angle from 0 to ±90° from horizontal. Each leg of the U was 56 ft (17 m) long with 22 ft (6.7 m) entrance lengths followed by 32- ft (9.8-m) long test sections on both uphill and downhill sides. Each test section could be isolated from the rest of the piping by pneumatically actuated ball valves that could be opened or closed simultaneously when calibrating holdup sensors. Pressure taps 30.5 ft (9.3 m) apart were located in each test section to permit measuring absolute and differential pressures using Validyne transducers. A 7-ft (2-m) long transparent Lexan pipe section was located in each test section to permit flow pattern observations and mounting of capacitance-type holdup sensors shown in Fig. 2. The outputs from these two sensors were recorded on an oscillograph as a time-varying trace. For obtaining an integrated value of holdup over a particular period of time, a digital multimeter was used to note the gain in voltage output over that particular time. This voltage gain can easily be converted to a liquid holdup fraction using a linear interpolation over the calibrated values of voltage gain for 0 and 100% oil in the pipe.

The oil and gas phases were carefully metered before mixing; turbine meters, orifice meters, or rotameters were used, depending on the phase and the flow rates. The two-phase mixture flowed through the test sections and into a horizontal separator. The gas (air) was vented to the atmosphere and the liquid passed through a filter and into a storage tank.

Kerosene and lube oil were used as the liquid phases. The surface tension, density, and viscosity of the kerosene at 60°F (15.56°C) were 26 dyne/cm (26 mN/m), 51 lbm/cu ft (816.9 kg/m^3) and 2 cp (0.002 Pa·s), respectively. Corresponding values for the lube oil were 35 dyne/cm (35 mN/m), 53 lbm/cu ft (849 kg/m^3) and 29 cp (0.029 Pa·s). Temperatures between 18 and 132°F (−7.8 and 55.56°C) were encountered during the tests.

Phase Slippage and Liquid Holdup

In inclined two-phase pipe flow, a substantial part of the total pressure losses may be contributed by the hydrostatic pressure difference. The relative contributions of friction gradient and hydrostatic gradient may be dictated by the prevailing flow patterns, angle of inclination, and direction of flow. Many of the current design procedures used for two-phase pipelines fail to account for these effects with any rigor. Part of the problem in most design procedures is the assumption that the void fraction is a unique function of quality and physical properties of the fluids. This is probably true where homogeneous flow can be assumed or during bubble flow at very low gas flow rates. Similar situations may also arise where the phase velocity is very high, so that friction pressure drop governs the total pressure loss. But in the remaining cases errors may arise from neglecting the slip velocity between phases. This concept of slip velocity comes from the physical phenomenon called "slippage."

The term "slippage" is used to describe a natural

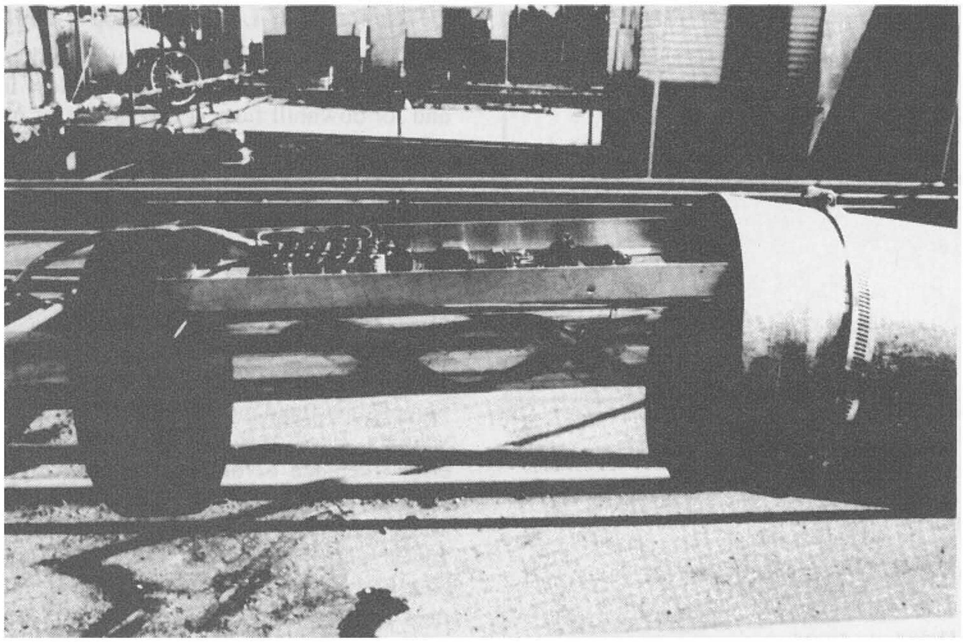

Fig. 2—Capacitance sensor test cell and local electronics board.

phenomenon of one phase slipping past the other in two-phase pipe flow. There are several causes for slippage between phases. Frictional resistances to flow or irreversible energy losses in the direction of flow are much less in the gas phase than in the liquid phase. This makes the gas more transmissible than liquid in two-phase flow, even in the absence of strong buoyancy effects such as in horizontal flow. This effect can be very pronounced in any segregated flow regime such as stratified flow. The large difference in compressibilities between gas and liquid causes the expanding gas to travel at a higher velocity and to slip past the liquid when pressure decreases in the direction of flow. Slippage between phases is also promoted by the difference in buoyant forces acting on the phases. In a static liquid medium, less-dense gas tends to rise with a velocity proportional to the density difference. Zukoski[11] studied the effect of pipe inclination angle on bubble rise velocity in a stagnant liquid. He concluded that, depending on the pipe diameter, surface tension and viscosity of fluids may appreciably affect the bubble rise velocity. His findings also showed that for some conditions an inclination angle as small as 1° from the horizontal can cause the bubble rise velocity to be more than 1.5 times the value obtained for horizontal pipes. This establishes a strong dependence between inclination angle and phase slippage. In the absence of any analytical formulation, the phenomenon of slippage caused by bubble rise velocity is studied empirically.

Greater gravitational forces on the more-dense liquid phase promotes fallback of liquid when shear forces and buoyant forces fail to support the liquid in upward flow. For downward flow it causes the liquid to travel faster than the gas. Thus, while buoyancy always causes the gas phase to rise relative to the liquid phase, gravity always tends to cause the liquid to fall faster than the gas.

A few important conclusions can be made from the preceding discussion. Except for homogeneous flow, the presence of slippage between phases in two-phase pipe flow is unavoidable at any angle of inclination. In both uphill and downhill bubble or slug flow, when the liquid phase is continuous and is capable of being supported by itself, buoyant forces generate bubble rise velocity causing slippage between phases. Near the slug and annular-mist flow transition or when the slug length becomes long [>1.5 to 3 ft (0.5 to 1 m)], the phases become discontinuous. During this type of flow, broken liquid slugs or ripples incapable of bridging the pipe are seen to fall back against the direction of uphill flow. Very similar flow phenomena occur in downhill stratified flow when the liquid falls back and accelerates until the liquid kinetic energy is balanced by the shear energy around the liquid layer. In stratified flow, large in-situ velocities attained by the liquid as a result of acceleration by gravity normally causes a very small liquid holdup. This phenomenon is shown in Fig. 3, where, in stratified flow at 0.363-ft/sec (0.11-m/s) liquid superficial velocity, void fraction rises rapidly to nearly 97% when very little air flows simultaneously. At higher liquid velocities in bubble or slug flow, the void fraction builds up more slowly. An important deduction at this point is that in uphill flow, slippage causes liquid velocity to slow down, resulting in a net accumulation of liquid in the flow channel or pipe and increasing the in-situ liquid fraction. The in-situ liquid fraction is commonly called "liquid holdup." In downhill flow, slippage causes the in-situ liquid velocity to increase, resulting in a decrease in liquid holdup. All these causes of phase slippage and the resulting flow patterns will occur as soon as one end of the pipe is raised about one pipe diameter from the other end, regardless of the angle of inclination. Thus, depending on the length of the pipe and direction of flow, characteristic flow patterns or liquid holdup for inclined flow should be observed even at extremely low angles. For example, at any low uphill angle, the stratified flow pattern should never be observed.

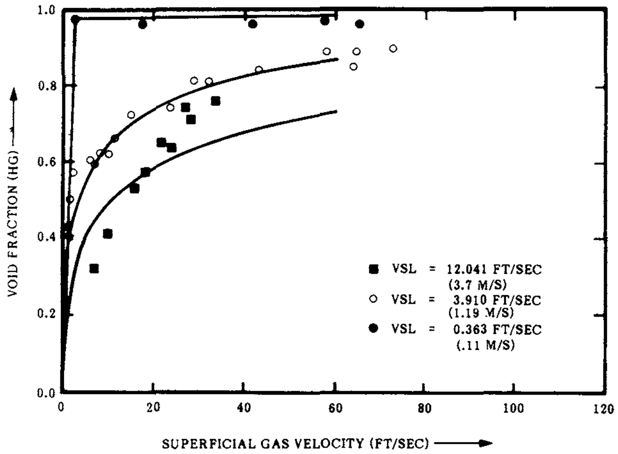

Fig. 3—Void fraction vs. v_{sg} at different values of v_{sL} for −30° angle.

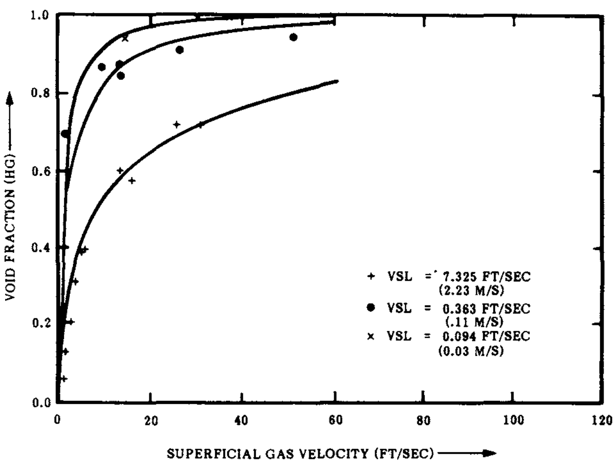

Fig. 4—Void fraction vs. v_{sg} at different values of v_{sL} for horizontal flow.

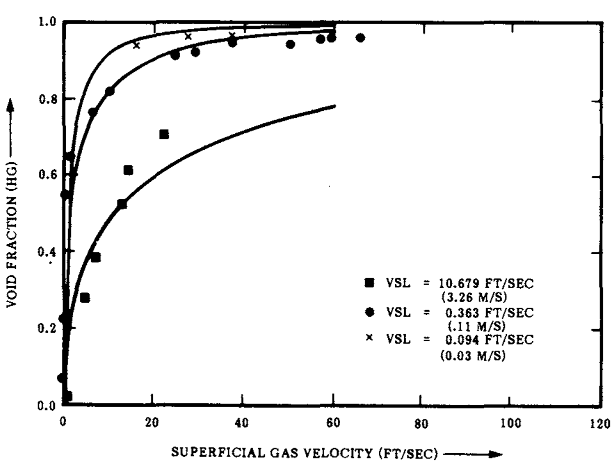

Fig. 5—Void fraction vs. v_{sg} at different values of v_{sL} for +90° angle.

Development of Liquid Holdup Correlation

Analytical expressions for liquid holdup have been attempted for uphill two-phase slug flow in vertical pipes and for downhill flow at low angles of inclination in the range of 0 to 15°. Considering the complex slippage mechanism, a global liquid holdup model for any pipe inclination has not been attempted before.

More than 1,500 liquid holdup measurements at uphill and downhill inclination angles from 0 to ±90° from horizontal were obtained in this study. Attempts to correlate these data into a global empirical liquid holdup correlation are presented in the following. At each uphill and downhill angle, void fraction was plotted as a function of superficial gas velocity for fixed superficial liquid velocity. Each of these plots was continuous within the error tolerance of the holdup measurements. Example plots are shown in Figs. 4 through 6. At very high gas rates, the curves almost become asymptotic with the 100% void fraction (0% liquid holdup). For downhill stratified flow at very low gas rates, the void fraction rises rapidly and then almost linearly increases with increased gas rates. However, the void fraction plot for horizontal flow is similar to the uphill plot, even in the stratified flow regime. The general shapes of these plots prompted selection of a nonlinear regression equation of the form

$$H_L = \exp\left[(c_1 + c_2 \sin\theta + c_3 \sin^2\theta + c_4 N_L^2) \cdot \frac{N_{gv}{}^{c_5}}{N_{Lv}{}^{c_6}}\right]. \quad (1)$$

Subsequently, three liquid holdup correlations were attempted, one for uphill and horizontal flow and the other two for downhill stratified flow and the other downhill flow patterns. The regression coefficients are given in Table 1. The coefficients were obtained by using the nonlinear BMDP regression programs.[12] In each of the regression analyses, the outliers in the residual plot were deleted from the data set and the analysis was repeated.

The selection of phase velocity numbers as the independent variables instead of the phase superficial velocities as shown in Figs. 4 through 6 was done to make the variables dimensionless. These numbers were also suggested as correlating parameters by Duns and Ros,[13] Hagedorn and Brown,[14] and Eaton *et al.*[1] The velocity numbers, together with the inclination angle, also formed the independent variables defining the flow patterns. Hence, inclusion of all these variables implicitly makes the holdup correlation flow-regime-dependent. The use of dimensionless numbers should not affect the shapes of curves shown in Figs. 4 through 6 since, for a fixed oil, converting superficial velocities to dimensionless form requires multiplication by a nearly constant quantity of approximately 2.5 for this study.

Effects of Inclination Angle and Viscosity

The second-degree polynomial function of the form $c_1 + c_2 \sin\theta + c_3 \sin^2\theta$ was selected by plotting liquid holdups for different angles of inclination at fixed liquid and gas velocity numbers. This relation was also confirmed by comparing results of other equation forms in trial runs of the regression analysis. The best error as indicated by the sum of squares was obtained using the second-degree relation. The equation is also consistent with the Beggs and Brill discovery that liquid holdup passes through maximums and minimums at fixed inclination angles of approximately +50° and −50°, respectively, for their data. Eq. 1 shows that the liquid holdup should increase as the uphill angle of inclination increases. This can be shown graphically by comparing liquid holdup values obtained from the plots in Figs. 7 and 8, where void fractions are plotted for the same oil at three similar superficial liquid velocities for horizontal and uphill 30° pipe inclination.

Intuitively, increased liquid viscosity should increase viscous shear, causing increased liquid holdup regardless of inclination angle. Positive coefficients of the liquid viscosity number in all the holdup correlations support this hypothesis.

In general, any force that creates drag on any phase against the direction of flow tends to increase the in-situ fraction of that phase. As a result, viscous drag on the liquid will always tend to increase the liquid holdup, irrespective of inclination angle. However, gravity forces on the more dense phase will tend to increase the holdup of that phase for uphill flow and decrease it for downhill flow. Similarly, buoyant forces will tend to decrease void fraction for uphill flow, while increasing it for downhill flow.

Discussion of Results

The proposed liquid holdup correlation was tested with the observed data to check the reproducibility of the observed holdup values. For both the oils at different angles of inclination, the relative percent errors were calculated for individual experimental observations. For each angle and each oil, the average percent errors and their standard deviations (Table 2) were also calculated. The capacitance sensor for liquid holdup measurement was found to be less accurate in the range of liquid holdups less than 10% or more than 90%. In these ranges of liquid holdups, very high percent errors were observed (more than 30%). More than 80% of these data points with high percent errors were found to be in annular or stratified flow regime. In both of these flow regimes, the contribution from hydrostatic head to the total pressure loss is quite insignificant. As such, those data points with more than 30% relative percent error were not used for the calculation of either average percent errors or the standard deviations.

In the development of these liquid holdup correlations, the BMDP nonlinear regression package was used. This regression method minimizes the residual sum of squares to calculate the regression coefficients. The observation corresponding to the outliers in the residual plot were excluded in the development of the holdup correlations. Normally, these outliers indicated erroneous observations. This criterion for culling data does not correspond to minimizing the average percent error. Hence, when these correlations were applied to the observed data, a further culling of data based on average percent error was required. Normally, depending on the value of the absolute relative error, the sensitivity of the holdup measurement techniques reflects a great deal on the average percent error. For very small values of observed holdup, even with acceptable absolute error, relative er-

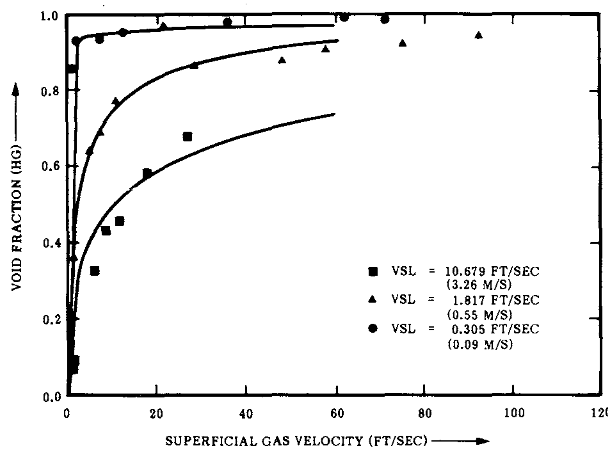

Fig. 6—Void fraction vs. v_{sg} at different values of v_{sL} for −90° angle.

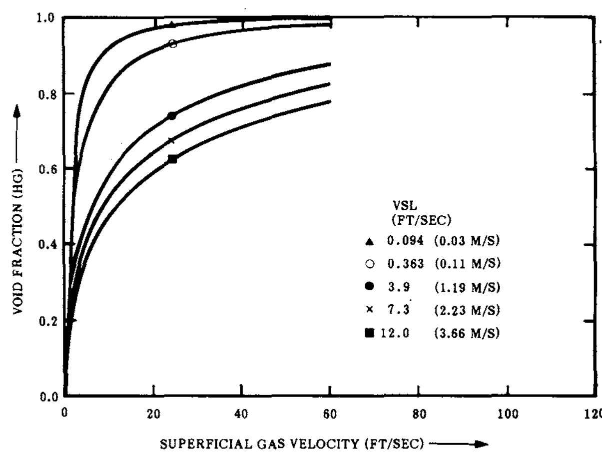

Fig. 7—Void fraction vs. superficial gas velocity at fixed superficial liquid velocity for horizontal flow.

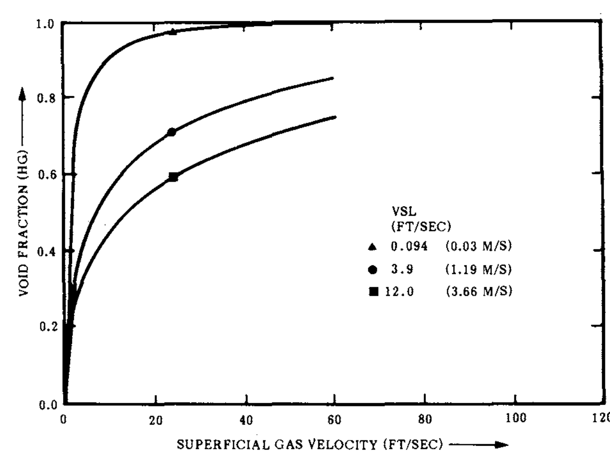

Fig. 8—Void fraction vs. superficial gas velocity at fixed superficial liquid velocity for uphill flow at 30°.

TABLE 1—COEFFICIENTS OF LIQUID HOLDUP EQUATION

Flow Direction	Flow Pattern	c_1	c_2	c_3	c_4	c_5	c_6
uphill flow	all	−0.380113	0.129875	−0.119788	2.343227	0.475686	0.288657
downhill flow	stratified	−1.330282	4.808139	4.171584	56.262268	0.079951	0.504887
	other	−0.516644	0.789805	0.551627	15.519214	0.371771	0.393952

TABLE 2—STATISTICAL PARAMETERS FOR HOLDUP CORRELATIONS APPLIED TO OBSERVED DATA

Angle (degrees)	Number of Data	Average Error (%)	Standard Deviation (%)
Kerosene			
5	35	2.79	13.64
20	48	0.04	14.25
30	57	4.71	13.40
45	4	5.49	1.77
50	43	−1.96	12.30
60	4	−2.17	3.16
70	63	4.98	11.92
80	49	−2.77	13.55
90	35	2.95	16.80
0	42	−1.86	13.95
−5	40	2.44	13.79
−20	41	−0.33	25.95
−30	42	6.81	18.73
−50	31	−1.71	20.32
−70	33	5.35	18.91
−80	29	−0.05	19.89
−90	29	6.19	21.20
Lube Oil			
30	33	−1.01	15.01
90	43	−7.52	8.22
0	38	−4.34	13.58
−30	23	−0.26	15.43
−90	37	−7.15	15.83

ror may be very large. This is often caused by division of a small quantity in the calculation of percent error. Values of average percent error and standard deviations for liquid holdup for each oil at different angles of inclination are shown in Table 2.

Conclusions

An empirical model for inclined two-phase flow liquid holdup is proposed. The proposed model enables the determination of liquid holdup regardless of the angle of inclination and the direction of flow. The set of holdup correlations is dependent on the same dimensionless parameters that control the flow pattern transitions in two-phase flow. Except for downhill stratified flow, the liquid holdup correlations are continuous across flow-pattern transitions.

Nomenclature

c = empirical constant
g = gravitational acceleration, ft/sec^2 (m/s^2)
H_L = liquid holdup
N_{gv} = gas velocity number, $v_{sg}[\rho_L/(g\sigma)]^{0.25}$
N_L = liquid viscosity number, $\mu_L[g/(\rho_L\sigma^3)]^{0.25}$
N_{Lv} = liquid velocity number, $v_{sL}[\rho_L/(g\sigma)]^{0.25}$
v_{sg} = superficial gas velocity, ft/sec (m/s)
v_{sL} = superficial liquid velocity, ft/sec (m/s)
μ = viscosity, cp (Pa·s)
ρ = density, lbm/cu ft (kg/m^3)
σ = surface tension, dyne/cm (mN/m)
θ = pipe inclination angle from horizontal, degree (rad)

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SI Metric Conversion Factors

in.	× 2.54*	E+00	=	cm
ft	× 3.048*	E−01	=	m

*Conversion factor is exact.

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