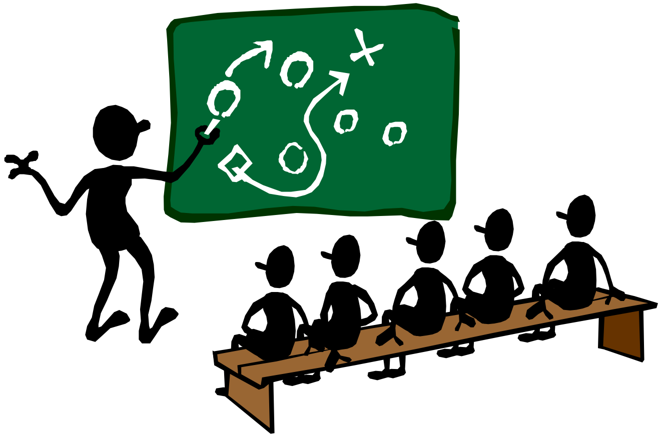
Algorithms – Chapter 12
Binary Search Trees



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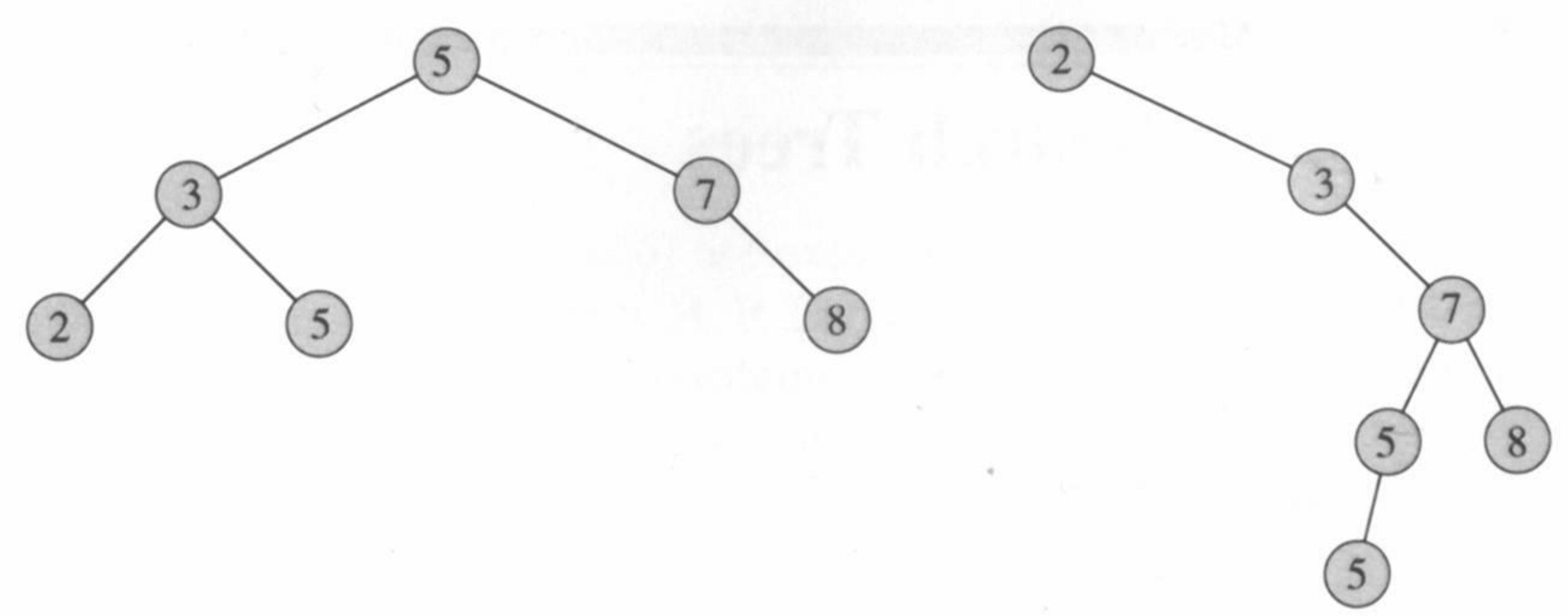
Binary Search Tree (BST)

- BSTs support
  - search, sort
  - minimum, maximum
  - predecessor, successor
  - insert, delete
- Basic operations on BSTs
  - time is proportional to the height of tree
- For a binary tree with n nodes
  - minimum height: $\Theta(\lg n)$ (e.g., complete binary tree)
  - maximum height: $\Theta(n)$ (e.g., skewed binary tree)

Property of BST

- Property
  - Let x be a node in a binary search tree
  - If y is a node in the left subtree of x ➔ key[y] $\leq$ key[x]
  - If y is a node in the right subtree of x ➔ key[x] $\leq$ key[y]



Tree Walks

- 3 types of tree walks
  - inorder: LVR
  - preorder: VLR
  - postorder: LRV
- Inorder walk on a BST produces a sorted list

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Inorder Tree Walks

```
INORDER-TREE-WALK(x)
if x ≠ nil
   then INORDER-TREE-WALK(left[x])
        print key[x]
        INORDER-TREE-WALK(right[x])
```

- If x is the root of n-node tree, then the call INORDER-TREE-WALK(x) takes $\Theta(n)$ time
  - because 2 calls for every node in the tree

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Recursive Search on BSTs

TREE-SEARCH(*x*, *k*)

1 **if** $x = nil$ **or** $k = key[x]$

2 **then return** *x*

3 **if** $k < key[x]$

4 **then return** TREE-SEARCH(left[*x*], *k*)

5 **else return** TREE-SEARCH(right[*x*], *k*)

Time complexity: O(h)

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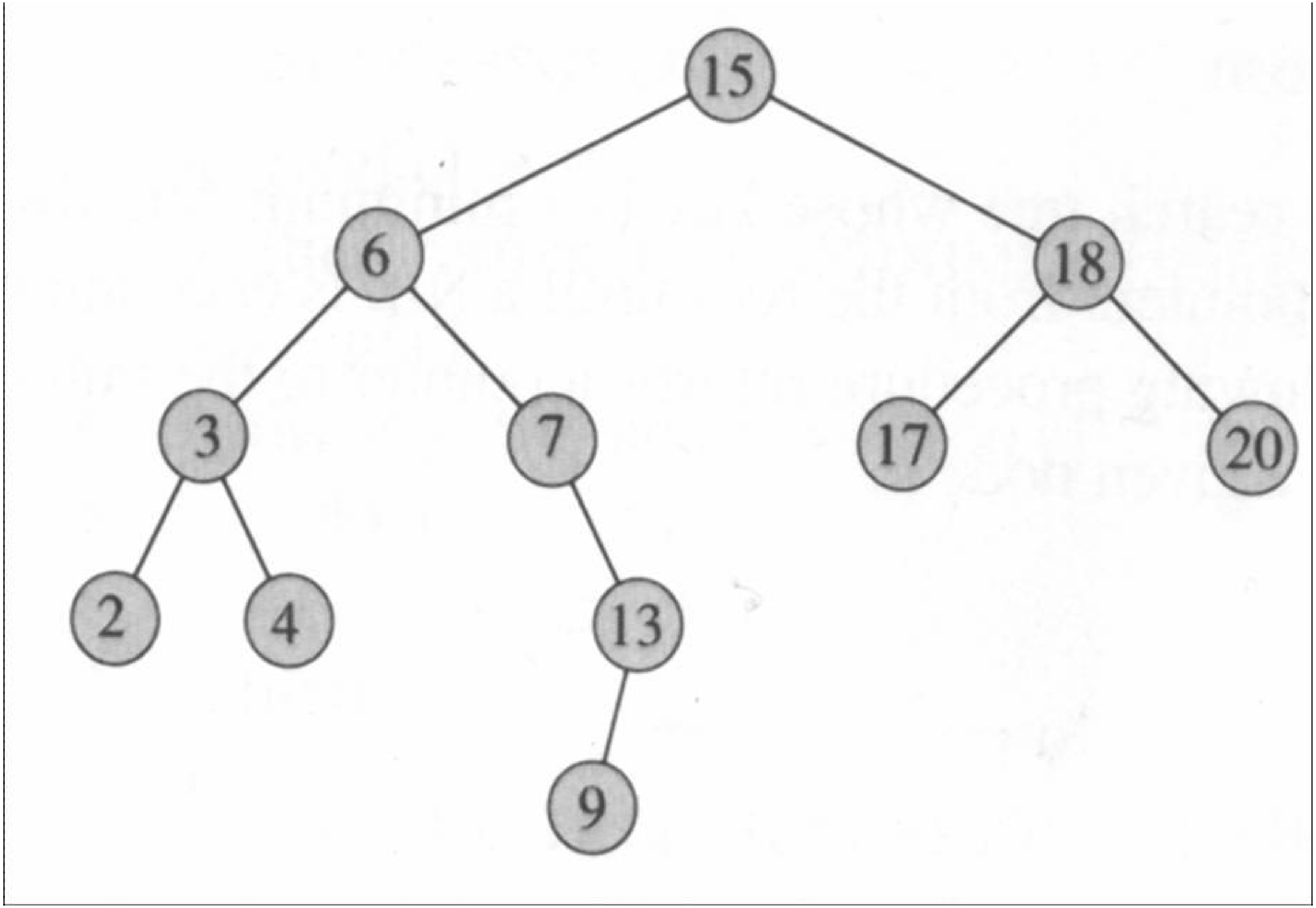
Iterative Search on BSTs

ITERATIVE-TREE-SEARCH(x,k)

While $x \neq nil$ **and** $k \neq key[x]$
    do if $k < key[x]$
        then $x \leftarrow left[x]$
        else $x \leftarrow right[x]$
return x

Time complexity: O(h)

BST Example



Minimum and Maximum on BSTs

TREE-MINIMUM(x)

while $left[x] \neq$ NIL
  do $x \leftarrow left[x]$
return x

leftmost element

TREE-MAXIMUM(x)

while $right[x] \neq$ NIL
  do $x \leftarrow right[x]$
return x

rightmost element

Time complexity: O(h)

Successor on BSTs (1/2)

- Successor of x
  - case 1: if x has a right subtree ➔ the minimum element in the right subtree
  - case 2: if x has no right subtree ➔ the lowest ancestor of x whose left child is x or an ancestor of x
  - case 3: x is the maximum element ➔ no successor

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Successor on BSTs (2/2)

TREE-SUCCESSOR(x)

```
if right[x] ≠ nil
    then return TREE-MINIMUM(right[x])
y ← p[x]
while y ≠ nil and x = right[y]
    do x ← y
       y ← p[y]
return y
```

Time complexity: O(h)

Try to develop TREE-PREDECESSOR(x)

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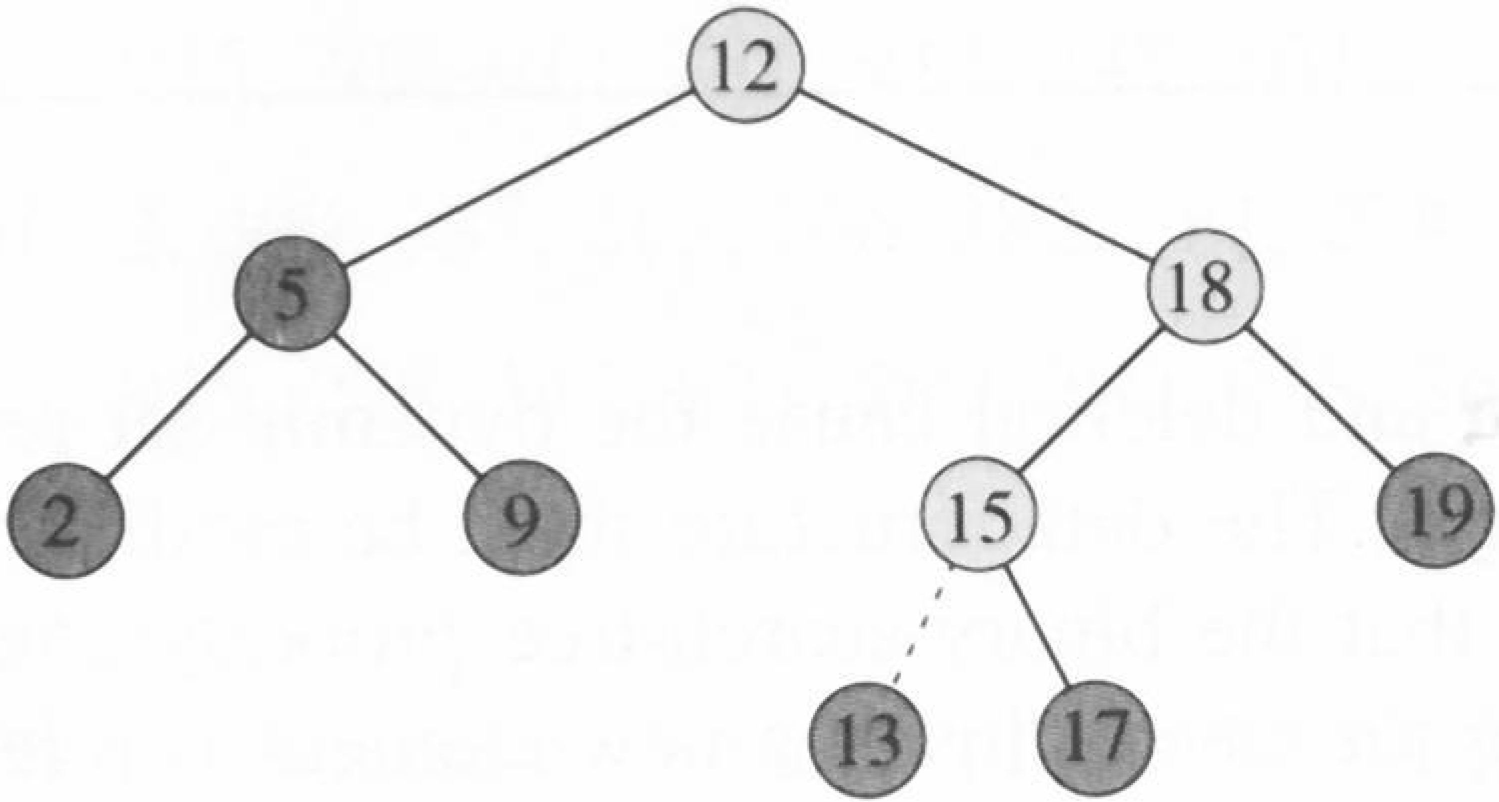
Insertion on BSTs

Time complexity: O(h)

TREE-INSERT(T, z)

```
y ← NIL
x ← root[T]
while x ≠ NIL
        do y ← x
                if key[z] < key[x]
                    then x ← left[x]
                    else  x ← right[x]
p[z] ← y
if  y = NIL
     then root[T] ← z        ▶ tree T was empty
     else if key[z] < key[y]
                then left[y] ← z
                else  right[y] ← z
```

Example



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Deletion on BSTs (1/2)

```
TREE-DELETE(T, z)
if left[z] = NIL or right[z] = NIL
    then y ← z   ▶ z has at most one child
    else y ← TREE-SUCCESSOR(z)   ▶ 2 children
if left[y] ≠ NIL
    then x ← left[y]
    else x ← right[y]
if x ≠ NIL
    then p[x] ← p[y]
```

Deletion on BSTs (2/2)

```
if  p[y] = NIL
        then root[T] ← x
        else  if y = left[p[y]]
                  then left[p[y]] ← x
                  else  right[p[y]] ← x
if y ≠ z
    then key[z] ← key[y]
          copy y's satellite data into z
return y
```

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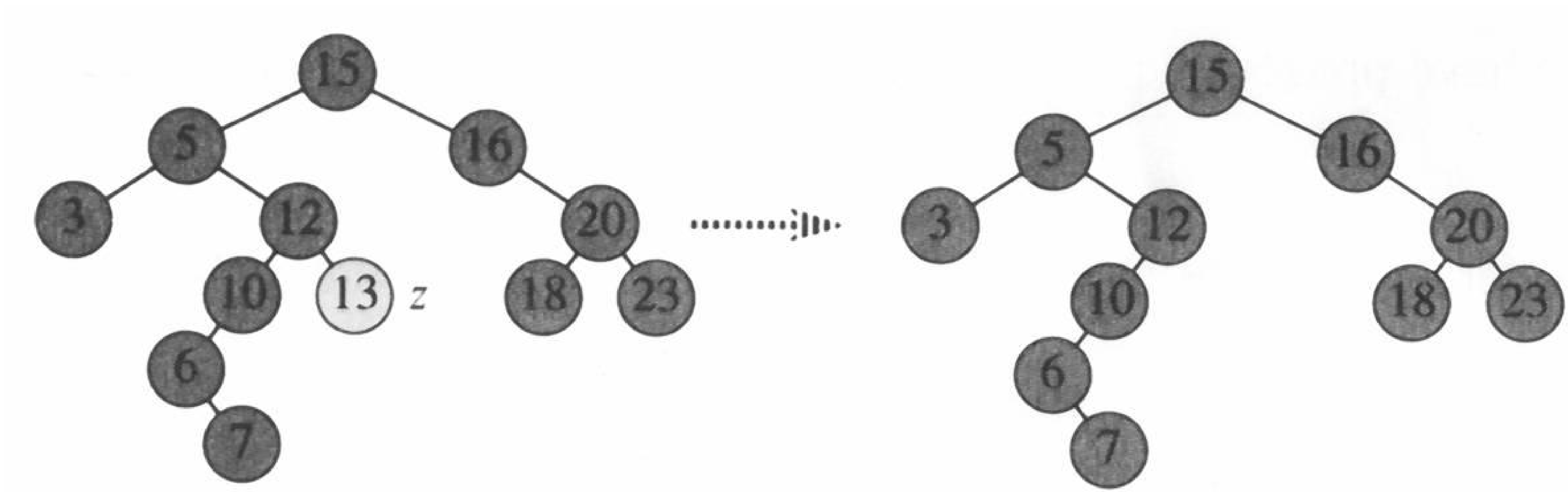
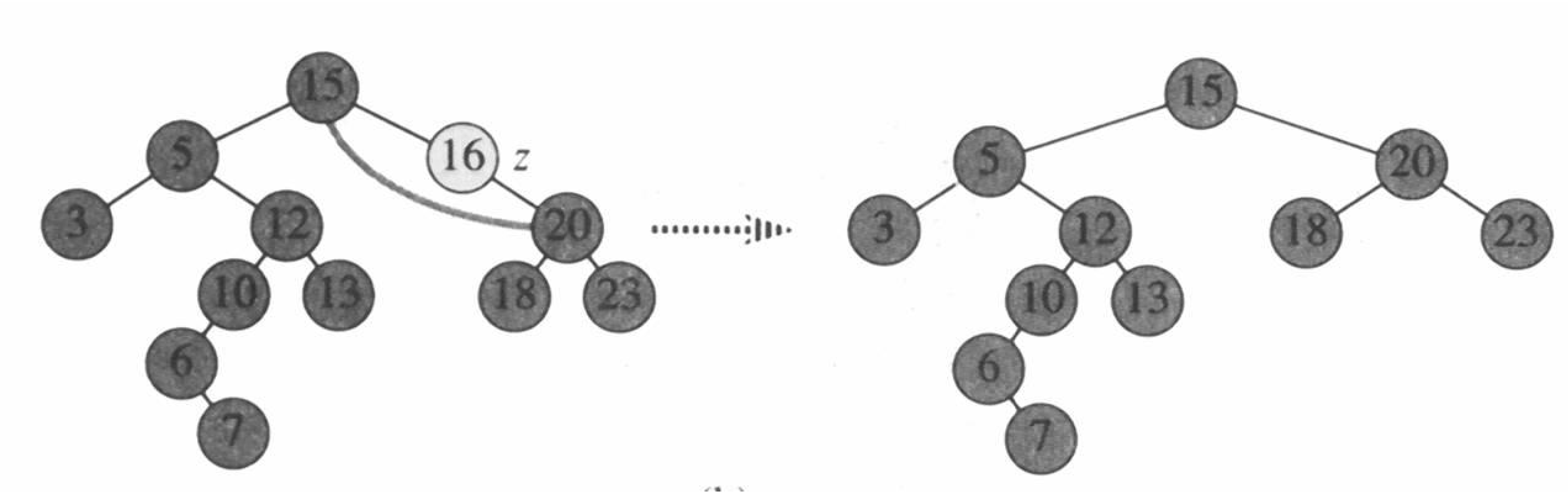
Case 1: z has no children



Case 2: z has one child



Case 3: z has 2 children