



CUDA Optimization with Parallel Transpose




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CUDA Optimization – Matrix Transpose

- I will use the case of “**Optimizing Parallel Transpose in CUDA**”

<https://devblogs.nvidia.com/efficient-matrix-transpose-cuda-cc/>

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}^T = \begin{bmatrix} 6 & 1 \\ 4 & -9 \\ 24 & 8 \end{bmatrix}$$


Matrix Transpose 1

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}^T = \begin{bmatrix} 6 & 1 \\ 4 & -9 \\ 24 & 8 \end{bmatrix}$$

- Consider an $n \times n$ matrix where 32 divides n .
- We focus on the device code:
 - the host code performs typical tasks: data allocation and transfer between host and device, the launching and timing of several kernels, result validation, and the deallocation of host and device memory.
- Benchmarks illustrate this section:
 - we **compare** our matrix transpose kernels against a **matrix copy kernel**,
 - for each kernel, we compute the **effective bandwidth**, calculated in **GB/s** as **twice the size of the matrix (once for reading the matrix and once for writing) divided by the time of execution**

Matrix Transpose 2

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}^T = \begin{bmatrix} 6 & 1 \\ 4 & -9 \\ 24 & 8 \end{bmatrix}$$

- Benchmarks illustrate this section:
 - for each kernel, we compute the [**effective bandwidth**], calculated in **GB/s** as **twice the size of the matrix (once for reading the matrix and once for writing) divided by the time of execution**

HBM2

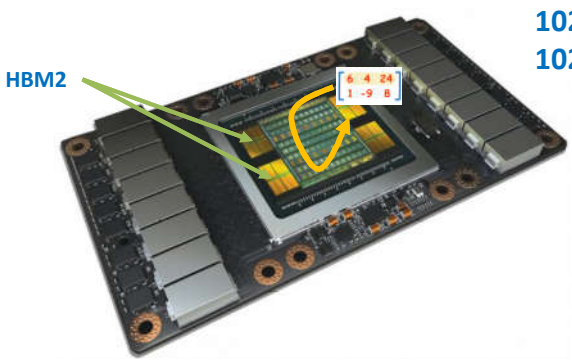
1024x1024 size float matrix
1024x1024 x4B x2 = 8MB

Effective Bandwidth
= 8MB/ t_{exe}
= ?

Matrix Transpose 2

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}^T = \begin{bmatrix} 6 & 1 \\ 4 & -9 \\ 24 & 8 \end{bmatrix}$$

1024x1024 size float matrix
 1024x1024 x4B x2 = 8MB



HBM2

Effective Bandwidth
 $= 8\text{MB}/t_{\text{exe}}$
 If ($t_{\text{exe}}=0.02\text{ ms}$) ?
 $= 8\text{MB}/0.02\text{ms} = 800\text{MB}/2\text{ms} = 400\text{GB/s}$

Transpose 'C' code: CPU version

O(n²)

```
void transpose_CPU(float in[], float out[])
{
    for(int j=0; j < N; j++)
        for(int i=0; i < N; i++)
            out[j + i*N] = in[i + j*N]; // out(j,i) = in(i,j)
}
```

<https://github.com/udacity/cs344/blob/master/Lesson%20Code%20Snippets/Lesson%20Code%20Snippets/transpose.cu>



Transpose 'C' code: GPU version 1

$O(n^2)$

```
// to be launched on a single thread
// transpose_serial<<<1,1>>>(d_in, d_out);
__global__ void transpose_serial(float in[], float out[])
{
    for(int j=0; j < N; j++)
        for(int i=0; i < N; i++)
            out[j + i*N] = in[i + j*N]; // out(j,i) = in(i,j)
}
```

<https://github.com/udacity/cs344/blob/master/Lesson%20Code%20Snippets/Lesson%205%20Code%20Snippets/transpose.cu>




Transpose 'C' code: GPU version 2

$O(n)$

```
// to be launched on a single thread
// transpose_parallel_per_row<<<1,N>>>(d_in, d_out);
__global__ void
transpose_parallel_per_row(float in[], float out[])
{
    int i = threadIdx.x;

    for(int j=0; j < N; j++)
        out[j + i*N] = in[i + j*N]; // out(j,i) = in(i,j)
}
```

<https://github.com/udacity/cs344/blob/master/Lesson%20Code%20Snippets/Lesson%205%20Code%20Snippets/transpose.cu>




Transpose 'C' code: GPU version 3

```
// dim3 blocks(N/K,N/K); // blocks per grid
// dim3 threads(K,K);    // threads per block
// transpose_parallel_per_element<<<blocks,threads>>>(d_in, d_out);
__global__ void
transpose_parallel_per_element(float in[], float out[])
{
    int i = blockIdx.x * K + threadIdx.x;
    int j = blockIdx.y * K + threadIdx.y;


    out[j + i*N] = in[i + j*N]; // out(j,i) = in(i,j)
}
```

O(1)

<https://github.com/udacity/cs344/blob/master/Lesson%20Code%20Snippets/Lesson%205%20Code%20Snippets/transpose.cu>



Deep Dive: Performance-Aware Opt.




Transpose 'C' code: GPU version 3

```
// dim3 blocks(N/K,N/K); // blocks per grid
// dim3 threads(K,K);    // threads per block
// transpose_parallel_per_element<<<blocks,threads>>>(d_in, d_out);
__global__ void
transpose_parallel_per_element(float in[], float out[])
{
    int i = blockIdx.x * K + threadIdx.x;
    int j = blockIdx.y * K + threadIdx.y;

    out[j + i*N] = in[i + j*N]; // out(j,i) = in(i,j)
}
```

O(1)

N = Height or width (square matrix: height == width)
K = size of a thread block in x-dimension or y-dimension





Deep Dive: *Performance-Aware Opt.*

```


ubuntu@tegra-ubuntu:~/TRANPOSE/cs344/Lesson Code Snippets/Lesson 5 Code Snippets$ ./transpose

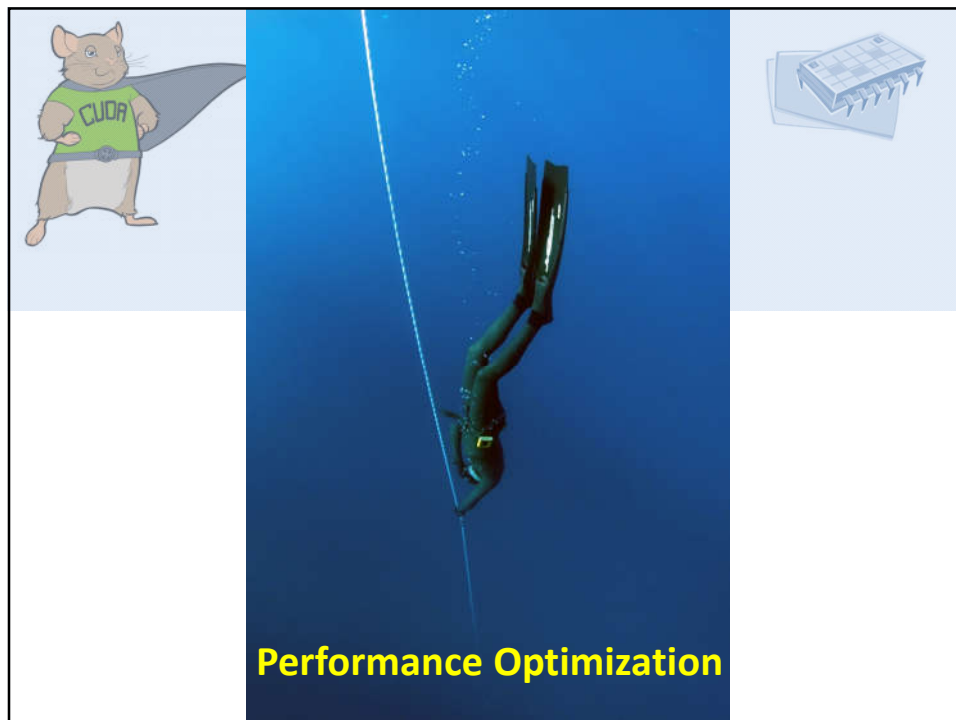
transpose_serial: 963.833 ms.
Verifying transpose...Success
transpose_parallel_per_row: 13.0229 ms.
Verifying transpose...Success
transpose_parallel_per_element: 11.4738 ms.
Verifying transpose...Success
transpose_parallel_per_element_tiled 32x32: 9.39575 ms.
Verifying ...Success
transpose_parallel_per_element_tiled 16x16: 4.44258 ms.
Verifying ...Success
transpose_parallel_per_element_tiled_padded 16x16: 4.073 ms.
Verifying...Success
  
```

~1/74 @ N threads

Performance Optimization





Matrix Transpose - CUDA

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}^T = \begin{bmatrix} 6 & 1 \\ 4 & -9 \\ 24 & 8 \end{bmatrix}$$

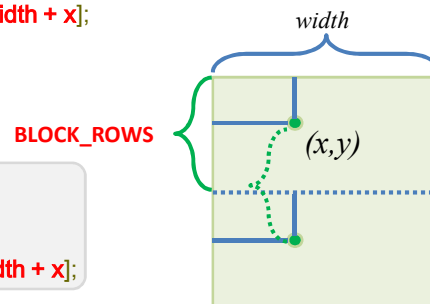
- Present different kernels called from the host code, each addressing different performance issues.
- All kernels launch **thread blocks** of dimension **“32x8”**,^{256 threads} where each block transposes (or copies) a tile of dimension 32x32. *launch blocks of 32x8 threads!*
- As such, the parameters `TILE_DIM` and `BLOCK_ROWS` are set to 32 and 8, respectively.

A simple copy kernel 1 – for Comparison

```
__global__ void copy(float *odata, float* idata)
{
    int x = blockIdx.x*TILE_DIM + threadIdx.x;
    int y = blockIdx.y*TILE_DIM + threadIdx.y;
    int width = gridDim.x * TILE_DIM;

    for (int j=0; j<TILE_DIM; j+=BLOCK_ROWS) {
        odata[(y+j)*width + x] = idata[(y+j)*width + x];
    }
}
```

```
j = 0
>> odata[y*width + x] = idata[y*width + x];
j = 1
>> odata[(y+1)*width + x] = idata[(y+1)*width + x];
```



<https://devblogs.nvidia.com/efficient-matrix-transpose-cuda-cc/>

A simple copy kernel 2 – for Comparison

- **odata** and **idata** are pointers to the input and output matrices,
- **width = height = gridDim.x*TILE_DIM**
- In this kernel, **xIndex** and **yIndex** are global 2D matrix indices and they used to calculate index, the 1D index used to access matrix elements.

```
__global__ void copy(float *odata, float* idata)
{
    int x = blockIdx.x*TILE_DIM + threadIdx.x;
    int y = blockIdx.y*TILE_DIM + threadIdx.y;
    int width = gridDim.x * TILE_DIM;

    for (int j=0; j<TILE_DIM; j+=BLOCK_ROWS) {
        odata[(y+j)*width + x] = idata[(y+j)*width + x];
    }
}
```

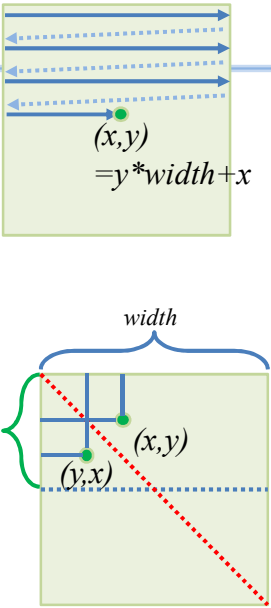

A naive transpose kernel

```

__global__ void transposeNaive(float *odata, float *idata)
{
    int x = blockIdx.x*TILE_DIM + threadIdx.x;
    int y = blockIdx.y*TILE_DIM + threadIdx.y;
    int width = gridDim.x*TILE_DIM;

    for (int j=0; j<TILE_DIM; j+=BLOCK_ROWS) {
        odata[x*width + (y+j)] = idata[(y+j)*width + x];
    }
}

```



(x,y)
 $=y*width+x$

(x,y)
 (y,x)


BLOCK_ROWS

<https://devblogs.nvidia.com/efficient-matrix-transpose-cuda-cc/>

Naive transpose kernel vs copy kernel

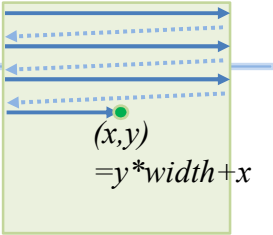
- The performance of these two kernels on a 1024x1024 matrix using a Tesla GPUs is given in the following table:

	Effective Bandwidth (GB/s, ECC enabled)	
Routine	Tesla M2050	Tesla K20c
copy	105.2	136.0
transposeNaive	18.8	55.3



<https://devblogs.nvidia.com/efficient-matrix-transpose-cuda-cc/>

Problem ?



```

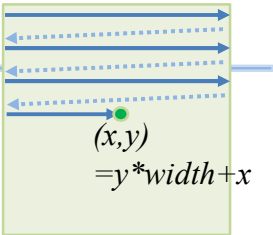
__global__ void transposeNaive(float *odata, float* idata)
{
    int x = blockIdx.x*TILE_DIM + threadIdx.x;
    int y = blockIdx.y*TILE_DIM + threadIdx.y;
    int width = gridDim.x*TILE_DIM;

    for (int j=0; j<TILE_DIM; j+=BLOCK_ROWS) {
        odata[x*width + (y+j)] = idata[(y+j)*width + x];
    }
}

```

(xindex, yindex): (0, 0) → (1, 0) → (2, 0) → (3, 0) → ... (31, 0)
 → (0, 1) → (1, 1) → (2, 1) → (3, 1) → ... (31, 1)
 → ...

Problem ?



```

__global__ void transposeNaive(float *odata, float* idata)
{
    int x = blockIdx.x*TILE_DIM + threadIdx.x;
    int y = blockIdx.y*TILE_DIM + threadIdx.y;
    int width = gridDim.x*TILE_DIM;

    for (int j=0; j<TILE_DIM; j+=BLOCK_ROWS) {
        odata[x*width + (y+j)] = idata[(y+j)*width + x];
    }
}

```

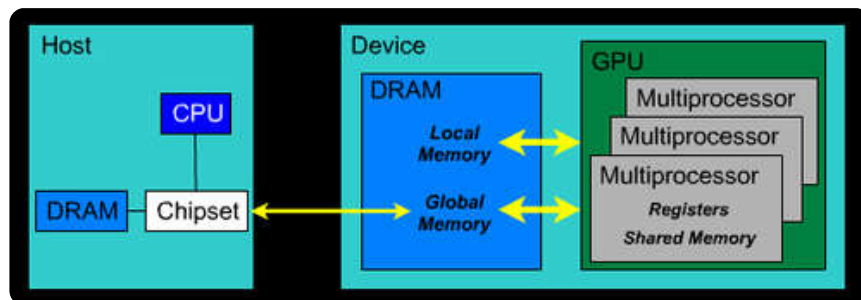
(xindex, yindex): (0, 0) → (1, 0) → (2, 0) → (3, 0) → ... (31, 0)
 → (0, 1) → (1, 1) → (2, 1) → (3, 1) → ... (31, 1)
 → ...

One Transaction .vs. 32 Transactions !



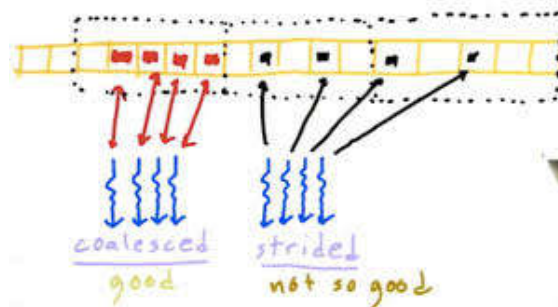
Coalesced Transpose 1

- Because **device memory [GPU DDR Memory]** has a much higher latency and lower bandwidth than **on-chip memory [shared memory]**, special attention must be paid to: **how global memory accesses are performed?**



Coalesced Transpose 2

- The **simultaneous global memory accesses** by each thread of a during the execution of a single read or write instruction will be **coalesced** into a single access if:
 - The size of the memory element accessed by each thread is either 4, 8, or 16 bytes.
 - The elements form a **contiguous block of memory**.
 - The i -th element is accessed by the i -th thread in the warp.



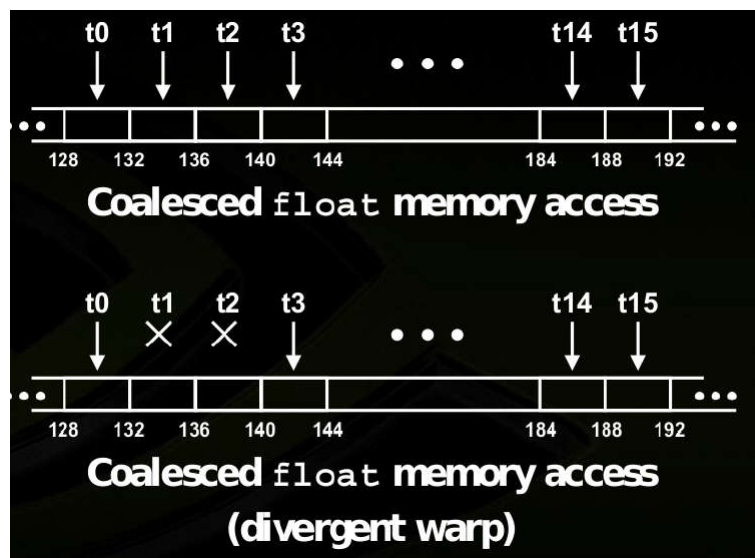


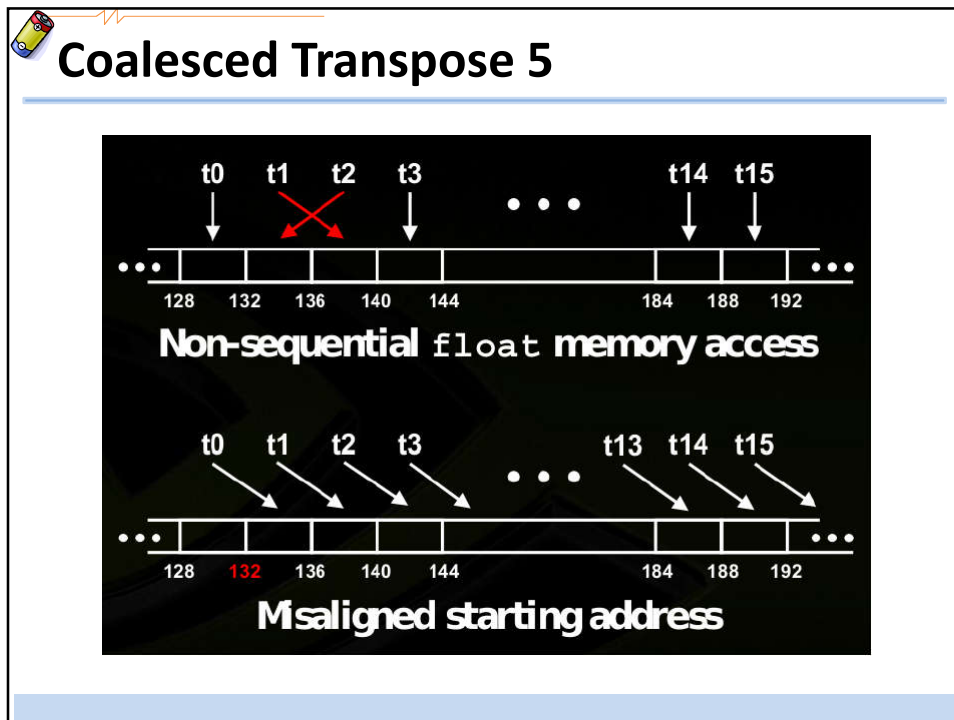
Coalesced Transpose 3

- The simultaneous global memory accesses by each thread of a during the execution of a single read or write instruction will be **coalesced** into a single access if:
 - The size of the memory element accessed by each thread is either 4, 8, or 16 bytes.
 - The elements form a **contiguous block of memory**.
 - The i-th element is accessed by the i-th thread in the warp.
- Last two requirements can be relaxed** (compiler optimization) with compute capabilities of 1.2.
- Coalescing happens even if some threads do not access memory (**divergent warp**)



Coalesced Transpose 4





Coalesced Transpose 6

- Allocating device memory through `cudaMalloc()` and choosing `TILE_DIM` to be a multiple of 16 ensures alignment with a segment of memory, therefore all loads from `idata` are coalesced.
- Coalescing behavior differs between the simple copy and naïve transpose kernels when writing to `odata`.



Coalesced Transpose 7

- The way to avoid **uncoalesced** global memory access is
 - to read the data into shared memory and,
 - have each warp access noncontiguous locations in shared memory in order to write contiguous data to odata.
- There is no performance penalty for noncontiguous access patterns in shared memory as there is in global memory.
- a **__syncthreads()** call is required to ensure that all reads from idata to shared memory have completed before writes from shared memory to odata.

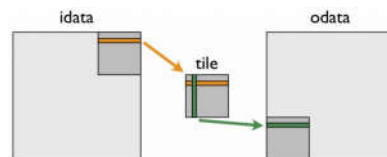


Coalesced Transpose 8

```
__global__ void transposeCoalesced(float *odata, const float *idata)
{
```

```
    __shared__ float tile[TILE_DIM][TILE_DIM];
```

```
    int x = blockIdx.x * TILE_DIM + threadIdx.x;
    int y = blockIdx.y * TILE_DIM + threadIdx.y;
    int width = gridDim.x * TILE_DIM;
```



```
    for (int j = 0; j < TILE_DIM; j += BLOCK_ROWS)
        tile[threadIdx.y+j][threadIdx.x] = idata[(y+j)*width + x];
```

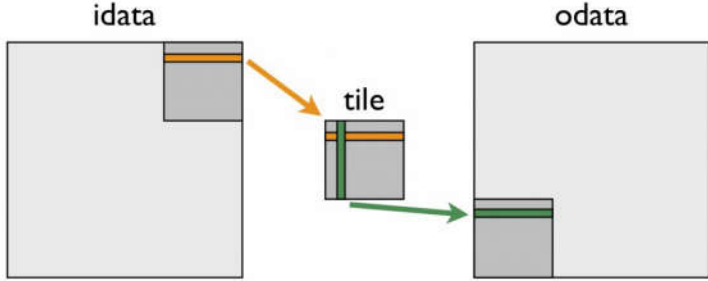
```
    __syncthreads();
```

```
    x = blockIdx.y * TILE_DIM + threadIdx.x; // transpose block offset
    y = blockIdx.x * TILE_DIM + threadIdx.y;
```

```
    for (int j = 0; j < TILE_DIM; j += BLOCK_ROWS)
        odata[(y+j)*width + x] = tile[threadIdx.x][threadIdx.y + j];
```

```
}
```

Coalesced Transpose 9



- a warp of threads reads contiguous data from `idata` into rows of the shared memory tile.
- After recalculating the array indices, **a column of the shared memory tile is written to contiguous addresses in `odata`.**
- Because threads write different data to `odata` than they read from `idata`, we must use a block-wise barrier synchronization `__syncthreads()`.

Coalesced Transpose 10

Effective Bandwidth (GB/s, ECC enabled)		
Routine	Tesla M2050	Tesla K20c
copy	105.2	136.0
copySharedMem	104.6	152.3
transposeNaive	18.8	55.3
transposeCoalesced	51.3	97.6

- There is a dramatic increase in effective bandwidth of the coalesced transpose over the naive transpose, but there still remains a large performance gap between the coalesced transpose and the copy:
 - One possible cause of this performance gap could be the **synchronization barrier** required in the coalesced transpose.
 - This can be easily assessed using the following copy kernel which utilizes shared memory and contains a `__syncthreads()` call.



Coalesced Transpose 11

```
__global__ void copySharedMem(float *odata, const float *idata)
{
    __shared__ float tile[TILE_DIM * TILE_DIM];

    int x = blockIdx.x * TILE_DIM + threadIdx.x;
    int y = blockIdx.y * TILE_DIM + threadIdx.y;
    int width = gridDim.x * TILE_DIM;

    for (int j = 0; j < TILE_DIM; j += BLOCK_ROWS)
        tile[(threadIdx.y+j)*TILE_DIM + threadIdx.x] = idata[(y+j)*width + x];

    __syncthreads();

    for (int j = 0; j < TILE_DIM; j += BLOCK_ROWS)
        odata[(y+j)*width + x] = tile[(threadIdx.y+j)*TILE_DIM + threadIdx.x];
}
```

<https://devblogs.nvidia.com/efficient-matrix-transpose-cuda-cc/>



Coalesced Transpose 12

Effective Bandwidth (GB/s, ECC enabled)		
Routine	Tesla M2050	Tesla K20c
copy	105.2	136.0
copySharedMem	104.6	152.3
transposeNaive	18.8	55.3
transposeCoalesced	51.3	97.6

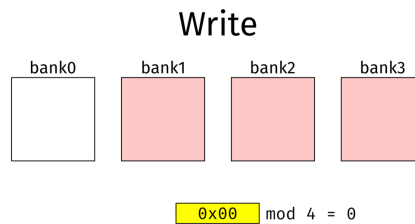
- The shared memory copy results seem to suggest that **the use of shared memory with a synchronization barrier has little effect on the performance.**





Shared memory bank conflicts 1

- Shared memory is divided into 32 equally-sized memory modules, called **banks**, which are organized such that successive 32-bit words are assigned to successive banks.

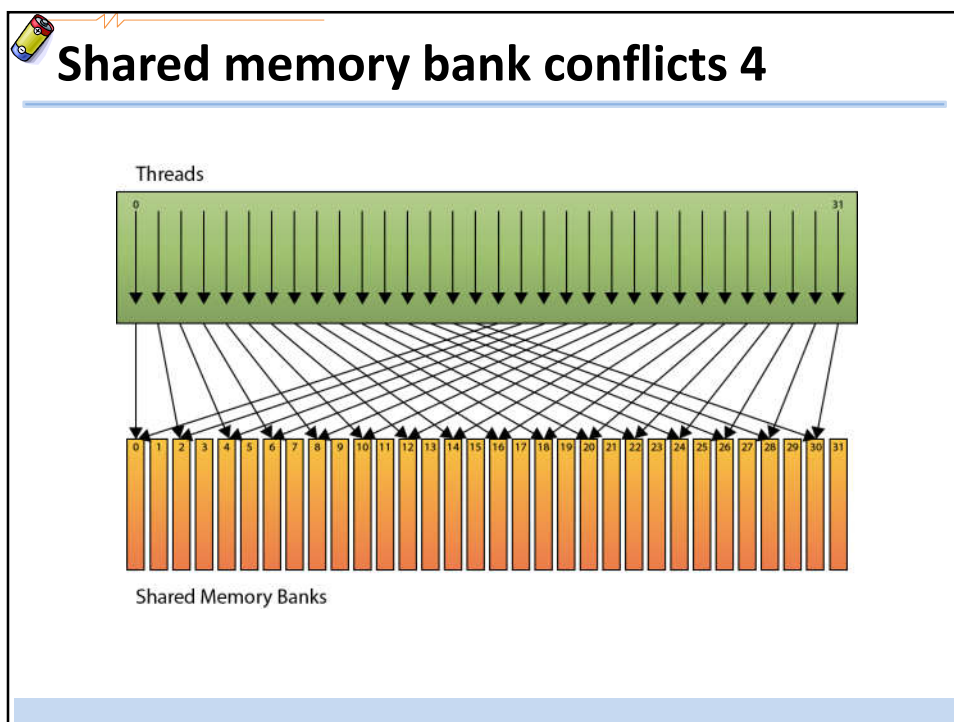
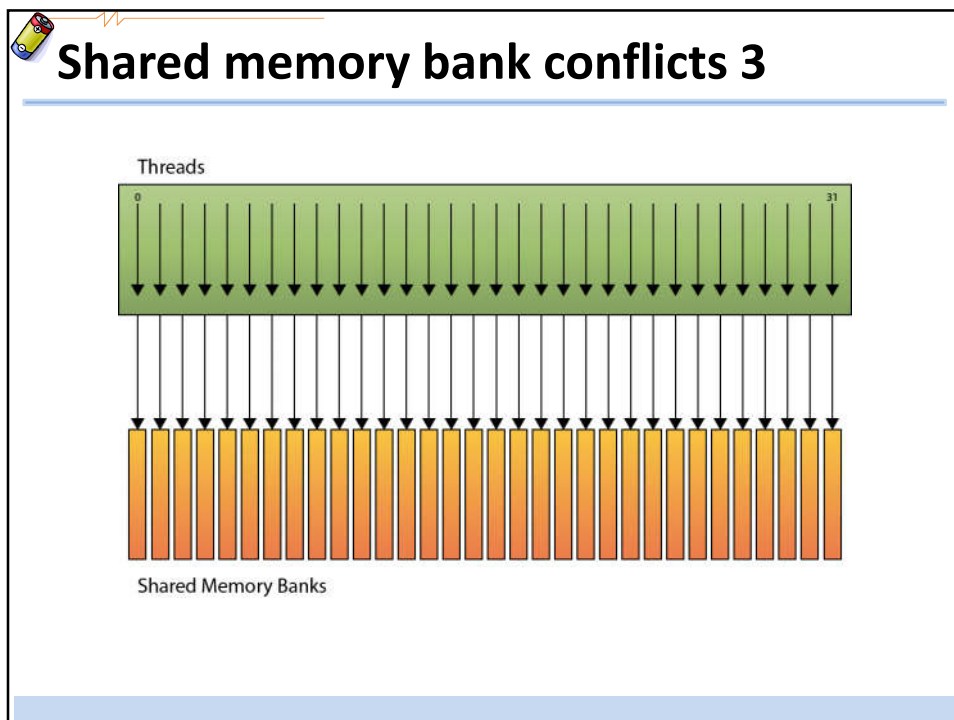


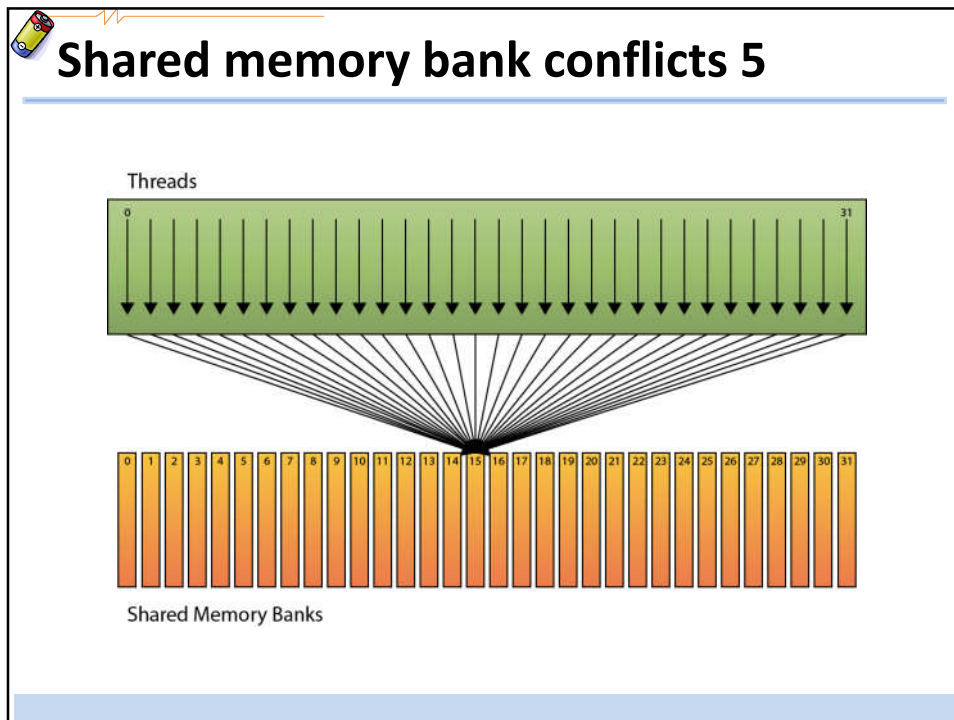
- These banks can be accessed simultaneously, and to achieve maximum bandwidth to and from shared memory, the **threads in a warp should access shared memory associated with different banks**.



Shared memory bank conflicts 2

- These banks can be accessed simultaneously, and to achieve maximum bandwidth to and from shared memory the **threads in a warp should access shared memory associated with different banks**.
- The **exception to this rule** is when **all threads in a half warp read the same shared memory address**, which results in a **broadcast** where the data at that address is sent to all threads of the half warp in one transaction.





Shared memory bank conflicts 6

- The coalesced transpose uses a 32×32 shared memory array of floats.
- For a shared memory tile of 32×32 elements, **all elements in a column of data map to the same shared memory bank**
 - Resulting in a worst-case scenario for memory bank conflicts: reading a column of data results in a **32-way bank conflict**.
- A simple way to avoid this conflict is to **pad the shared memory array by one column**:

```
__shared__ float tile[TILE_DIM][TILE_DIM+1];
```



Shared memory bank conflicts 7

- A simple way to avoid this conflict is to **pad the shared memory array by one column**:

```
__shared__ float tile[TILE_DIM][TILE_DIM+1];
```

bnk1	bnk2	bnk3	bnk4
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

bnk1	bnk2	bnk3	bnk4
1	2	3	4
p	5	6	7
8	p	9	10
11	12	p	13
14	15	16	p

tile[tid % 4]:

→ 1, 5, 9, 13 threads access a bank1



Shared memory bank conflicts 8

Effective Bandwidth (GB/s, ECC enabled)		
Routine	Tesla M2050	Tesla K20c
copy	105.2	136.0
copySharedMem	104.6	152.3
transposeNaive	18.8	55.3
transposeCoalesced	51.3	97.6
transposeNoBankConflicts	99.5	144.3



SATISFIED!

<https://devblogs.nvidia.com/efficient-matrix-transpose-cuda-cc/>



Granularity of Parallelism

- Size of a Tile ?
 - We do test with a block of 32x8 threads with config. of “(32,8)”
 - What about 32x32 ?
 - “1024 threads wait at a barrier”
 - High Parallelism (?) but high synchronization overhead
 - What about 16x16 ?
 - “256 threads wait at a barrier”
 - Lower Parallelism (?) but lower synchronization overhead

Minimize timing waiting at a barrier !

<https://github.com/jeonggunlee/cs344/blob/master/Lesson%20Code%20Snippets/Lesson%205%20Code%20Snippets/transpose.cu>



Conclusion - Transpose

- Understand CUDA performance characteristics
 - **Memory coalescing**
 - **Bank conflicts**
 - **Granularity of parallelism**
- Use peak performance metrics to guide optimization