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Two-Phase Flow Through Vertical, Inclined, or Curved Pipe

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Introduction

Problems related to two-phase flow are frequently encountered in the design and operation of oil and gas production or storage fields. In addition, two-phase flow technology is of great interest to chemical engineers in dealing with such areas as boilers, condensers, heat exchangers, reactors, and process piping. Many of the concepts and correlations developed originally for petroleum operations are being extended to other fluids and new applications. The energy shortage, currently so much the focus of public attention, has recently stimulated interest in new areas, many of which closely relate to the current understanding of two-phase flow. Simultaneous long-distance pipelining of crude oil and natural gas, harnessing of geothermal energy in the form of steam and hot water, production from offshore locations, pipelining of LNG, and mining and dredging from the bottom of the oceans are among the new areas where recently developed technology in two-phase flow is being evaluated in the light of economic feasibility.

The basic engineering problem in calculating the pressure distributions in conduits subject to two-phase flow may be spelled out as follows: Knowing the geometry of conduit, physical properties of the two-phase flow system, and conditions prevailing at one end, predict pressure profiles along the pipe.

Contributions to solving various aspects of two-phase flow problems have been numerous in the literature. This paper will be limited to an investigation of those pressure-drop models that are currently available for vertical and inclined flow. We shall consider both the quantitative and the qualitative aspects of the models and discuss their extension to inclined and curvilinear flow.

Through the years, a number of investigators in vertical two-phase flow chose to correlate both slippage and friction losses by a unique and single energy-loss factor.[1-6] The results of their efforts fell short of their desired goal because their correlations did not include the effects of all pertinent variables, and, more importantly, they did not reflect the effect of various flow regimes.

Another school of thought is represented by other investigators[7-14] who chose to define, measure, and predict slip or holdup as an intermediate parameter leading to the calculation of pressure drop. This approach, along with considerations of energy balance, led to an interpretation of pressure gradient as a sum of three individual gradients: density, acceleration, and friction.

One of the principal reasons for the failure of most vertical two-phase flow correlations was the coexistence of several flow regimes along the same pipe for a given set of operating conditions. Indeed, if one started at the bottom and proceeded up the tubing in an oil well, one could possibly observe, in succession at various depths, (1) single-phase liquid flow, (2) two-phase bubble flow, (3) slug flow, (4) transi-

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A model for predicting pressure distribution in two-phase flow through vertical, inclined, or curved pipes combines the best available correlations for predicting pressure gradients for each flow regime. It has been evaluated statistically against literature data and directly against field data from directionally drilled offshore wells. On the whole, the model performs commendably.

tion (possibly representing annular or froth flow), and finally, (5) mist flow. Any correlation developed specifically for any one of those flow regimes would probably be inadequate to describe flow in the entire pipe.

In 1967, Orkiszewski[15] re-examined all existing correlations for comparison with data he had gathered on 22 heavy-oil wells in Venezuelan fields. He proposed that correlations by Duns and Ros[14] for transition and mist flow, and by Griffith and Wallis[9] for bubble flow could be used with discretion in predicting pressure drops in the respective flow regimes. This approach permitted Orkiszewski to combine the best part of each into one algorithm for calculating pressure drops.

In summary, it may be fairly stated that all earlier work in two-phase flow spanning the period from 1935 to 1972 may be classified into three groups:

1. Work that considers no-slip and no-flow regimes,
2. Work that considers slip concept but not flow regime, and
3. Work that considers both the slip concept (as related to holdup) and the flow regime.

The papers discussed above are mostly on vertical two-phase flow systems. There are very few studies on inclined two-phase flow and practically none on two-phase flow through curvilinear pipe except the recent papers by Gould and Tek,[16] and by Beggs and Brill,[34] and some recent Master's thesis work at the U. of Tulsa, referred to in the Appendix. On inclined flow, the works of Baker[17] and Flannigan,[18] as well as the thesis by Sevigny,[19] belong to the first group. A number of investigators studied slug flow in inclined pipe.[20-23] Their works deal with only one flow regime and with a limited extent of inclination.

The problem of inclined down-flow has been treated by Sevigny[19] without regard to flow regimes. A recent study by Greskovich[20] in segregated down-flow is concerned with perdiction of holdup.

Studies of effect of physical parameters and operating conditions on the flow regimes in two-phase flow have been extensive.[24-28] Some of these studies discern the flow regimes from trends on pressure-drop curves, others from direct flow pattern observations.

Against the background of a voluminous existing literature only partially cited above, the work presented here involves a computerized method designed and developed first to determine the flow regime likely to prevail at a given point in the pipe, then to use the best correlation available in the literature to predict the density, friction, and acceleration gradients for that particular flow regime before iterating to the next pipe-length increment. The procedure originally introduced by Orkiszewski[15] has been generally followed in this investigation. The resulting model, including changes, improvements, and added data on flow regime predictions was cast in finite-difference form, suitable for quick implementation by a computer algorithm. The method so developed predicts pressure drops and holdups for two-phase flow systems in vertical, inclined, or curvilinear pipe.

The generalized model is evaluated on the basis of available laboratory and field data. Maps for predicting flow regime are extended and modified with new direct laboratory data. Also included is a new computer graphics procedure for perspective display of three-dimensional pressure gradient or holdup surfaces — a valuable tool in evaluating correlations and discovering discontinuities.

The Generalized Pressure-Drop Model

In spite of the overwhelming amount of literature on the subject, there exists no perfectly general model capable of representing vertical, horizontal, and inclined flow. However, there are a number of reliable analytical relations or empirical correlations, usually limited to particular flow regimes or certain ranges of geometry, pressure level, and physical properties. The pressure-drop model investigated during this work starts with the thermodynamic energy balance and uses separate and selected correlations for each of the main ingredients of the over-all pressure drop for each likely flow regime.

Fig. 1 represents the geometry used for inclined and curvilinear pipe to develop the pressure-drop model.

A thermodynamic energy balance, written in differential form between Points 1 and 2, gives

$$\frac{dp}{\rho} + d\left(\frac{v^2}{2g_c}\right) + d\left(\frac{gh}{g_c}\right) = -l_w \quad . \quad . \quad . \quad (1)$$

The lost-work term, l_w, in this equation is usually written for pipe flow in terms of a friction factor as follows:

$$l_w = \frac{fv^2}{2\,g_c\,d}\,dL \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For flow in a curvilinear pipe, the length is simply related to the height above datum by $dh = dL\cos\theta$. In oil and gas operations, the wellhead is usually taken as the point of reference that requires the definition of depth $dD = -dh$. Substituting these relations into Eq. 1 gives

$$\frac{dp}{dD} = \frac{\dfrac{\rho_m g}{g_c} + \dfrac{f\rho_m v^2}{2g_c d\cos\theta}}{1 - \dfrac{W_T q_g}{g_c A^2 \bar{p}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The denominator of this equation assumes that $d\left(\frac{v^2}{2g_c}\right)$ is significant in nearly all gas or mist flow only at conditions of low pressure. Wallis[28] has shown that the denominator is actually $1 - M^2$ where M is the Mach number, or ratio of flow velocity to sonic velocity.

For computational purposes, Eq. 3 is often approximated by taking a finite difference of the pressure gradient. In addition, the friction gradient can be represented as a single term, τ_f, which yields the following:

$$\frac{\Delta p}{\Delta D} = \frac{\bar{\rho}_m g/g_c + \tau_f/\cos\theta}{1 - M^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Although Eq. 4 was developed by considering a homogeneous fluid, we are left with two gradients, $\overline{\rho_m}$ and τ_f, that could represent the flowing properties of a multiphase mixture. Bridging the gap between homogeneous flow and nonhomogeneous or two-phase flow must then be an empirical process.

The friction gradient has not received a great deal of attention recently for vertical two-phase flow. This gradient is dominant only at high flow rates, which are difficult to obtain in laboratory-scale equipment. Most investigators simply base the friction gradient on the single phase that is in continuous contact with the pipe wall. Of course Eq. 4 shows that under inclined-flow conditions this gradient will be magnified. In a truly horizontal pipe, the two-phase friction gradient has been shown[34, 35] to be as much as two to three times the magnitude of the homogeneous flow gradient. One wonders if this would also be true for vertical two-phase flow. However, to date the vast majority of work has been concerned with the empirical correlation of the term $\overline{\rho_m}$ in Eq. 4.

The term $\overline{\rho_m}\, g/g_c$ may be referred to as the density gradient where $\overline{\rho_m}$ is average mixture density. The density of the mixture may be computed as

$$\rho_m = \rho_l H_l + \rho_g(1 - H_l) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Where H_l is the actual flowing volume fraction liquid referred to as holdup. It can be shown that the holdup is directly related to the relative or "slip" velocity between the phases. Based on the superficial velocity of each phase, the slip velocity is given by

$$v_s = \frac{v_{sg}}{1 - H_l} - \frac{v_{sl}}{H_l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

In liquid-phase-continuous flow, or bubble flow, the slip velocity may be taken as the terminal velocity of bubble rise. For most two-phase flow problems in the bubble regime, it is recognized that small bubbles coalesce into larger ones and large bubbles, which do not survive the usual turbulence, split into smaller ones, resulting in a size-invariant bubble-rise velocity. In this investigation, the following equation has been used in calculating the bubble-rise velocity, v_∞, given by Harmathy.[30]

$$v_\infty = v_s = 1.53 \left[\frac{\sigma\, g\, g_c\, (\rho_l - \rho_g)}{\rho_l^2}\right]^{1/4} \quad . \quad . \quad (7)$$

During late stages of this investigation, another method has also been successfully implemented to estimate holdup in the bubble regime. That method, credited to Wallis,[28] also gave answers comparable with those obtained by Eq. 6. It is called the "Drift-Flux" method.

When a large number of bubbles are present, the swarm does not always rise with the terminal velocity of the single bubble. The factor for correcting the value of v_∞ for a single bubble for the condition of hindered ascent is in turn a function of the holdup, H_l. There are several correlations available in the literature to evaluate this effect, such as those of Marucci[31] and Bhatia.[32] It was not quantitatively evaluated during this investigation.

Referring to Eq. 4 and Fig. 1, we can develop an iterative finite-difference algorithm to calculate length or depth increments corresponding to selected values of Δp, provided that the holdup and friction gradients can be evaluated at the pressure, temperature, and flow conditions in the increment. The procedure is applied incrementally up or down the pipe until the sum of the increments equals the total depth. This iterative calculation permits the evaluation of the flow regime in the next pipe segment. If the flow regime is known, the best correlations available for ρ_m and τ_f can then be used. This procedure at the same time allows for the presence of several different flow regimes at various depths in the same pipe.

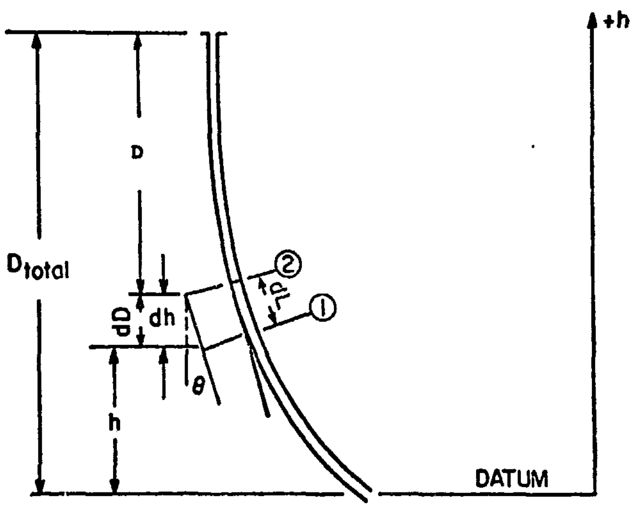

Fig. 1—Well geometry for generalized pressure-drop model.

Determinations of Flow Regimes

Using the physical properties of vapor and liquid phases determined in-situ, the individual flow rates, pipe geometry, and pressure level, one can predict the flow regime likely to occur at that particular point. One of the principal shortcomings of most vertical two-phase flow models in the literature has been their failure to account for the coexistence of several flow regimes along the pipe for a given set of operating conditions.

The flow regime map used earlier in this study is shown in Fig. 2. It was developed from data of Wallis[6] in bubble and slug flow and from the work of Ros[7] in the area of slug and annular mist flow. The flow regime quadrant is based on dimensionless parameters suggested by Ros[14] — superficial gas velocity and superficial liquid velocity influence numbers. In Fig. 2, the solid lines of demarcation indicate the approximate loci of transition from one regime to another. The dotted straight line between bubble and slug flow indicates a proposed transition region that will be discussed later. We have indicated on this diagram that the flow regime boundary between plug and slug flow varies with diameter when calculated from Wallis' original work. The question of whether or not these flow regime boundaries should vary with diameter has not to date been sufficiently investigated.

If a point representing two-phase slug flow is spotted on the flow regime map, the following may

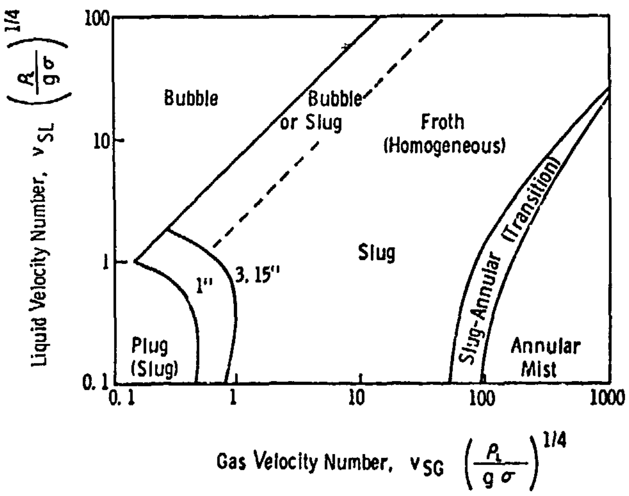

Fig. 2—The vertical two-phase flow regime map used for the pressure-drop model (developed from Refs. 7 and 8).

be deduced.

If the gas rate is held constant but liquid rate is gradually decreased, the image point would move vertically downward. As the liquid rate becomes smaller and smaller, it is logical to expect that at some point the flow regime would have to revert to annular mist. Accordingly, it may be reasoned that the boundaries between slug, transition, and mist flow that appear to be nearly vertical at a liquid velocity number of 0.1 must turn to the left to reflect this behavior if they are extended to lower values of the liquid velocity influence number.

Using the above rationale, and some quantitative data for cases known to apply to mist flow condition, Gould and Tek[25] proposed an extension of the flow regime map. On the basis of their recent results, observations on literature data, and direct laboratory experiments, a vertical flow regime map was developed (Fig. 3). This extended map is very similar to Fig. 2 except for the new region designated "heading." The region might also be classified as both phases continuous, as discussed below. Fig. 3, which has been developed by combining direct laboratory observations and analyses of laboratory and field pressure-drop data, is used to arrive at the best possible locations of liquid-phase-continuous, alternating phases, and gas-phase-continuous boundaries. The description of the procedure and the data in support of this map are included in the Appendix.

Effect of Inclination on Flow Regimes

Throughout the investigation reported here, the basic geometry has been that of the vertical pipe. However, as the model developed for pressure drop was extended and generalized to oblique and even curvilinear pipe, there arose the question of the effect of inclination on the flow regime map. This effect was investigated by direct observations on a two-phase flow apparatus designed to operate at various angles of inclination.

The results of our observations are shown in Figs. 4, 5, and 6. Fig. 4 is for totally vertical flow and Figs. 5 and 6 are for 45° and 90° inclination from vertical, respectively. The flow pattern data have been classified on the basis of the observed continuous phase.[24]

As the flow regime quadrant is traversed in both directions, various observations on liquid-phase continuous (L), alternating phases (A), transition from liquid-phase continuous to alternating (AL), and from alternating to gas-phase continuous (AG), have

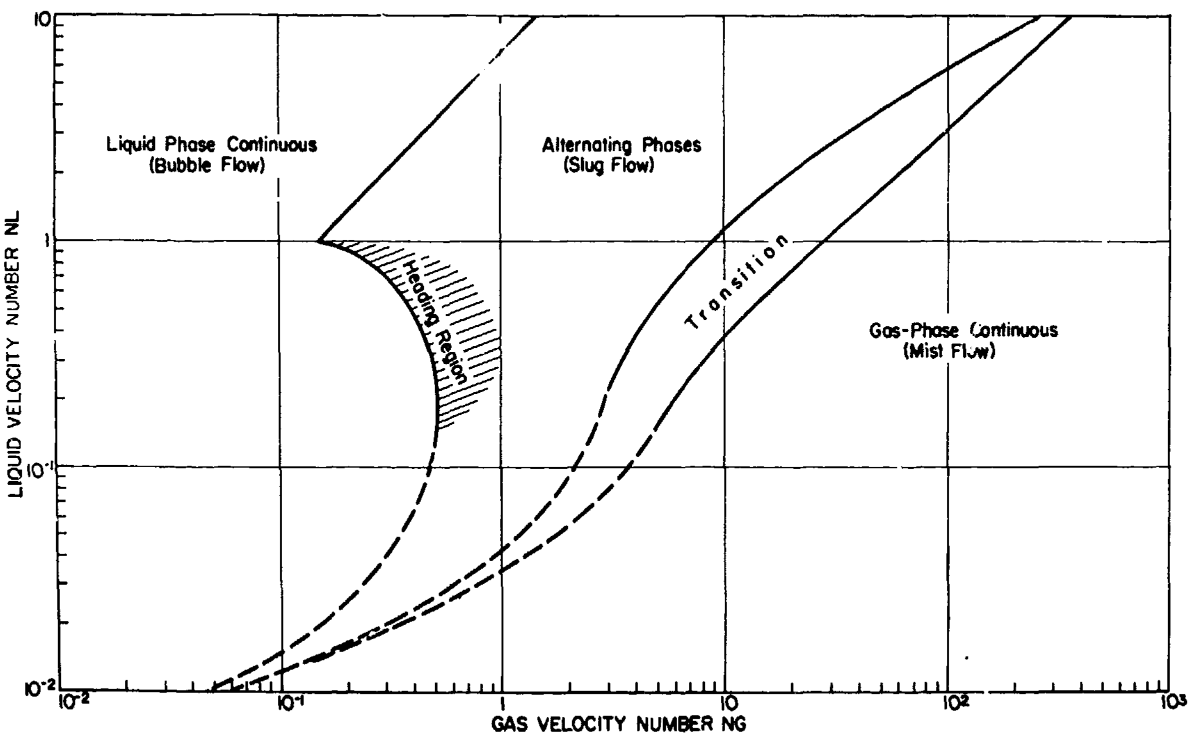

Fig. 3—Extended flow regime map for vertical two-phase flow.

been spotted at appropriate locations. The letter B designates both phases continuous, and BA designates the transition between both phases continuous and alternating phases. Note that the data for Fig. 4 bear out the same approximate locations of previously established bubble/slug, slug/transition, and transition/mist boundaries for vertical flow.

The region where both phases are continuous is newly defined and is a region for vertical two-phase flow where no known pressure-drop correlations exist. In this region, gas is separating from the liquid and allowing liquid fallback. Even though the net mixture flow is upward, individual pipe segments contain countercurrent liquid downflow for short periods of time. The phenomenon of liquid fallback was called "heading" by Ros.[14] Fig. 5 shows that this region occupies a significant portion of the flow regime quadrant.

For horizontal flow, the region where both phases are continuous occupies a significant portion of the flow regime quadrant, as shown on Fig. 6. The region corresponds to the conventional classification of stratified flow for horizontal pipe. This conclusion is substantiated when the data of Hoogendoorn[29] for air and water are replotted on a flow diagram (Fig. 7). By combining Froth and Slug in the alternating-phase classification and combining Bubble and Plug in the liquid-phase-continuous classification, there is almost perfect agreement between Figs. 6 and 7.

Comparing Figs. 4, 5, and 6 leads to a conclusion that will be important in the development of pressure-drop correlations for inclined flow. The liquid-phase-continuous and gas-phase-continuous flow regime boundaries do not appear to vary significantly with inclination. This implies that it should be possible to develop an alternating-phase pressure-drop correlation that is generalized for all inclinations and that is a continuous function at the flow regime boundaries.

Calculation of Acceleration Gradient

The acceleration gradient given previously in Eq. 3 as a fraction of total pressure gradient has been derived for the case $q_l << q_g$ or $q_g \cong q_T$:

$$\left(\frac{dp}{dL}\right)_A = \frac{W_T q_g}{g_c A^2 p}\left(\frac{dp}{dL}\right)_T \quad . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

In this study for all two-phase flow regimes, Eq. 8 has been used to evaluate the acceleration gradient. This implies a tacit simplifying assumption that for

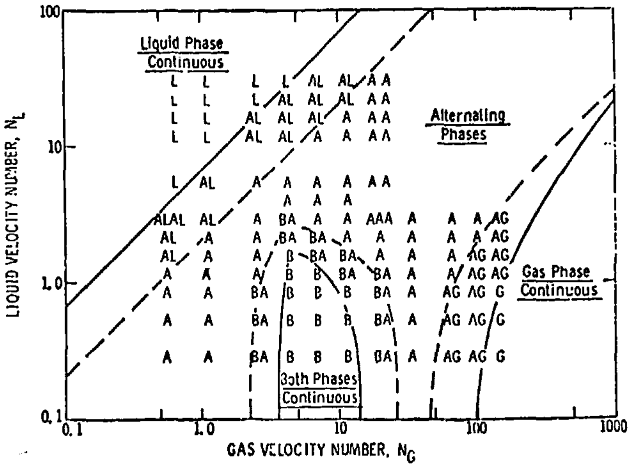

Fig. 4—Flow regime map for inclination of 0° (vertical).

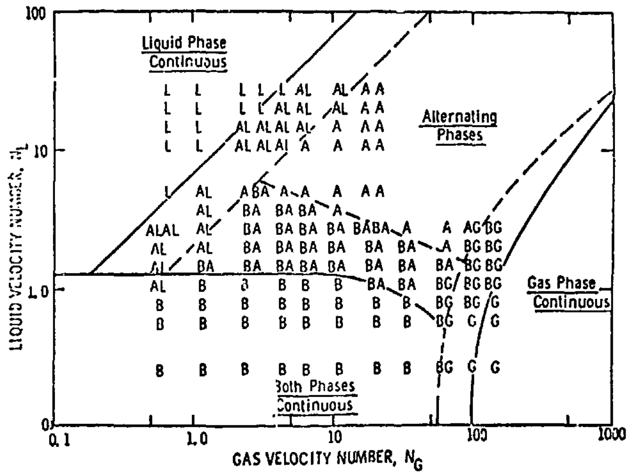

Fig. 6—Flow regime map for inclination of 90° (horizontal).

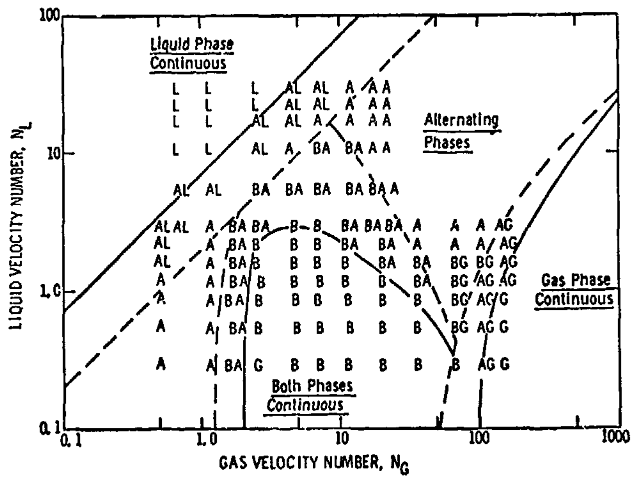

Fig. 5—Flow regime map for inclination of 45°.

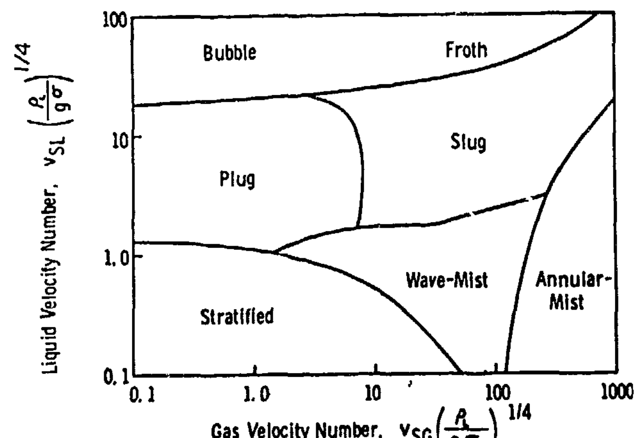

Fig. 7—The horizontal two-phase flow regime map (Hoogendoorn data[29]).

bubble, slug, or transition regimes the acceleration gradient, which is small anyway, may be computed from a procedure more rigorously applicable for mist flow. Dukler[35] gives a procedure that does not require the assumption of homogeneous mist flow, but he does assume a constant ratio between the liquid and gas velocities.

Since Eq. 4 was implemented by finite difference techniques on the computer, the size of Δp increments as related to the current value of $\bar{p}$ occasionally resulted in computational instability wherever the numerical value of the coefficient $\left(\dfrac{W_T\, q_g}{g_c A^2 \bar{p}}\right)$ was close to unity. Further examination of these cases disclosed that under those conditions the sonic velocity was being approached by the two-phase mixture in the pipe. In fact, if this coefficient is exactly unity, the pressure gradient from Eq. 4 is infinite, corresponding to a shock front or choked flow.

Logically then, we can determine directly the flowing pressure at which sonic flow occurs by

$$\bar{p} = \frac{W_T\, q_g}{g_c A^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

as long as the volumetric gas flow rate is much greater than the liquid flow rate. Fig. 8 shows a plot of this pressure in field units for various flow conditions. Operating at pressures lower than those predicted from the curves will result in choked flow in the wellbore. This type of analysis can be crucially important to safety equipment design.

Evaluation of the Pressure-Drop Model

Field and laboratory data available from various literature sources have been examined, computed, and compared with results of procedures developed in this work.

Comparisons With Data From Literature

Fig. 9 shows data from five such sources plotted in pairs representing terminal conditions in relation to the flow regime map. Scrutiny reveals that nearly all the field data fall within the alternating phases or transition regimes. A few points from Orkiszewski's data and Poettmann and Carpenter's data cross over into the bubble and transition region and to annular-mist flow. During this study we encountered very few points in the published field data that cross over into the annular-mist region, more recently located as shown in Fig. 3.

Using the literature data available from five sources, we have compared statistically the procedures developed during this investigation with reported results of the Orkiszewski and Duns and Ros methods. This comparison is given in Table 1. On the basis of 148 data points so sampled, this study shows good agreement with Orkiszewski, as expected. The slug-flow correlation used was essentially the same, but the bubble-flow holdup was modified from his method as noted above. Since the data used in this comparison are primarily located in the slug-flow regime, we did not expect the results to be too different from those reported by Orkiszewski. Calculated and measured pressure drops are plotted in Fig. 10. Note that there appears to be more deviation at the lower pressure drops than at higher values. This may indicate possible measurement errors. Because the sample size is too small, any quantitative generalization of the results of Table 1 may be hazardous. On the other hand, the magnitude and the range of average and standard deviations observed confirm the over-all validity of the methods used.

Table 2 shows the range of variables involved in this study. The most restricted variable was diameter, whereas the gas/liquid ratios available covered a fairly wide range. Since there are not a great deal of data available on large-diameter two-phase flow, it is interesting and relevant to report the few large-diameter points where our procedure has been tested against data. (See Table 3.) The comparison is indeed encouraging from the standpoint of hydrodynamic scale-up, with average error being within acceptable limits for all three cases.

Comparisons With Data From Offshore Platforms

Pressure profile data from four directionally drilled offshore wells were made available by Marathon Oil Co. The wells were producing crude oil by gas-lift through 4.0-in. ID plastic-coated tubing. The data

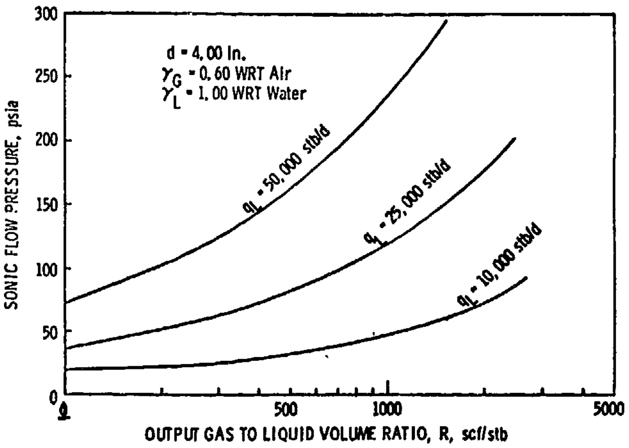

Fig. 8—Sonic flow boundaries for various flow rates in a 4.0-in.-diameter pipe.

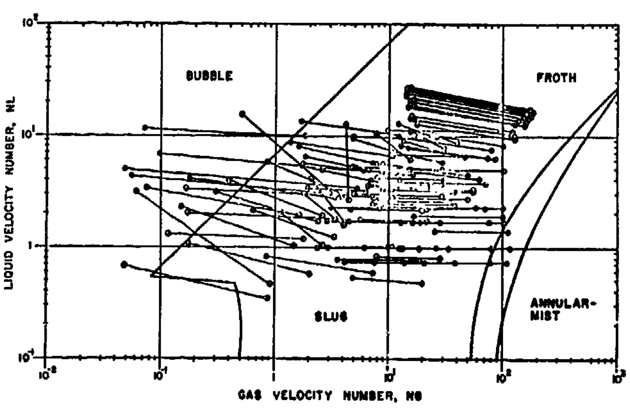

Fig. 9—Plot of wellbore flow regime profiles for the published field data summarized in Table 1.

TABLE 1—STATISTICAL COMPARISON OF PREDICTED VS MEASURED PRESSURE DROPS

Data Source	This Method	Orkiszewski Method	Duns and Ros Method
Baxendell-Thomas[1] (25 points, d = 2.992 and 2.441 in.)			
Average error, percent	−1.5	−2.1	+2.3
Standard deviation, percent	7.5	11.1	20.0
Fancher-Brown[2] (20 points, d = 1.995 in.)			
Average error, percent	+4.4	+0.3	+1.7
Standard deviation, percent	12.9	11.8	32.1
Hagedorn-Brown[9] (32 points, d = 1.38 in.)			
Average error, percent	−3.9	+0.1	−16.9
Standard deviation, percent	12.7	8.2	36.6
Orkiszewski[15] (22 wells, d = 2.992 in.)			
Average error, percent	+1.3	−1.2	+22.7
Standard deviation, percent	10.7	10.4	18.7
Poettmann-Carpenter[5] (49 wells, d = 1.995 to 2.992 in.)			
Average error, percent	−1.1	−1.0	+5.8
Standard deviation, percent	10.4	12.0	12.4
Over-all results (148 data points)			
Average error, percent	−0.096	−0.8	+2.4
Standard deviation percent	11.3	10.8	27.0

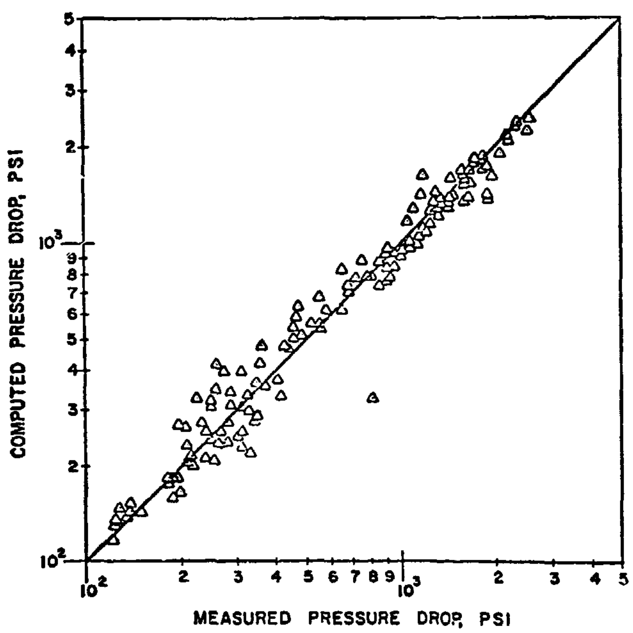

Fig. 10—Comparison of the pressure-drop model with published field data.

TABLE 2—RANGE OF VARIABLES TESTED AGAINST FIELD DATA FOR THE PRESSURE-DROP MODEL

Variable	Range
Diameter (d), in.	1.380 to 2.992
Viscosity (μ_l), cp	0.7 to 2,000
Gas-liquid ratio (R), scf/bbl	200 to 100,000
Pressure level, psia	30 to 4,000
Length of flow (L), ft	1,000 to 10,000
Flow regime	Slug-froth
Inclination, degrees from vertical	0.

TABLE 3 STATISTICAL COMPARISON OF PREDICTED VS MEASURED PRESSURE DROPS FOR LARGE-DIAMETER PIPES[24]

d_h (in.)	Number of Data Points	Average Error (percent)
10.05	3	− 9.8
6.398	4	+11.0
6.0	2	+ 1.5

TABLE 4—COMPARISON OF MEASURED AND CALCULATED PRESSURE DROPS IN DIRECTIONAL WELLS

Well	True Depth	Oil Rate (STB/D)	GOR (scf/STB)	Percent Difference, Directional	Percent Difference Vertical
D-4	8,010	5,063	710	−1.7	−7.7
D-7	7,020	4,445	466	+1.4	−8.0
D-12	8,280	3,667	713	−0.6	−6.8
D-17	6,812	5,250	548	+3.2	−0.5

from these wells provided a limited basis for testing Eq. 4 in directional wells. We fully realize that the simple angular correction in the friction term of Eq. 4 is probably not sufficient to describe all inclination effects. In particular, this correction does not account for the variation of holdup and flow regime with inclination. However, for the data available, the approach did improve the accuracy of directional well pressure-drop calculations.

Measured and calculated pressure drops for the four wells are compared in Table 4, which also lists the measured rates. Figs. 11 and 12 show the entire pressure traverse comparison for Wells D-4 and D-7. The GOR's measured cover a range of 466 to 713 scf/STB. The pressure drops calculated on the basis of true vertical depth are consistently lower than the measured pressure drops. By contrast, the pressure drops computed for curved pipe are in excellent agreement with measurements.

Examination of the Pressure-Drop Model By Computer Graphics

A technique developed during this study permitted perspective displays of the pressure gradient as a three-dimensional surface, as a function of both liquid and gas velocity numbers.

Fig. 13 shows such a plot representing the pressure gradient surface for an air-water system at 500 psia through 2.5-in.-diameter vertical pipe. The surface is fairly smooth and descends with increasing gas number from bubble to slug to transition and finally ascends to mist flow region increasingly dominated by friction. When these surfaces are generated at larger diameters, at higher pressures, or for other fluids, discontinuities begin to appear between flow regimes and within individual correlations.

Although some of the discontinuities are understandably due to having different correlations calculated for each flow regime, others may indicate singularities and behavior not observable in laboratory or field conditions. Fig. 14 shows a plot of the holdup surface as a function of liquid and gas velocity numbers for air-water flow in a 4.0-in.-ID pipe at 500 psia. The liquid-phase-continuous correlation is both higher and lower than the Orkiszewski alternating-phase correlation at the flow regime boundary. The second discontinuity at a gas velocity number of about 20 corresponds to the change of correlations prescribed by Orkiszewski at a total velocity of 10 ft/sec. The degree and extent of these discontinuities sharply increase with larger diameter. Fig. 15 shows this increase when the flowing conditions used for Fig. 13 are extended to 24.0 in. ID. Although this diameter is much larger than that for most vertical two-phase flow applications, it clearly shows the pitfalls that can occur across a flow regime boundary if hydrodynamic scale-up has ignored such effects.

The use of computer-generated surface displays for investigating areas where correlations may fail and for general exploratory studies in hydrodynamic scale-up was effective, and worthy of presentation and further investigation.

Applications

The generalized model for pressure drop in straight, inclined, or curvilinear pipe developed during this work has found numerous applications; for example, gas wells subject to liquid accumulation, oil wells produced by gas lift, design of risers in offshore platforms, removal of condensate from long-distance pipelines, dredging by gas lift, and mining from the bottom of the ocean. Some of the computational effort involved in translating Eq. 4 and various correlations into a finite-difference algorithm resulted in the concept of "lifting potential."[24] This concept proved to be quite useful for design and performance analyses of gas wells subject to two-phase flow and particularly helpful in evaluating modes of stability and unsteady-state effects.

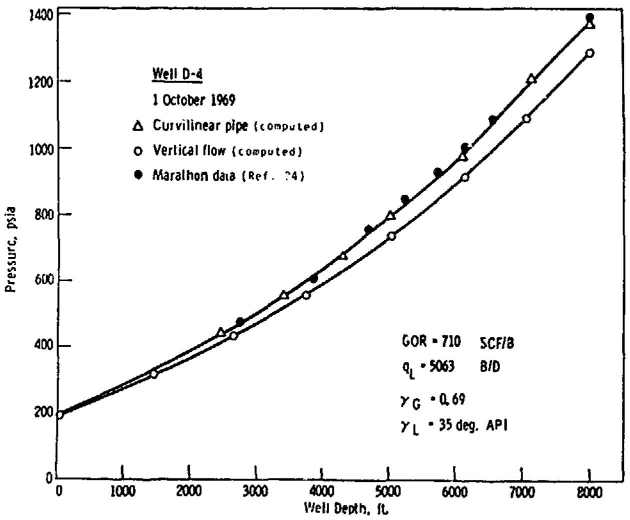

Fig. 11—Two-phase pressure-drop data from directionally drilled offshore wells.

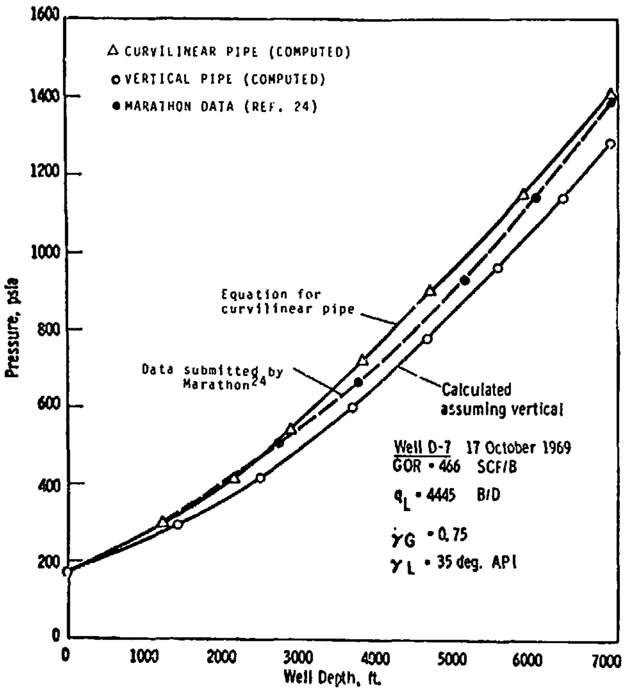

Fig. 12—Two-phase pressure-drop data from directionally drilled offshore wells.

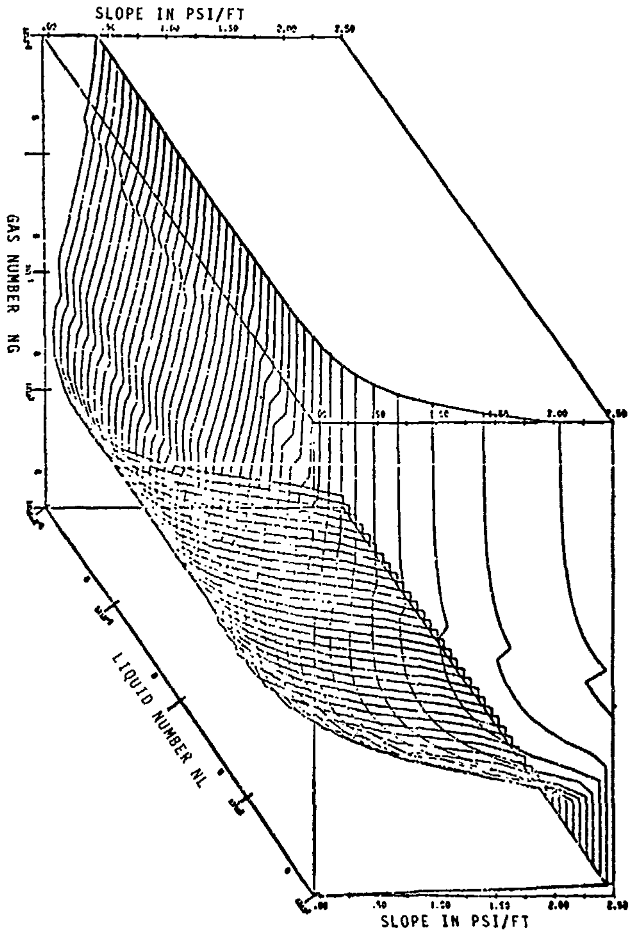

Fig. 13—Pressure gradient surface for air-water flow in a 2.5-in.-ID pipe at 500 psia.

Conclusions

The research work documented here confirmed some aspects and views of two-phase flow that had been previously established and known. It explained some semi-empirical observations and permitted the proposal of some new ideas.

On the basis of our study we confirm that stepwise calculation procedures implemented on the computer and coupled with reliable diagnostics on flow regimes permit a reasonably accurate prediction of pressure drops through vertical pipe. Further, the results we obtained support the idea that broadly classifying flow regimes into categories such as liquid-phase continuous, gas-phase continuous, alternating phases, and both phases continuous, is indeed sound and useful. It is general and can be applied to inclined pipe as well as vertical.

On the basis of new data and work presented in our study we specifically conclude the followin' .

1. The flow regimes in vertical or inclined pipe are predictable from individual phase flow rates, physical properties of two-phase flow system, and pipe geometry. The boundaries between liquid-phase-continuous and gas-phase-continuous regions are approximately independent of pipe inclination. On the proposed flow regime map, the boundary delineating the region where both phases are continuous changes in location and shape with the inclination of the pipe.

2. Two-phase pressure profiles can be more accurately predicted for directional wells if the friction gradient is based on curvilinear pipe length rather than on true vertical depth.

3. The total pressure gradient is affected mostly by density gradient in bubble flow, friction gradient in mist flow, and acceleration gradient in mist flow if the pressure level at the pipe terminal becomes too low.

4. Computer-generated surface display in two-phase flow can be quite useful in locating discontinuities and matching different correlations across flow regime boundaries, and is applicable in other studies generally related to hydrodynamic scale-up.

Nomenclature

A = cross-sectional area of pipe, ft^2
D = depth from surface, ft
d = diameter, ft
f = Moody friction factor, dimensionless
g = acceleration of gravity, ft/sec^2
$g_c = 32.17 \dfrac{\text{lb}_m}{\text{lb}_f} \dfrac{\text{ft}}{\text{sec}^2}$
H_l = volume fraction liquid (holdup)

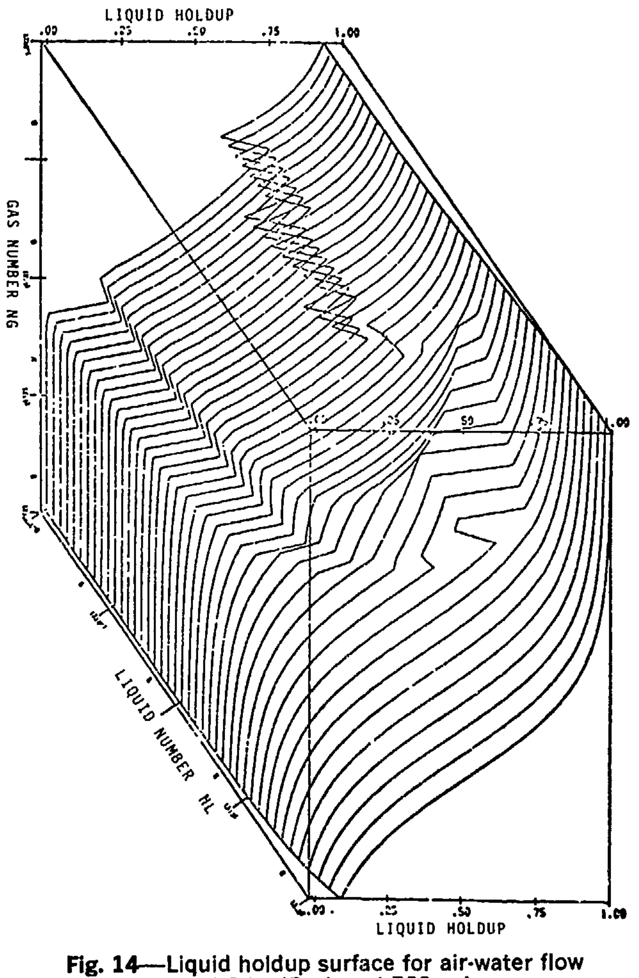

Fig. 14—Liquid holdup surface for air-water flow in a 4.0-in.-ID pipe at 500 psia.

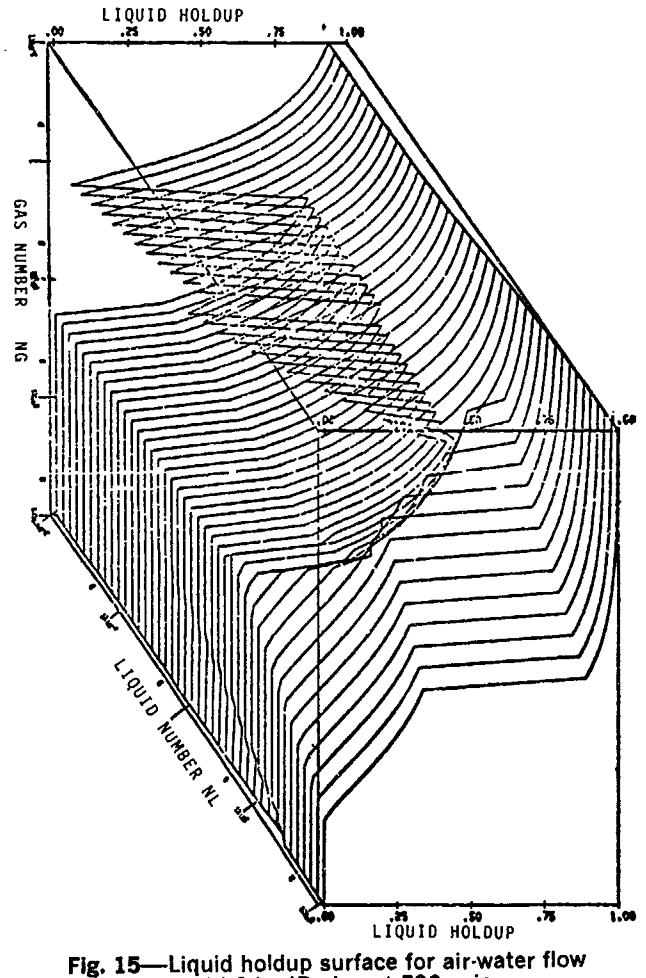

Fig. 15—Liquid holdup surface for air-water flow in a 24.0-in.-ID pipe at 500 psia.

h = height above datum, ft
L = length of pipe, ft
l_w = lost work, $\frac{ft \times lb_f}{lb_m}$
M = ratio of flow velocity to sonic velocity, Mach number
$N_{gv} = V_{sg} \sqrt[4]{\frac{\rho_l}{g\, g_c\, \sigma}}$, gas velocity influence number, dimensionless
$N_{lv} = V_{sl} \sqrt[4]{\frac{\rho_l}{g\, g_c\, \sigma}}$, liquid velocity influence number, dimensionless
p = pressure, lb_f/ft^2
$\bar{p}$ = average pressure, lb_f/ft^2
q = volumetric rate of flow, cu ft/sec
v = velocity of mixture, ft/sec
v_S = slip velocity, ft/sec
v_∞ = terminal velocity of single bubble, ft/sec
W_T = total mass flow rate, lb_m/sec
Δ = increment or decrement
γ = fluid gravity
ρ = density, lb_m/cu ft
ρ_m = mixture density, lb_m/cu ft
σ = surface tension, lb_f/ft
τ_f = friction gradient, $\frac{lb_f/ft^2}{ft}$
θ = angle with vertical, radians

Subscripts

A = acceleration
b = bubble
g = gas
l = liquid
S = slip
sg = superficial gas
sl = superficial liquid
T = total

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TABLE 5—RESULTS OF FLOW REGIME STUDY

Code Number This Work	Brill	Pressure Drop Measured	Wellhead Rates: Gas Number	Wellhead Rates: Liquid Number	Apparent Flow Regime
1*	500	989.0	132.00	9.560	Slug
2	501	1001.0	104.00	8.380	Trans
3	502	683.0	87.80	6.350	Slug
4	503	519.0	64.60	4.150	Trans
5**	507	850.0	16.00	4.440	Trans
6	511	650.0	30.00	8.250	Mist
7	514	740.0	34.50	9.930	Trans
8	516	950.0	93.40	3.800	Slug
9	521	720.0	20.30	7.370	Trans
10	523	1100.0	12.10	3.670	Slug
11	524	750.0	28.50	5.830	Trans
12***	529	2635.0	4.29	1.610	Slug
13	533	1857.0	2.34	0.820	Slug
14	534	2292.0	3.85	1.210	Trans
15	536	1044.0	4.72	0.493	Slug
16	539	1118.0	3.88	0.281	Slug
17	547	1547.0	7.77	1.920	Slug
18	562	1250.0	2.69	0.440	Mist
19	564	3800.0	0.01	0.695	Slug
20	566	1857.0	5.26	2.150	Slug
21	570	1483.0	11.40	1.620	Slug
22†	580	2302.0	8.85	0.934	Slug
23	581	266.0	25.40	0.487	Trans
24	582	356.0	3.52	0.710	Mist
25	591	539.0	15.50	0.550	Trans
26	600	1568.0	6.20	2.150	Mist
27	601	684.0	4.30	0.389	Slug
28	602	1766.0	3.17	0.994	Trans
29	609	1584.0	4.37	1.330	Trans
30	611	1288.0	6.22	1.100	Trans
31	614	966.0	7.52	0.240	Trans
32	626	1547.0	1.82	0.689	Slug
33‡	627	1498.0	43.30	4.640	Trans
34	628	1449.0	36.60	4.100	Trans
35	629	1375.0	28.70	3.340	Trans
36	629	1322.0	20.70	2.540	Trans
37	630	1101.0	32.90	2.450	Trans
38	633	978.0	17.90	1.390	Trans
39	636	928.0	33.60	2.110	Mist
40	637	1162.0	47.00	2.630	Mist
41	638	1493.0	42.80	1.150	Slug
42	639	1277.0	32.00	0.979	Slug
43	640	983.0	27.20	0.869	Trans
44	643	706.0	15.80	0.374	Mist
45	646	760.0	28.10	0.729	Mist
46	647	926.0	37.10	0.893	Mist
47	648	1323.0	53.60	1.070	Slug
48	650	637.0	11.50	0.228	Mist
49	651	655.0	17.60	0.336	Mist
50	652	671.0	21.30	0.400	Mist
51	654	669.0	11.50	0.166	Mist
52	655	687.0	17.90	0.249	Mist
53	656	666.0	21.90	0.301	Mist
54	657	681.0	25.00	0.340	Mist
55	660	734.0	31.90	0.420	Mist

Code Number This Work	Brill	Pressure Drop Measured	Wellhead Rates: Gas Number	Wellhead Rates: Liquid Number	Apparent Flow Regime
56	661	909.0	47.10	0.572	Mist
57	662	1484.0	37.70	5.910	Trans
58	663	1140.0	33.70	4.350	Mist
59	664	841.0	10.50	0.561	Trans
60	665	1047.0	34.70	3.080	Mist
61	666	969.0	11.30	0.865	Trans
62	669	1019.0	29.40	2.040	Trans
63	670	891.0	25.80	1.730	Mist
64	672	1016.0	20.10	1.090	Trans
65	673	1029.0	26.00	1.400	Trans
66	674	1002.0	19.20	0.950	Trans
67	676	1038.0	40.60	1.930	Mist
68	677	1144.0	50.80	2.300	Mist
69	678	919.0	17.90	0.761	Trans
70	679	941.0	36.90	1.530	Mist
71	680	1042.0	49.80	1.970	Mist
72	681	1233.0	65.10	2.320	Mist
73	682	642.0	27.90	0.673	Mist
74	683	828.0	40.00	0.797	Mist
75	684	655.0	15.50	0.415	Mist
76	685	625.0	11.10	0.313	Mist
77	686	963.0	31.80	0.665	Trans
78	687	715.0	34.20	0.336	Trans
79	688	646.0	26.30	0.288	Trans
80	689	540.0	15.20	0.198	Mist
81	690	508.0	10.20	0.142	Mist
82	692	618.0	19.80	0.260	Mist
83	693	585.0	17.00	0.232	Mist
84	696	959.0	34.50	0.496	Trans
85	697	789.0	12.60	0.497	Mist
86	698	793.0	18.70	0.710	Mist
87	701	742.0	11.10	0.376	Mist
88	703	684.0	21.50	0.537	Mist
89	706	638.0	15.30	0.281	Mist
90	708	709.0	29.50	0.499	Mist
91	709	782.0	40.00	0.632	Mist
92	710	1008.0	16.30	2.800	Mist
93	711	906.0	13.50	1.600	Mist
94	712	690.0	28.40	1.180	Trans
95	713	670.0	22.60	0.960	Trans
96	716	618.0	12.30	0.636	Mist
97	717	440.0	32.00	0.241	Mist
98	718	382.0	18.80	0.154	Mist
99	719	363.0	12.60	0.107	Mist
100	720	393.0	30.50	0.081	Mist
101	721	331.0	24.90	0.069	Mist
102	722	360.0	25.00	0.055	Mist
103	723	443.0	34.30	0.041	Mist
104	724	391.0	22.40	0.028	Mist
105	725	369.0	17.60	0.022	Mist
106	726	345.0	12.90	0.016	Mist

*Code Numbers 1 through 4 — Ref. 1.
**Code Numbers 5 through 11 — Ref. 15.
***Code Numbers 12 through 21 — Ref. 36.
†Code Numbers 22 through 32 — Ref. 37.
‡Code Numbers 33 through 74 — Ref. 38.

APPENDIX

Use of the Computer To Determine Proposed Flow Regime Boundaries Based on Field Data

Fig. 3, recommended for predicting prevailing flow regimes, has evolved from continuing studies at the U. of Michigan during the last 4 years. Besides making some direct, though limited, laboratory observations, we analyzed extensive literature data with our computer model to determine the likely location of flow regime boundaries.

The procedure merely consisted in calculating on the computer the pressure drop for each run by assuming one (say plug flow), then another (say transition flow). The particular flow regime that agrees most closely with the observed pressure drop is carefully determined, then the corresponding data point is plotted on N_{gv} vs N_{lv} Quadrant as plug, transition, or mist. Having plotted a large number of such points, we could arrive at reasonable approximate boundaries for plug, transition, and annular-mist flow regimes. This plot is shown on Fig. 16.

The data used during the exploration of flow regime boundaries included the early works of Baxendell and Thomas,[1] the work of Orkiszewski,[15] and more recent master's theses at the U. of Tulsa[38] by Espanol Herrera[36] and Camacho,[37] as well as the more recent papers by Brill and Lawson[39] and by Chierici *et al.*[40]

It must be observed that with the limited data on hand and the spread of data points probably caused randomly by errors, the flow regime boundaries are in reality bands, and therefore their location can only be approximated.

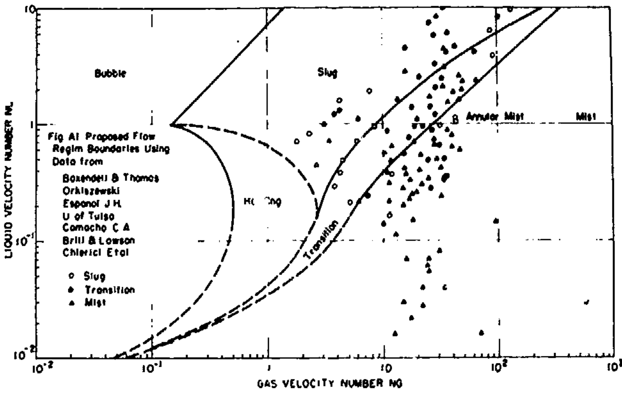

Fig. 16—Proposed flow regime boundaries using data from Refs. 1, 15, and 36 through 40.

Gas and liquid velocity numbers, coordinates on the flow regime map, and a determination of the flow as being plug, transition, or mist, are given in Table 5.

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