Image Processing
INT3404 20
Week 9:
Geometric transformations

Lecturer: Nguyen Thi Ngoc Diep, Ph.D.

Email: ngocdiep@vnu.edu.vn

Slide & code: https://github.com/chupibk/INT3404_20

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Schedule

Week	Content	Homework
1	Introduction	Set up environments: Python 3, OpenCV 3, Numpy, Jupyter Notebook
2	Digital image – Point operations Contrast adjust – Combining images	HW1: adjust gamma to find the best contrast
3	Histogram - Histogram equalization – Histogram-based image classification	Self-study
4	Spatial filtering - Template matching	Self-study
5	Feature extraction Edge, Line, and Texture	Self-study
6	Morphological operations	HW2: Barcode detection → Require submission as mid-term test
7	Filtering in the Frequency domain Announcement of Final project topics	Final project registration
8	Color image processing	HW3: Conversion between color spaces, color image segmentation
9	Geometric transformations	Self-study
10	Noise and restoration	Self-study
11	Compression	Self-study
12	Final project presentation	Self-study
13	Final project presentation Class summarization	Self-study

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Week 8: Supplementary

- Color appearance model
- Color attributes

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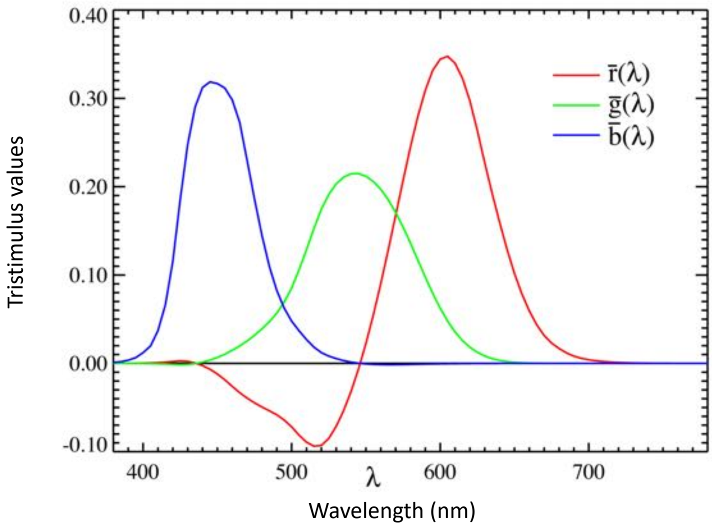
Color matching RGB



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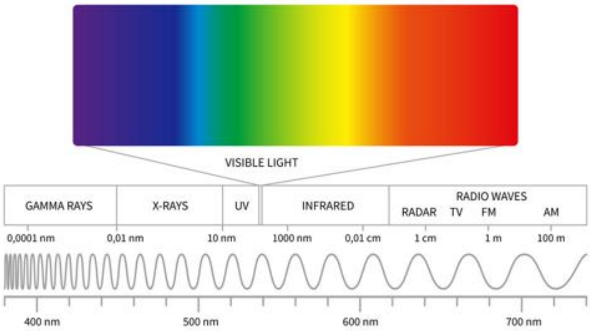
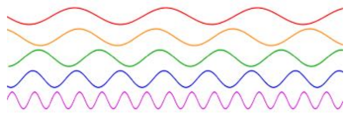
Visible spectrum





Amplitude tells about the intensity of the light

Wavelength tells the type of light

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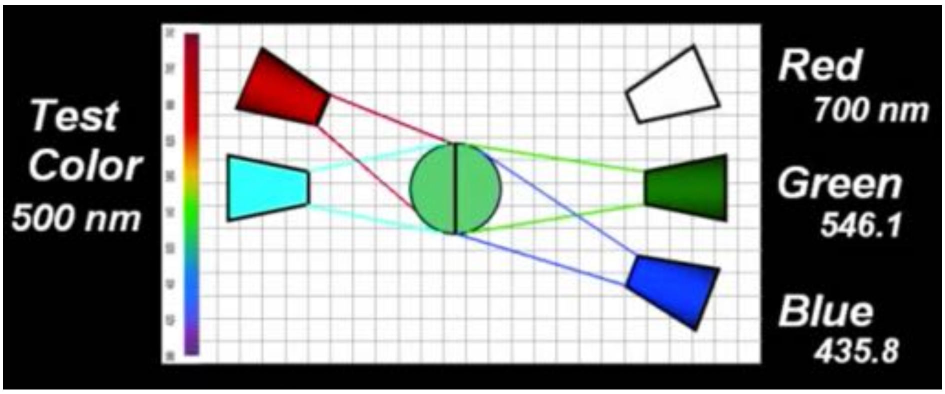

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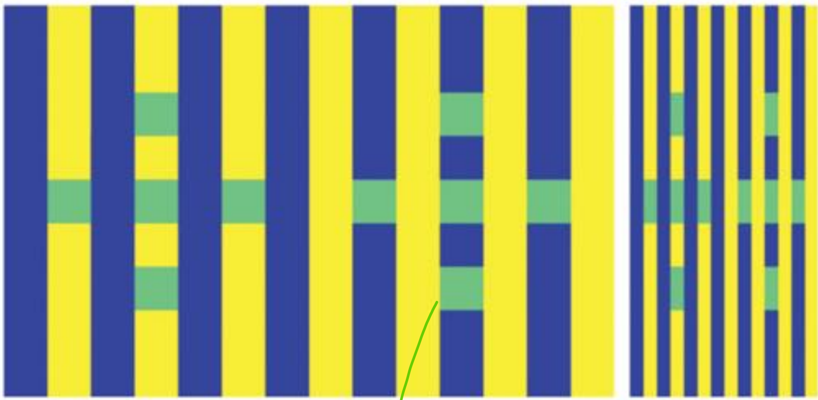
Chromatic white effect



Lighter and yellower

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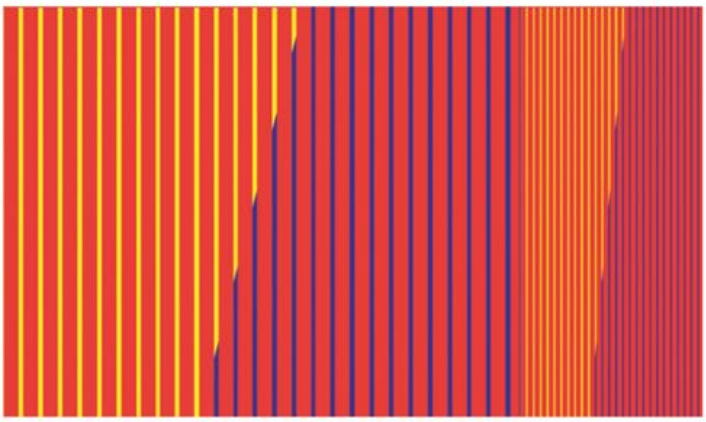
Neon spreading phenomenon



The red bars look bluer on the blue background and yellower on the yellow background

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Hunt effect

colorfulness increases with luminance

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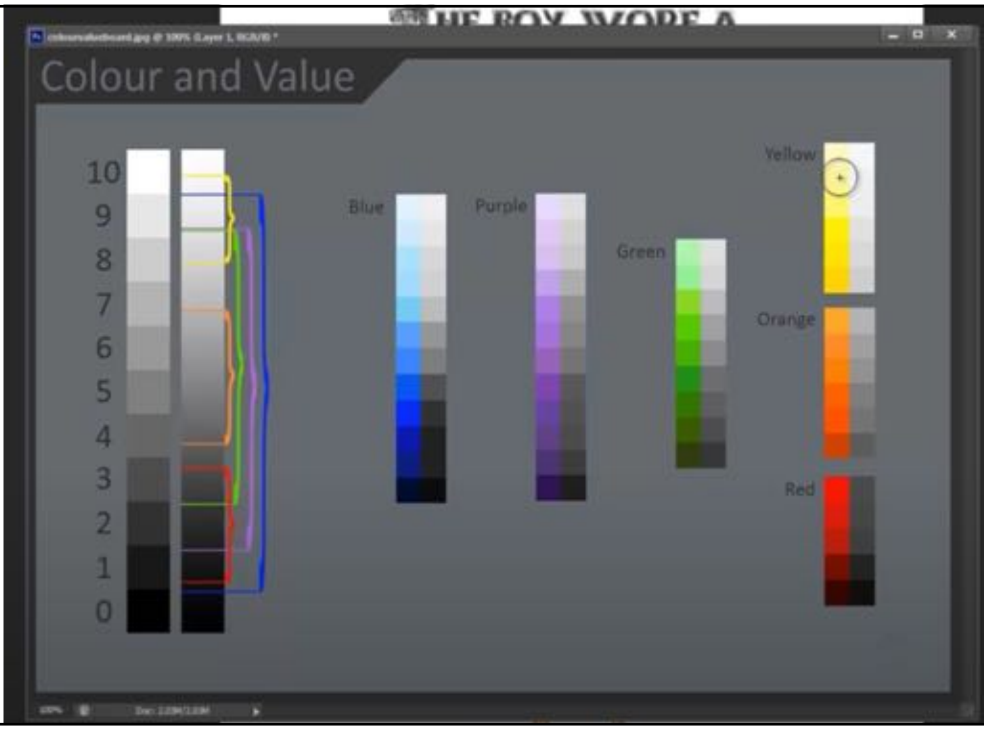

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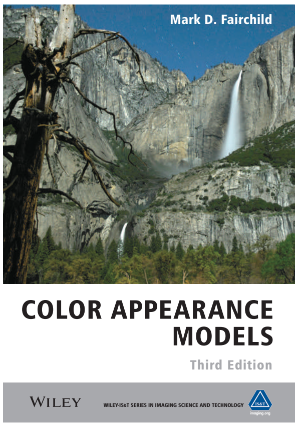
Terminology

From this book:



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Hue
Attribute of a visual perception according to which an area appears to be similar to one of the colours – red, yellow, green, and blue – or to a combination of adjacent pairs of these colours considered in a closed ring.

Achromatic Colour
Perceived color devoid of hue.

Chromatic Colour
Perceived colour possessing hue.

Once again, it is difficult, if not impossible, to define hue without using examples. This is due, in part, to the nature of the hue perception. It is a natural interval scale as illustrated by the traditional description of a “hue circle.” There is no natural “zero” hue. Color without hue can be described, but there is no perception that corresponds to a meaningful hue of 0 units. Thus the color appearance models described in later chapters never aspire to describe hue with more than an interval scale.

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Absolute level of the perception

Brightness
Attribute of a visual perception according to which an area appears to emit, or reflect, more or less light.

Lightness Relative brightness – normalized for changes in the illumination and viewing conditions
The brightness of an area judged relative to the brightness of a similarly illuminated area that appears to be white or highly transmitting.

Note
Only related colours exhibit lightness.

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Colorfulness
Attribute of a visual perception according to which the perceived colour of an area appears to be more or less chromatic.

Note
For a color stimulus of a given chromaticity and, in the case of related colors, of a given luminance factor, this attribute usually increases as the luminance is raised, except when the brightness is very high.

Chroma
Colorfulness of an area judged as a proportion of the brightness of a similarly illuminated area that appears white or highly transmitting.

Note
For given viewing conditions and at luminance levels within the range of photopic vision, a colour stimulus perceived as a related colour, of a given chromaticity, and from a surface having a given luminance factor exhibits approximately constant chroma for all levels of illuminance except when the brightness is very high. In the same circumstances, at a given level of illuminance, if the luminance factor increases, the chroma usually increases.

As was discussed in Chapters 1 and 3, color perception is generally thought of as being three dimensional. Two of those dimensions (hue and brightness/lightness) have already been defined. Colorfulness and chroma define the remaining dimension of color. Colorfulness is to chroma as brightness is to lightness. It is appropriate to think of chroma as relative colorfulness just as lightness can be thought of as relative brightness. Colorfulness describes the intensity of the hue in a given color stimulus. Thus, achromatic colors exhibit zero colorfulness and chroma, and as the amount of color content increases (with constant brightness/lightness and hue), colorfulness and chroma increase.

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Saturation
Colorfulness of an area judged in proportion to its brightness.

Note
For given viewing conditions and at luminance levels within the range of photopic vision, a colour stimulus of a given chromaticity exhibits approximately constant saturation for all luminance levels, except when brightness is very high.

saturation is the colorfulness of a stimulus relative to its own brightness, while chroma is colorfulness relative to the brightness of a similarly illuminated area that appears white.

While one can be derived from the other (*i.e.*, both are derived from colorfulness), it does not seem possible to completely eliminate either of them. Saturation is more fundamental for the description of colour appearance while chroma is more fundamental for the measurement and specification of color differences

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Chroma can be thought of as colorfulness relative to the brightness of a similarly illuminated white as shown in Equation 4.1.

$$\text{Chroma} = \frac{\text{Colorfulness}}{\text{Brightness (white)}} \tag{4.1}$$

Saturation can be described as the colorfulness of a stimulus relative to its own brightness as illustrated in Equation 4.2.

$$\text{Saturation} = \frac{\text{Colorfulness}}{\text{Brightness}} \tag{4.2}$$

Finally, lightness can be expressed as the ratio of the brightness of a stimulus to the brightness of a similarly illuminated white stimulus as given in Equation 4.3.

$$\text{Lightness} = \frac{\text{Brightness}}{\text{Brightness (white)}} \tag{4.3}$$

$$\text{Saturation} = \frac{\text{Chroma}}{\text{Lightness}}$$

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While color is typically thought of as three dimensional and color matches can be specified by just three numbers, it turns out that three dimensions are not enough to completely specify color appearance. In fact, five perceptual dimensions are required for a complete specification of color appearance:

- Brightness
- Lightness
- Colorfulness
- Saturation
- Hue.

Imagine viewing a yellow school bus outside on a sunny day. The yellow bus will exhibit its typical appearance attributes of hue (yellow), brightness (high), lightness (high), colorfulness (high), and chroma (high). Now imagine viewing a printed photographic reproduction of the school bus in the relatively subdued lighting of an office or home. The image of the bus could be a perfect match to the original object in hue (yellow), lightness (high), and chroma (high). However, the brightness and colorfulness of the print viewed in subdued lighting could never equal that of the original school bus viewed in bright sunlight.

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Munsell color order system

The basic premise of the system is to specify color appearance according to three attributes:
- Hue (*H*)
- Value (*V*) --> referring to lightness
- Chroma (*C*).

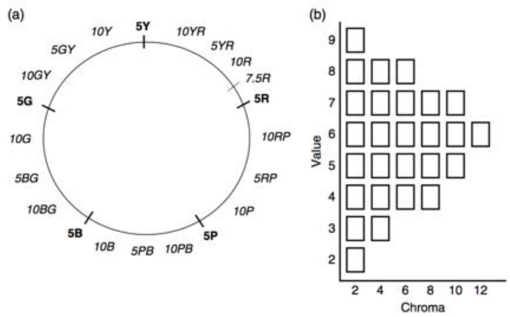

Figure 5.2 A graphical representation of (a) the hue circle and (b) a value/chroma plane of constant hue in the Munsell system

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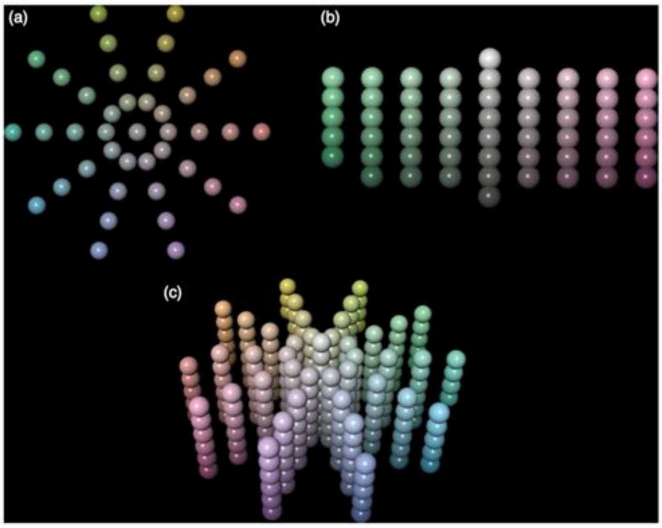

Figure 5.3 A color rendering of samples from the Munsell system in (a) a constant value plane, (b) a pair of constant-hue planes, and (c) a three-dimensional perspective

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Geometric transformations

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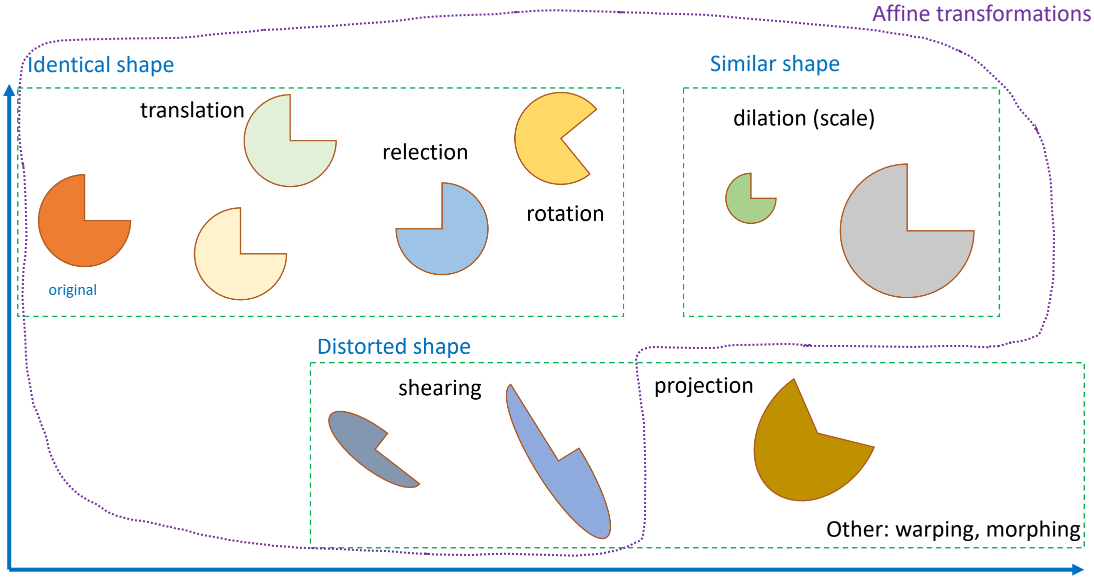
Geometric transformations



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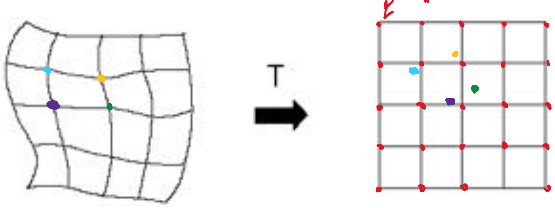
Two basic operations of geometric transformation

1. Spatial transformation of coordinates

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{T} \begin{bmatrix} x \\ y \end{bmatrix}$$

2. Intensity interpolation that assigns intensity values to the spatially transformed pixels

?



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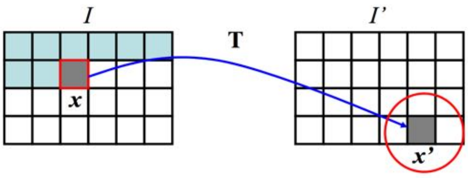
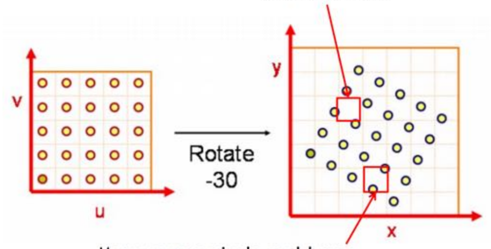
Forward vs backward (inverse) mapping

- Forward mapping: $\begin{bmatrix} x' \\ y' \end{bmatrix} = T \begin{bmatrix} x \\ y \end{bmatrix}$
  - Iterates over pixels of the input image (x,y), computes new location (x', y'), and copies its values to the new location





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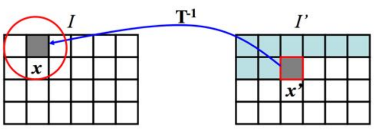
Forward vs backward (inverse) mapping

- Backward mapping: $\begin{bmatrix} x \\ y \end{bmatrix} = T^{-1} \begin{bmatrix} x' \\ y' \end{bmatrix}$
  - Iterates over pixels of the output image and uses the inverse transformation to determine the position in the input image from which a value must be sampled



- What if pixel comes from "between" two pixels?
- Answer: *resample* color value from *interpolated* (*prefiltered*) source image

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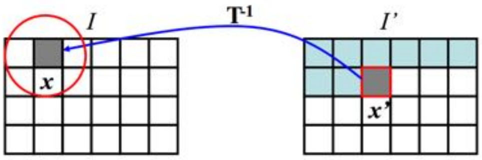
Inverse warping

```
iwarp(I, I', T)
{
  for (y=0; y<I'.height; y++)
    for (x=0; x<I'.width; x++) {
      (x,y)=T^-1(x',y');
      I'(x',y')=Reconstruct(I,x,y,kernel);
    }
}
```



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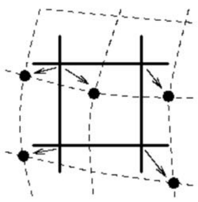
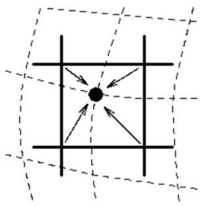
Intensity interpolation

Nearest neighbor



Assigns to the new location the intensity of its nearest neighbor

Bilinear

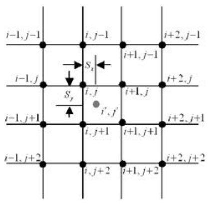


Use four nearest neighbors to estimate the intensity at a given location

$$v(x,y) = ax + by + cxy + d$$

-> solve for a, b, c, d

Bicubic



Use sixteen nearest neighbors to estimate the intensity at a given location

$$v(x,y) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij}x^i y^j$$

-> solve for a_{ij}

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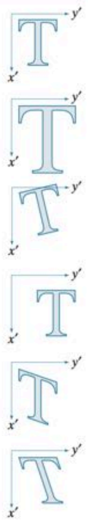
Affine transformation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{T}\begin{bmatrix} x \\ y \end{bmatrix}$$

Expressed using a 3x3 matrix

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{A}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Transformation Name	Affine Matrix, A	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x$ $y' = y$	
Scaling/Reflection (For reflection, set one scaling factor to −1 and the other to 0)	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = c_x x$ $y' = c_y y$	
Rotation (about the origin)	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x\cos\theta - y\sin\theta$ $y' = x\sin\theta + y\cos\theta$	
Translation	$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x + t_x$ $y' = y + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & s_v & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x + s_v y$ $y' = y$	
Shear (horizontal)	$\begin{bmatrix} 1 & 0 & 0 \\ s_h & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x$ $y' = s_h x + y$	



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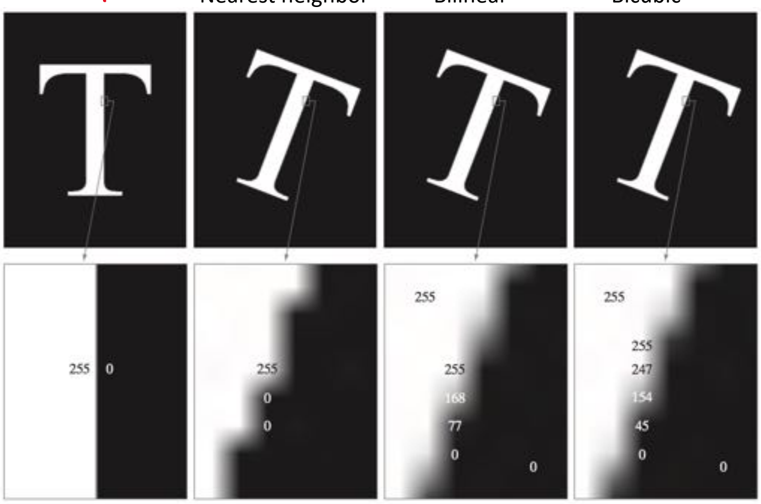
Example of interpolation

Rotated -21º

Nearest neighbor Bilinear Bicubic



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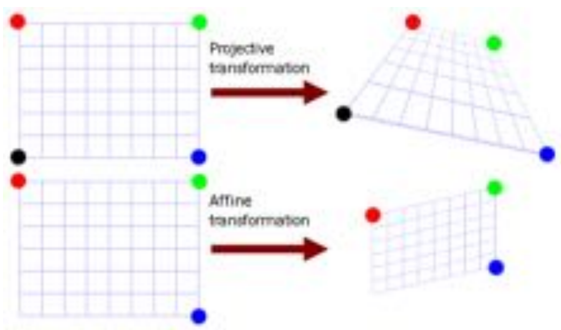
Linear transformation

- Two types of linear transformation:
  - Affine transformation:
    - Preserve parallelism, length, and angle
  - Projective transformation:
    - Preserve collinearity, and incidence



Source image: https://www.graphicsmill.com/docs/gm5/Transformations.htm

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General tranformation matrix

Translation vector

Rotation matrix

$$\begin{bmatrix} a1 & a2 & b1 \\ a3 & a4 & b2 \\ c1 & c2 & 1 \end{bmatrix}$$

Projection vector

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \cong \begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a1 & a2 & b1 \\ a3 & a4 & b2 \\ c1 & c2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

More on projective transformation: https://mc.ai/part-ii-projective-transformations-in-2d/

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General transformations

- Control points
- Projective transformation
- Image warp and image morphing

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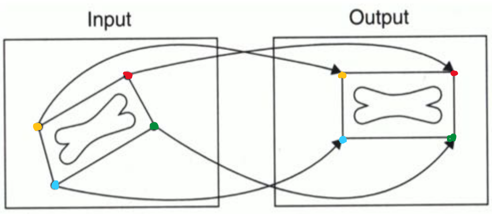
General transformations

- Till now: rotation, scaling, translation (shift), projective transformation are all linear in x and y
- More general transformations: polynormial transformations
  - Order is higher than 1
- Extract transformation is derived from control points
  - a point in the input image and its corresponding point in the output image



Ref: http://www.iup.uni-bremen.de/~melsheim/dip/dipSS17-L3.pdf

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Polynomial transformation

- Assume a(x,y) and b(x, y) as polynomials of order N with unknown coefficients

$$x' = a(x,y) = \sum_{i=0}^{N}\sum_{j=0}^{N-i} a_{ij}x^i y^j$$
$$y' = b(x,y) = \sum_{i=0}^{N}\sum_{j=0}^{N-i} b_{ij}x^i y^j$$

Determining the coefficients requires at least as many control points as the polynomials have coefficients.

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Polynomial transformation with oder N=2

$$x' = a(x,y) = \boxed{a_{00}} + a_{10}x + a_{01}y + a_{11}xy + a_{20}x^2 + a_{02}y^2$$
$$y' = b(x,y) = \boxed{b_{00}} + b_{10}x + b_{01}y + b_{11}xy + b_{20}x^2 + b_{02}y^2$$

a_{00}, b_{00}: Shift vector
a_{10}, b_{01}: Linear scaling in x, y direction
a_{01}, b_{10}: Shear in x, y direction[1]
a_{11}, b_{11}: y-dependent scale in x, x-dependent scale in y
a_{20}, b_{02}: non-linear (quadratic) scale in x, y

[1] A rotation can be described as a combination of shear and linear scaling first in one, then the other coordinate: Any Rotation by angle $\theta \neq \pm 90°$ can be decomposed in the following way:

$$\begin{bmatrix} a_{10} & a_{01} \\ b_{10} & b_{01} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1/\cos\theta & \sin\theta/\cos\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\sin\theta & \cos\theta \end{bmatrix} \quad (4.18)$$

The first (the rightmost one) is a 1D scale and shear in y, the second (the left one) is a 1D scale and shear in x.

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Example of polynomial geometric warps

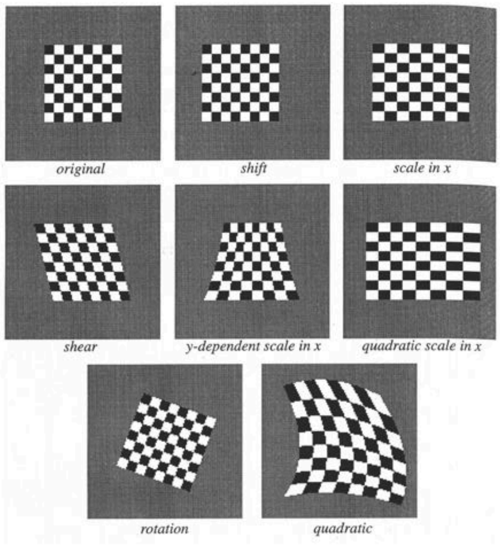

Fig. 4.9: *Some polynomial geometric warps (Fig 7-30 in Schowengerdt, 1997)*

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Applications of geometric transformation

- Geometric calibration/Image rectification
  - Remove camera-induced distortion, i.e., convert non-rectangular pixel coordinates to rectangular coordinates
- Image registration
  - Geometrically match two images or an image and a map; stationary objects should have same position in both images
- Map projections

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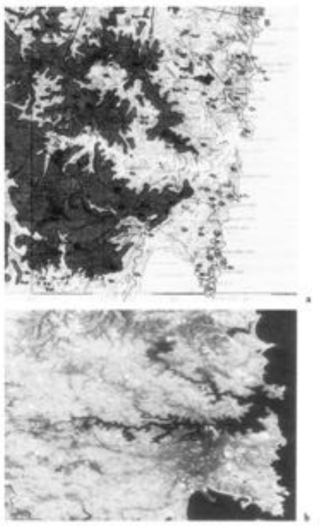
Image registration example



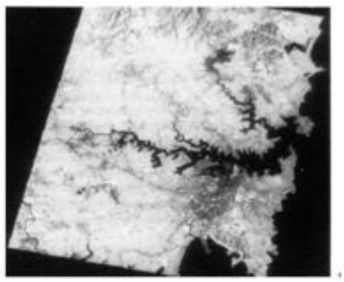


Fig. 4.10: *Image registration. (a) Map; (b) Landsat MSS image to be registered; (c) Landsat image registered to map using 2nd order polynomials (Fig. 2.16 from Richards, 1986)*

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Example of image rectification

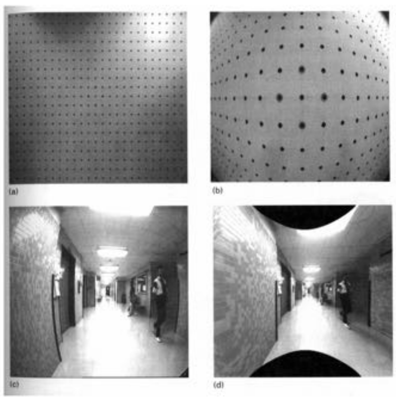

Fig. 4.8: *Geometric rectification of an image taken with a fish-eye lens: (a) test target, (b) fish-eye image of test target, (c) fish-eye image (d) rectified image (Fig 8.9 from Castleman, 1996)*

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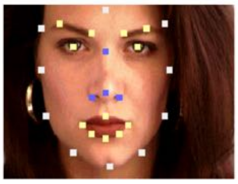
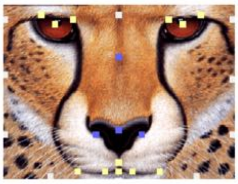
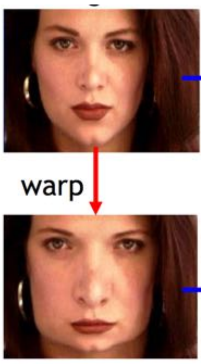
Mesh warping

Specify corresponding points





Interpolate to a complete warping function



Convert to mesh warping

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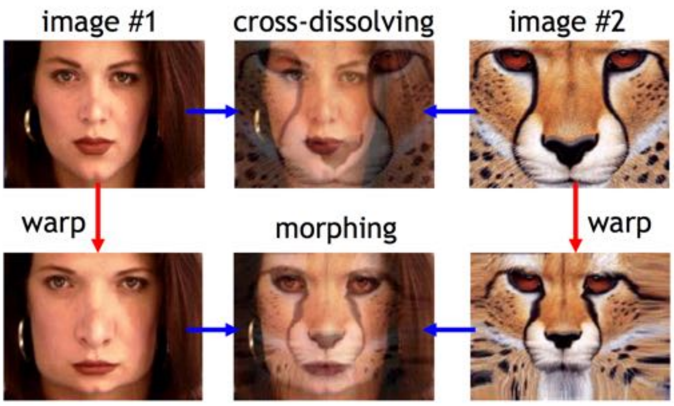
Image morphing

- To synthesize a fluid transformation from one image to another



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References

- Gonzalez
- (Chap 4) Geometric transformations
  - http://www.iup.uni-bremen.de/~melsheim/dip/dipSS17-L3.pdf
- Image warping/morphing
  - https://www.csie.ntu.edu.tw/~cyy/courses/vfx/18spring/lectures/handouts/lec05_morphing.pdf