■ Taylor series.

$$f(x+h) = C_0 + C_1h + C_2h^2 + C_3h^3 + \cdots + C_nh^n \quad (n \to \infty)$$

$$f'(x+h) = C_1 + 2C_2h + 3C_3h^2 + \cdots + nC_nh^{n-1}$$

$$f''(x+h) = 2C_2 + 3 \cdot 2C_3h + \cdots + n(n-1)C_nh^{n-2}$$

$$C_0 = f(x)$$

$$C_1 = f'(x)$$

$$C_2 = \frac{f''(x)}{2}$$

$$\therefore\ f(x+h) = f(x) + f'(x)h + \frac{f''(x)}{2}h^2 + \cdots + \frac{f^n(x)}{n!}h^n \qquad ①$$

$$h = -h \to f(x-h) = f(x) - f'(x)h + \frac{f''(x)}{2}h^2 - \cdots + \frac{f^n(x)}{n!}(-h)^n \quad ②$$

① - ②

$$f(x+h) - f(x-h) = 2f'(x)h + R \to\ f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$$

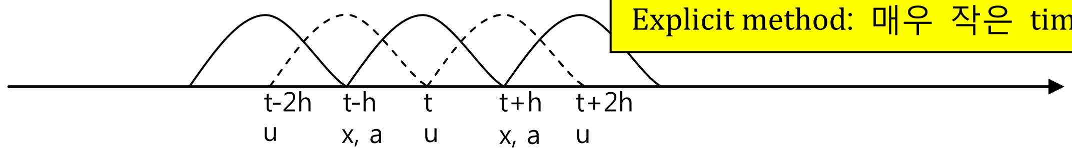
① + ②

$$f(x+h) + f(x-h) = 2f(x) + f''(x)h^2 + R \to\ f''(x) \approx \frac{f(x+h)+f(x-h)-2f(x)}{h^2}$$

ex) $F = m\ddot{x} + c\dot{x} + kx = ma + cu + kx \to\ x(t), u(t), a(t)$

$$u(t) \approx \frac{x(t+h)-x(t-h)}{2h}\ ,\quad a(t-h) \approx \frac{u(t)-u(t-2h)}{2h}$$

Explicit method: 매우 작은 time step



■ Implicit method.

$$Ma'_{n+1} + Cu'_{n+1} + Kx'_{n+1} = F^{ext}_{n+1}$$

$$x'_{n+1} = x_n + u_n\Delta t + \frac{((1-2\beta)a_n\Delta t^2)}{2} + \beta a'_{n+1}\Delta t^2 \ \rightarrow \quad x'_{n+1} = x^*{}_n + \beta a'_{n+1}\Delta t^2$$

$$u'_{n+1} = u_n + (1-\gamma)a_n\Delta t + \gamma a'_{n+1}\Delta t \ \rightarrow \quad u'_{n+1} = u^*{}_n + \gamma a'_{n+1}\Delta t$$

$$Ma'_{n+1} + C(u^*{}_n + \gamma a'_{n+1}\Delta t) + K(x^*{}_n + \beta a'_{n+1}\Delta t^2) = F^{ext}_{n+1}$$

$$[M + C\gamma\Delta t + K\beta\Delta t^2)]a'_{n+1} = F^{ext}_{n+1} - Cu^*{}_n - Kx^*{}_n$$

$$M^* a'_{n+1} = F^{residual}_{n+1}$$

$$a'_{n+1} = M^{*-1} F^{residual}_{n+1}$$

Trapezoidal rule: $\gamma = \frac{1}{2}, \beta = \frac{1}{4}$

Explicit method:

장점: time step당 연산시간 빠름.

단점: 큰 time step=> unstable=>매우 작은time step요구=> 최소 element 크기와 음속의 비율로 결정(⊿t=S*L/C).

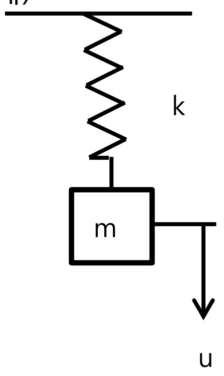
Implicit method:

장점: stable, 큰 time step, adaptive time step control.

단점: non-linearity 및 time step 변화=>matrix 재구성 및 연산 필요=> time step당 연산시간이 길다.

*** 충돌 등 매우 짧은 시간에 일어나는 물리적 현상의 해석 => explicit method

예제)

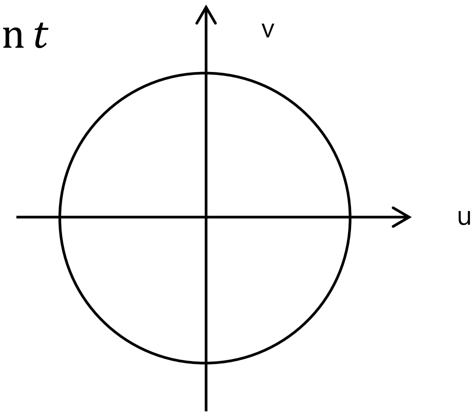


$$m\ddot{u} + ku = 0, \qquad if)\ m = 1,\ \ k = 1$$

$$\ddot{u} + u = 0, \qquad v = \frac{du}{dt}$$

$$u(t) = \cos t\,, \qquad v(t) = -\sin t$$

$$u^2 + v^2 = 1$$



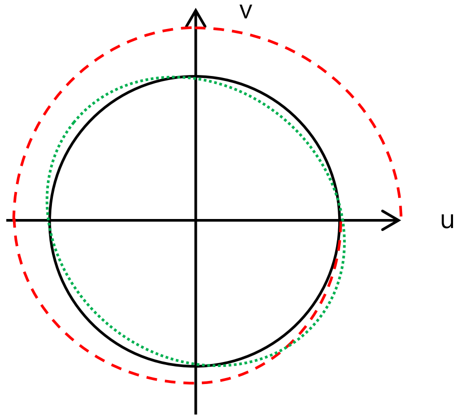
$$\frac{dv}{dt} = -u,\ \frac{du}{dt} = v$$

$$u_{n+1} = u_n + \Delta t \cdot v_n$$
$$v_{n+1} = v_n - \Delta t \cdot u_n$$

Forward Euler

$$u_{n+1} = u_{n-1} + 2\Delta t \cdot v_n$$
$$(u_{n+1} = u_n + \Delta t \cdot v_n)$$
$$v_{n+2} = v_n - 2\Delta t \cdot u_{n+1}$$
$$(v_{n+1} = v_n - \Delta t \cdot u_{n+1})$$

Leap frog



$$u_{n+1} = u_n + \Delta t \cdot v_{n+1}$$
$$v_{n+1} = v_n - \Delta t \cdot u_{n+1}$$

Backward Euler

$$u_{n+1} = u_n + \Delta t \cdot \frac{v_{n+1} + v_n}{2}$$
$$v_{n+1} = v_n - \Delta t \cdot \frac{u_{n+1} + u_n}{2}$$

(Phase error)

Trapezoidal rule