PHY526 PS9

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Code is attached at the end of this file.

Problem 1

$$
\begin{aligned}
\frac{\mathrm{d}n}{\mathrm{d}\ln M} =& \frac{\rho_{M,0}}{M}\left|\frac{\mathrm{d}F(M)}{\mathrm{d}\ln M}\right| = \frac{\rho_{M,0}}{M}\frac{1}{\sqrt{2\pi}}\left|\int_{\delta_c}^{\infty}\frac{\partial}{\partial \ln M}\frac{\exp\left[\frac{-\delta^2}{2\sigma(M)^2}\right]}{\sigma(M)}\mathrm{d}\delta\right| \\
=& \frac{\rho_{M,0}}{M}\frac{1}{\sqrt{2\pi}}\left|\int_{\delta_c}^{\infty}\frac{\mathrm{d}\sigma}{\mathrm{d}\ln M}\frac{1}{\sigma^2}\left\{\frac{-\delta^2}{2}\frac{-2}{\sigma^2}\exp\left[-\frac{\delta^2}{2\sigma^2}\right] - \exp\left[-\frac{\delta^2}{2\sigma^2}\right]\right\}\mathrm{d}\delta\right| \\
=& \frac{\rho_{M,0}}{M}\frac{1}{\sqrt{2\pi}}\left|\frac{\mathrm{d}\ln\sigma}{\mathrm{d}\ln M}\int_{\delta_c/\sigma}^{\infty}\left[\left(\frac{\delta}{\sigma}\right)^2 - 1\right]\exp\left[-\frac{\delta^2}{2\sigma^2}\right]\mathrm{d}\frac{\delta}{\sigma}\right| \\
=& \frac{\rho_{M,0}}{M}\frac{1}{\sqrt{2\pi}}\left|\frac{\mathrm{d}\ln\sigma}{\mathrm{d}\ln M}\int_{\delta_c/\sigma}^{\infty}(x^2-1)e^{-\frac{x^2}{2}}\mathrm{d}x\right| \\
=& \frac{\rho_{M,0}}{M}\left|\frac{\mathrm{d}\ln\sigma}{\mathrm{d}\ln M}\frac{1}{\sqrt{2\pi}}\left(-xe^{x^2/2}\Big|_{\delta_c/\sigma}^{\infty} + \int_{\delta_c/\sigma}^{\infty}e^{x^2/2}\mathrm{d}x - \int_{\delta_c/\sigma}^{\infty}e^{x^2/2}\mathrm{d}x\right)\right| \\
=& \frac{\rho_{M,0}}{M}\frac{1}{\sqrt{2\pi}}\left|\frac{\mathrm{d}\ln\sigma}{\mathrm{d}\ln M}\right|\frac{\delta_c}{\sigma}\exp\left[\frac{-\delta^2}{2\sigma(M)^2}\right].
\end{aligned}
\tag{1}
$$

Problem2,3,4,5a See the figures below.

Problem 5

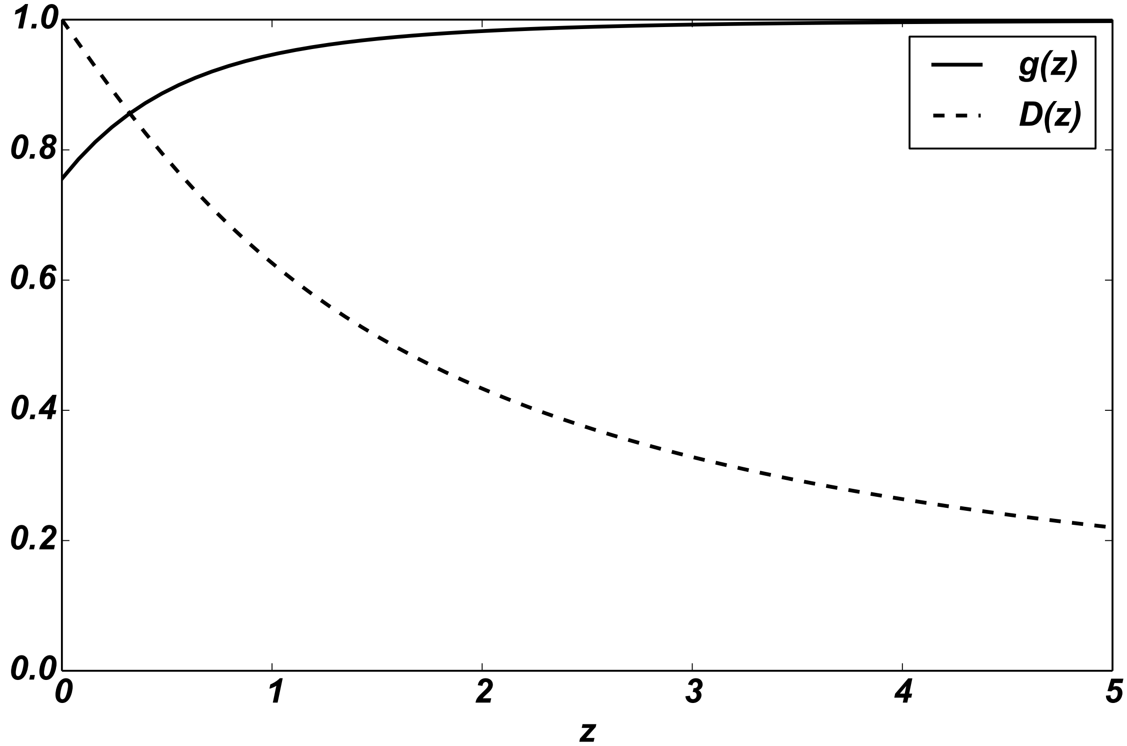
(b)(c) The number of halos expected in the Dark Energy Survey writes,

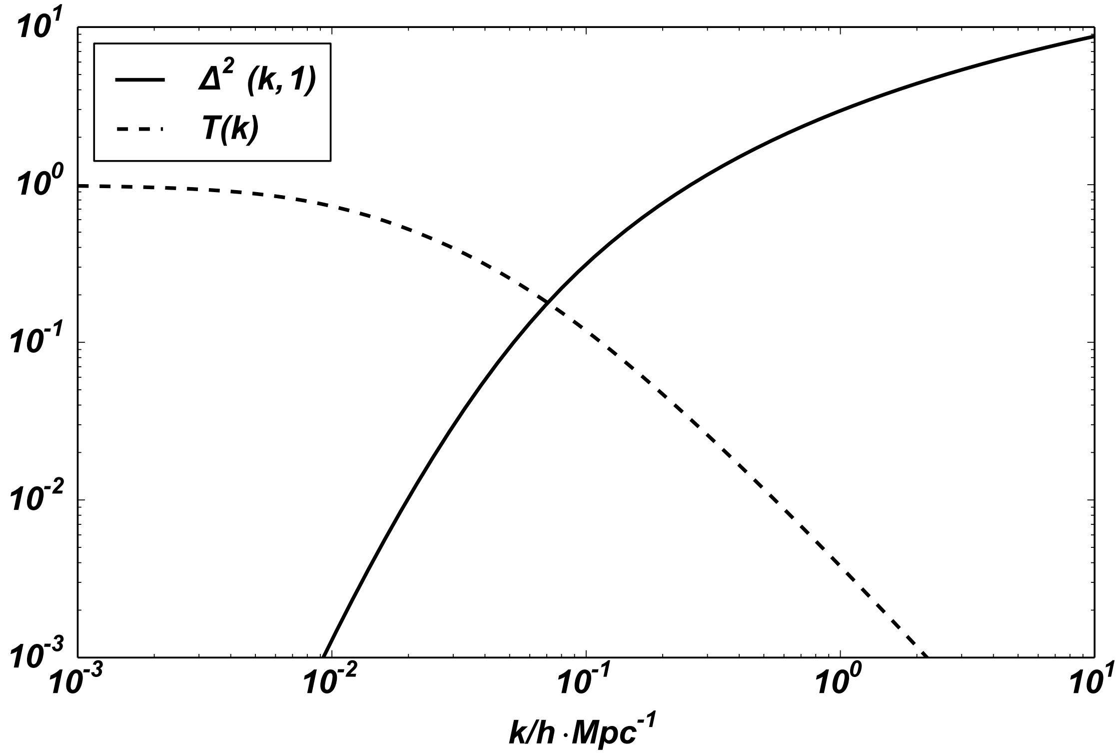
$$
N = \int_{\Omega}\int_{z=0}^{1}\int_{M=10^{14}}^{\infty}\frac{\mathrm{d}n}{\mathrm{d}\ln M}\mathrm{d}\ln M\frac{\mathrm{d}^2V}{\mathrm{d}z\mathrm{d}\Omega}\mathrm{d}z\mathrm{d}\Omega, \tag{2}
$$

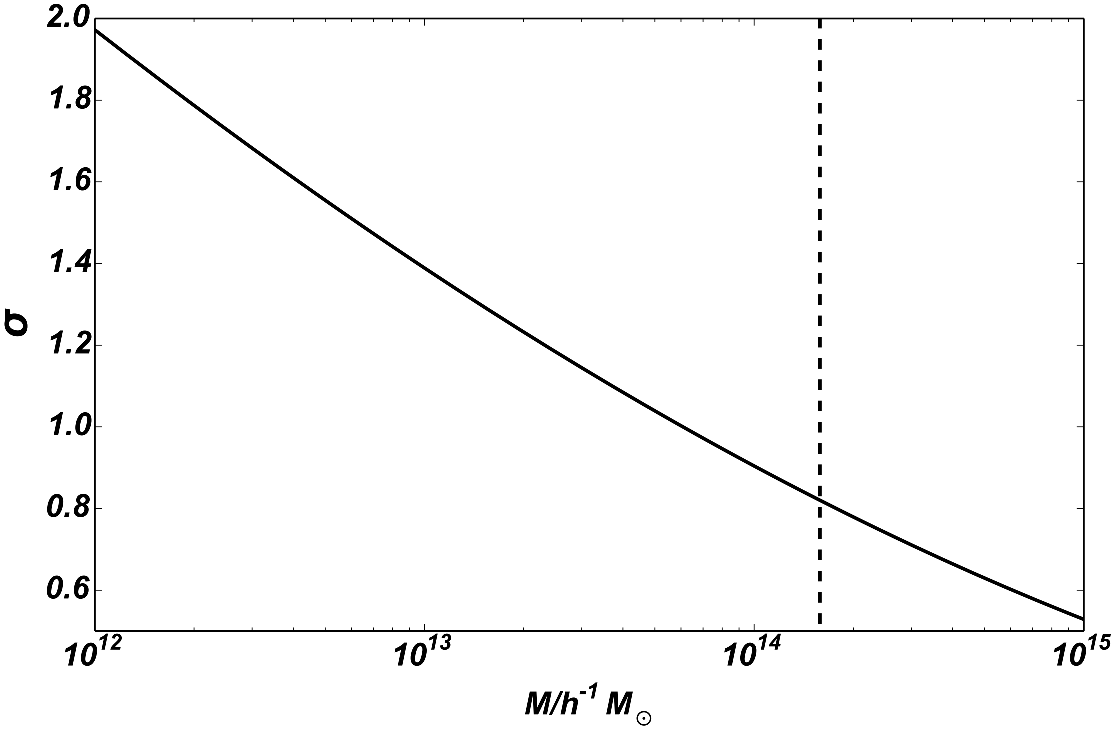
where the integration over solid angle gives a factor of $4\pi\times\frac{5000}{41253}$ since the whole sky covers 41253 square degree. To calculate this integration, one needs to tabulate the integrand to save computation time, and the infinity limit is changed to 10^{20} in Eq.(2) to make the integration computer friendly. Indeed there is no halo ($\frac{\mathrm{d}n}{\mathrm{d}\ln M}\frac{\mathrm{d}^2V}{\mathrm{d}z\mathrm{d}\Omega} < 10^{-100}$) when $M > 10^{20}$. It turns out that:

$$
\begin{aligned}
N(\Omega_M = 0.3, h = 0.71) &\approx 6437 \\
N(\Omega_M = 1, h = 0.5) &\approx 71083.
\end{aligned}
\tag{3}
$$

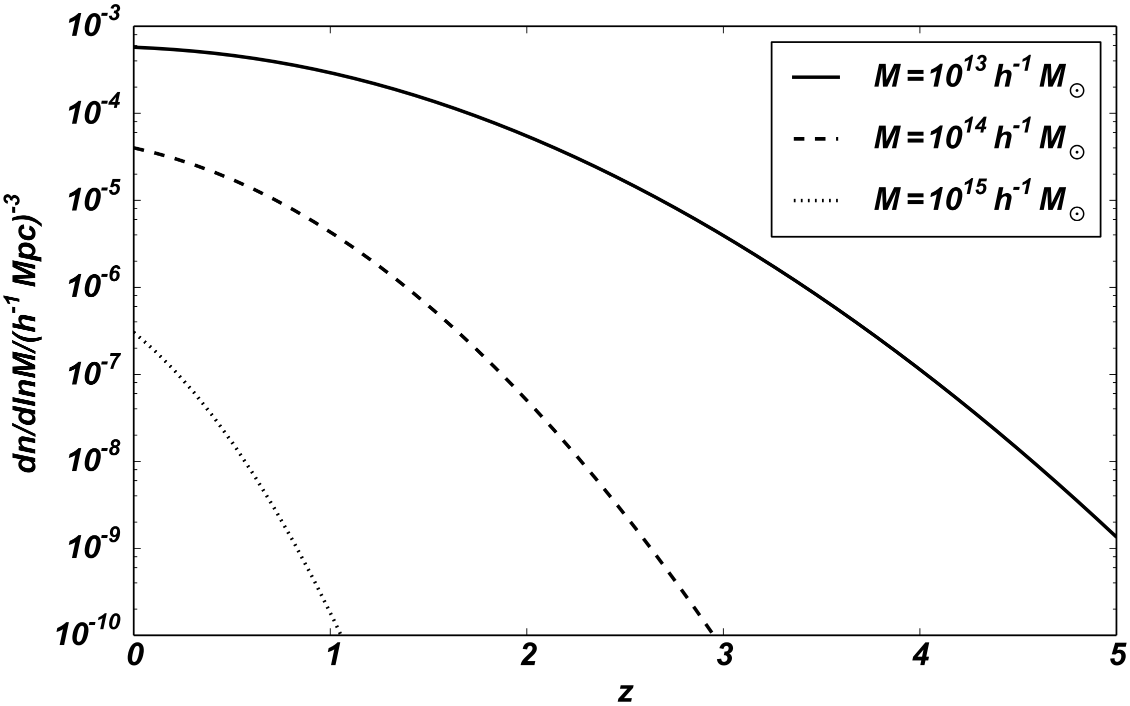
It is reasonable that the matter dominant Universe generates 1 order of magnitude more massive halos since no dark nergy exsits to suppress the contraction of matter driven by gravity.


g(z)
D(z)
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
z


Δ^2 (k, 1)
T(k)
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
$k/h \cdot Mpc^{-1}$



σ
M/h^-1 M_⊙



dn/dlnM/(h^-1 Mpc)^-3
z
M = 10^13 h^-1 M_⊙
M = 10^14 h^-1 M_⊙
M = 10^15 h^-1 M_⊙