

8주차(3/3)

역전파 1

파이썬으로 배우는 기계학습

한동대학교
김영섭 교수

역전파 1

- 학습 목표

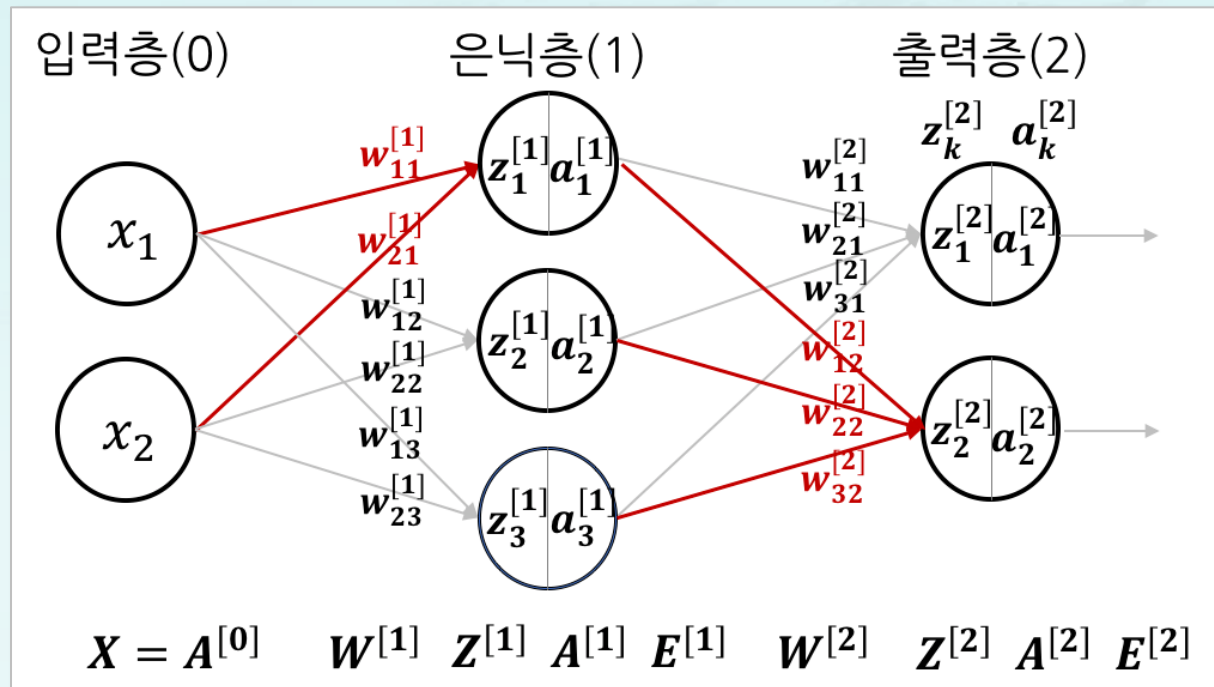
- 역전파 개념의 이해 한다
- 다층 신경망에서 은닉층 오차를 계산하는 방법을 배운다.
- 은닉층 오차 계산을 단순화하고 일반화 하는 방법을 배운다.

- 학습 내용

- 출력층의 오차를 역전파하기
- 역전파로 은닉층의 오차 계산하기
- 은닉층 오차 계산을 단순화하고 일반화하기

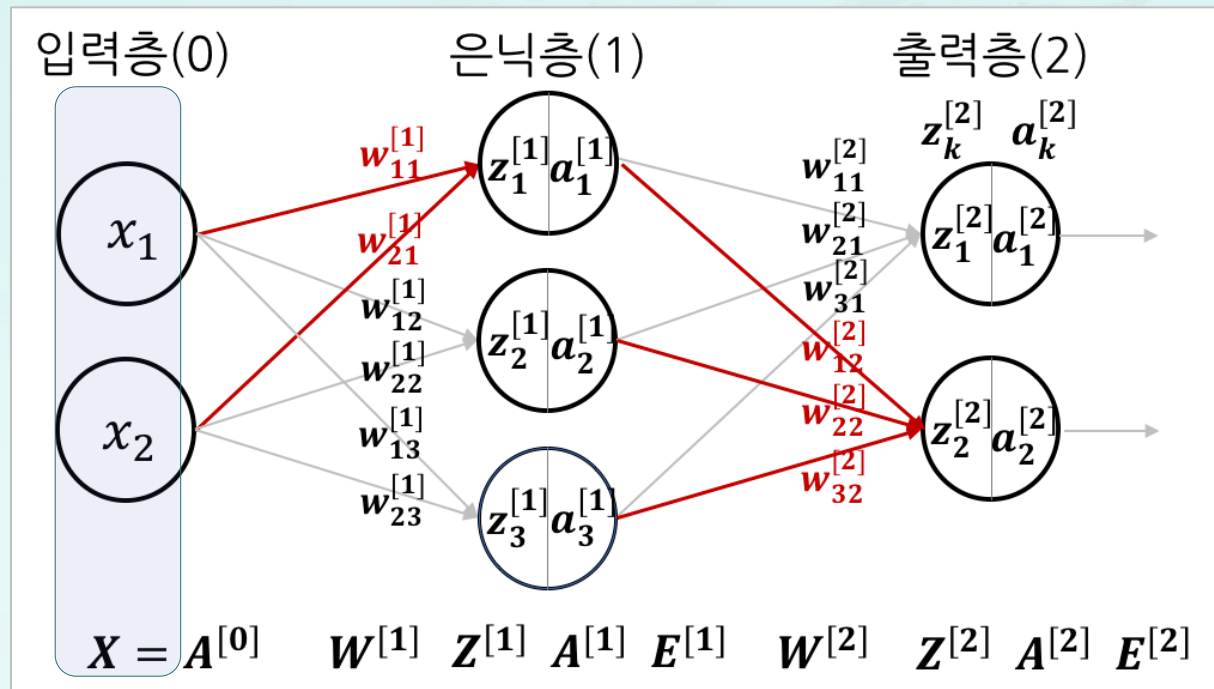
역전파 1: 신호 처리(복습)

- 다층 구조 인공신경망



역전파 1: 신호 처리(복습)

- 다층 구조 인공신경망
- 신호처리
 - X : 입력

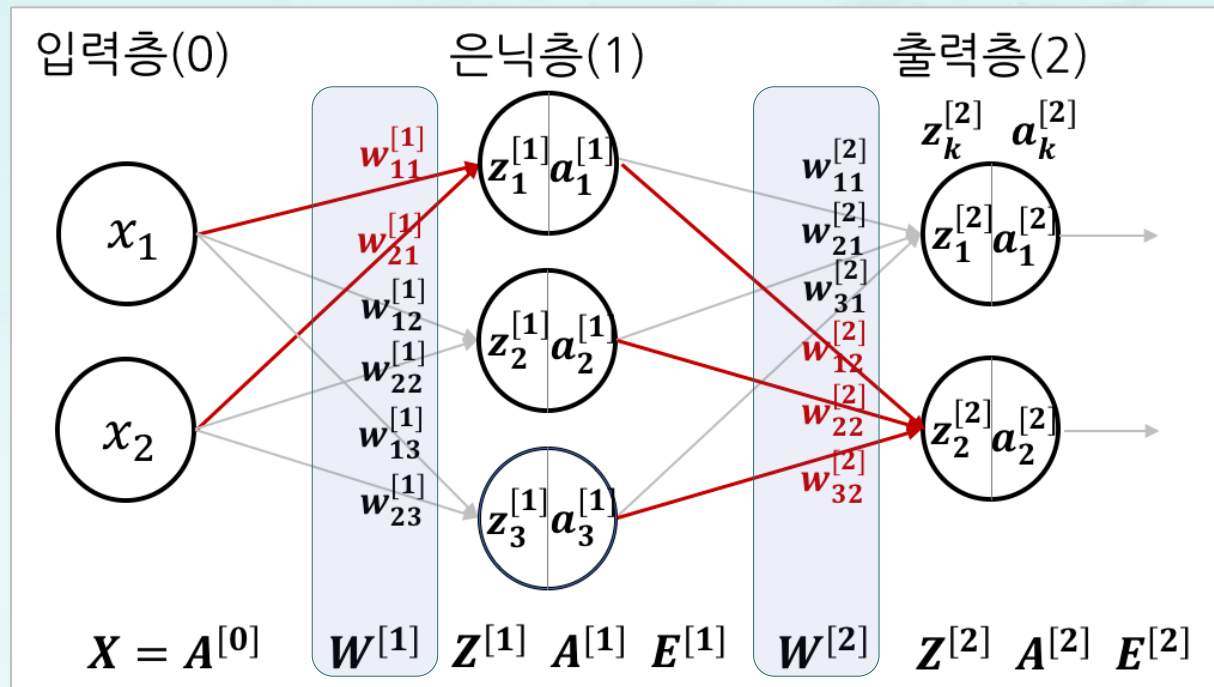


역전파 1: 신호 처리(복습)

- 다층 구조 인공신경망

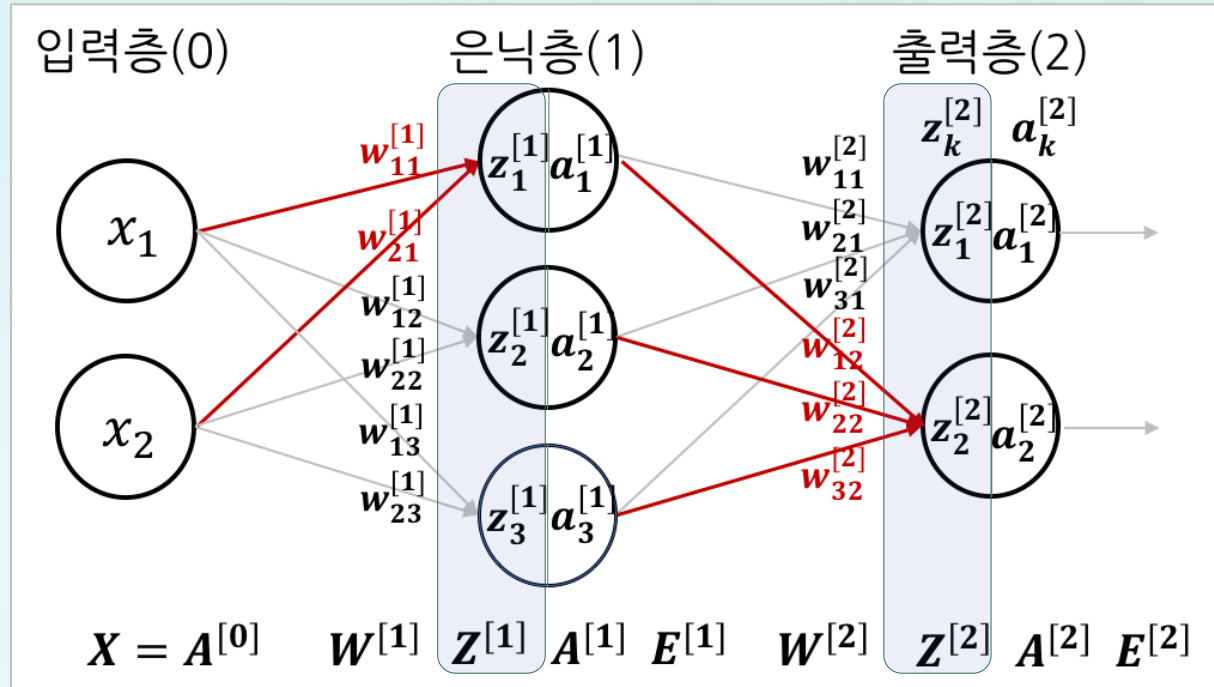
- 신호처리

- \mathbf{X} : 입력
- \mathbf{W} : 가중치



역전파 1: 신호 처리(복습)

■ 다층 구조 인공신경망



■ 신호처리

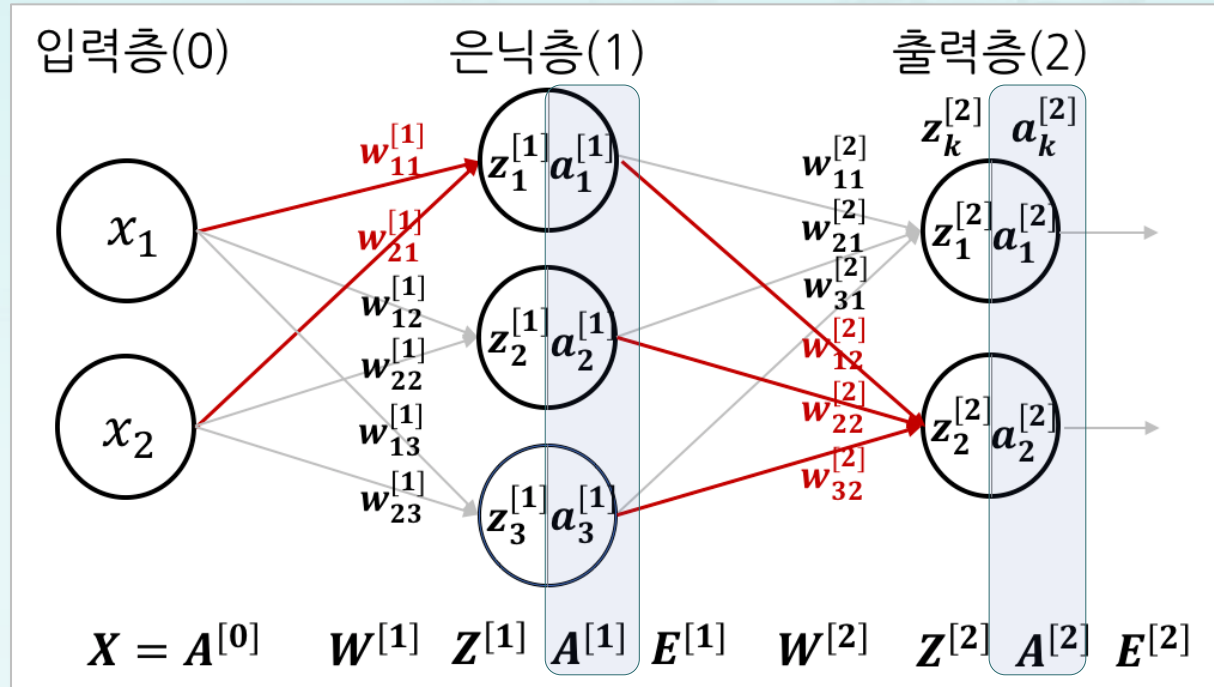
- X : 입력
- W : 가중치
- Z : 순입력

$$Z^{[l]} = W^{[l]T} A^{[l-1]}$$

$$Z^{[2]} = W^{[2]T} A^{[1]}$$

역전파 1: 신호 처리(복습)

■ 다층 구조 인공신경망



■ 신호처리

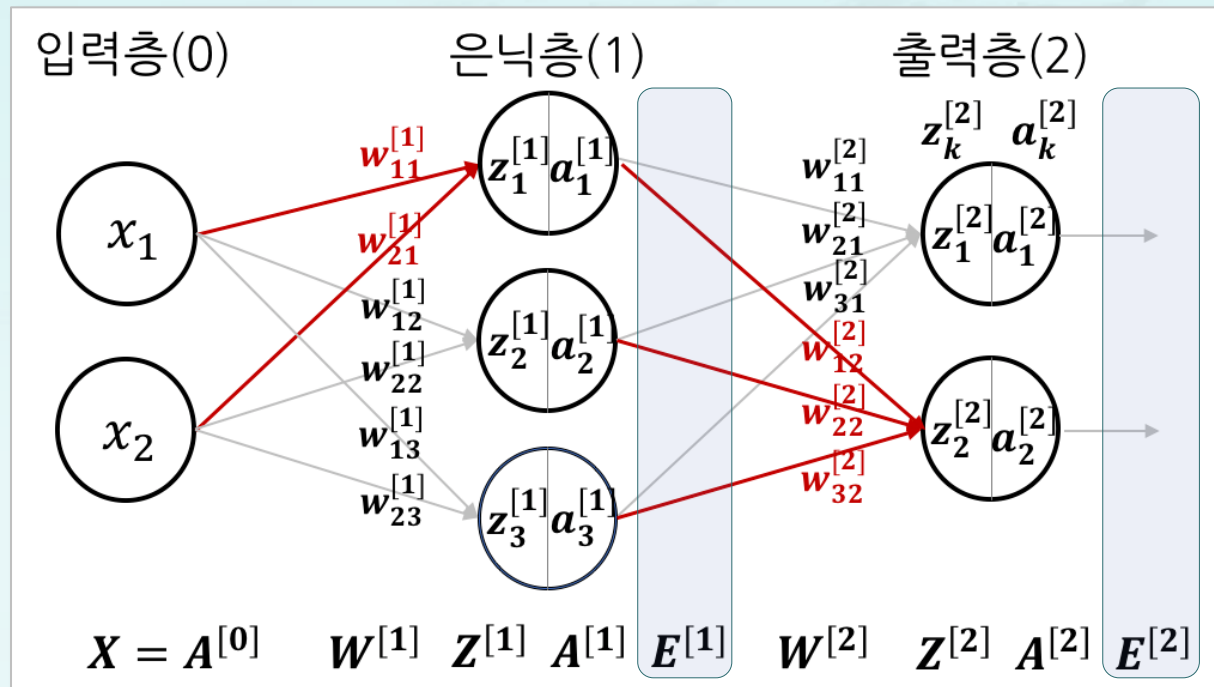
- X : 입력
- W : 가중치
- Z : 순입력
- A : 출력

$$A^{[l]} = g(Z^{[l]})$$

$$A^{[2]} = \text{sigmoid}(Z^{[2]})$$

역전파 1: 신호 처리(복습)

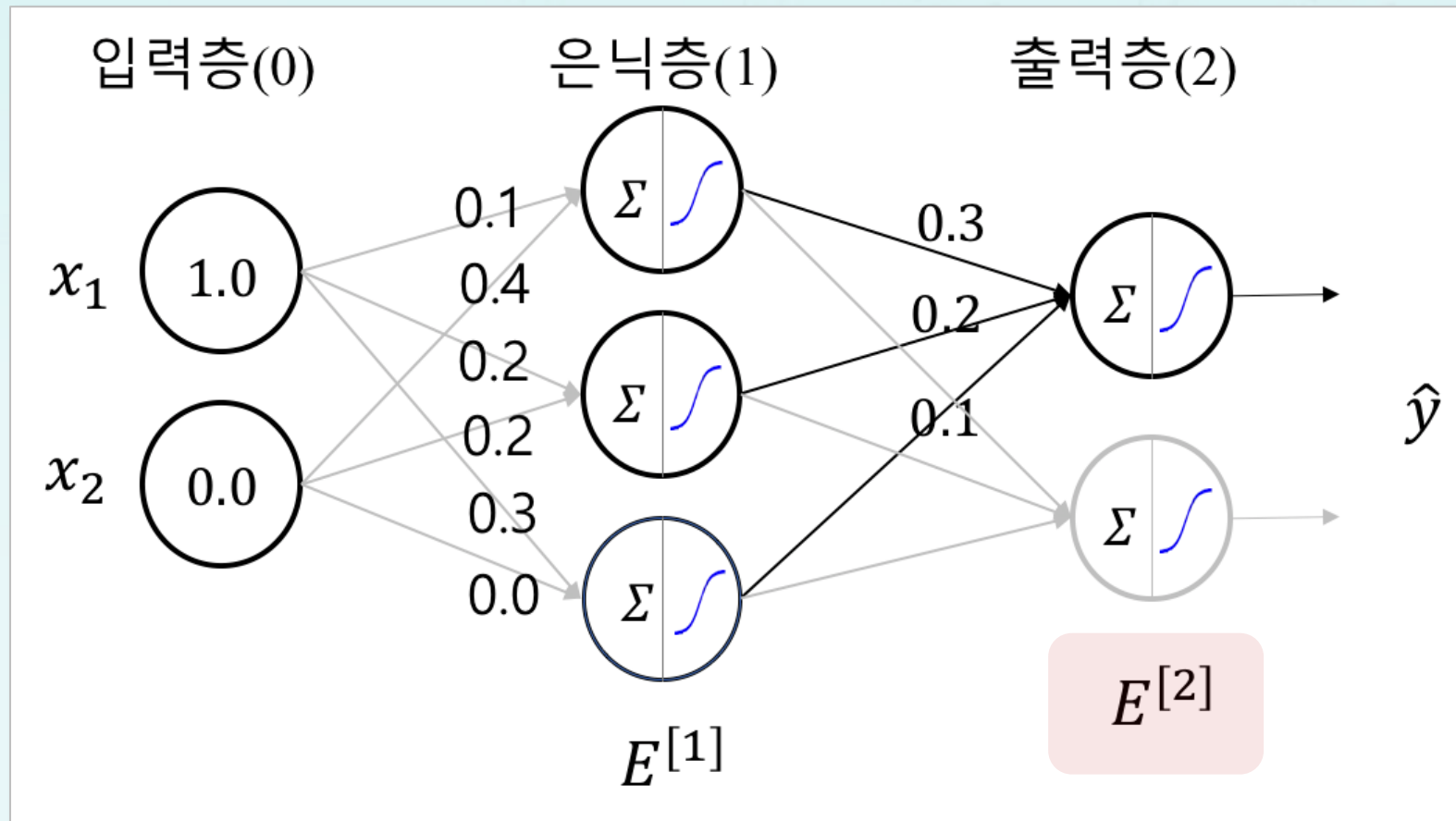
- 다층 구조 인공신경망



- 신호처리

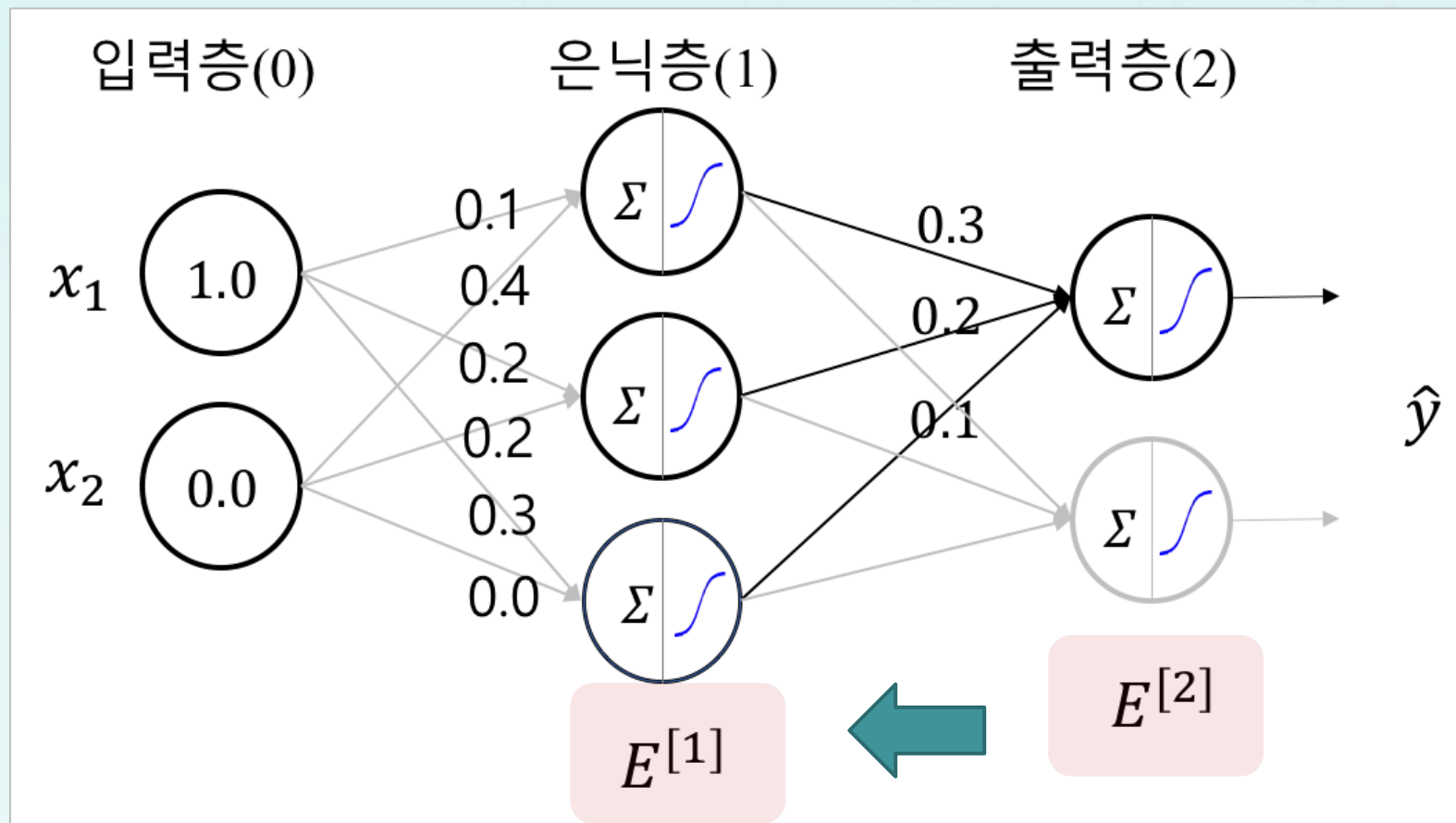
- X : 입력
- W : 가중치
- Z : 순입력
- A : 출력
- E : 오차

역전파 1: 은닉층의 오차 계산



역전파 1: 은닉층의 오차 계산

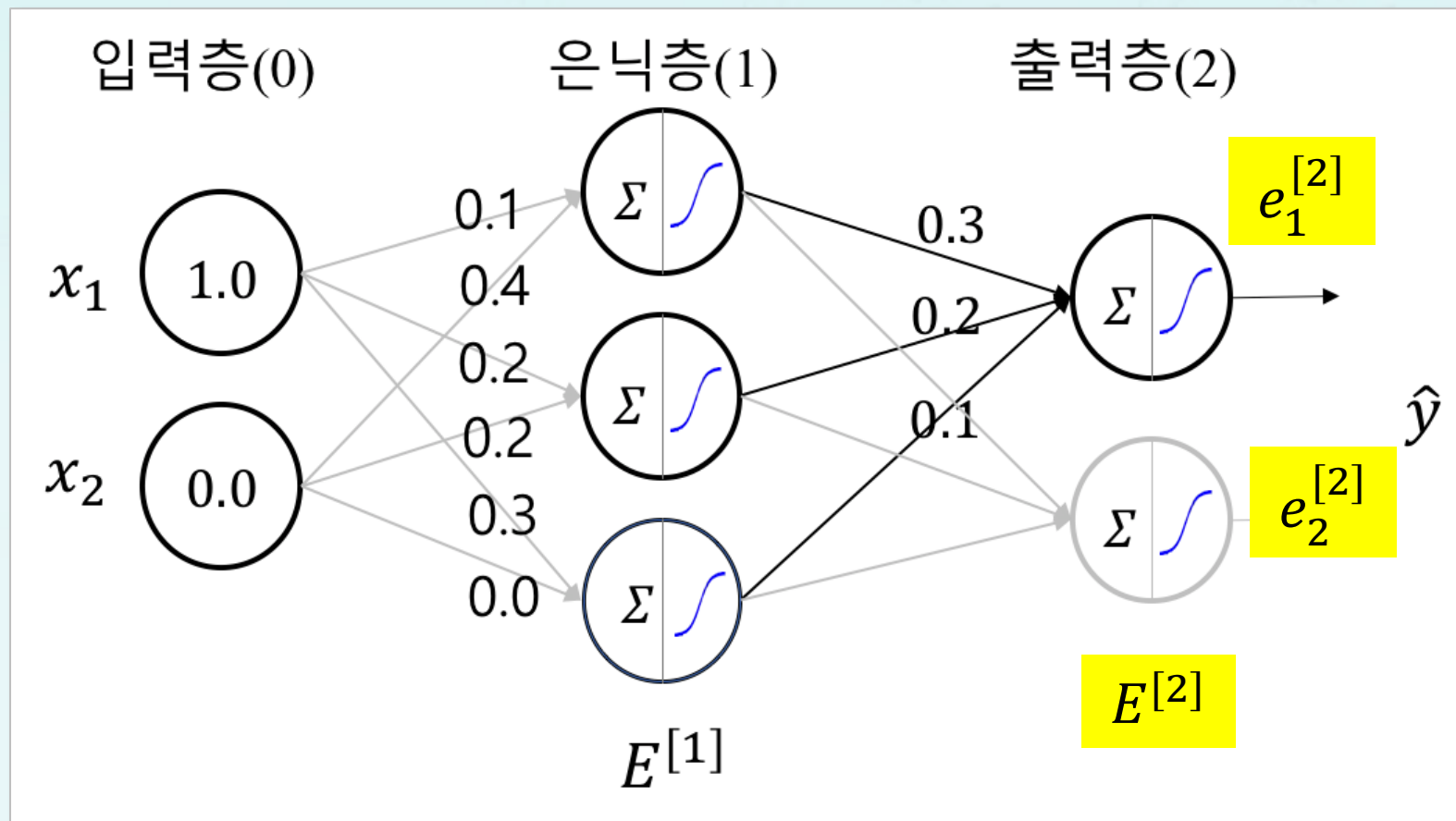
- 가정: $y_1 = 1, \hat{y}_1 = 0.58$
- 오차: $e_1 = y_1 - \hat{y}_1 = 0.42$



$$\begin{aligned} e_1 &= y_1 - \hat{y}_1 \\ &= 1.0 - 0.58 \\ &= 0.42 \end{aligned}$$

역전파 1: 은닉층의 오차 계산

- 가정: $y_1 = 1, \hat{y}_1 = 0.58$
- 오차: $e_1 = y_1 - \hat{y}_1 = 0.42$

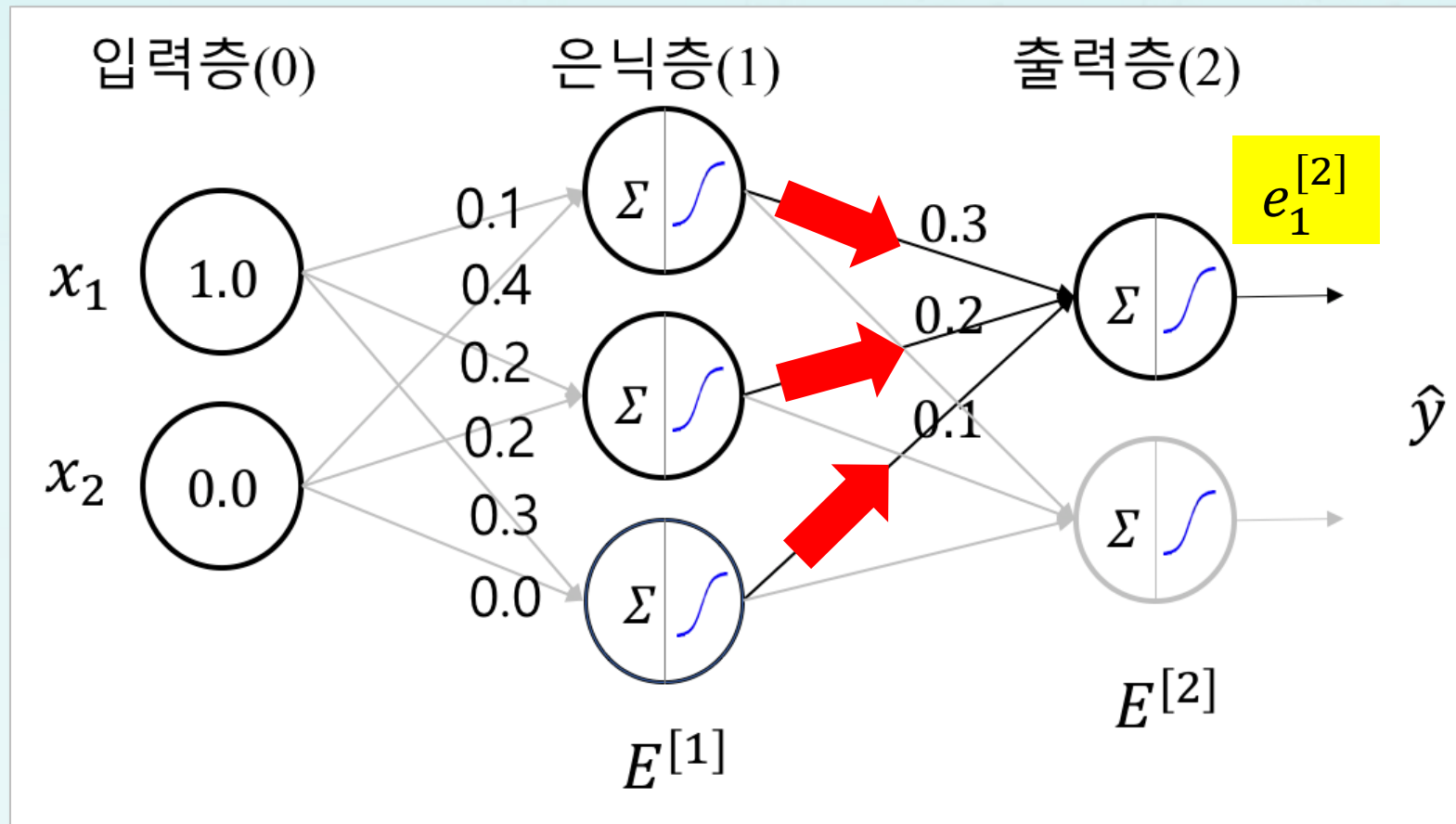


$$\begin{aligned} e_1 &= y_1 - \hat{y}_1 \\ &= 1.0 - 0.58 \\ &= 0.42 \end{aligned}$$

$$E^{[2]} = \begin{pmatrix} e_1^{[2]} \\ e_2^{[2]} \end{pmatrix} = \begin{pmatrix} 0.42 \\ \text{미정} \end{pmatrix}$$

역전파 1: 은닉층의 오차 계산

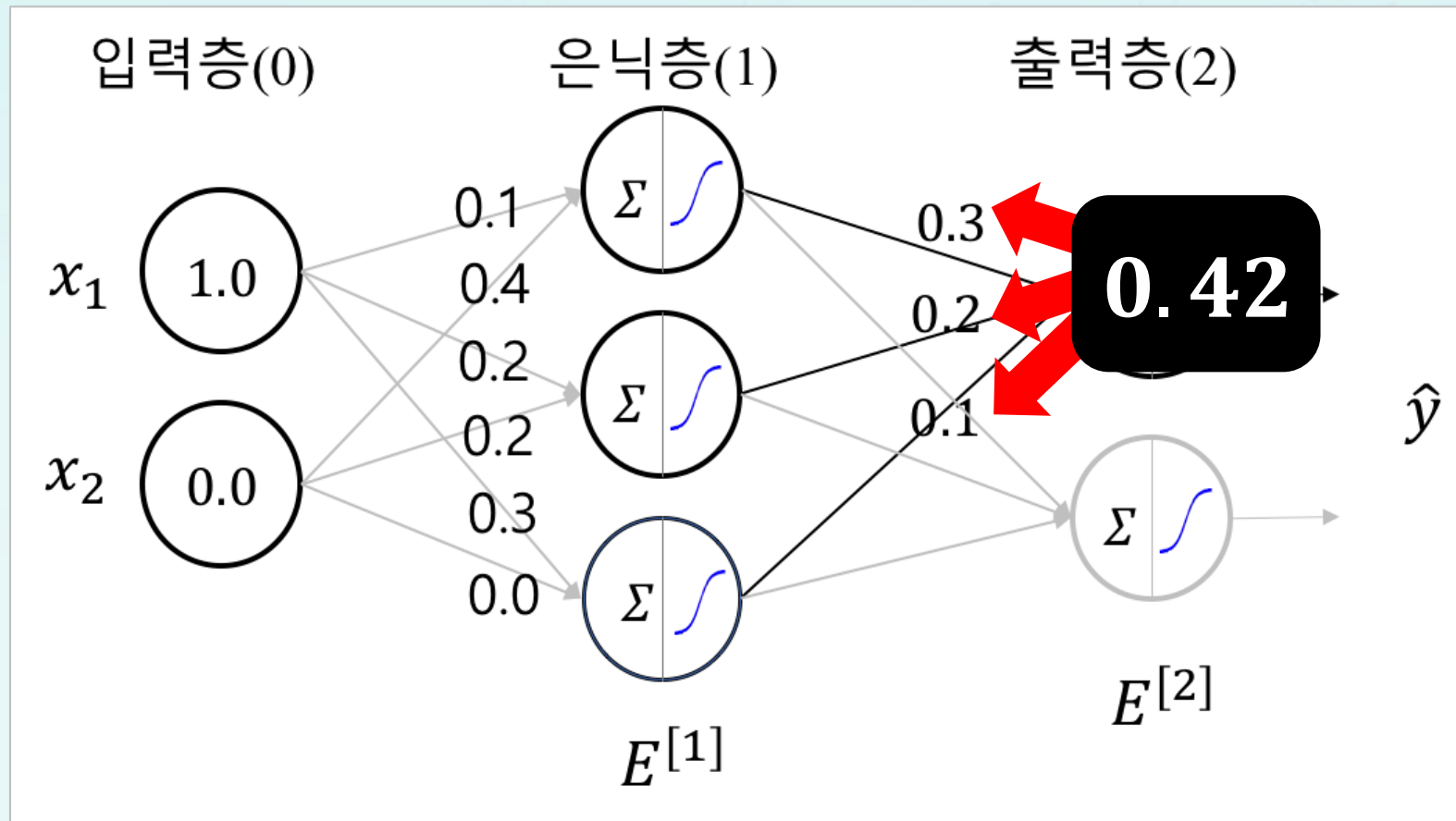
- 가정: $y_1 = 1$, $\hat{y}_1 = 0.58$
- 오차: $e_1 = y_1 - \hat{y}_1 = 0.42$



$$\begin{aligned} e_1 &= y_1 - \hat{y}_1 \\ &= 1.0 - 0.58 \\ &= 0.42 \end{aligned}$$

역전파 1: 은닉층의 오차 계산

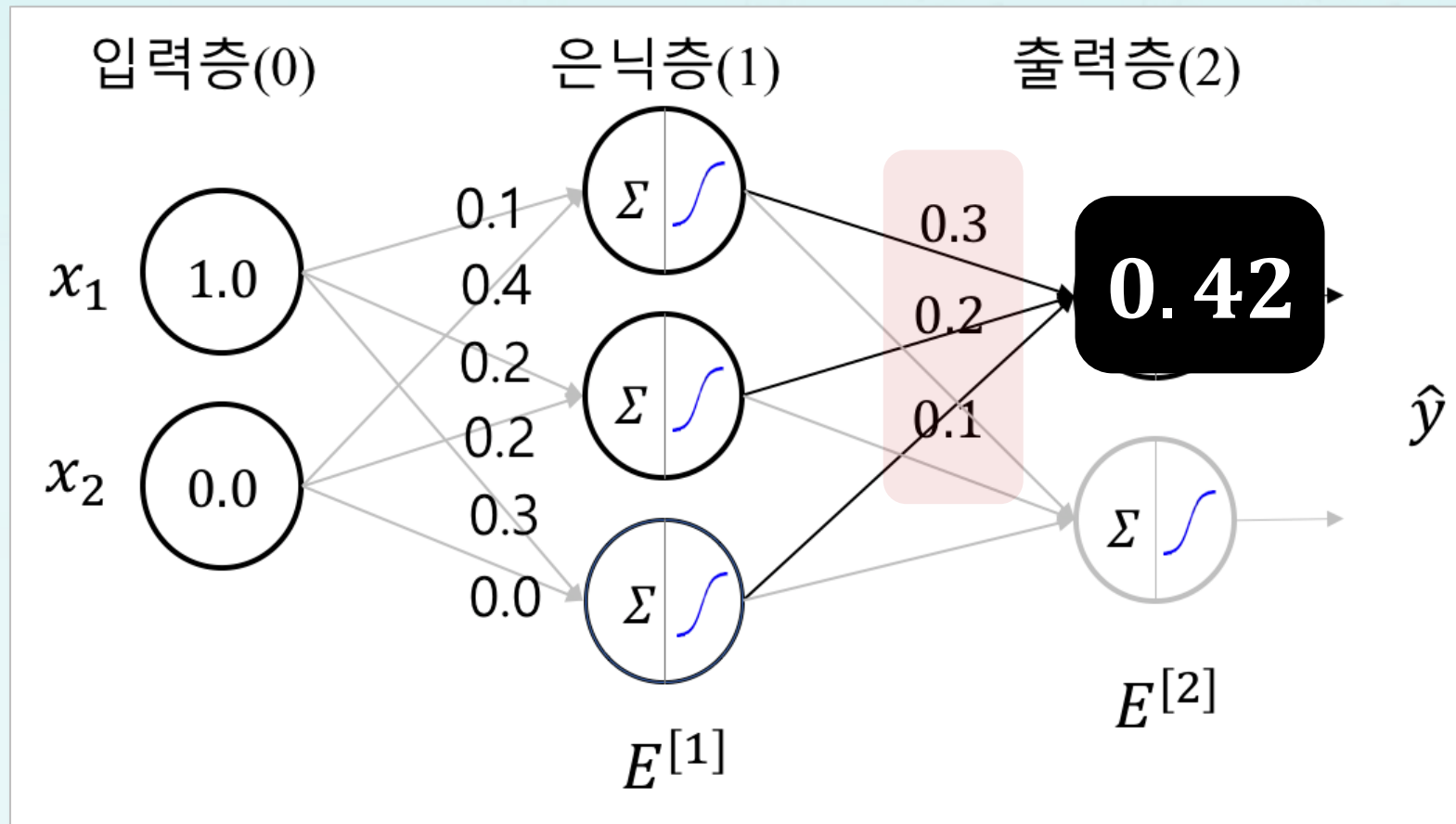
- 가정: $y_1 = 1, \hat{y}_1 = 0.58$
- 오차: $e_1 = y_1 - \hat{y}_1 = 0.42$



$$\begin{aligned} e_1 &= y_1 - \hat{y}_1 \\ &= 1.0 - 0.58 \\ &= 0.42 \end{aligned}$$

역전파 1: 은닉층의 오차 계산

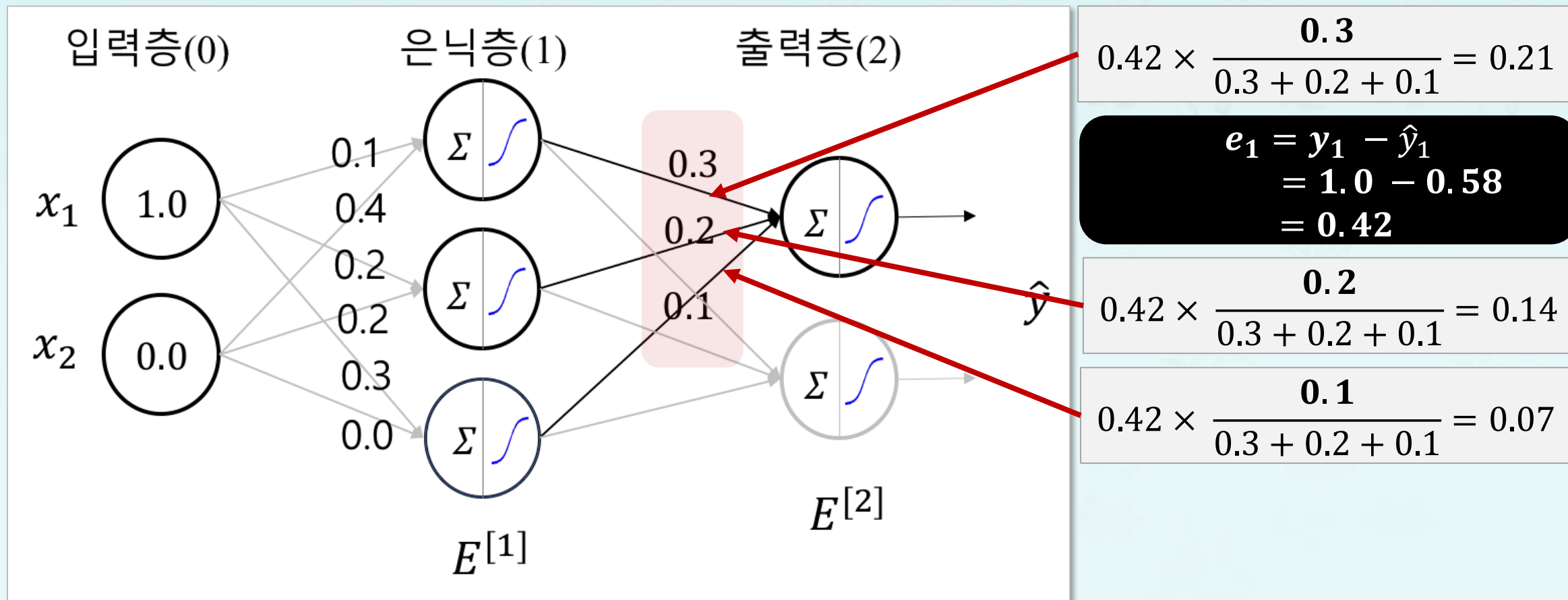
- 가정: $y_1 = 1, \hat{y}_1 = 0.58$
- 오차: $e_1 = y_1 - \hat{y}_1 = 0.42$



$$\begin{aligned} e_1 &= y_1 - \hat{y}_1 \\ &= 1.0 - 0.58 \\ &= 0.42 \end{aligned}$$

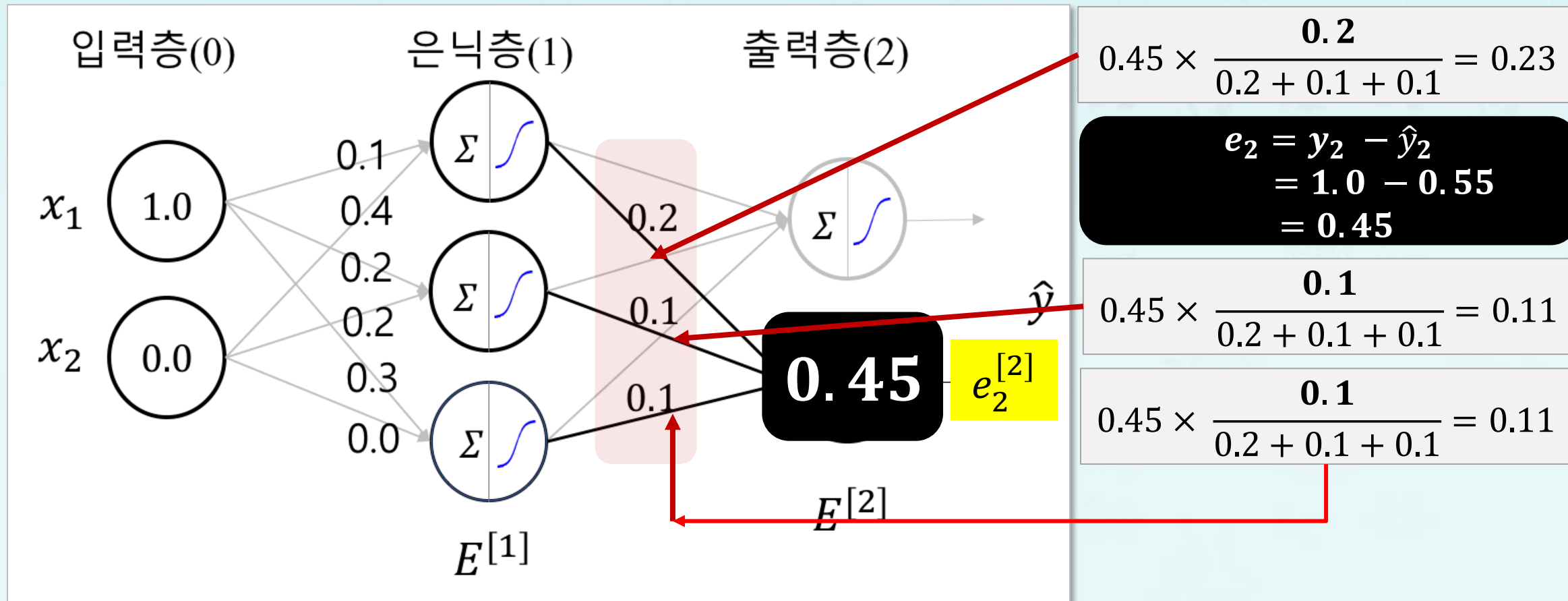
역전파 1: 은닉층의 오차 계산

- 가정: $y_1 = 1$, $\hat{y}_1 = 0.58$
- 오차: $e_1 = y_1 - \hat{y}_1 = 0.42$

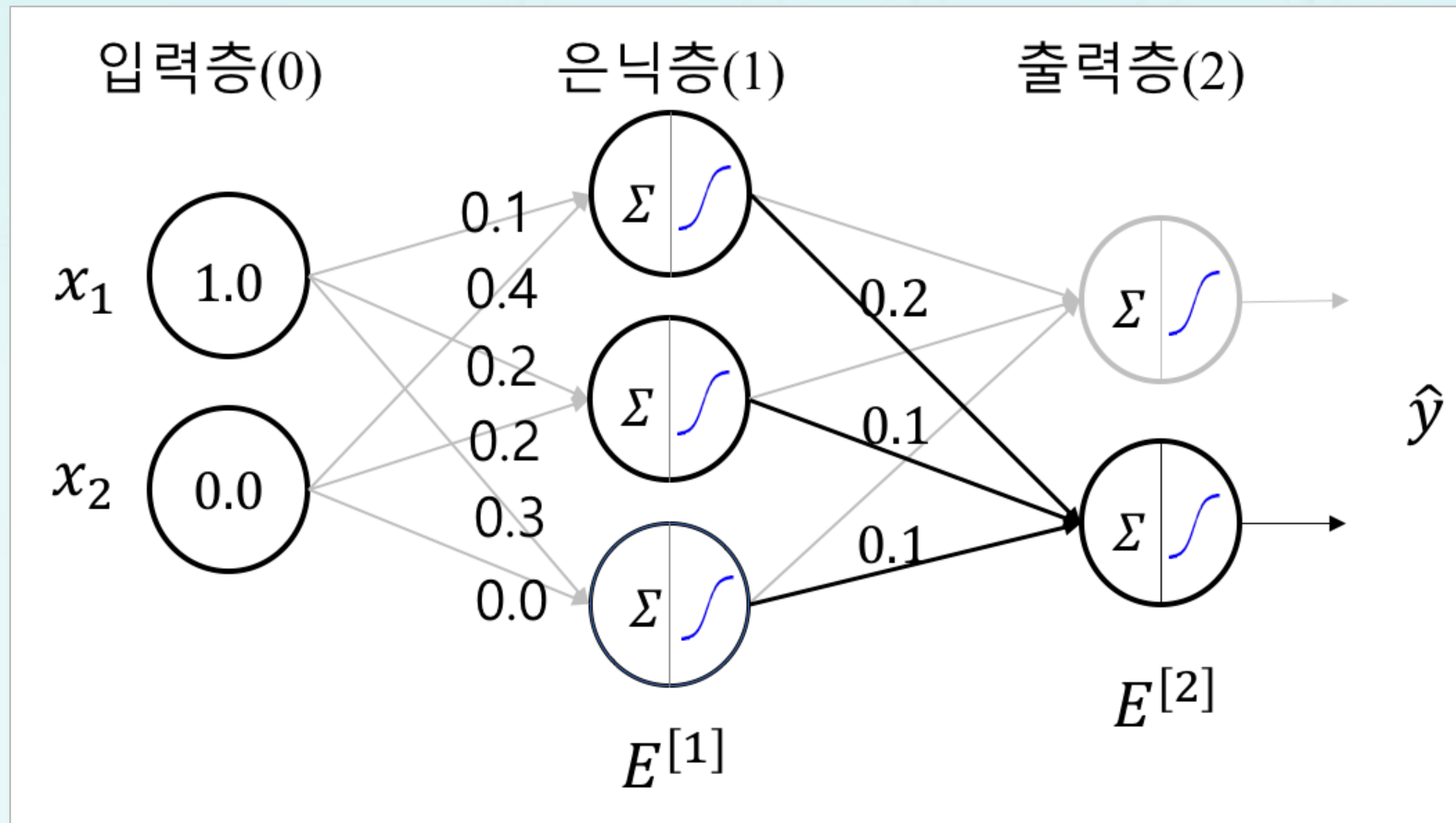


역전파 1: 은닉층의 오차 계산

- 가정: $y_2 = 1, \hat{y}_2 = 0.55$
- 오차: $e_2 = y_2 - \hat{y}_2 = 0.45$



역전파 1: 은닉층의 오차 계산



$$\begin{aligned} e_1^{[2]} &= y_1 - \hat{y}_1 \\ &= 1 - 0.58 = 0.42 \\ e_2^{[2]} &= y_2 - \hat{y}_2 \\ &= 1 - 0.55 = 0.45 \end{aligned}$$

$$E^{[2]} = \begin{pmatrix} e_1^{[2]} \\ e_2^{[2]} \end{pmatrix} = \begin{pmatrix} 0.42 \\ 0.45 \end{pmatrix}$$

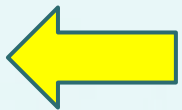
역전파 1: 은닉층의 오차 계산

$$E^{[1]} = \begin{pmatrix} e_1^{[1]} \\ e_2^{[1]} \\ e_3^{[1]} \end{pmatrix} = \begin{pmatrix} e_1^{[2]} \cdot \frac{w_{11}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{12}}{\sum_{i=1}^3 w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{21}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{22}}{\sum_{i=1}^3 w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{31}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{32}}{\sum_{i=1}^3 w_{i2}} \end{pmatrix}$$

역전파 1: 은닉층의 오차 계산

$$E^{[1]} = \begin{pmatrix} e_1^{[1]} \\ e_2^{[1]} \\ e_3^{[1]} \end{pmatrix} = \begin{pmatrix} \overset{0.42}{e_1^{[2]}} \cdot \frac{w_{11}}{\sum_{i=1}^3 w_{i1}} + \overset{0.45}{e_2^{[2]}} \cdot \frac{w_{12}}{\sum_{i=1}^3 w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{21}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{22}}{\sum_{i=1}^3 w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{31}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{32}}{\sum_{i=1}^3 w_{i2}} \end{pmatrix}$$

$E^{[1]}$

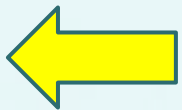


$E^{[2]}$

역전파 1: 은닉층의 오차 계산

$$E^{[1]} = \begin{pmatrix} e_1^{[1]} \\ e_2^{[1]} \\ e_3^{[1]} \end{pmatrix} = \begin{pmatrix} \overset{0.42}{e_1^{[2]}} \cdot \frac{w_{11}}{\sum_{i=1}^3 w_{i1}} + \overset{0.45}{e_2^{[2]}} \cdot \frac{w_{12}}{\sum_{i=1}^3 w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{21}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{22}}{\sum_{i=1}^3 w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{31}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{32}}{\sum_{i=1}^3 w_{i2}} \end{pmatrix}$$

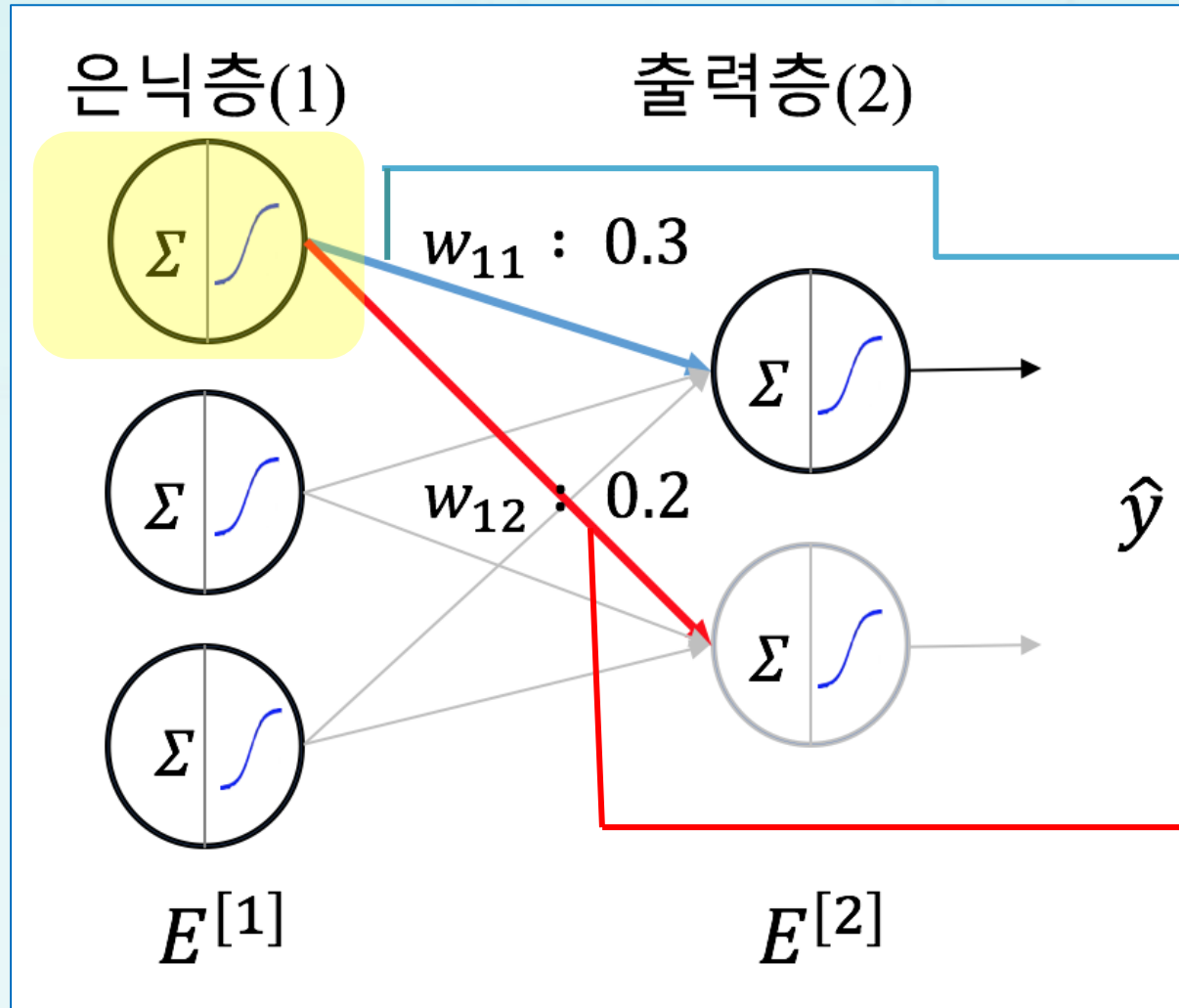
$E^{[1]}$



$$E^{[2]} = \begin{pmatrix} e_1^{[2]} \\ e_2^{[2]} \end{pmatrix} = \begin{pmatrix} 0.42 \\ 0.45 \end{pmatrix}$$

역전파 1: 은닉층의 오차 계산

- 은닉층의 오차 계산



$$E^{[1]} = \begin{pmatrix} e_1^{[2]} \cdot \frac{w_{11}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{12}}{\sum_{i=1}^3 w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{21}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{22}}{\sum_{i=1}^3 w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{31}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{32}}{\sum_{i=1}^3 w_{i2}} \end{pmatrix}$$

역전파 1: 은닉층의 오차 계산

- 은닉층의 오차 계산

$$\begin{aligned} E^{[1]} = \begin{pmatrix} e_1^{[1]} \\ e_2^{[1]} \\ e_3^{[1]} \end{pmatrix} &= \begin{pmatrix} e_1^{[2]} \cdot \frac{w_{11}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{12}}{\sum_{i=1}^3 w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{21}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{22}}{\sum_{i=1}^3 w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{31}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{32}}{\sum_{i=1}^3 w_{i2}} \end{pmatrix} \\ &= \begin{pmatrix} 0.42 \cdot \frac{0.3}{0.3 + 0.2 + 0.1} + 0.45 \cdot \frac{0.2}{0.2 + 0.1 + 0.1} \\ 0.42 \cdot \frac{0.3}{0.3 + 0.2 + 0.1} + 0.45 \cdot \frac{0.2}{0.2 + 0.1 + 0.1} \\ 0.42 \cdot \frac{0.3}{0.3 + 0.2 + 0.1} + 0.45 \cdot \frac{0.2}{0.2 + 0.1 + 0.1} \end{pmatrix} \end{aligned}$$

역전파 1: 은닉층의 오차 계산

- 은닉층의 오차 계산

$$\begin{aligned} E^{[1]} = \begin{pmatrix} e_1^{[1]} \\ e_2^{[1]} \\ e_3^{[1]} \end{pmatrix} &= \begin{pmatrix} e_1^{[2]} \cdot \frac{w_{11}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{12}}{\sum_{i=1}^3 w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{21}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{22}}{\sum_{i=1}^3 w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{31}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{32}}{\sum_{i=1}^3 w_{i2}} \end{pmatrix} \\ &= \begin{pmatrix} 0.42 \cdot \frac{0.3}{0.3 + 0.2 + 0.1} + 0.45 \cdot \frac{0.2}{0.2 + 0.1 + 0.1} \\ 0.42 \cdot \frac{0.2}{0.3 + 0.2 + 0.1} + 0.45 \cdot \frac{0.1}{0.2 + 0.1 + 0.1} \\ 0.42 \cdot \frac{0.1}{0.3 + 0.2 + 0.1} + 0.45 \cdot \frac{0.1}{0.2 + 0.1 + 0.1} \end{pmatrix} = \begin{pmatrix} 0.44 \\ 0.25 \\ 0.18 \end{pmatrix} \end{aligned}$$

역전파 1: 오차 계산의 단순화

- 오차 계산의 단순화와 일반화

- 단순화

$$E^{[1]} = \begin{pmatrix} e_1^{[1]} \\ e_2^{[1]} \\ e_3^{[1]} \end{pmatrix} = \begin{pmatrix} e_1^{[2]} \cdot \frac{w_{11}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{12}}{\sum_{i=1}^3 w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{21}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{22}}{\sum_{i=1}^3 w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{31}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{32}}{\sum_{i=1}^3 w_{i2}} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{w_{11}}{\sum_{i=1}^3 w_{i1}} & \frac{w_{12}}{\sum_{i=1}^3 w_{i2}} \\ \frac{w_{21}}{\sum_{i=1}^3 w_{i1}} & \frac{w_{22}}{\sum_{i=1}^3 w_{i2}} \\ \frac{w_{31}}{\sum_{i=1}^3 w_{i1}} & \frac{w_{32}}{\sum_{i=1}^3 w_{i2}} \end{pmatrix} \cdot \begin{pmatrix} e_1^{[2]} \\ e_2^{[2]} \end{pmatrix}$$

역전파 1: 오차 계산의 단순화

- 오차 계산의 단순화와 일반화

- 단순화

$$E^{[1]} = \begin{pmatrix} e_1^{[1]} \\ e_2^{[1]} \\ e_3^{[1]} \end{pmatrix} = \begin{pmatrix} e_1^{[2]} \cdot \frac{w_{11}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{12}}{\sum_{i=1}^3 w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{21}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{22}}{\sum_{i=1}^3 w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{31}}{\sum_{i=1}^3 w_{i1}} + e_2^{[2]} \cdot \frac{w_{32}}{\sum_{i=1}^3 w_{i2}} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{w_{11}}{\sum_{i=1}^3 w_{i1}} & \frac{w_{12}}{\sum_{i=1}^3 w_{i2}} \\ \frac{w_{21}}{\sum_{i=1}^3 w_{i1}} & \frac{w_{22}}{\sum_{i=1}^3 w_{i2}} \\ \frac{w_{31}}{\sum_{i=1}^3 w_{i1}} & \frac{w_{32}}{\sum_{i=1}^3 w_{i2}} \end{pmatrix} \cdot \begin{pmatrix} e_1^{[2]} \\ e_2^{[2]} \end{pmatrix}$$

역전파 1: 오차 계산의 단순화

- 오차 계산의 단순화와 일반화
 - 단순화

$$E^{[1]} = \begin{pmatrix} \frac{w_{11}}{\sum_{i=1}^3 w_{i1}} & \frac{w_{12}}{\sum_{i=1}^3 w_{i2}} \\ \frac{w_{21}}{\sum_{i=1}^3 w_{i1}} & \frac{w_{22}}{\sum_{i=1}^3 w_{i2}} \\ \frac{w_{31}}{\sum_{i=1}^3 w_{i1}} & \frac{w_{32}}{\sum_{i=1}^3 w_{i2}} \end{pmatrix} \cdot \begin{pmatrix} e_1^{[2]} \\ e_2^{[2]} \end{pmatrix}$$

역전파 1: 오차 계산의 단순화

- 오차 계산의 단순화와 일반화
 - 단순화

$$E^{[1]} = \begin{pmatrix} \frac{w_{11}}{\sum_{i=1}^3 w_{i1}} & \frac{w_{12}}{\sum_{i=1}^3 w_{i2}} \\ \frac{w_{21}}{\sum_{i=1}^3 w_{i1}} & \frac{w_{22}}{\sum_{i=1}^3 w_{i2}} \\ \frac{w_{31}}{\sum_{i=1}^3 w_{i1}} & \frac{w_{32}}{\sum_{i=1}^3 w_{i2}} \end{pmatrix} \cdot \begin{pmatrix} e_1^{[2]} \\ e_2^{[2]} \end{pmatrix} \Rightarrow E^{[1]} = \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{pmatrix} \cdot \begin{pmatrix} e_1^{[2]} \\ e_2^{[2]} \end{pmatrix}$$

역전파 1: 오차 계산의 단순화

- 오차 계산의 단순화와 일반화
 - 일반화

$$E^{[1]} = \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{pmatrix} \cdot \begin{pmatrix} e_1^{[2]} \\ e_2^{[2]} \end{pmatrix}$$

$$\mathbf{E}^{[1]} = \mathbf{W}^{[2]T} \cdot \mathbf{E}^{[2]}$$

역전파 1: 오차 계산의 일반화

$$E^{[1]} = \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{pmatrix} \cdot \begin{pmatrix} e_1^{[2]} \\ e_2^{[2]} \end{pmatrix}$$



$$E^{[l]} = \begin{pmatrix} w_{11} & \cdots & w_{1m} \\ \vdots & \ddots & \vdots \\ w_{n1} & \cdots & w_{nm} \end{pmatrix}^T \cdot \begin{pmatrix} e_1^{[l+1]} \\ \vdots \\ e_n^{[l+1]} \end{pmatrix}$$

$$\mathbf{E}^{[1]} = \mathbf{W}^{[2]T} \cdot \mathbf{E}^{[2]}$$



$$E^{[l]} = W^{[l+1]T} \cdot E^{[l+1]}$$

역전파 1

- 학습 정리
 - 출력층으로부터 은닉층의 오차를 계산하기
 - 복잡한 은닉층 오차 계산을 일반화 하기
- 역전파 2

8주차(3/3)

역전파 1

파이썬으로 배우는 기계학습

한동대학교
김영섭 교수