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Performance of a two-phase gas/liquid flow model in vertical wells

C.S. Kabir and A.R. Hasan
Chevron Oil Field Research Co, P.O. Box, La Habra, CA 90633-0446 (U.S.A.)
Chemical Engineering Dep., University of North Dakota, Grand Forks, ND 58201 (U.S.A.)

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ABSTRACT

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This paper presents application of a recently developed method for predicting two-phase gas/oil pressure-drop in vertical oil wells. The new method, which is flow-pattern based, is capable of handling flow in both circular and annular channels. Five principal flow regimes–bubbly, dispersed bubbly, slug, churn and annular – are recognized while developing appropriate correlations for predicting void fraction and pressure-drop in each flow regime.

Standard oilfield correlations are used for estimating *PVT* properties of oil and gas: Standing's correlation for solution gas–oil ratio; Katz's correlation for oil formation volume factor; Standing's, and Chew and Connally's correlations for dead and live oil viscosities, respectively; and Lee et al.'s correlation for gas viscosity. A finite-difference algorithm is developed to compute pressure gradient in a wellbore.

Computations performed on 115 field tests, involving all the two-phase flow regimes, suggest that the new method performs better than the Aziz et al. correlation. Further comparison of the new method's performance with other standard methods, such as, Orkiszewski, Duns and Ros, Beggs and Brill, Hagedorn and Brown, and Chierici et al., reveals its consistency and improved performance. The test data bank used in this study is that previously used by other authors; thus, validation of the new method is demonstrated with an independent data set.

Introduction

Predicting vertical multiphase flow behavior in oil and gas-condensate wells is of great practical significance and importance. Pressure losses encountered during concurrent vertical flow of two or three phases enter into a wide array of design calculations. Such design considerations include: tubing size and operating wellhead pressure in a flowing well; well completion or recompletion scheme; artificial lift during either gas-lift or pump operation (submersible, sucker-rod, etc.) in a low-energy reservoir; liquid unloading in gas wells; direct input for surface flow-line and equipment design calculations.

Efforts to predict the pressure drop in an oil well can be traced back to the early 1950s when Poettmann and Carpenter (1952) published their predictive scheme. Since then many attempts have been made to predict the complex flow behavior starting from modifications of the Poettmann–Carpenter correlation to more complex mathematical models capable of handling flow hydrodynamics. To date, no single correlation or model has emerged to successfully predict pressure drop under the wide range of operating conditions encountered in wells around the world.

The published correlation models can be classified into three categories based upon the way the behavior is modeled. These models are briefly discussed in the order of increasing complexity.

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Nomenclature

C_o	distribution parameter, dimensionless
dp/dz	pressure gradient, kPa m^{-1} [psi ft^{-1}]
D	pipe diameter, m [ft]
D_c	casing inside diameter, m [ft]
D_t	tubing outside diameter, m [ft]
E	entrainment of liquid in gas-ore, dimensionless
E_g	in-situ gas volume-fraction, dimensionless
E_{gc}	in-situ gas volume-fraction in gas-core, dimensionless
f_m	Fanning friction factor for the gas/oil mixture
g	gravitational acceleration, m s^{-2} [ft s^{-1}]
g_c	conversion factor, 32.2 lbm ft lbf^{-1} s^{-2} [1 in SI]
N_{Re}	Reynolds number ($=Dv_m\rho_m/\mu_m$), dimensionless
p	pressure at any location in flow string, kPa [lbf ft^{-2}]
v_{cgs}	critical gas velocity in the gas-core, m s^{-1} [ft s^{-1}]
v_{gs}	superficial gas velocity, m s^{-1} [ft s^{-1}]
v_m	mixture ($v_{gs}+v_{os}$) superficial velocity, m s^{-1} [ft s^{-1}]
v_o	in-situ oil velocity, m s^{-1} [ft s^{-1}]
v_{os}	superficial oil velocity, m s^{-1} [ft s^{-1}]
v_∞	terminal rise-velocity of a gas bubble, m s^{-1} [ft s^{-1}]
$v_{\infty T}$	terminal rise-velocity of a Taylor bubble, m s^{-1} [ft s^{-1}]
x	mass-fraction of the lighter phase, dimensionless
X	Lockhart-Martinelli parameter for turbulent flow, dimensionless
ϵ/D	relative pipe roughness, dimensionless

Greek symbols

Δ	difference
μ_g	gas viscosity, Pa s [cP]
μ_o	oil viscosity, Pa s [cP]
ρ_g	gas density, kg m^{-3} [lbm ft^{-3}]
ρ_o	oil density, kg m^{-3} [lbm ft^{-3}]
σ	surface tension, N m^{-1} [dynes cm^{-1}]

Subscripts

c	core-fluid in annular flow
f	fluid, single or multiphase
g	gas
l	liquid
m	mixture of phases

Homogeneous flow model

This model assumes that a multiphase mixture behaves much like a homogeneous single-phase fluid, with property values that are some type of average (weighted, volumetric, etc.) of the constituent phases. Thus the model implicitly assumes no slip or difference between in-situ phase velocities.

The pioneering work of Poettmann and Carpenter (1952) and the subsequent modifications of that model made by Baxendell and Thomas (1961), Tek (1961), Fancher and Brown (1963) and Hagedorn and Brown (1964) fall into this category. Each modification of the Poettmann–Carpenter correlation improved its applicability over the range of subsequent investigations. At the same time, these studies revealed that the underlying assumptions of the original work are severly limiting. More importantly, the effects of gas/liquid ratio, total well flow-rate, liquid viscosity and tubing diameter are not properly handled in the model.

Govier and Aziz (1972) report other correlations that are developed from laboratory experiments on air–water systems. These correlations have not found much application in the oil industry.

Separated flow or slip model

This model assumes the phases to be segregated with unequal velocities, known as slip. Thus the slip velocity or the in-situ void fraction of each phase needs to be known along with the frictional interaction of the phases with the wall and amongst themselves.

Hagedorn and Brown (1965) reported a correlation that requires estimation of an effective average in-situ void fraction. This correlation has found wide application in the industry despite its empirical origin.

The Beggs and Brill (1973) correlation is based on extensive laboratory data and their own flow pattern map developed for horizontal flow. This flow pattern map was used only as a correlating parameter at other pipe orientations. The Mukherjee–Brill (1983) correlation was also developed empirically from their large experimental database. A single equation for liquid holdup or void fraction was proposed for al inclination angles from horizontal. The liquid and gas velocity numbers together with the inclination angle implicitly accounted for the flow regimes observed in their experiments. In general, both the Beggs–Brill and Mukherjee–Brill correlations perform similarly. Thus, although flow patterns were recognized in these two correlations, no independent equations were proposed to predict gas void fraction in each flow pattern as it actually exists in the given conditions.

A large number of correlations have been developed using the slip model for horizontal multiphase flow. The pioneering work of Lockhart and Martinelli (1949) is based on the slip model. A detailed work of Idsinga et al. (1977) indicates that, for steam–water systems, the Lockhart–Martinelli correlation is applicable even for vertical upflow and downflow when certain modifications are incorporated. However, application of such an approach has not been documented as yet.

Flow pattern approach

In this approach, an attempt is made to delineate the flow pattern at each pipe segment and then use the appropriate void fraction and pressure-drop correlations for each flow pattern. Although, in principle, this technique promises to be the most rigorous of all, the difficulty of identifying each flow regime has led to different flow-pattern maps and hence to different correlations (Ros, 1961; Duns and Ros, 1963; Orkiszewski, 1967; Aziz et al., 1972; Chierici et al., 1974a; Gould et al., 1974).

In the early work of Ros (1961) and Duns and Ros (1963), a flow pattern map with dimensionless gas and liquid velocity numbers as coordinates was used. Other flow pattern maps published in the 1960s were those of Griffith and Wallis (1961) and Govier et al. (1961). These maps were subsequently used by others while developing pressure-drop correlations.

Orkiszewski (1967) proposed a semi-mechanistic approach in which bubbly to slug transition is predicted by the Griffith–Wallis (1961) criterion, while the transitions from slug to churn, and churn to annular follow the Duns–Ros (1963) criteria. Orkiszewski's method has proven to be one of the reliable correlations in the oil industry.

In the 1970s, four more correlations appeared in the literature (Aziz et al., 1972; Chierici et al., 1974a; Gould et al., 1974). Of these, Aziz et al.'s semi-mechanistic model based on the Govier–Aziz (1972) flow pattern map proved to be one of the better methods.

The remaining two correlations of Chierici et al. (1974a) and Gould et al. (1974) were both derived from the Orkiszewski (1967) model with minor modifications. For example, Chierici et al. made slight changes in the void-fraction prediction during slug flow, while Gould et al. made changes in predicting void fraction in bubbly flow. Neither correlation has found wide application, however.

Although all these correlations, based on the flow-pattern maps, are being used with varying degrees of success, no single method has emerged as the most reliable. This uncertainty stems partially from the flow-pattern maps. These maps, based on experimental data, are plotted on two-dimensional coordinate systems delineating the transition boundaries. The dimensionless coordinate variables are generally chosen arbitrarily, thereby limiting the accuracy and scope of application of these maps beyond the original data base.

In their classical work, Taitel et al. (1980) showed that the transition from one flow pattern to another could be modeled from first principles by properly incorporating the fluid properties and conduit size. Recently, Hasan and Kabir (1988a,b) adopted an approach very similar to Taitel et al. to model each flow-pattern transition. Subsequently, the void fraction and pressure drop in each flow regime was modeled.

In this work, we first briefly discuss the flow pattern transition boundaries. In particular, the departure from our previous approach for delineating the slug/churn transition boundary is described. Second, we present both the computational and application aspects of the model (Hasan and Kabir, 1988a) using field examples.

Flow pattern determination

The chaotic nature of two-phase flow presents a considerable challenge to classify the flow regimes and ascribe transition criteria to them. However, with the recent attempts of physical modeling (Taitel et al., 1980, for example), a consensus appears to have emerged with respect to the principal flow regimes. These flow regimes are identified as bubbly, dispersed bubbly, slug, churn and annular.

In bubbly flow, the gas phase tends to rise through the continuous liquid column as small, discrete bubbles. Because pure bubbly flow is associated with low gas velocities, its occurrance is confined to low gas volume fractions. At higher gas rates, the smaller bubbles collide with each other, resulting in coalesence and formation of elongated (Taylor) bubbles. These agglomerated bubbles are separated by aerated liquid slugs and the flow regime is called slug flow. With the rise in liquid velocity the turbulence increases, leading to the breakup of large bubbles and formation of fully dispersed bubbly flow as dispersive forces overcome interfacial tension.

As the flow rates of both phases are further increased, the shear stress between the Taylor bubbles and the liquid film separating them from the pipe wall increases, finally causing a breakdown of these bubbles. The liquid slug also collapses and this liquid forms a new bridge after accumulating, only to be lifted again by the oncoming, narrow Taylor bubbles. This oscillatory motion of the liquid slugs leads to the transition to churn flow. The final flow regime, annular flow, occurs at extremely high gas-flow rates, which cause the entire gas phase to flow through the central portion of the pipe carrying some liquids as droplets. The remaining liquid flows up the wall through the annulus formed by the pipe wall and the gas core.

In the following we briefly discuss the criteria for transition from one flow regime to another. The approach adopted in this work is the one pioneered by Taitel et al. (1980). This approach examines each flow-pattern transition individually and is more reliable than other empirical methods because it allows physical modeling of transition boundaries.

Bubbly/slug flow transition

Various experimental and theoretical investigations suggest that this transition occurs when the gas void fraction reaches about 25 percent. This criterion translates into the following relationship in terms of the superficial velocities of the gas and liquid phases (Hasan and Kabir, 1988a):

$$v_{gs} = 0.429 v_{ls} + 0.546 [g\sigma(\rho_l - \rho_g)/\rho_l^2]^{0.25} \quad (1)$$

At superficial gas velocities greater than that given by Eq. 1, transition to slug flow takes place.

Transition to dispersed bubbly flow

When large bubbles are dispersed into small bubbles at high liquid rates, transition to slug flow is inhibited even though the gas void fraction exceeds 25%. An examination of the shear forces have led several investigators (Taitel et al., 1980; Barnea et al., 1982, 1985) to propose the following equation for mixture velocity for transition to dispersed bubbly flow:

$$v_m^{1.12} = 4.68 (D)^{0.48} [g(\rho_l - \rho_g)/\sigma]^{0.5} (\sigma/\rho_l)^{0.6} (\rho_l/\mu_l)^{0.08} \quad (2)$$

When the gas void fraction exceeds 52%, bubble coalescence cannot be prevented and transition to either slug, churn or annular flow must occur.

Slug/churn flow transition

A very few reliable works exist to model this transition. For instance, the approach of Taitel et al. (1980) to view this transition as an entrance effect may not be applicable in long oil wells. In this work, we have adopted the physical model of Barnea and Brauner (1985) and Brauner and Barnea (1986). They argued that the transition from slug to churn flow occurs when the liquid slug trailing a Taylor bubble attains the maximum possible gas void fraction of 52% as given by Eq. 2. In other words, this transition boundary is a locus of *constant* mixture velocity, v_m, where the turbulent intensity is maintained at the same level as that in the dispersed bubbly flow. We clarify this point further while discussing the flow pattern maps.

Transition to annular flow

In annular flow, the high gas velocities keep the liquid droplets in suspension. This transition is modeled by balancing the drag forces on the liquid droplets and the gravitational forces acting on them. The resulting expression in terms of superficial gas velocity is given by (Taitel et al., 1980; Hasan and Kabir, 1988a,b):

$$v_{gs} = 3.1\,[\sigma g(\rho_l - \rho_g)/\rho_g^2]^{0.25} \qquad (3)$$

Equation 3 clearly suggests that this transition criterion is independent of the liquid rate.

Flow pattern maps

The equations presented earlier can be used to construct the familiar flow-pattern maps, with gas and liquid superficial velocities as the coordinates. Two such maps were prepared for a 30°API oil flowing in a 3-inch ID pipe. The first map was made by evaluating the fluid properties at the standard conditions (14.7 psia, 60°F). The fluid properties used are: σ=0.0685 lbm s^{-2}, ρ_l=54.67 lbm ft^{-3}, ρ_g=0.0534 lbm ft^{-3} and μ_g=0.0103 cP.

Figure 1 shows the flow pattern map at the standard conditions. Note that the dispersed

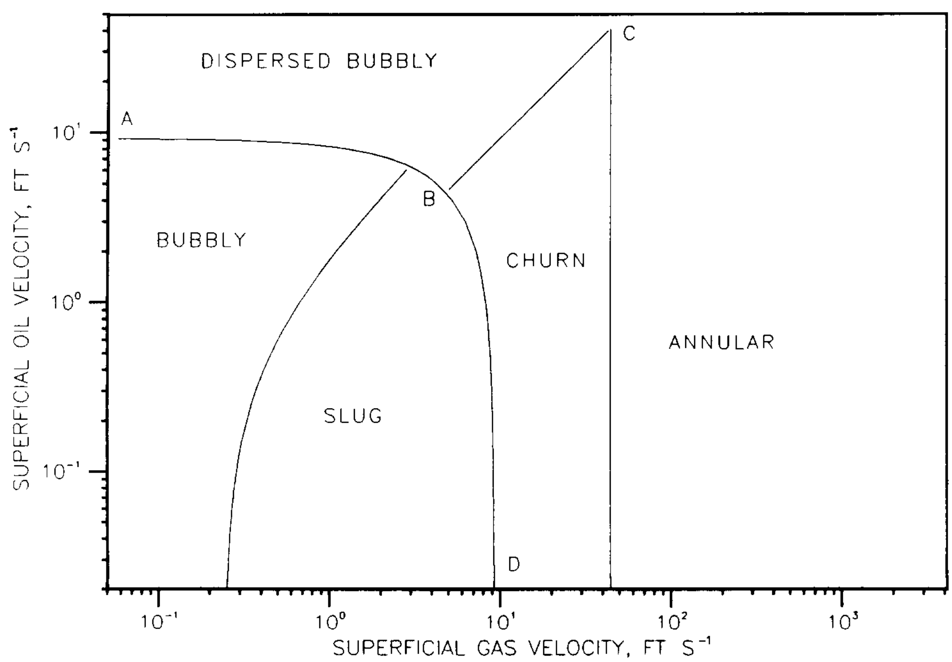

Fig. 1. Flow pattern map for a gas/oil system at standard conditions.

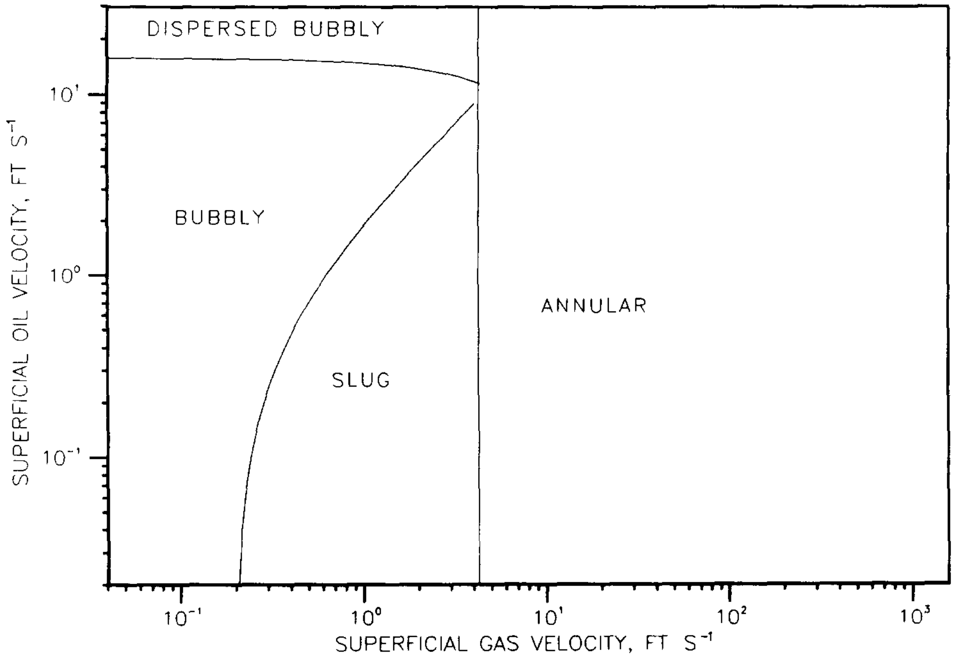

Fig. 2. Flow pattern map for a gas/oil system at 1000 psia, 100°F.

bubbly flow pattern is represented by two transition lines. The segment represented by AB is given by Eq. 2, while the BC segment is based on the fact that the dispersed bubbly flow is limited by the maximum gas void fraction criterion of 52%. The segment BD, representing the slug/churn transition, is indeed a continuation of the AB line. As discussed earlier, the BD line represents the locus of constant mixture velocity for a given set of fluid properties. Physically, this line implies that the agglomeration of bubbles in the liquid slug (present in between two Taylor bubbles) reaches 52% leading to the destruction of the liquid bridge, and the resulting churning motion. The BD line corresponds to the l_e/D ratio of about 130 as given by the Taitel et al. (1980) equation:

$$L_e/D = 40.6\left(\frac{v_m}{\sqrt{gD}} + 0.22\right) \tag{4}$$

To examine the map at a condition likely to be prevalent in an oil well, we constructed Fig. 2 at 1000 psia and 100°F. The property values used are: $\sigma = 0.03164$ lbm s^{-2}, $\rho_l = 54$ lbm ft^{-3}, $\rho_g = 3.77$ lbm ft^{-3} and $\mu_g = 0.0127$ cP. At higher pressures, when the gas density is significant, the transition to annular flow occurs at a much lower superficial gas velocity of 4.2 ft s^{-1} compared to 44.4 ft s^{-1} at the atmospheric pressure. Note also that the dispersed bubbly flow area shrinks and the churn flow regime vanishes. Consequently, there appears to be a transition between bubbly and annular flow regime. Physical explanation for a possible existence of such a transition is beyond the scope of this work.

Pressure gradient estimation

The overall pressure gradient, dp/dz, in two-phase flow can be written as the sum of the gravitational or hydrostatic head $(dp/dz)_H$, frictional $(dp/dz)_F$ and accelerational $(dp/dz)_A$ components:

$$dp/dz = (dp/dz)_H + (dp/dz)_F + (dp/dz)_A = (g/g_c)\rho_m + \frac{2f_m v_m^2 \rho_m}{g_c D} + \rho_m v_m \frac{dv_m}{dz} \quad (5)$$

where:

$$\rho_m = \rho_g E_g + \rho_l (1 - E_g) \quad (6)$$

In general, the accelerational component could be neglected during all but the annular flow regime. Equation 6 clearly suggests that an accurate estimation of the gas void fraction is essential to the hydrostatic head computation that accounts for most of the pressure drop occurring in the bubbly and slug flow regimes.

Because of different hydrodynamics in each flow regime, estimations of void fraction, E_g, in-situ mixture density, ρ_m, and the friction factor, f_m, are made separately.

Void fraction estimation

Here, we present the appropriate equations as given by Hasan and Kabir (1988a) for performing void fraction calculations in each of the five flow regimes.

Bubbly/dispersed bubbly flow

In bubbly flow the expression for void fraction, E_g, is given by:

$$E_g = \frac{v_{gs}}{C_o v_m + v_\infty} \quad (7)$$

where:

$$C_o = 1.2 + 0.371 \frac{D_t}{D_c} \quad (8)$$

$$v_\infty = 1.50 [g\sigma(\rho_l - \rho_g)/\rho_l^2]^{0.25} \quad (9)$$

and, D_t/D_c is the tubing to casing diameter ratio when flow occurs in a tubing-casing annulus. This term vanishes when the flow is in a circular pipe; thus, retaining the C_o value of 1.2.

In the dispersed bubbly flow, where larger bubbles coexist without coalescing, the same bubbly flow treatment could be used for estimating the gas void fraction.

Slug and churn flow

The drift-flux model or Eq. 7 still applies in slug flow with different values for C_o and v_∞ as given by:

$$C_o = 1.18 + 0.90 \frac{D_t}{D_c} \quad (10)$$

$$v_{\infty T} = [0.30 + 0.22(D_t/D_c)] [\sqrt{g(D_c - D_t)(\rho_l - \rho_g)/\rho_l}] \quad (11)$$

For churn flow, C_o assumes a values of 1.15 to reflect the flatter velocity profile, whereas the expression for $v_{\infty T}$ remains unchanged.

Annular flow

In annular flow, estimation of gas void fraction in the central core is pertinent rather than the entire pipe cross-sectional area. The gas void fraction for the gas-core, E_{gc}, is given by:

$$E_{gc} = \frac{v_{gs}}{v_{gs} + E v_{ls}} \quad (12)$$

where the vapor entrainment, E, is estimated from the following conditions.

If $v_{cgs} \times 10^4 \leq 4$:

$$E = 0.0055 (v_{cgs} \times 10^4)^{2.86} \quad (13)$$

If $v_{cgs} \times 10^4 \geq 4$:

$$E = 0.857 \log[v_{cgs} \times 10^4] - 0.20 \quad (14)$$

where the critical vapor velocity, v_{cgs}, is calculated from:

$$v_{cgs} = \frac{v_{gs} \mu_g \sqrt{\rho_g/\rho_l}}{\sigma} \quad (15)$$

Frictional head estimation

The friction factor correlation used here is that of Chen (1979). In a recent study, Gregory and Fogarasi (1985) observed that the explicit form of f given by Chen is the best over a wide range of Reynolds number and relative pipe roughness. Chen's equation is given as:

$$\frac{1}{\sqrt{f}}=4\log\left[\frac{\epsilon/D}{3.7065}-\frac{5.0452\log A}{Re}\right] \tag{16}$$

where:

$$A=\frac{(\epsilon/D)^{1.1098}}{2.8257}+(7.149/Re)^{0.8981} \tag{17}$$

ϵ/D is the relative pipe roughness and Re is the Reynolds number ($=Dv\rho/\mu$). While evaluating the Reynolds number in two-phase flow, the density and viscosity of the *liquid* phase needs to be used (Aziz et al., 1972). This friction factor correlation is used for computing frictional head in the bubbly, slug, and churn flow regimes. Use of the explicit form of the Chen equation significantly enhances the computation speed when compared with other implicit form of f-correlations, such as given by Colebrook (1939).

Bubbly flow

In bubbly flow we assume that the two-phase frictional pressure drop is the same as the single-phase liquid flow. Thus we can write:

$$\Delta p_f=\frac{2f_m v_m^2\rho_m}{g_c D} \tag{18}$$

In Eq. 18, f_m indicates that the term should be evaluated for the gas–liquid mixture Reynolds numbers.

Slug and churn flow

In these two flow regimes, the wall friction is essentially experienced by the liquid film flowing up the pipe with large bubbles occupying most of the cross-sectional area of flow. This fact requires a modification of the density term as given by Eq. 18 given below:

$$\Delta p_f=\frac{2f_m v_m^2\rho_l(1-E_g)}{g_c D} \tag{19}$$

Annular flow

In annular flow, fine liquid droplets flow in the gas core with a velocity the same as the gas phase, while a thin liquid film creeps up the wall of the pipe. Thus the frictional pressure drop pertains to the gas interacting with the wavy liquid film. The frictional pressure drop, which is a large component of the total pressure drop, can be written as:

$$\Delta p_f=\frac{2f_c\rho_c(v_{gs}/E_g)^2}{g_c D} \tag{20}$$

where:

$$f_c=0.079[1+75(1-E_g)]/Re^{0.25} \tag{21}$$

$$\rho_c=\frac{v_{sg}\rho_g+E\rho_l v_{ls}}{v_{gs}+Ev_{gs}} \tag{22}$$

The expressions for entrainment, E, are given by Eqs. 13 and 14.

The total pressure gradient, including the accelerational component, is given by:

$$-\mathrm{d}p/\mathrm{d}z=\frac{1}{g_c}\frac{[g\rho_c+(2f_c\rho_c v_g^2/d)]}{[1-(\rho_c v_g^2/pg_c)]} \tag{23}$$

The gas void fraction in the gas core, E_{gc}, in Eq. 12 can be estimated from the Lockhart–Martinelli (1949) parameter, X, which can be written in terms of the gas mass fraction, x, as:

$$X=[(1-x)/x]^{0.9}(\rho_g/\rho_l)^{0.5}(\mu_l/\mu_g)^{0.1} \tag{24}$$

Fluid *PVT* properties computation

Well-known empirical correlations were used to obtain the pressure- and temperature-dependent physical and transport properties of the two phases. For the oil phase, Katz's (1942) correlation is used to calculate the formation volume factor, while Standing's (1947) methods predicted the solution gas–oil ratio and bubblepoint pressure. For dead-oil viscosity we used Standing's (1962) correlation and the Chew–Connally (1959) correlation for live-oil viscosity.

The gas viscosity was calculated from the Lee et al. (1964) method, whereas the gas-law deviation factor or z-factor was obtained from the Standing's (1977) modification of Beggs–Brill curve-fit of the Standing–Katz (1959) z-factor chart. In computing surface tension, we used the Baker–Swerdloff (1956) correlation for gas/oil system and the Hough et al. (1951) correlation for gas/water system.

We recognize that the Beggs and Robinson (1974) correlations for the dead- and live-oil viscosities are perhaps superior to those used in this work. However, two reasons prevent us from using the Beggs and Robinson correlations. First, the correlations have a lower limit of 15°API oil gravity, whereas we treated tests having as low as 8.3°API oil. Second, we compared our results directly with the published results, which make use of the classical correlations (Standing, 1962; Chew and Connally, 1959). In any case, we point out that the oil viscosity plays a minor role in pressure drop computation.

Similar comments apply for not using the Vazquez–Beggs (1980) correlations for estimating oil formation volume factor and solution gas–oil ratio. Besides the lower limit of oil gravity (15° API), the correlations require the knowledge of gas gravity at the separator conditions and of the operating pressure and temperature of the separator. Unfortunately, separator conditions are rarely reported together with other data for two-phase flow calculations.

Computational method

An iterative finite-difference algorithm is developed to calculate depth increments corresponding to the specified values of pressure drop, Δp. Gas void fraction and hence the hydrostatic head together with the frictional head are calculated after evaluation of the fluid properties and volumetric flow rates of the phases at the mid-point of the depth increment. When the calculated value of Δp is within ± 0.10 psi of the assumed value, the calculation proceeds to the next step. Otherwise, the depth increment is halved and the procedure repeated. This iterative calculation is continued until the sum of the depth increments equals the length of the flow string.

During the pressure traverse calculation, a "flow regime" sub-routine identifies the flow regime prevalent in a certain pipe segment, thereby permitting the use of an appropriate set of equations for calculating the total Δp.

The flow-regime transition equations presented earlier pose some problems during computation. The problems stem from sharp rather than a smooth change at the transition boundaries. This apparent discontinuity problem is pronounced at the bubbly/slug and churn/annular boundaries. For the bubbly/slug transition, a weighted-average rise velocity is calculated using the terminal rise velocities of a single and a Taylor bubble spanning over $\pm 15\%$ of the transition velocity itself. The void fraction in the transition zone is estimated from Eq. 7 by simply replacing the terminal rise velocity with the weighted-average rise velocity. This scheme ensures a smooth transition across the bubbly/slug boundary. Treatment of the churn/annular boundary is somewhat different, however. A zone of transition is arbitrarily defined by assuming a $\pm 15\%$ of the velocity given by Eq. 3. Void fractions are directly computed by assuming both the churn and annular flow, and weighted by the relative "position" of the velocity with respect to the transition velocity.

Model validation and comparison

The validity of our model is examined by comparing with measured pressure-drop data, together with other methods. Some 115 test data points are considered for statistical comparison.

In this study, the computation involved the proposed method and that of Aziz et al. (1972): the remaining results of other methods are taken from various sources. The other predictive schemes considered for comparison are those of Orkiszewski (1967), Duns and Ros (1963), Beggs and Brill (1973), Hagedorn and Brown (1965) and Chierici et al. (1974a).

TABLE 1

Range of well test parameters used in this study

	Field Units	SI units
Oil producing rate	44 – 11 623 STB/D	7 – 1838 $m^3 d^{-1}$
Gas–oil ratio	143 – 9975 SCF/STB	25 – 1776 $m^3 m^{-3}$
Water–oil ratio	0 – 1.38	–
Oil gravity	8.3 – 46° API	–
Gas gravity (air = 1)	0.57 – 1.7	–
Well Depth	3890 – 13 123 ft	1186 – 4000 m
Internal tubing diameter	1.995 – 4.488 in	50.67 – 114 mm
Wellhead pressure	20 – 2780 psig	138 – 19 168 kPa
Bottomhole pressure	640 – 6630 psig	4413 – 45 714 kPa
Observed pressure-drop	474 – 4566 psi	3268 – 31 482 kPa
Wellhead temperature	40 – 241 °F	277.5 – 389 K
Bottomhole temperature	95 – 297 °F	308 – 420 K

Data sets from four independent sources are used for the comparative study. Table 1 lists the range of well-test parameters used in this study. We discuss each data set separately.

Data of Aziz et al.

In their original work, Aziz et al. (1972) performed computation on 48 well tests, using their own method and that of Orkiszewski (1967). They reported Espanol's (1968) calculated results on 38 wells for Hagedorn and Brown (1965) and Duns and Ros (1963) correlations. Table 2 displays those results together with calculated results from the proposed method showing percentage error in predicting flowing bottomhole pressures.

The calculated percentage error in this study for both the total pressure drop and the flowing bottomhole pressure are very close to the reported values of Aziz et al. when their method was used. Therefore, they are not repeated here.

Table 3 compares the statistical results of five methods for predicting the total pressure drop. The proposed method performs just as well as the methods of Aziz et al. and Orkiszewski for this data group, dominated by the single-phase, bubbly and slug flow regimes. In most cases, the hydrostatic head contributed largely to the total pressure drop; the frictional head's contribution was no greater than 2% of the total. Exception to this observation occurs in some high flow rate wells. For example, well no. 47 indicates a Δp_f of 16.4% of the total.

As pointed out by Aziz et al., some of the data points are obviously suspect. For example, although flow in well nos. 1 and 16 is essentially single phase, large errors are predicted by *all* the methods. These and other dubious data points are excluded from the statistical analysis.

Data of Chierici et al.

This data group is perhaps one of the most reliable and complete sets reported in the literature. Comparison of the various methods for the total pressure-drop is shown on Table 4.

Flow patterns in most wells are restricted to single phase, bubbly and slug. Churn and annular flow regimes are rarely observed in these wells. The proposed method appears to perform quite satisfactorily compared to the Chierici et al. and Aziz et al. methods. Note that the Chierici et al. method is expected to perform the best because their correlation was "tuned" to this data set.

Our calculations with the Aziz et al. (1972) method are in good agreement with those reported by Gregory et al. (1980). As indicated

TABLE 2

Comparison of results using Aziz et al. data

Well No.	Aziz et al.	Orkiszewski	Hagedorn–Brown	Duns–Ros	Proposed method
	Error in calculated flowing bottomhole pressure (%)				
1	29.7	29.7	–	–	33.70
2	2.0	2.0	2.1	1.7	2.10
3	−6.2	−6.3	−5.9	−7.0	−5.7
4	−4.9	−4.9	−4.7	−7.8	−4.9
5	16.6	16.2	–	–	17.0
6	−8.8	−8.8	−8.6	−10.9	−8.7
7	−6.4	−6.4	−6.3	−8.7	−6.3
8	−3.3	−3.3	−3.2	−8.4	−3.2
9	9.3	7.4	–	–	10.4
10	−11.9	−11.9	−11.7	−14.1	−11.8
11	40.0	36.7	23.6	37.8	41.8
12	−3.7	−7.6	−11.0	−5.0	−2.1
13	−11.1	−9.5	−51.3	−14.3	−9.6
14	−1.7	−1.7	–	–	−1.6
15	−8.7	−7.2	−73.3	−13.2	−7.8
16	55.3	53.4	33.0	4.1	56.0
17	−11.5	−12.1	−37.3	−14.4	−11.1
18	4.1	13.7	−59.3	−2.4	–
19	−8.4	−8.5	−8.4	−10.2	−8.0
20	5.9	−4.8	−24.6	−0.2	4.1
21	−4.5	−10.7	−30.0	−10.0	−7.2
22	−13.6	−24.9	−41.6	17.9	−18.0
23	4.8	2.6	–	–	1.03
24	14.8	1.9	–	–	–
25	19.5	14.9	−9.3	15.2	17.80
26	12.9	12.5	13.3	10.7	13.0
27	−1.3	−3.6	−1.0	1.0	0.80
28	10.2	9.6	10.4	8.0	10.8
29	−10.3	−7.2	−84.2	−16.5	−8.5
30	−14.7	−14.8	−15.2	−15.4	−14.2
31	−15.1	−15.8	−31.6	−19.0	−15.6
32	−5.7	−6.8	–	–	−4.7
33	12.3	11.7	12.7	11.0	12.4
34	−14.0	−14.6	−29.9	−17.2	−13.1
35	19.0	6.8	−27.5	46.0	13.8
36	−6.3	−9.5	−7.8	−6.4	−5.9
37	13.9	13.1	−23.7	11.1	12.1
38	7.2	6.5	2.4	4.3	7.1
39	−7.7	−5.2	−38.4	−12.4	−9.6
40	−1.8	−2.6	−3.6	−6.6	−1.8
41	−5.1	−5.1	−6.6	−6.5	−5.0
42	−7.1	−7.6	−8.7	−8.4	−6.9
43	−6.2	−6.4	−8.6	−8.4	−6.0
44	−11.1	−17.9	−3.4	−19.6	−16.4
45	9.5	−0.6	–	–	10.6
46	2.0	–	–	–	1.3
47	15.8	16.8	–	–	17.1
48	21.0	23.3	5.0	23.1	18.2

TABLE 3

Statistical results using Aziz et al. data

Prediction method	Average error (%)	Standard deviation (%)	Remarks
Aziz et al.	−0.57	10.45	44 points used (well nos. 1, 11, 16, 46 discarded)
Orkiszewski	−1.98	10.69	Same as above
Hagedorn and Brown	−17.52	22.98	36 points used as indicated on Table 2
Duns and Ros	−4.13	13.52	Same as above
*Proposed method	−1.02	10.37	43 points used (well nos. 1, 11, 16, 18,24 discarded)

*Computation performed for this study: the remaining results are from Aziz et al.

TABLE 4

Statistical results using Chierici et al. data

Prediction method	Average error (%)	Standard deviation (%)	Remarks
Beggs and Brill	3.97	13.76	All 31 points considered in all cases
Aziz et al.	−2.82	5.75	
Orkiszewski	−2.18	14.70	
Duns and Ros	6.94	15.43	
Hagedorn and Brown	1.06	17.01	
+Chierici et al.	0.12	5.51	
*Aziz et al.	−2.45	7.01	
*Proposed method	−0.86	9.27	

*Computation performed for this study: the remaining results are from Gregory et al. and Chierici et al.
+Results from Chierici et al.

in Table 4, the top five entries are reproduced from Gregory et al.'s work for top-down calculations using the experimental bubblepoint pressure.

We observed an improvement in pressure-drop prediction when the reported bubble-point pressure was used instead of the Standing's predicted value. The same conclusion was reached earlier by Gregory et al. (1980). Thus, whenever the producing bottomhole pressure is above the oil's bubblepoint pressure – meaning single-phase flow prevails in some segment of the pipe – pressure-drop prediction becomes sensitive to the chosen bubblepoint pressure. Obviously, the input of measured bubblepoint pressure is desirable in such a computational scheme.

Data of Orkiszewski

Results of statistical analysis are presented in Table 5 for 22 heavy-oil (9.5–18.7° API) wells. The proposed method appears to perform reasonably well when compared with the methods of Orkiszewski and Aziz et al. As indicated on Table 4, Lawson and Brill's (1974) calculated results are used for comparison.

Lawson and Brill reported a large discrepancy that exists between their own results and those of Orkiszewski (1967). They attributed this anomaly to the definition of percentage error. However, the confusion resurfaces when the same data are reexamined by Gould et al. (1974), who achieved good agreement with Orkiszewski's calculated values.

Because of the lack of necessary data on oil and gas gravities, top and bottomhole temperatures and the like, we assumed the same values as reported for well 22 in Orkiszewski's original paper for all the wells. These necessary assumptions could conceivably lead to disagreements in results. In addition we found that some of the wells, five to be specific, behave erratically and were excluded from the statistical analysis.

TABLE 5

Statistical results using Orkiszewski data

Prediction method	Average error (%)	Standard deviation (%)	Remarks
Orkiszewski	−12.0	13.1	All 22 points considered
Duns and Ros	−42.0	19.7	Same as above
Hagedorn and Brown	−29.7	42.0	Same as above
*Aziz et al.	24.92	13.3	17 points considered
*Proposed method	13.27	18.44	Same as above

*Computation performed for this study: the remaining results are from Lawson and Brill.

TABLE 6

Statistical results using Baxendell–Thomas data

Prediction method	Average error (%)	Standard deviation (%)	Remarks
Orkiszewski	−3.7	41.1	All 25 points used
Duns and Ros	−13.9	39.5	All 25 points used
Hagedorn and Brown	6.0	11.6	All 25 points used
*Axis et al.	−38.8	26.8	24 points used, test # 14 discarded
*Proposed method	−8.8	27.3	

*Computation performed for this study: the remaining results are from Lawson and Brill.

Data of Baxendell and Thomas

This data set, obtained from a series of high-rate experiments (upto 5082 STB/D), indicates presence of annular flow regime throughout the pipe in many cases. Results of the statistical analysis reflecting the total pressure-drop prediction are presented in Table 6. Test number 14 is discarded from analysis because both Aziz et al. and the proposed method predict over 200% error in pressure drop: experimental error is suspected.

The proposed method appears to perform better than the other flow-pattern approaches when both the average error and standard deviation are compared. This result probably reflects the improved recent knowledge gained in the annular flow regime as implemented in the model. We however note that the Hagedorn–Brown (1965) slip model performs surprisingly well for this data set.

Discussion

In this work we tested and compared results of the proposed model with those routinely used in the industry. As indicated in the previous section, the proposed model performs just as well as the Aziz et al. (1972) and Orkiszewski (1967) models when the flow is predominantly in the bubbly and slug flow regimes. This observation is not very surprising because the proposed model does not substantially differ from the Aziz et al. model. Bubbly-slug transition criterion and the consideration of dispersed bubbly flow are the major differences between the two models when bubbly and slug flow regimes are considered. However, because of different transition criteria in the two models, the proposed method predicts a longer pipe segment experiencing bubbly flow than that of Aziz et al.

When most of the pressure drop occurs in the churn and annular flow regimes, Aziz et al.'s (1972) model appears to yield large errors, as Table 6 indicates. We point out that the transition criteria for the churn and annular flow are adopted from the work of Duns and Ros (1963), although not stated explicitly. Here, the proposed model appears to perform well because of improved treatment of the transition to annular flow and of the frictional pressure drop in annular flow.

Thus when we consider the 115 test data group spanning all the four flow regimes, the proposed model indicates an improvement over the Aziz et al. method. Table 7 shows the comparative statistical data, while Figs. 3 and 4 exhibit the graphical comparison of the two models with the measured data.

Despite the apparent improvement in pressure-drop prediction, many uncertainties remain. The *first* question concerns the relative pipe roughness. Certain crude constituents, wax for example, are likely to be deposited as a thin film on the pipe wall, resulting in a smooth pipe. In all of our computations, a ϵ/D of 0.0008 was used to retain the smooth pipe definition. Additionally, estimation of friction factor from the knowledge of Reynolds number and relative pipe roughness still retains empirical origin, especially in two-phase flow. Thus frictional pressure-drop prediction is not rigorous.

TABLE 7

Overall comparison of proposed method with Aziz et al., 115 tests

Prediction method	Average error (%)	Standard deviation (%)
Aziz et al.	−5.34	25.42
Proposed method	1.25	19.05

Fortunately, the frictional pressure-drop component is only a small fraction of the total pressure drop in many cases. For example, Gregory et al. (1980) showed that the frictional head amounted to only a maximum of 5% of the total head in 85 of their 105 well tests.

The preceding observation implies that the hydrostatic head is by far the most dominant component when the flow pattern can be described as either single phase, bubbly and slug or a combination of these. However, at higher superficial fluid velocities, for example in annular flow regime, the dominating influence of the frictional head warrants a good handle on the friction factor itself.

The *second* question pertains to the reliability of data against which a model or a correlation is judged. Barring some obvious reporting errors from the field as eluded to earlier, published data do not state the well/reservoir flow condition. In other words, if the surface flow measurements are made during the unsteady-state flow period in the reservoir, serious problems may arise. These problems relate to the reported values of flowing bottomhole pressure (p_{wf}) and the corresponding flow rates. The measured p_{wf} would change minimally when the pseudosteady-state flow is attained in the reservoir. ideally, one would like to observe no change in the p_{wf} when steady-state flow occurs in the reservoir. We point out that stable flow condition in the entire flow string is required for application of the steady-state pressure-drop correlations. Recognizing the difficulty of maintaining a constant wellhead rate and given the current surface flow-rate measurement practice, we recommend gathering p_{wf} and flow-rate data during either pseudosteady- or steady-state flow in the reservoir.

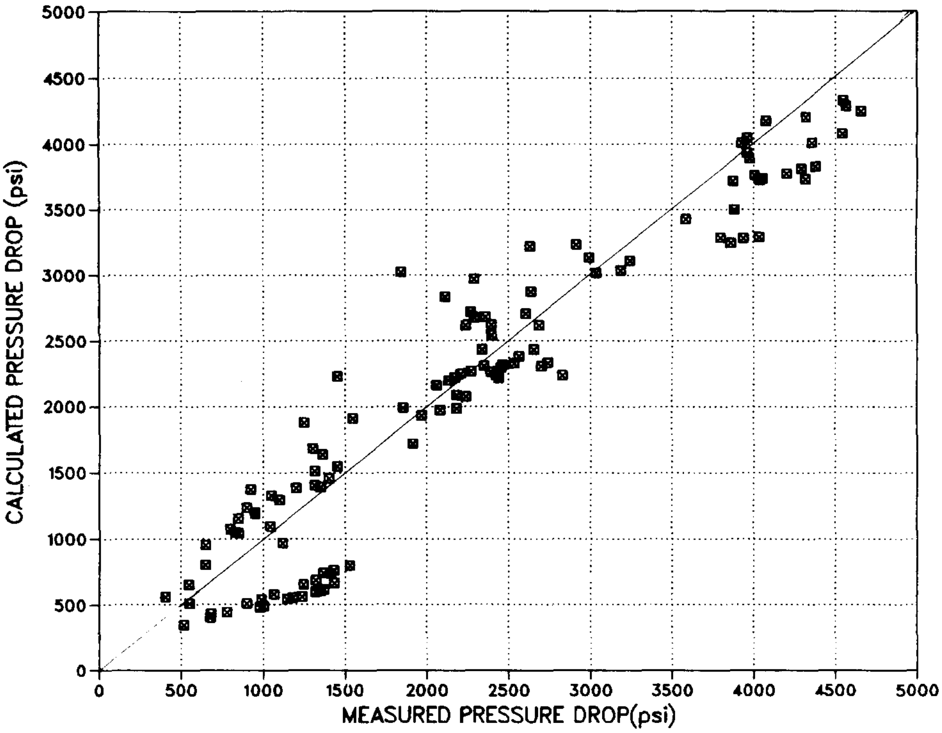

Fig. 3. Comparison of measured and calculated pressure drops: Aziz et al. method.

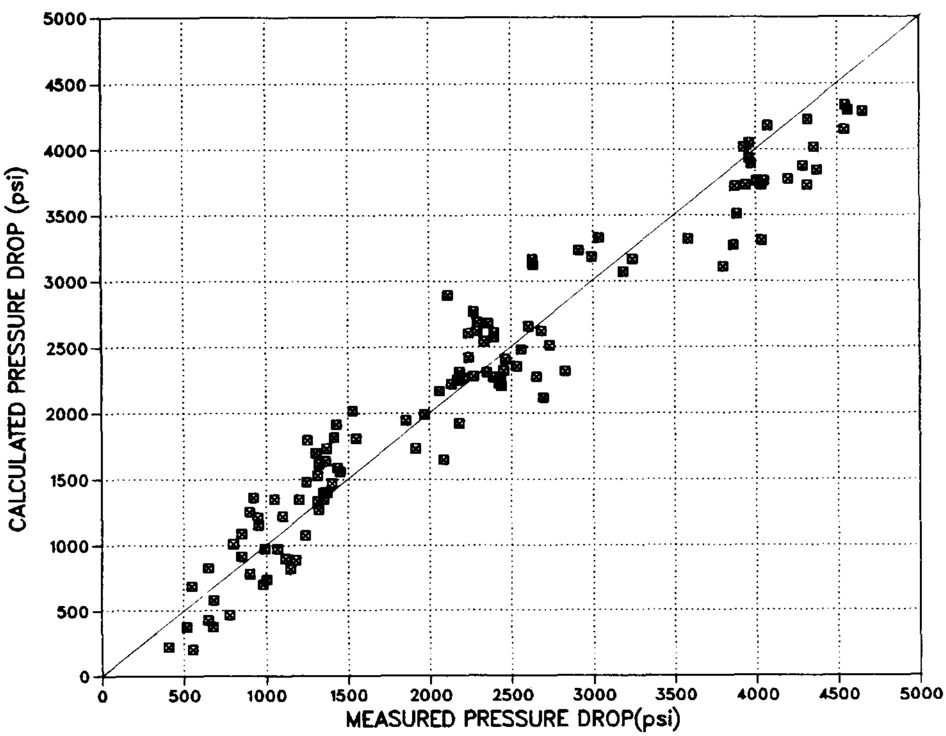

Fig. 4. Comparison of measured and calculated pressure drops: proposed method.

Flow rate measurement at the surface over a test duration gives only a time-average value, with no indication of instantaneous variations that might have occurred. Experience indicates that instantaneous *downhole* measurements of flow rate together with pressure, density and temperature can largely overcome the limitations associated with surface measurements. While the oil flow-rate measurement is accurate, the same cannot be stated for gas and water. Our calculations indicate that some of the discrepancies between the predicted and the observed pressure drops could be explained by only $\pm 15\%$ change in gas-oil ratio.

Another source of uncertainty in the reported data stems from the tubing diameter. A nominal diameter of, say $2\frac{3}{8}$ inch, does not specify the internal diameter required for calculations. Consequently, we observe a 5% difference in the tubing diameter translating into a large error in the pressure-drop prediction – upto 8% in some cases. Such an error is likely to be significant when the overall pressure drop is dominated by the frictional head.

Additionally, our implicit assumption is that all the test data pertain to truly vertical wells. This assumption is seldom strictly correct, especially for deep wells. In a deviated well, considerable problems may arise when slug flow is present in the flow string. For example, the increase in Taylor bubble rise velocity compared to that observed in a vertical well (Hasan–Kabir, 1988b) will translate into a lower gas void fraction and hence a higher overall Δp. Thus, ignoring the true well deviation is a potential source for error.

Even though a number of uncertainties inherent in field data may potentially prevent us from rigorously testing a model, comparison with other models largely removes the validation problem. In this work, we considered some 115 well tests. Because a larger database is desirable from model validation viewpoint, we do not claim universality of the proposed model.However, we point out that the *quality* of data is of greater importance than the *quantity* whenever a meaningful validation and comparative study is attempted.

One of the major applications of a vertical multiphase flow model is to generate the tubing performance curves under various operating conditions of gas–oil ratio, tubing head and bottomhole pressures, and tubing sizes. Under a set of operating variables, even an empirical correlation may indicate the least error compared to a mechanistically based model, such as the one described here. However, with a change in operating variables, an empirical correlation is unlikely to perform well even though a good match was indicated previously – a typical problem with an empirical correlation. For example, the Lawson and Brill (1974) study and the subsequent discussion by Chierici et al. (1974b) clearly expose the limitations associated with the homogeneous model of Poettmann and Carpenter (1952) and other methods (Baxendell and Thomas, 1961; Fancher and Brown, 1963; Duns and Ros, 1963). Thus, for any design calculations, we strongly recommend use of a mechanistic model for reliable performance over a wide operating conditions.

Conclusions

The general observations from this study are in good agreement with those reported in the literature. In the following, we summarize the specific conclusions reached.

(1) When vertical multiphase flow is dominated by any combination of single-phase, bubbly and slug flow regimes, the new method is consistent with the other mechanistic models of Aziz et al. and Orkiszewski.

(2) In the churn and annular flow regimes, the new method appears superior to the existing models.

(3) For the 115 well tests considered in this work, encompassing all the four two-phase flow regimes, the new method appears to be a letter overall performer than that of Aziz et al. Comparison with other methods also indicates the

new method's improved performance.

(4) A number of uncertainties may cloud a model validation study. These uncertainties include relative pipe roughness of the flow channel, gas–oil ratio, water–oil ratio, well deviation, oil bubblepoint pressure, and pressure and flow-rate data (that is whether gathered during stable well and reservoir flow condition).

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