Vertical Multiphase Flow Correlations for High Production Rates and Large Tubulars

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Summary

Numerous correlations exist for predicting pressure drop in vertical multiphase flow. These correlations, however, were all developed and tested under limited operating conditions that do not match the high production rates and large tubulars normally found in the Middle East fields. This paper presents a comprehensive evaluation of existing correlations and modifications of some correlations to determine and recommend the best correlation or correlations for various field conditions. More than 400 field data sets covering tubing sizes from 2$^3/_8$ to 7 in., oil rates up to 23,200 B/D, water cuts up to 95%, and gas/oil ratio (GOR) up to 927 scf/STB were used in this study. Considering all data combined, the Beggs and Brill correlation provided the best pressure predictions. However, the Hagedorn and Brown correlation was better for water cuts above 80%, while the Hasan and Kabir model was better for total liquid rates above 20,000 B/D. The Aziz correlation was significantly improved when the Orkiszewski flow-pattern transition criteria were used.

Introduction

A reliable and accurate way of predicting pressure drop in vertical multiphase flow is essential for the proper design of well completions and artificial-lift systems and for optimization and accurate forecast of production performance. Because of the complexity of multiphase flow, mostly empirical or semiempirical correlations have been developed for prediction of pressure drop. Many correlations have been developed since 1914. However, the correlations that have seen the most application and testing are Aziz *et al.*,[1] Hagedorn and Brown,[2] Duns and Ros,[3] Beggs and Brill[4] and Hasan and Kabir.[5] Each of these correlations was developed and/or tested for a specific range of operating conditions.

Several studies have been performed to evaluate various multiphase-flow correlations. Orkiszewski[6] was the first to perform such a study using 148 well cases, resulting in the development of his own correlation. A later study by Espanol[7] on 44 wells confirmed the conclusions of Orkiszewski. Camacho[8] used data from 111 high gas/liquid ratio (GLR) wells to test five correlations. He concluded that no method was sufficiently accurate for all ranges of GLR. He reported that the best results were obtained from the Fancher and Brown[9] correlation, followed by the correlation of Poettmann and Carpenter.[10] A study by Messulam[11] using data from 434 wells concluded that the Hagedorn and Brown[2] correlation was the most accurate, followed by Orkiszewski's correlation.[6] None of the correlations, however, had consistent accuracy for all flow conditions. Similar conclusions were also obtained by Lawson and Brill.[12] Vohra *et al.*[13] evaluated more recent pressure-drop-prediction methods against the same data used by Lawson and Brill. They obtained the best predictions from the correlation of Aziz *et al.*[1] followed by the Beggs and Brill[4] and Chierici *et al.*[14] correlations. Kabir and Hasan[15] used 115 data sets to evaluate several correlations. They concluded that their model[5] performs as well as the Aziz *et al.*[1] and Orkiszewski[6] correlations when the flow is predominantly in the bubble and slug flow regimes. They also reported that, in the churn and annular flow regimes, their correlation is superior to all existing models.

Experience, therefore, has shown that the accuracy of any correlation depends on the range of variables under which it is used. Therefore, no correlation should be used in any field before first testing and evaluating its accuracy under the specific field conditions. None of the previous studies used data that cover the range of operating conditions normally experienced in the Middle East fields, particularly the high production rates and large tubing sizes. The purpose of this study was, therefore, to evaluate the most commonly used correlations against data from Middle East fields and to recommend the best correlation(s) for specific ranges of operating conditions.

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Present Data

Data from 414 wells from five different fields were used in the study. **Table 1** lists the range of values for the various variables covered by the data. Wellhead pressures were measured with calibrated pressure gauges or dead-weight testers. Bottomhole pressures (BHP's) were measured with Amerada recorders. Normally, more than one pressure recorder was used. The readings from the recorders were first corrected to a certain depth and then averaged. Bottomhole temperatures (BHT's) were measured in a similar manner. The phase flow rates were measured at the separator conditions and corrected to standard conditions.

Method of Analysis

The data were classified according to tubing size, total liquid rate, water cut, and GLR. Each of the six correlations was tested against the individual groups of data and against all data combined. A crossplot of the predicted and measured pressures was prepared for each case to illustrate the general trend of the correlation for a particular group of data. A statistical analysis of the predictions was then performed with the following statistical parameters: the percent relative error, e; the average absolute percent relative error, $\bar{e}$; and the correlation coefficient, r. A definition of these parameters is given in the Appendix. A comparison of all correlations was then performed illustrating the effects of tubing size, water cut, liquid rate, and GLR on the accuracy of the correlations.

Results and Discussion

The six most commonly used correlations were tested in this study; these are the correlations of Aziz *et al.*,[1] Hagedorn and Brown,[2] Duns and Ros,[3] Beggs and Brill,[4] Hasan and Kabir,[5] and Orkiszewski.[6] Computer programs were written for calculation of the total pressure drops for each correlation. For each correlation, the specific friction-factor correlation recommended by the respective author was used in the determination of the frictional-pressure drops. The friction factors were determined based on a tubing surface roughness of 0.00015 (for new commercial steel). We should mention that the surface roughness, and consequently the friction factor, may influence the frictional-pressure-drop calculations, especially at high flow rates. The roughness of a new tubing, however, was used in all calculations because most data were collected from wells with relatively new tubings.

For all correlations, the oil-formation-volume factor was determined from the Frick[16] correlation, the solution GOR was determined from the Standing[17] correlation, the oil viscosity was determined from the Beggs and Robinson[18] correlation, the gas viscosity was determined from the Lee *et al.*[19] correlation, and the gas compressibility was determined from the Katz *et al.*[20] correlation.

The calculated total pressure drops from the various correlations were compared to the measured values. **Tables 2 through 7** summarize the results of the statistical analysis for the six correlations, respectively. **Figs. 1 through 6** illustrate the error distribution histo-

TABLE 1—RANGE OF VARIABLES FOR THE PRESENT DATA				
Variable	Minimum	Maximum	Average	Standard Deviation
Total production rate, STB/D	280	24,900	10,525	267
Oil production rate, STB/D	220	23,200	8,952	281
Gas production rate, Mscf/D	3.374	16,685	5,131	190
Water production rate, STB/D	0	7,705	1,573	102
GLR, scf/STB	6	927	437	10.6
Measured (vertical) depth, ft	4,550	8,580	6,353	28.5
ID, in.	1.995	6.366	3.937	0.038
Measured BHP, psia	1,115	3,308	2,425	17.6
Measured wellhead pressure, psia	10	800	314	7.6
Wellhead temperature, °F	76	160	151	0.7
BHT, °F	157	215	204	0.8

TABLE 2—STATISTICAL ANALYSIS RESULTS FOR AZIZ *et al.* CORRELATION[1]										
Tubing Size				Data Points (% of total) lying in the specified range of deviation						
(in.)	Condition	$\bar{e}$	r	Points	±5%	±10%	±15%	±20%	±25%	±>25%
$2^3/_8$	WC = 0%	23.7	0	7	14	28	42		71	100
	0 < WC < 20%	19.4	0	8	37		50	63		100
and	20% < WC < 100%	18.3	0.62	8	37	50	62			100
$2^7/_8$	0 < GLR < 350	10.2	0	5	40	60	80			100
	350 < GLR < 750	26.4	0	11	18		27	36	45	100
	750 < GLR < 950	17.9	0	7	44	58	72		86	100
	All Data	20.3	0	23	31	40	53	57	65	100
$3^1/_5$	All Data	16.5	0	10	20	40	60	80		100
4½	WC = 0%	14.6	0	117	38	48	58	67	77	100
	0 < WC < 20%	22.6	0	114	9	21	32	47	57	100
	20 < WC < 40%	9.81	0.64	56	37	57	80	89	93	100
	40 < WC < 60%	8.60	0.55	37	38	65	87	92	97	100
	60 < WC < 80%	8.35	0.58	17	42	65	88		94	100
	80 < WC < 100%	8.03	0.70	11	9	82	100			
	0 < GLR < 350	6.72	0.79	126	50	77	92	96	98	100
	350 < GLR < 750	21.3	0	194	16	25	38	49	60	100
	750 < GLR < 950	12.7	0	32	12	37	65	87	97	100
	All Data	15.3	0	352	28%	45%	59	69	77	100
7	WC = 0%	10.3	0.70	12	50	67		75	83	100
	0 < WC < 100%	16.0	0	17	18	43	62		80	100
	0 < GLR < 350	7.66	0	6	50		100			
	350 < GLR < 950	15.2	0	23	24	48	53	58	72	100
	All Date	13.6	0	29	31	52	66	69	79	100

WC=water cut

gram and the normal distribution for the six correlations. For the Aziz *et al.*[1] correlation, Fig. 1 shows a shift of the mean of the errors toward the positive side of the plot (at about 10%), indicating that the correlation in general underpredicts the pressure. Table 2 shows that, with the exception of the 7-in. tubing data, the correlation provides better predictions for higher water cuts and lower GLR values. The correlation is, in general, better for larger tubing sizes.

Fig. 2 shows that, for the Hagedorn and Brown[2] correlation, the errors are normally distributed with a mean approximately equal to 3%. This indicates a good correlation. Table 3 shows that the correlation provides better predictions for higher water cuts and lower GLR.

The error distribution for the Duns and Ros[3] correlation, Fig. 3, indicates a generally good correlation. Table 4 shows that the correlation has, in general, the same degree of accuracy for all tubing sizes. The accuracy of the correlation tends to improve as the GLR decreases, while the water cut has no significant influence.

Fig. 4 shows the error distribution for the Beggs and Brill[4] correlation. The errors are normally distributed and the mean is almost at zero, indicating a very good correlation. Table 5 lists the statistical parameters for the correlation. The table shows that the correlation predicts about 80% of the data within ±10%. The accuracy of the correlation is, in general, better for the smaller tubing sizes; it improves as the water cut increases and the GLR decreases.

The error distribution histogram of Hasan and Kabir's[5] model in Fig. 5 shows normal distribution with a mean at about 4%. Table 6 shows that the accuracy of the correlation is better for the larger tubings. There is no specific trend for the effect of water cut on the accuracy of the correlation; however, the accuracy improves at lower GLR values.

Fig. 6 shows that the Orkiszewski[6] correlation has a near normal error distribution with a mean at approximately 3%, indicating a good correlation. Table 7 shows that the accuracy of the correlation improves as the water cut increases, the GLR decreases, and the tubing size increases. The correlation, in general, provides acceptable predictions of pressure.

A comparison of all six correlations was made using all 414 data sets. The results are shown in **Figs. 7 through 10,** illustrating the effects of GLR, water cut, production rate, and tubing size on the accuracy of the correlations, respectively.

Fig. 7 shows that for a GLR < 350 scf/STB, all correlations predict the pressure within ±10% errors. For GLR > 350 scf/STB, the

TABLE 3—STATISTICAL ANALYSIS RESULTS FOR HAGEDORN AND BROWN CORRELATION[2]

Tubing Size (in.)	Condition	$\bar{e}$	r	Data Points (% of total) lying in the specified range of deviation						
				Points	±5%	±10%	±15%	±20%	±25%	±>25%
2 3/8	WC = 0%	10.1	0.61	7	29	58	71			100
	0 < WC < 20%	5.76	0	8	50	75	100			
and	20% < WC < 100%	6.77	0.96	8	50	87	100			
2 7/8	0 < GLR < 350	5.20	0.96	5	60	100				
	350 < GLR < 750	6.90	0.88	11	55	73	82	91	100	
	750 < GLR < 950	9.85	0.62	7	14	57	86		100	
	All Data	7.43	0.85	23	44	74	87	91	100	
3 1/5	All Data	9.15	0.81	10	50	70	80	90	100	
4½	WC = 0%	7.13	0.68	117	50	73	91	98		100
	0 < WC < 20%	8.59	0.22	114	47	68	80	89	95	100
	20 < WC < 40%	6.20	0.87	56	54	79	98	99	100	
	40 < WC < 60%	7.27	0.65	37	43	81	87	98	100	
	60 < WC < 80%	8.21	0.47	17	35	64	82	94	100	
	80 < WC < 100%	4.00	0.23	11	82	91		100		
	0 < GLR < 350	6.31	0.79	126	57	79	90	97	99	100
	350 < GLR < 750	8.28	0.48	194	43	69	84	92	96	100
	750 < GLR < 950	6.59	0.71	32	50	72	94	100		
	All Data	7.42	0.66	352	49	73	87	94	97	100
7	WC = 0%	7.45	0.70	12	58	83	91			100
	0 < WC < 100%	7.05	0.77	17	53	82	94			100
	0 < GLR < 350	1.96	0.94	6	100					
	350 < GLR < 950	8.59	0.73	23	43	78	91			100
	All Data	7.22	0.74	29	55	83	93			100

TABLE 4—STATISTICAL ANALYSIS RESULTS FOR DUNS AND ROS CORRELATION[3]

Tubing Size (in.)	Condition	$\bar{e}$	r	Data Points (% of total) lying in the specified range of deviation						
				Points	±5%	±10%	±15%	±20%	±25%	±>25%
2 3/8	WC = 0%	14.0	0	7	29	58		71		100
	0 < WC < 20%	5.96	0	8	37	87	100			
and	20% < WC < 100%	13.7	0.83	8	12	62	87			100
2 7/8	0 < GLR < 350	6.85	0.91	5	20	80	100			
	350 < GLR < 750	12.2	0.65	11	36	72	81			100
	750 < GLR < 950	12.5	0.09	7	14	58	72	86		100
	All Data	11.1	0.67	23	26	70	83	87		100
3 1/5	All Data	11.4	0.69	10	30	60	70		90	100
4½	WC = 0%	6.81	0.69	117	52	74	92	96	98	100
	0 < WC < 20%	8.34	0.26	114	48	68	82	89	92	100
	20 < WC < 40%	5.95	0.88	56	54	86	93	98	100	
	40 < WC < 60%	6.66	0.69	37	49	79	87	98	100	
	60 < WC < 80%	7.84	0	17	53	76	82	88	94	100
	80 < WC < 100%	6.91	0	11	27	82	100			
	0 < GLR < 350	5.67	0.83	126	59	83	92	97	99	100
	350 < GLR < 750	8.15	0.48	194	45	72	87	92	94	100
	750 < GLR < 950	7.53	0.62	32	47	66	85	97	100	
	All Data	7.21	0.67	352	50	75	88	94	96	100
7	WC = 0%	8.90	0.66	12	58	75		92		100
	0 < WC < 100%	7.97	0.73	17	41	79	94			100
	0 < GLR < 350	3.0	0.86	6	100					
	350 < GLR < 950	9.7	0.70	23	38	61	80	90		100
	All Data	8.3	0.70	29	48	69	86	93		100

Beggs and Brill correlation followed by Hagedorn and Brown correlation are superior to the rest of the correlations.

Fig. 8 shows that all correlations, with the exception of Aziz *et al's*., provide predictions within ±10% error at all water cuts. The Beggs and Brill correlation provides the best predictions for water cuts below 80%, while the Orkiszewski and the Hagedorn and Brown correlations are better for water cuts >80%.

Fig. 9 shows that, with the exception of the Aziz *et al*. correlation, the accuracy of all correlations is within ±12% at all production rates. The correlations of Beggs and Brill and Hagedorn and Brown are the best for rates up to about 20,000 B/D, while the model of Hasan and Kabir is the best at higher rates.

Finally, Fig. 10 shows that the accuracy of all correlations is better for the larger tubings, and the Beggs and Brill and Hagedorn and

TABLE 5—STATISTICAL ANALYSIS RESULTS FOR BEGGS AND BRILL CORRELATION[4]										
Tubing Size (in.)	Condition	$\bar{e}$	r	Data Points (% of total) lying in the specified range of deviation						
				Points	±5%	±10%	±15%	±20%	±25%	±>25%
$2\frac{3}{8}$	WC = 0%	9.88	0.57	7	29	71			100	
	0 < WC < 20%	5.18	0	8	50	87	100			
and	20% < WC < 100%	7.16	0.95	8	25	87		100		
$2\frac{7}{8}$	0 < GLR < 350	7.78	0.90	5	20	80		100		
	350 < GLR < 750	5.50	0.92	11	55	91			100	
	750 < GLR < 950	9.78	0.59	7	14	72	86		100	
	All Data	7.30	0.85	23	35	83	87	91	100	
$3\frac{1}{5}$	All Data	8.73	0.83	10	50	70	80	90		100
$4\frac{1}{2}$	WC = 0%	6.87	0.74	117	50	77	92	97	98	100
	0 < WC < 20%	6.79	0.61	114	54	77	88	94	96	100
	20 < WC < 40%	5.22	0.92	56	57	92	94	98		100
	40 < WC < 60%	6.37	0.77	37	51	81	92	97		100
	60 < WC < 80%	7.55	0	17	53	82	88		94	100
	80 < WC < 100%	7.43	0	11	36	72	100			
	0 < GLR < 350	6.27	0.81	126	58	82	91	95	97	100
	350 < GLR < 750	6.82	0.69	194	50	80	91	96	97	100
	750 < GLR < 950	6.36	0.72	32	47	75	91	100		
	All Data	6.58	0.76	352	52	80	91	96	97	100
7	WC = 0%	8.10	0.78	12	50	67	84	92		100
	0 < WC < 100%	6.74	0.82	17	35	82	94			100
	0 < GLR < 350	3.41	0.79	6	83	100				
	350 < GLR < 950	8.32	0.81	23	38	71	85	90		100
	All Data	7.30	0.81	29	45	80	90	93		100

TABLE 6—STATISTICAL ANALYSIS RESULTS FOR HASAN AND KABIR CORRELATION[5]										
Tubing Size (in.)	Condition	$\bar{e}$	r	Data Points (% of total) lying in the specified range of deviation						
				Points	±5%	±10%	±15%	±20%	±25%	±>25%
$2\frac{3}{8}$	WC = 0%	19.9	0	7	14	43	57			100
	0 < WC < 20%	8.22	0	8	25	75	88	100		
and	20% < WC < 100%	12.1	0.85	8	25	50	75	87		100
$2\frac{7}{8}$	0 < GLR < 350	9.3	0.86	5	40	60	80	100		
	350 < GLR < 750	13.2	0.52	11	18	55	73	82		100
	750 < GLR < 950	15.5	0	7	14	57	71			100
	All Data	13.1	0.49	23	22	57	74	83		
$3\frac{1}{5}$	All Data	14.8	0	10	20	40	60	70	80%	100
$4\frac{1}{2}$	WC = 0%	8.42	0.58	117	44	65	81	93	97%	100
	0 < WC < 20%	10.5	0	114	29	56	72	83	87%	100
	20 < WC < 40%	8.25	0.78	56	35	67	83	92	96%	100
	40 < WC < 60%	7.74	0.64	37	35	78	86	94	97%	100
	60 < WC < 80%	8.09	0	17	48	65	82	88	94%	100
	80 < WC < 100%	7.76	0	11	9	82	100			
	0 < GLR < 350	6.25	0.81	126	52	81	91	96	98%	100
	350 < GLR < 750	11.5	0	194	25	51	72	86	91%	100
	750 < GLR < 950	7.99	0.61	32	34	78	88	94	97%	100
	All Data	9.30	0.44	352	36	65	80	90	94%	100
7	WC = 0%	10.2	0.49	12	58	75		83		100
	0 < WC < 100%	7.33	0.72	17	47	82	94			100
	0 < GLR < 350	4.53	0.64	6	50	10%				
	350 < GLR < 950	9.55	0.63	23	52	74	83	88		100
	All Data	8.5	0.64	29	52	80	87	90		100

Brown correlations provide the best predictions among all correlations.

Table 8 summarizes the results of the statistical analysis of the correlations using all 414 data sets. It is clear from the table that the Beggs and Brill correlation provides the best overall accuracy; the correlations of Hagedorn and Brown, Duns and Ros, and Orkiszewski are the next best.

An attempt was made to improve the accuracy of the correlations by using various available flow-pattern-transition criteria. No improvement was obtained, except for the Aziz correlation, when the Orkiszewski's flow-pattern transition criteria were used. The modification reduced the average absolute error from 15.5% to 8.7%, and predicted 85% of the data within ±15% error.

TABLE 7—STATISTICAL ANALYSIS RESULTS FOR ORKISZEWSKI CORRELATION[6]

Tubing Size (in.)	Condition	$\bar{e}$	r	Data Points (% of total) lying in the specified range of deviation						
				Points	±5%	±10%	±15%	±20%	±25%	±>25%
$2^3/_8$	WC = 0%	18.3	0	7	29	29	43	57	71	100
	0 < WC < 20%	7.20	0	8	37	74	100			
and	20% < WC < 100%	15.4	0	8	12	25	62	87		100
$2^7/_8$	0 < GLR < 350	11.2	0.74	5	20	40	80	100		
	350 < GLR < 750	13.2	0.55	11	36	45	72	81		100
	750 < GLR < 950	15.3	0	7	14	44	58	72	86	100
	All Data	13.4	0.44	23	26	44	70	83	87	100
$3^1/_5$	All Data	12.4	0.51	10	3	30	40	80		100
$4½$	WC = 0%	7.60	0.63	117	45	71	88	97	98	100
	0 < WC < 20%	9.38	0	114	44	64	80	86	90	100
	20 < WC < 40%	6.43	0.87	56	48	82	92	96	100	
	40 < WC < 60%	6.16	0.75	37	54	81	86	100		
	60 < WC < 80%	7.35	0.25	17	35	82	88	94	100	
	80 < WC < 100%	4.84	0	11	73	91		100		
	0 < GLR < 350	6.03	0.84	126	54	81	90	98	100	
	350 < GLR < 750	8.97	0.26	194	42	68	83	91	94	100
	750 < GLR < 950	7.06	0.64	32	47	72	91	97	100	
	All Data	7.74	0.61	352	46	72	85	93	96	100
7	WC = 0%	8.32	0.73	12	50	75	84	93		100
	0 < WC < 100%	8.06	0.75	17	24	76	94			100
	0 < GLR < 350	5.90	0	6	50	83	100			
	350 < GLR < 950	8.70	0.76	21	33	71	85	90		100
	All Data	8.10	0.74	29	35	76	90	93		100

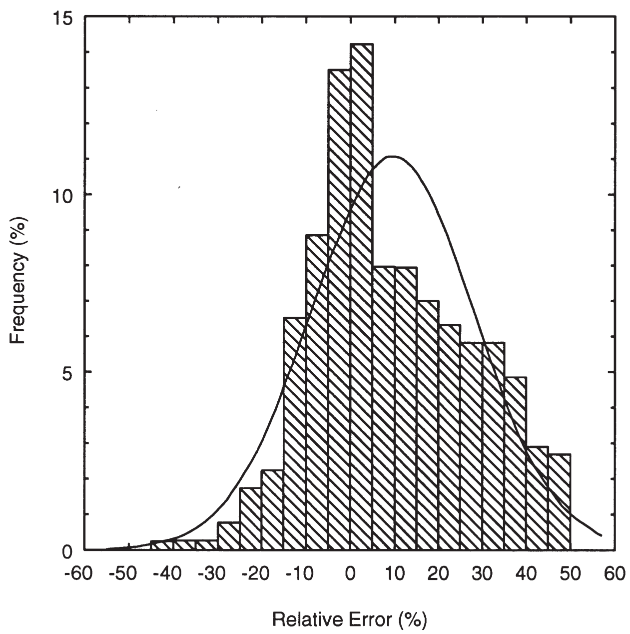

Fig. 1—Error distribution for Aziz *et al.* correlation.[1]

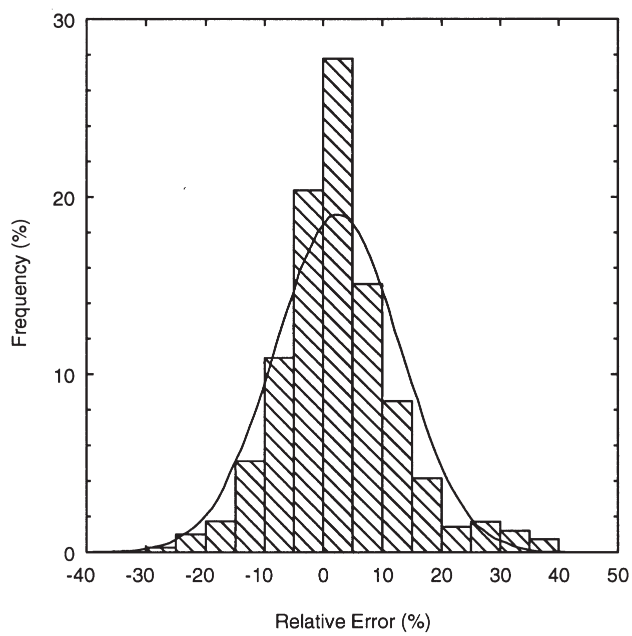

Fig. 2—Error distribution for Hagedorn and Brown correlation.[2]

Conclusions

Based on the present 414 data sets that cover a very wide range of tubing size, production rate, water cut, and GLR, the following conclusions are made.

1. No single correlation was found to provide good accuracy of pressure predictions for all ranges of variables.
2. The Hagedorn and Brown correlation provides the best predictions for wells producing with more than 80% water cuts.
3. The Hasan and Kabir model provides the best results for wells producing more than 20,000 B/D of total liquid.
4. The Beggs and Brill correlations followed by the Hagedorn and Brown correlation are found to be the best of the six correlations considering all present data combined.
5. The accuracy of the correlation of Aziz *et al.* may be greatly improved by implementing Orkiszewski's flow-pattern transition criteria.

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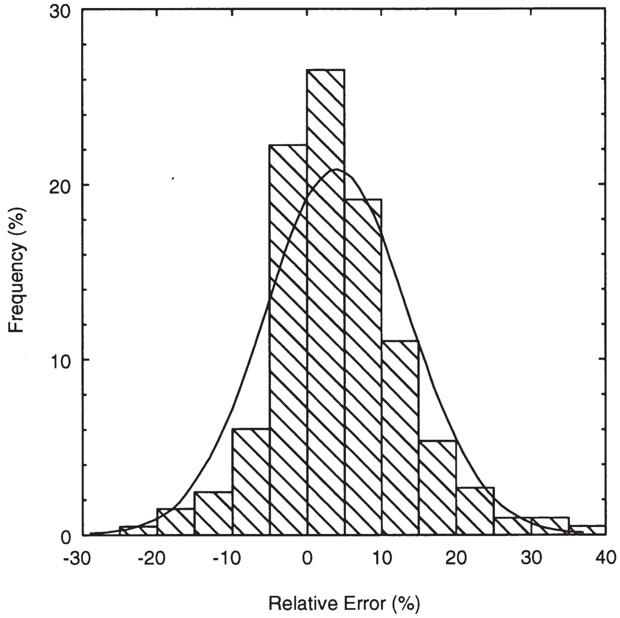

Fig. 3—Error distribution for Duns and Ros correlation.[3]

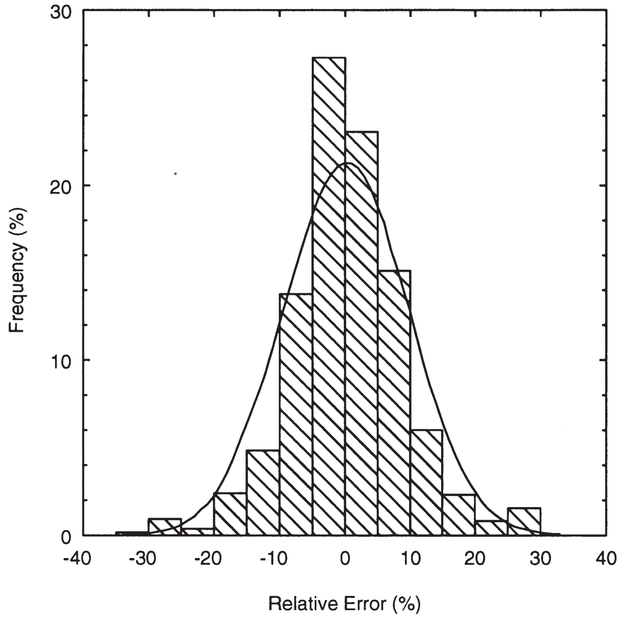

Fig. 4—Error distribution for Beggs and Brill correlation.[4]

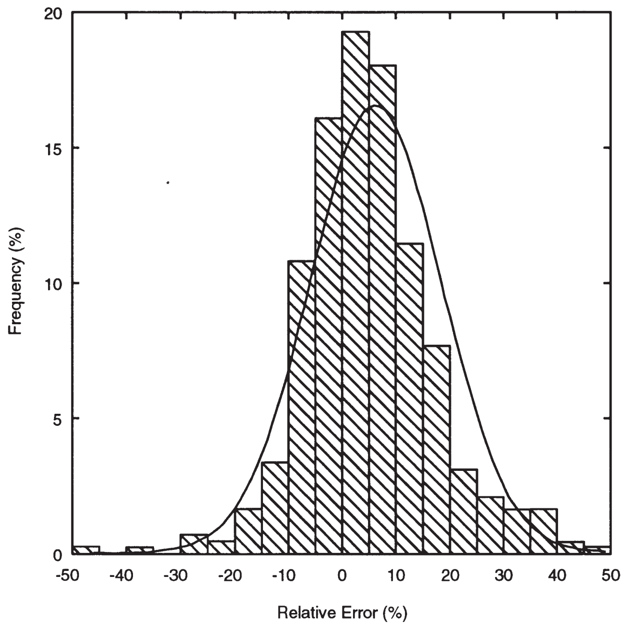

Fig. 5—Error distribution for Hasan and Kabir correlation.[5]

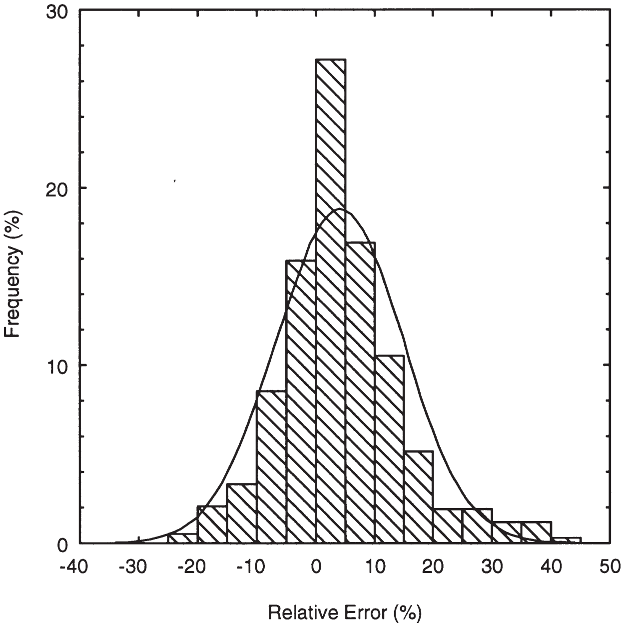

Fig. 6—Error distribution for Orkiszewski correlation.[6]

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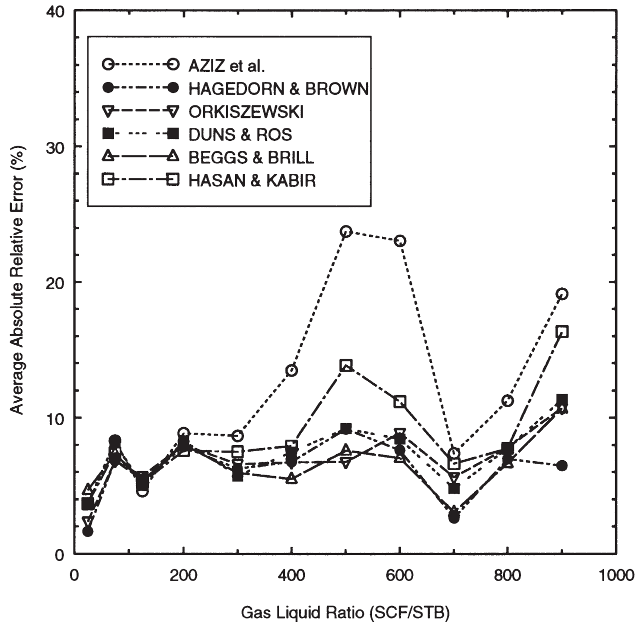

Fig. 7—Effect of GLR on accuracy of correlations.

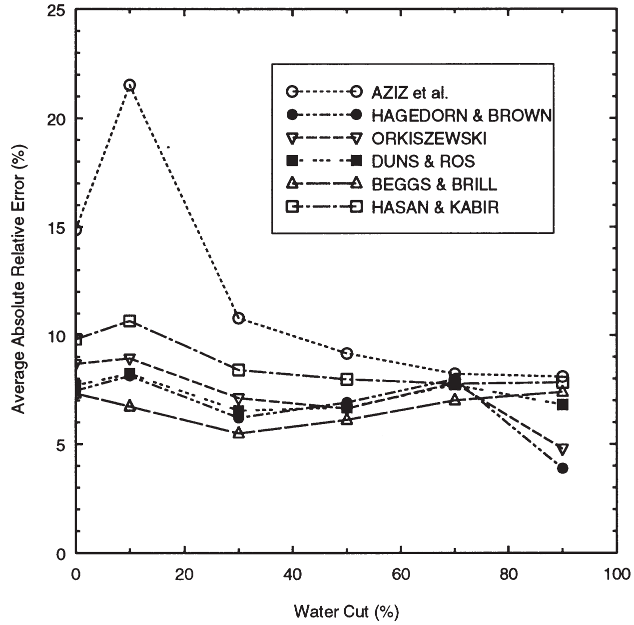

Fig. 8—Effect of water cut on accuracy of correlations.

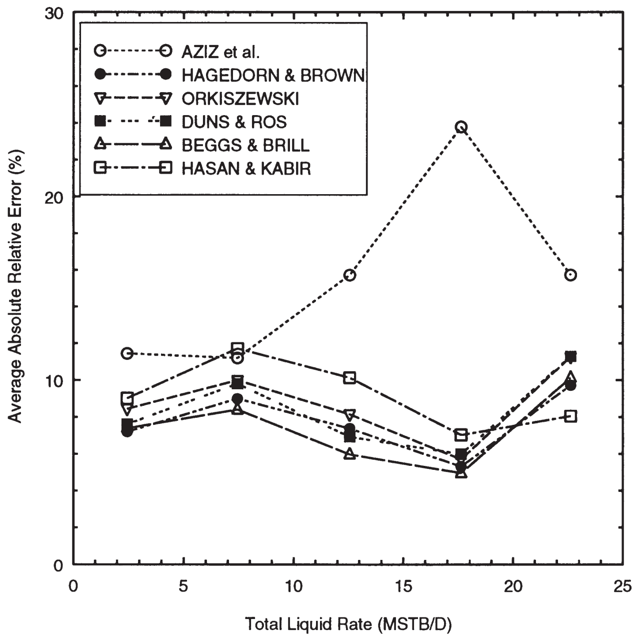

Fig. 9—Effect of rate on accuracy of correlations.

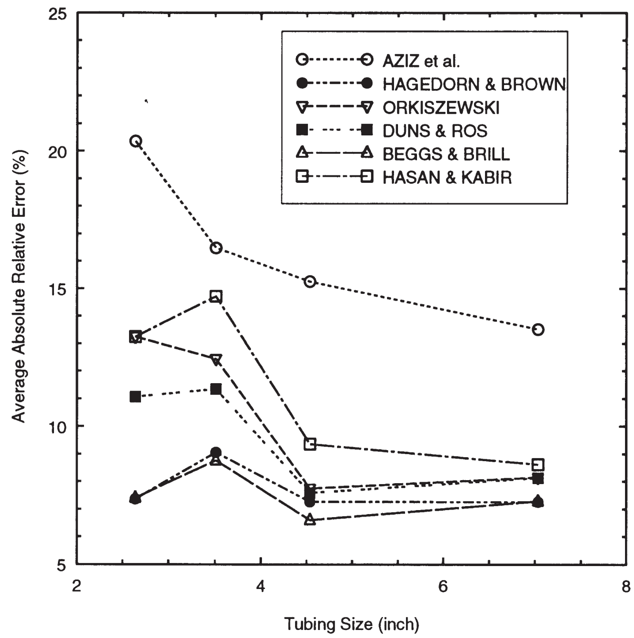

Fig. 10—Effect of tubing size on accuracy of correlations.

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Appendix—Definition of Statistical Parameters

The statistical parameters used in the present study are defined below.

Percent Relative Error, *e*. The percent relative error, e, is the relative deviation of a calculated value, P_c, from a corresponding measured value, P_m, and is given by

$$e = \frac{P_m - P_c}{P_m} \times 100. \quad \text{(A-1)}$$

Average Absolute Percent Error, $\bar{e}$. The average absolute percent error, $\bar{e}$, is the average of the absolute values of the relative errors and is given by

$$\bar{e} = \frac{1}{n}\sum_{i=1}^{n} |e_i|, \quad \text{(A-2)}$$

where n is the number of data points. A lower value of $\bar{e}$ indicates a good correlation.

TABLE 8—STATISTICAL ANALYSIS RESULTS FOR ALL CORRELATIONS USING ALL DATA

Condition	$\bar{e}$	r	Data Points (% of total) lying in the specified range of deviation						
			Points	±5%	±10%	±15%	±20%	±25%	±>25%
Aziz *et al.*[1]	15.5	0	414	28%	45%	60%	69%	77%	100%
Hagedorn and Brown[2]	7.49	0.70	414	49%	74%	88%	95%	98%	100%
Duns and Ros[3]	7.60	0.68	414	48%	74%	88%	94%	96%	100%
Beggs and Brill[4]	6.72	0.78	414	51%	80%	91%	96%	97%	100%
Hasan and Kabir[5]	9.59	0.48	414	36%	65%	80%	90%	93%	100%
Orkiszewski[6]	8.20	0.62	414	44%	70%	85%	93%	95%	100%

Correlation Coefficient, *r*. The correlation coefficient, r, represents the degree of success in reducing the standard deviation and is calculated from

$$r^2 = 1 - \frac{\sum_{i=1}^{n}(P_c - P_m)_i^2}{\sum_{i=1}^{n}(P_m - P_a)_i^2}, \quad \text{(A-3)}$$

where $P_a = \frac{1}{n}\sum_{i=1}^{n}(P_m)_i. \quad \text{(A-4)}$

The value of the correlation coefficient ranges from zero to one. The closer the value to one, the better the correlation.

Nomenclature

e = relative error, %
$\bar{e}$ = average absolute error, %
i = index
n = number of data points
P_a = mean of measured values
P_c = calculated value
P_m = measured value
r = correlation coefficient

SI Metric Conversion Factors

bbl	× 1.589 873	E − 01	= m^3
ft	× 3.048*	E − 01	= m
ft^3	× 2.831 685	E − 02	= m^3
°F	(°F − 32)/1.8		= °C
in.	× 2.54*	E + 00	= cm
psi	× 6.894 757	E + 00	= kPa

*Conversion factor is exact.

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