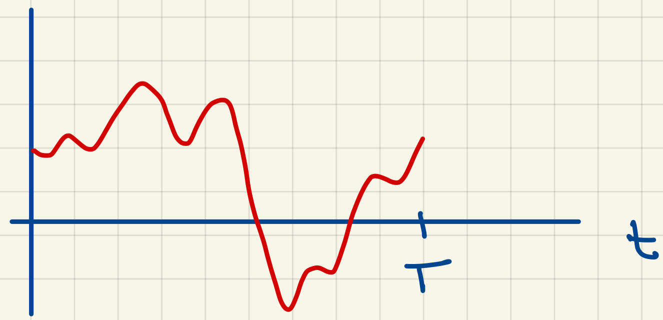
Finite period T

Continuous

f(t) Fourier Series



Discrete

f(t) Discrete Fourier Transform (DFT)

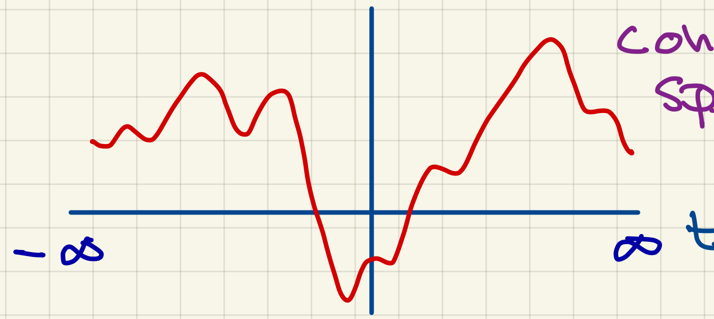
	Finite period T	Infinite periode
Continuous	Fourier Series Discrete spectrum	Fourier Transform continuous spectrum

f(t) Fourier Series

Discrete spectrum

f(t) Fourier Transform

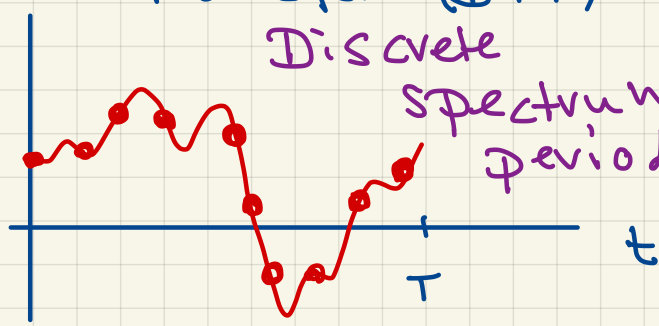
continuous spectrum



	Finite period T	Infinite periode
Discrete	Discrete Fourier Transform (DFT) Discrete spectrum, periodic	Discrete Time Fourier Transform continuous spectrum, periodic

f(t) Discrete Fourier Transform (DFT)

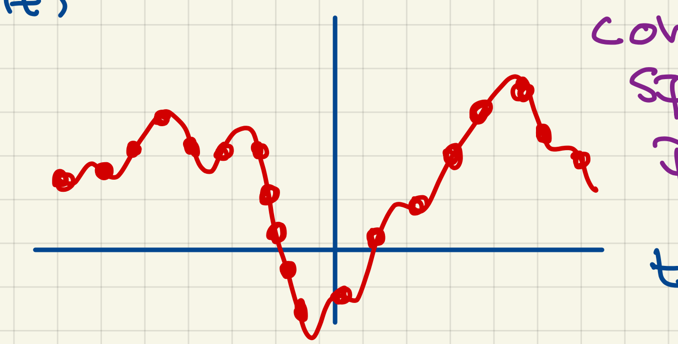
Discrete spectrum, periodic



Discrete Time Fourier Transform

f(t)

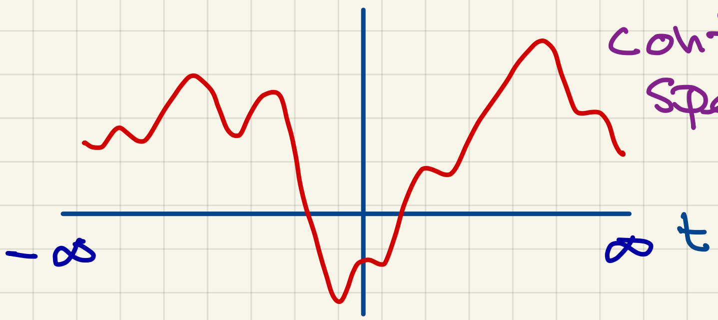
continuous spectrum, periodic



In finite periode

f(t)

Fourier Transform

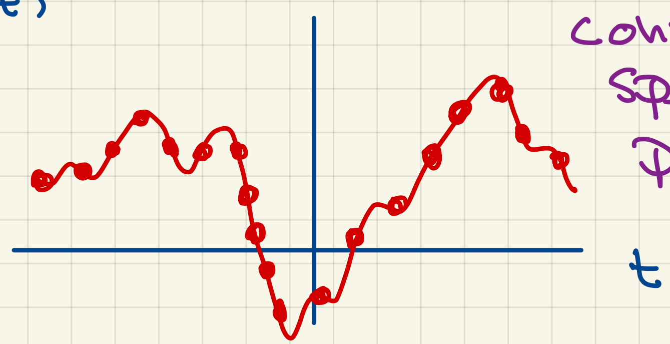


continuous spectrum

$\xrightarrow{\text{Extension}}$ Laplace-Transform

Discrete Time Fourier Transform

f(t)



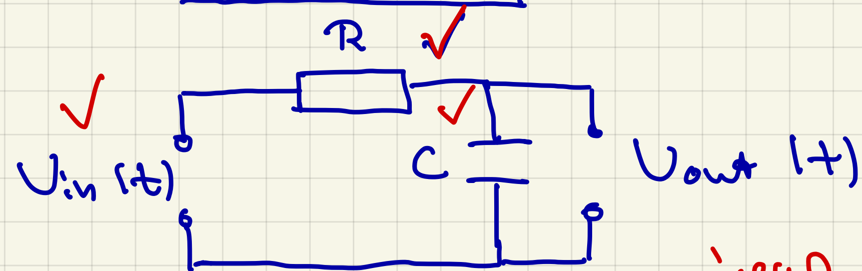
continuous spectrum periodic

$\longrightarrow$ z-Transform

Fourier Transform → mainly used in linear system theory, control engineering

$x(t) \longrightarrow$ Linear System $\longrightarrow y(t)$?

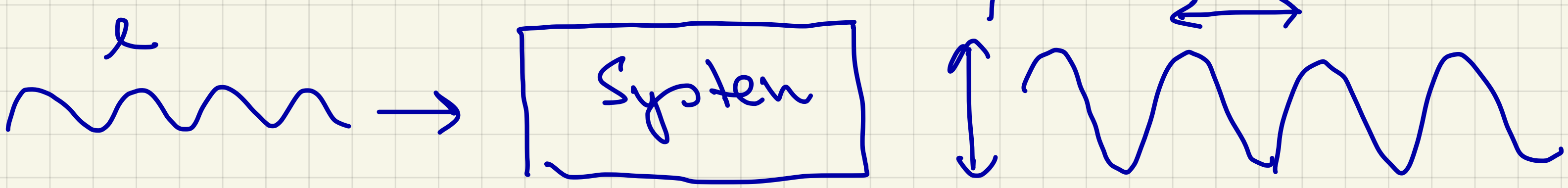
decompose into Fourier modes



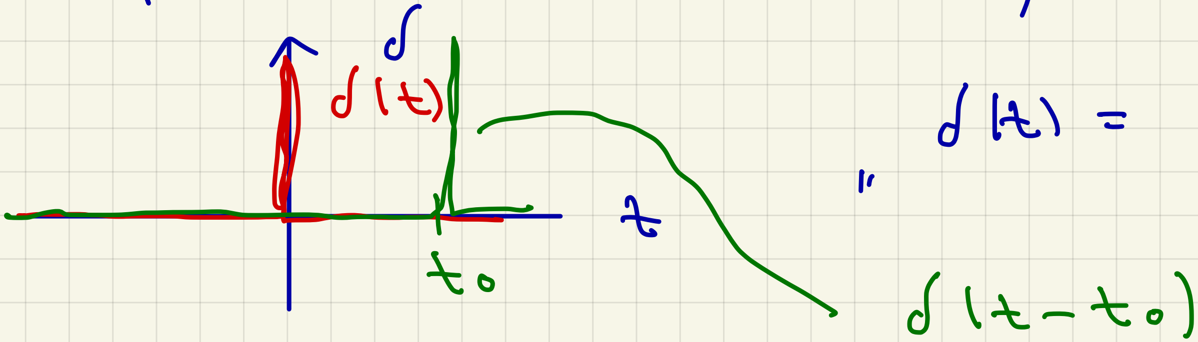
$|H(\omega)| e^{i\varphi(\omega)}$ complex transfer functions

$e^{i\omega t}$ 10

$H(\omega) e^{i\omega t}$ 10

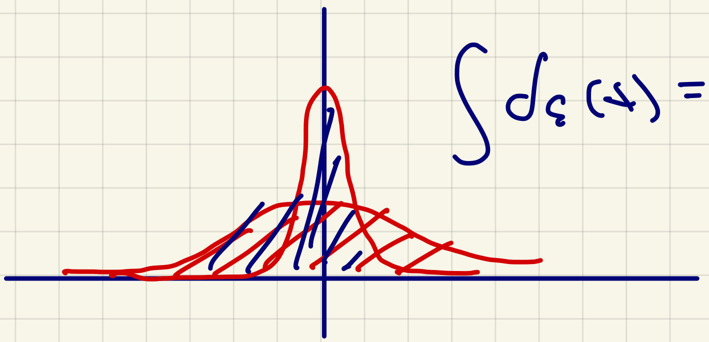


- δ- function (Dirac Pulse)



$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases}$$

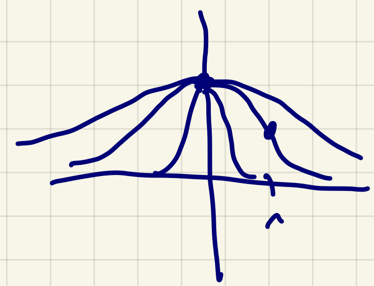
$\varepsilon \to 0$



$$\int \delta_\varepsilon(t) = 1$$

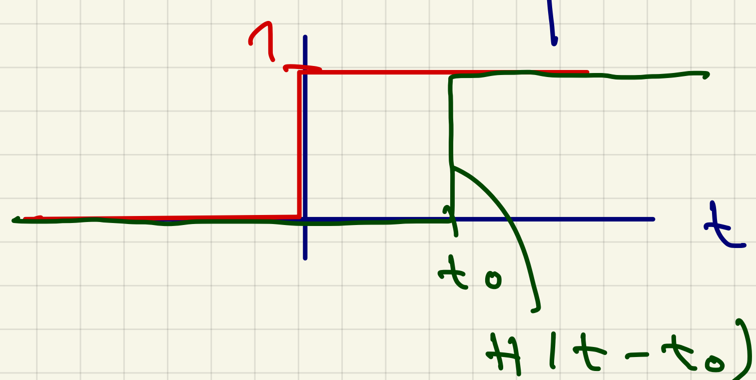
$$\frac{1}{\sqrt{\pi \varepsilon}} e^{-\frac{t^2}{\varepsilon}} = \delta_\varepsilon(t)$$

$$e^{-\frac{t^2}{\varepsilon}}$$



- Heaviside / Step-function

$$H(t) = \begin{cases} 0 & t \leq 0 \\ 1 & t > 0 \end{cases}$$



$$H'(t) = \delta(t)$$

- $\int f(t)\cdot\delta(t-t_0)\cdot dt = f(t_0)$

Examples:

- $\int \cos(t)\ \overset{t-0}{\delta(t)}\ dt = \cos(0) = 1$

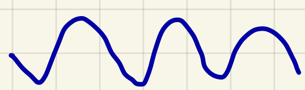
- $\int e^{-t^2}\cdot\delta(\underbrace{t+1}_{=0\ \to\ t=-1})\,dt = e^{-(-1)^2} = e^{-1} = \frac{1}{e}$

- $\int_{-\infty}^{\infty} e^{i(\overset{3}{\alpha}-\overset{1}{\beta})t}\,dt = 2\pi\,\delta(\alpha-\beta)$

$\alpha=\beta$: ∞

$\alpha\neq\beta$: 0

$e^{i2t} = \cos(2t) + i\sin(2t)$

Fourier-Transform (FT)

Fourier Series

$$c_n = \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\, e^{-i\omega_n t}\, dt$$

$$\omega_n = \frac{2\pi}{T}\cdot n = 2\pi \underbrace{\frac{1}{T}\cdot n}_{f_n}$$

$T \to \infty$:

$$\boxed{F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt}$$

$F(\omega)$ is FT of $f(t)$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{i\omega_n t}$$

$$\boxed{f(t) = \int F(\omega)\, e^{i\omega t}\, d\omega}$$

inverse FT

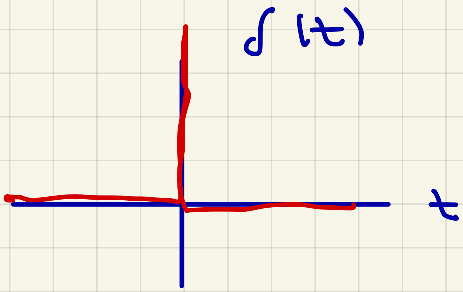
$$\boxed{F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt}$$

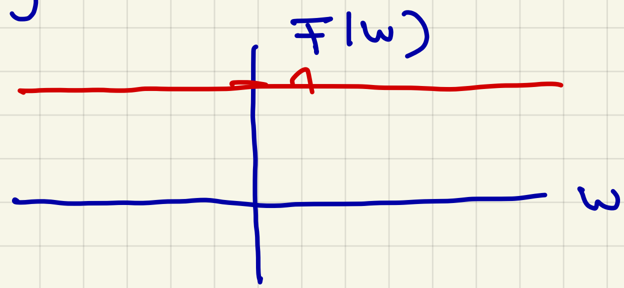
Examples:

- $\int f(t) \cdot \delta(t-t_0) \cdot dt = f(t_0)$

- $\delta(t) = f(t)$

$$F(\omega) = \int e^{-i\omega t} \cdot \delta(t)\, dt = e^{-i\omega \cdot 0} = 1$$

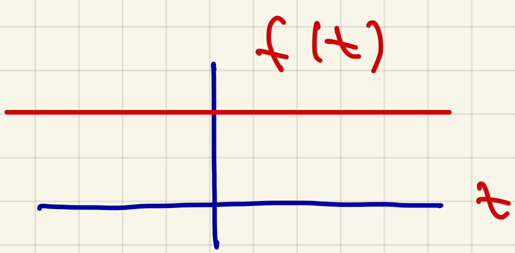


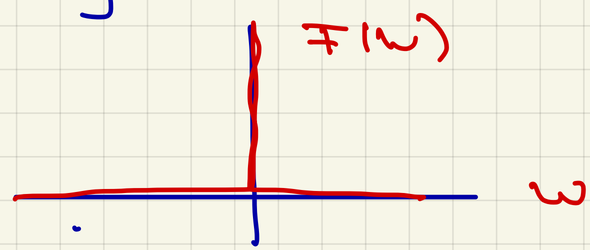


Time domain $\quad \delta(t) \quad$ ○——● $\quad 1 \quad$ frequency domain

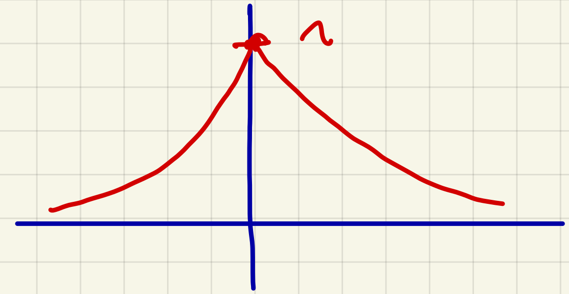
- $f(t) = 1$

$$F(\omega) = \int e^{-i\omega t}\, d\omega = 2\pi\, \delta(\omega)$$





- $f(t) = e^{-\alpha |t|}$ $\quad \alpha > 0$



$\int e^{5t} = \frac{1}{5} e^{5t}$

$$F(\omega) = \int_{-\infty}^{0} e^{+\alpha t} e^{-i\omega t} \, dt + \int_{0}^{\infty} e^{-\alpha t} e^{-i\omega t} \, dt$$

$|-5| = -(-5)$

$|5| = 5$

$$= \int_{-\infty}^{0} e^{(\alpha - i\omega) t} \, dt + \int_{0}^{\infty} e^{-(\alpha + i\omega) t} \, dt$$

$$= \left[\frac{1}{(\alpha - i\omega)} e^{(\alpha - i\omega t)} \right]_{-\infty}^{0} + \left[\frac{1}{-(\alpha + i\omega)} e^{-(\alpha + i\omega) t} \right]_{0}^{\infty}$$

$$= \frac{1}{\alpha - i\omega} - 0 + 0 + \frac{1}{\alpha + i\omega}$$

$$= \frac{2\alpha}{\omega^2 + \alpha^2}$$

12pm