Finite period T

Infinite periode

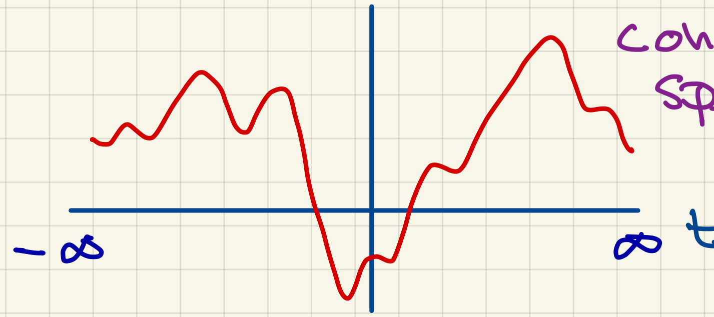
Continuous

f(t) Fourier Series

Discrete spectrum

f(t) Fourier Transform

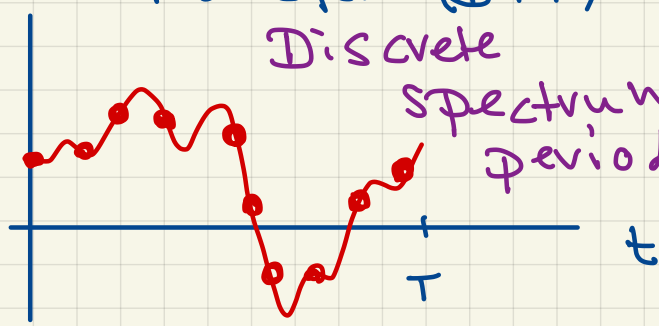
continuous spectrum



Discrete

f(t) Discrete Fourier Transform (DFT)

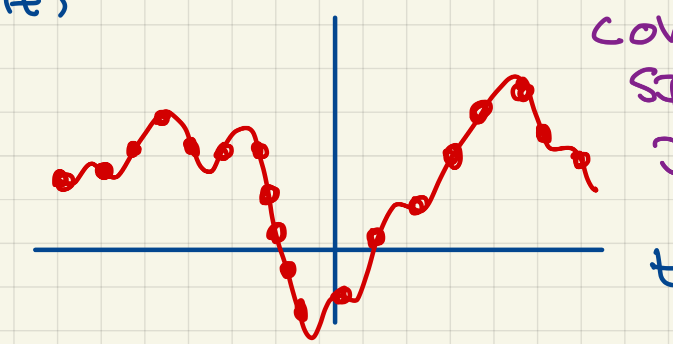
Discrete spectrum, periodic



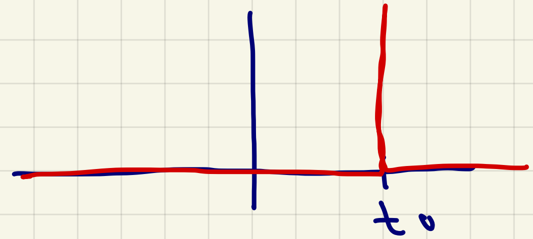
Discrete Time Fourier Transform

f(t)

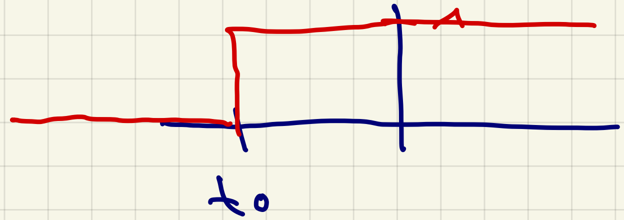
continuous spectrum, periodic



• $\delta(t-t_0)$



• $H(t-t_0)$



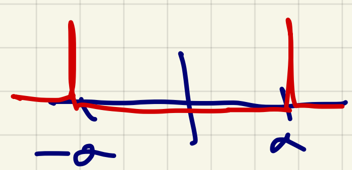
• $\int f(t)\cdot\delta(t-t_0)\,dt = f(t_0)$

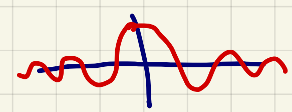
• $\int e^{\pm i(\omega_1-\omega_2)t}\,dt = 2\pi\,\delta(\omega_1-\omega_2)$

time domain $f(t) \circ\!\!-\!\!\bullet\ F(\omega)$ frequency domain

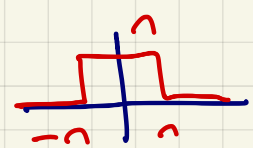
FT

$$F(\omega)=\int_{-\infty}^{\infty} f(t)\,e^{-i\omega t}\,dt \;;\quad f(t)=\frac{1}{2\pi}\int F(\omega)\,e^{i\omega t}\,d\omega$$

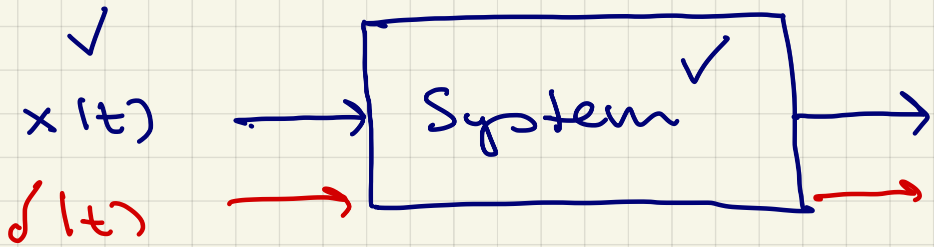




• $\cos(\alpha t) \circ\!\!-\!\!\bullet \sim \delta(\omega+\alpha)+\delta(\omega-\alpha)$;



$\circ\!\!-\!\!\bullet \sim \frac{\sin\omega}{\omega}$

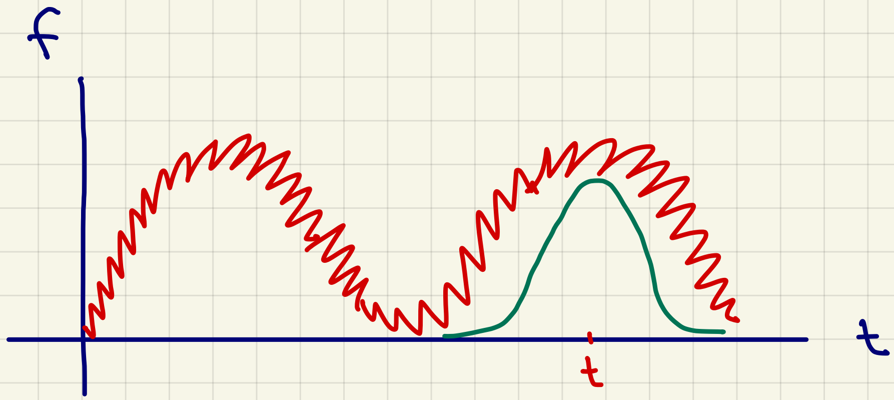


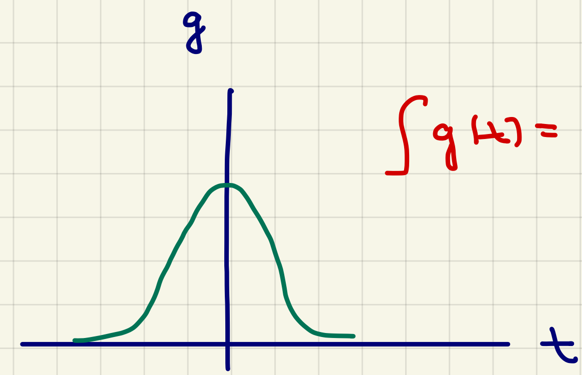
$y(t) = h(t) * x(t)$

$h(t)$

impulse response function

Convolution / Faltung





$\int g(t) = 1$

Für alle Stellen t

- Setze g an die Stelle t, und flippe
- Multipliziere g mit f
- Integrieren

$g(t-\tau)$

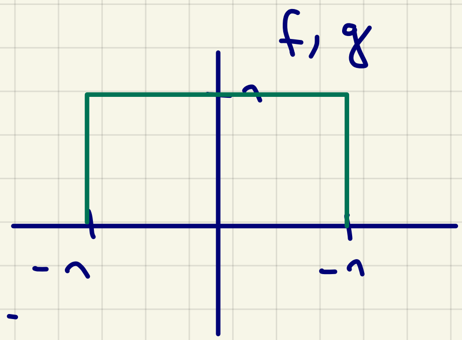
$g(-(\tau - t))$

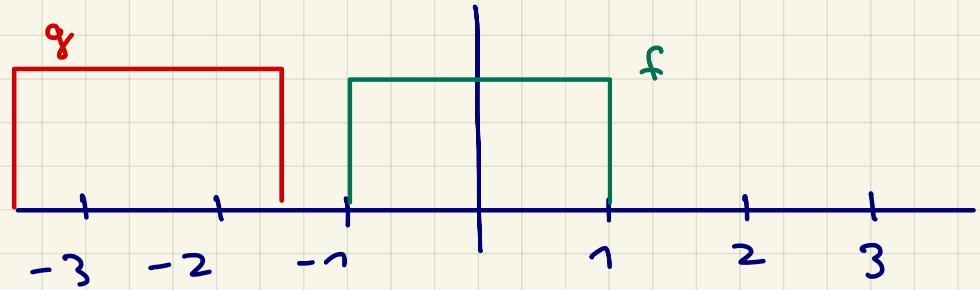
$$(f * g)(t) = \int f(\tau) \cdot g(t-\tau)\, d\tau$$

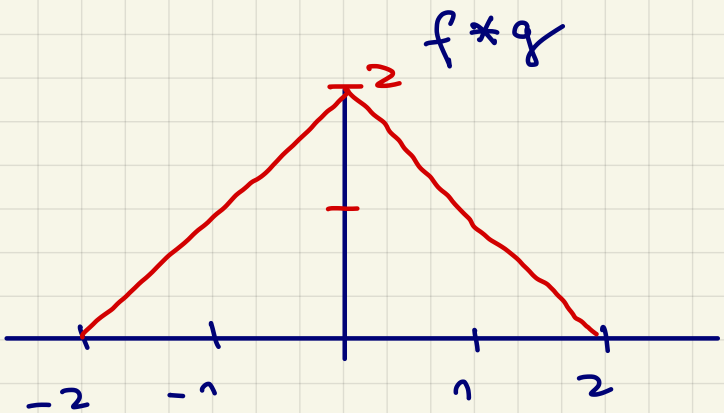
$$\|$$

$$(g * f)(t) = \int g(\tau) \cdot f(t-\tau)\, d\tau$$

$$(f * g)(t) = \int f(\tau) \cdot g(t-\tau)\, d\tau$$







Numerisch mit Python:

$$(f * g)(t) = \int f(\tau) \cdot g(t-\tau)\, d\tau$$

Arrays: f_i, g_i

$$c_n = (f * g)_n = \sum_i f_i \, g_{n-i}$$

$$\shortparallel$$

$$c_n = (g * f)_n = \sum_i g_i \, f_{n-i} \leftarrow$$

$g_0 \quad g_1$

$g = [\frac{1}{2} \quad \frac{1}{2}]$

$f = [1 \quad 2 \quad 3 \quad 4 \quad 5]$

$f_0 \quad f_1 \quad f_2 \quad f_3 \quad f_4$

$$c_0 = g_0 f_0 = \frac{1}{2}$$

$$c_1 = g_0 f_1 + g_1 f_0 = 1.5$$

$$c_2 = g_0 f_2 + g_1 f_1 = 2.5$$

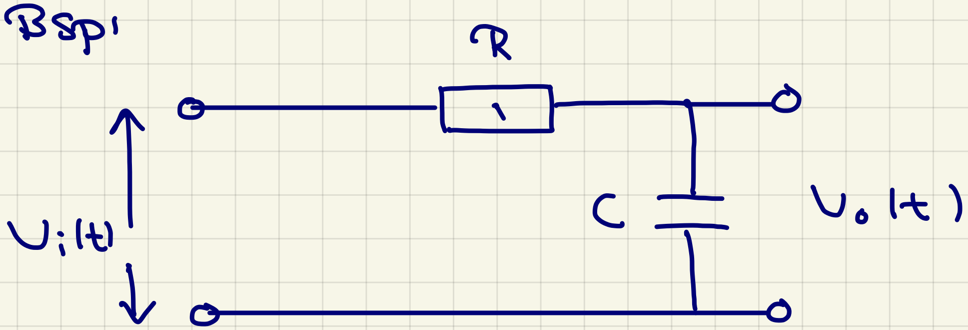
$$c_3 = \ldots = 3.5$$

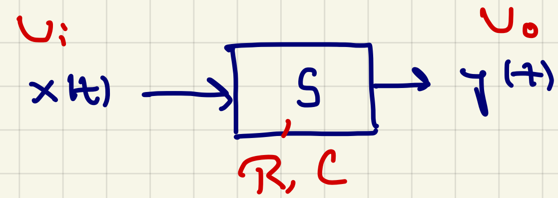
$$c_4 = \ldots = 4.5$$

$$c_5 = g_1 f_4 = 2.5$$

Wie beschreibt man „Systeme" mathematisch?

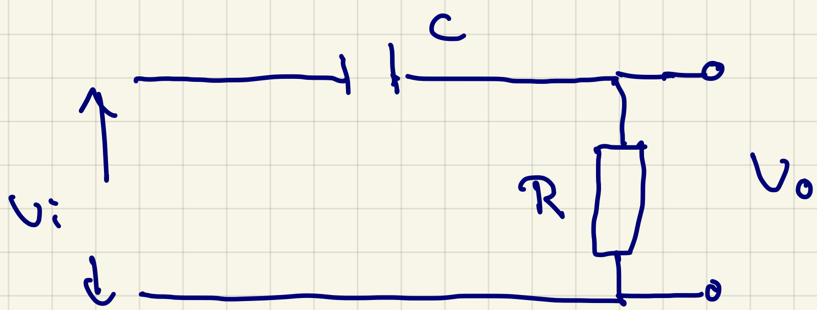
Bsp:





$$C = \frac{Q}{U_o} \qquad \dot{Q} = C \cdot \dot{U}_o$$
$$\dot{Q} = I$$

$$U_i = U_R + U_o = R I_C + U_o = R \cdot C \dot{U}_o + U_o$$



$$\dot{U}_C = \frac{\dot{Q}_C}{C} = \frac{I}{C} = \frac{U_o}{RC}$$

$$\dot{U}_i = \dot{U}_C + \dot{U}_o = \frac{1}{RC} U_o + \dot{U}_o$$

$$\boxed{a_2 \ddot{y}(t) + a_1 \dot{y}(t) + a_0 y(t) = b_1 \dot{x}(t) + b_0 x(t)}$$

Wie werden Ableitungen Fourier transformiert?

$$f(t) \circ\!\!-\!\!\bullet F(\omega)$$

$$f(t) = \int F(\omega)\, e^{(i\omega)t}\, d\omega$$

$$f^{(n)}(t) = \int \left((i\omega)^n F(\omega) \right) e^{i\omega t}\, d\omega$$

$$\boxed{f^{(n)}(t) \circ\!\!-\!\!\bullet (i\omega)^n F(\omega)}$$

1, 2 Faltung

1d 2d

3, 4 Systeme