Chapter 2

Indexing Structures for Files

Adapted from the slides of “Fundamentals of Database Systems” (Elmasri et al., 2011)

Chapter outline

- Types of Single-level Ordered Indexes
  - Primary Indexes
  - Clustering Indexes
  - Secondary Indexes
- Multilevel Indexes
- Dynamic Multilevel Indexes Using B-Trees and B^+-Trees
- Indexes in Oracle

Indexes as Access Paths

- A single-level index is an auxiliary file that makes it more efficient to search for a record in the data file.
- The index is usually specified on one field of the file (although it could be specified on several fields)
- One form of an index is a file of entries <**field value, pointer to record>**, which is ordered by field value
- The index is called an access path on the field.

Indexes as Access Paths (cont.)

- The index file usually occupies considerably less disk blocks than the data file because its entries are much smaller.
- A binary search on the index yields a pointer to the file record.
- Indexes can also be characterized as dense or sparse:
  - A **dense index** has an index entry for every search key value (and hence every record) in the data file.
  - A **sparse (or nondense) index**, on the other hand, has index entries for only some of the search values

Example 1: Given the following data file:
EMPLOYEE(NAME, SSN, ADDRESS, JOB, SAL, ...)

Suppose that:
record size R=150 bytes
block size B=512 bytes
r=30000 records
SSN Field size V_{SSN}=9 bytes, record pointer size P_R=7 bytes

Then, we get:
blocking factor: *bfr*= $\lfloor B/R \rfloor = \lfloor 512/150 \rfloor$ = 3 records/block
number of blocks needed for the file: b= $\lceil r/bfr \rceil = \lceil 30000/3 \rceil$ = 10000 blocks

For an dense index on the SSN field:
index entry size: $R_I=(V_{SSN}+P_R)=(9+7)=16$ bytes
index blocking factor $bfr_I = \lfloor B/R_I \rfloor = \lfloor 512/16 \rfloor$ = 32 entries/block
number of blocks for index file: $b_i = \lceil r/bfr_I \rceil$ = (30000/32)= 938 blocks
binary search needs $\lceil \log_2 b_i \rceil + 1 = \lceil \log_2 938 \rceil + 1$ = 11 block accesses

This is compared to an average linear search cost of:
(b/2)= 10000/2 = 5000 block accesses
If the file records are ordered, the binary search cost would be:
$\lceil \log_2 b \rceil = \lceil \log_2 10000 \rceil$ = 13 block accesses

Types of Single-level Ordered Indexes

- Primary Indexes
- Clustering Indexes
- Secondary Indexes

Primary Index

- Defined on an **ordered data file.**
  - The data file is ordered on a ***key field.***
- One index entry *for each block* in the data file
  - *First record* in the block, which is called the *block anchor*
- A similar scheme can use the *last record* in a block.

Index file
(<*K*(*i*), *P*(*i*)> entries)

Primary key value	Block pointer
1	
4	
8	
12	

Primary key field

ID	Name	DoB	Salary	Sex
1				
2				
3				
4				
6				
7				
8				
9				
10				
12				
13				
15				

Primary Index

- Number of index entries?
  - Number of blocks in data file.
- Dense or Nondense?
  - Nondense
- Search/ Insert/ Update/ Delete?

Clustering Index

- Defined on an **ordered data file.**
  - The data file is ordered on a *non-key field.*

- One index entry *each distinct value* of the field.
  - The index entry points to the *first data block* that contains records with that field value

Clustering field

Index file

(<*K(i)*, *P(i)*> entries)

Clustering field value	Block pointer
1	
2	
3	
4	
5	

Dept_No	Name	DoB	Salary	Sex
1				
1				
2				
2				
2				
2				
2				
3				
3				
4				
4				
5				

Clustering field

Index file

(<*K(i)*, *P(i)*> entries)

Clustering field value	Block pointer
1	
2	
3	
4	
5	

Dept_No	Name	DoB	Salary	Sex
1				
1				
2				
2				
2				
2				
2				
3				
3				
4				
4				
5				

Clustering Index

- Number of index entries?
  - Number of distinct indexing field values in data file.
- Dense or Nondense?
  - Nondense
- Search/ Insert/ Update/ Delete?
- At most **one primary index or one clustering index but not both.**

Secondary index

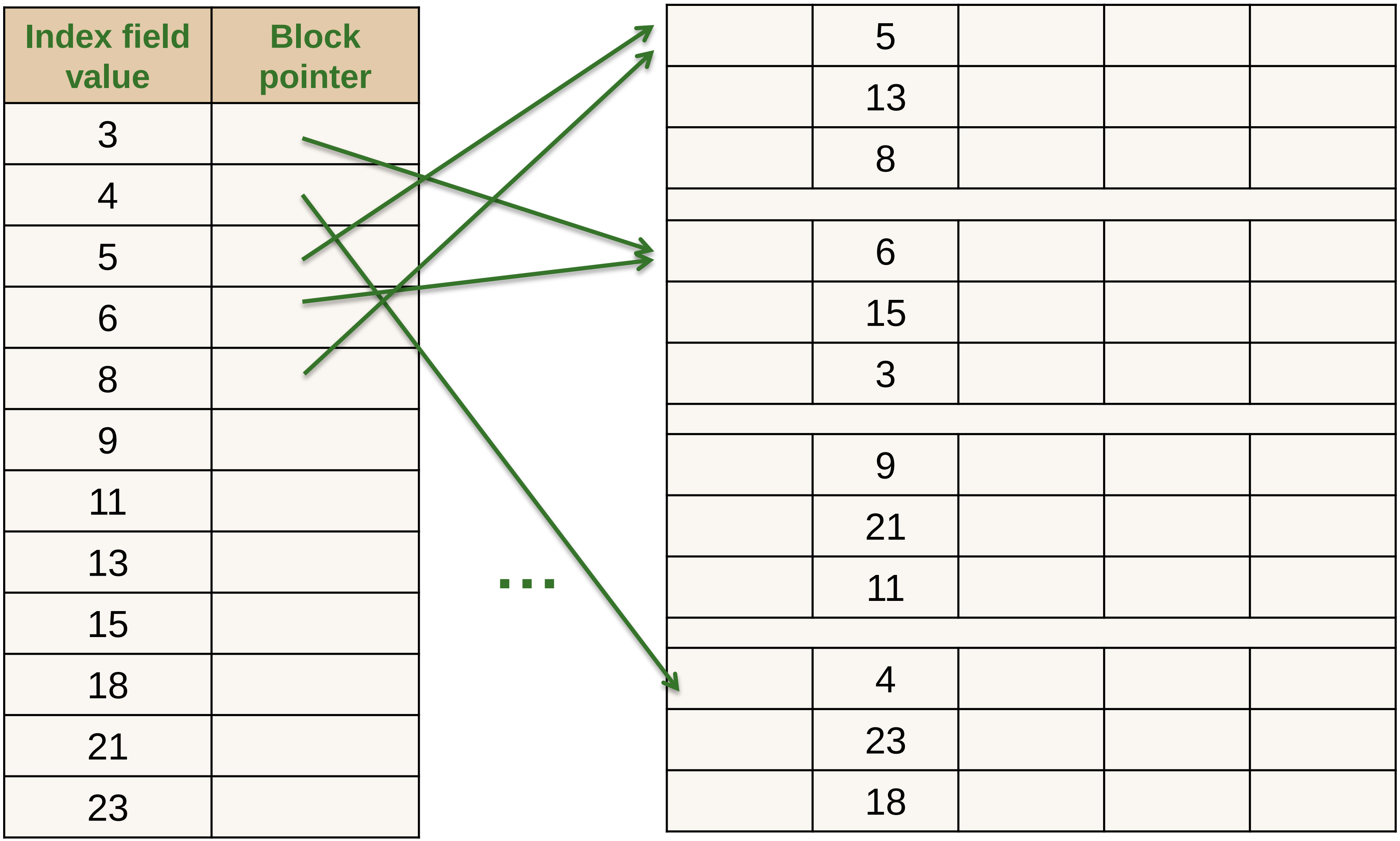
- A secondary index provides a secondary means of accessing a file**.**
  - The data file is unordered on indexing field*.*
- Indexing field:
  - secondary key (unique value)
  - nonkey (duplicate values)

- The index is an ordered file with two fields:
  - The first field: *indexing field.*
  - The second field: *block* pointer or *record* pointer.

- There can be ***many*** secondary indexes for the same file.

Index file
(<*K(i)*, *P(i)*> entries)

Secondary key field



Secondary index on key field

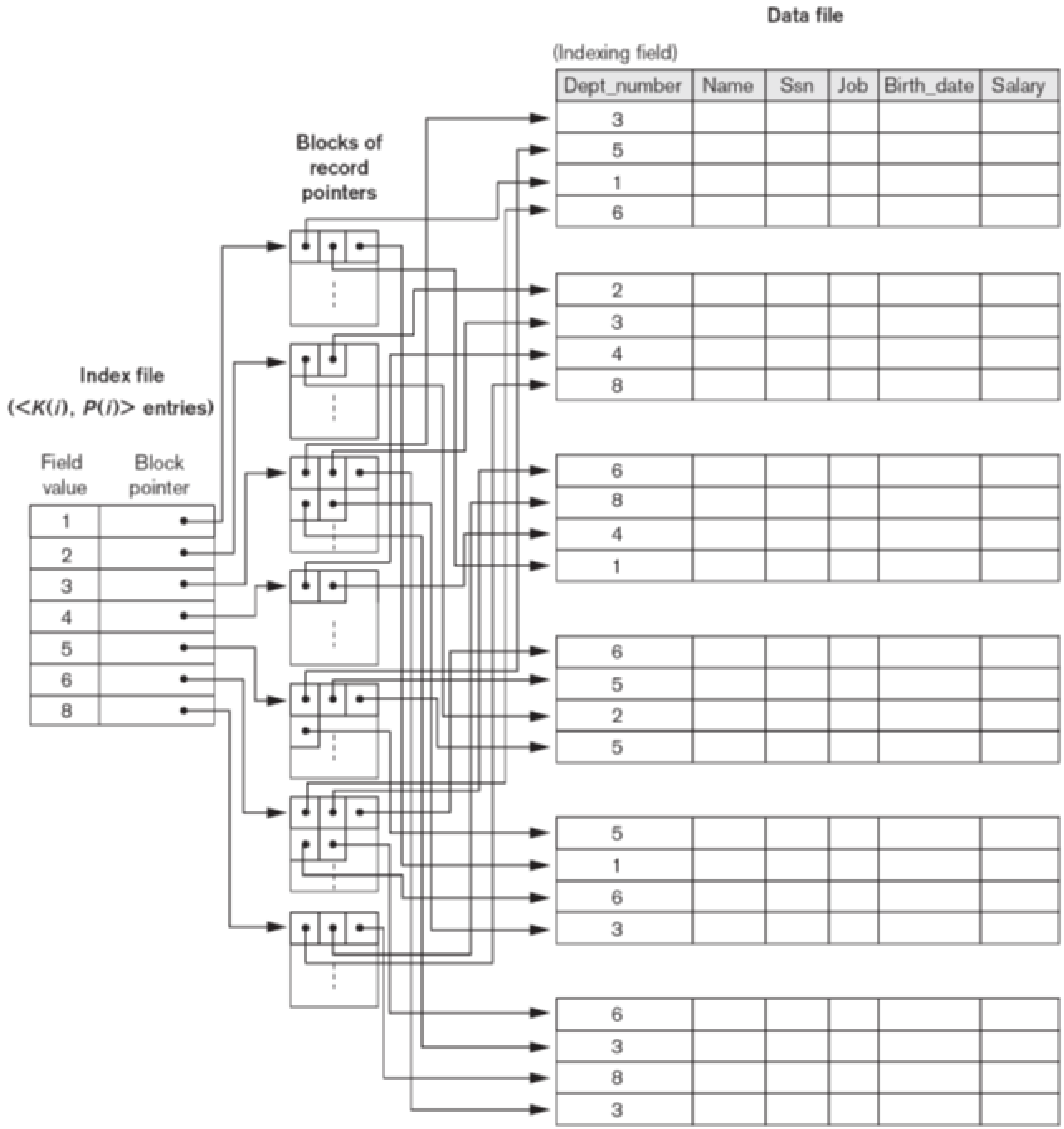
Secondary index on key field

- Number of index entries?
  - Number of record in data file
- Dense or Nondense?
  - Dense
- Search/ Insert/ Update/ Delete?

Secondary index on non-key field

- **Discussion:** Structure of Secondary index on non-key field?
- Option 1: include **duplicate index entries** with the same $K(i)$ value - one for each record.
- Option 2: keep a **list of pointers** $<P(i, 1), ..., P(i, k)>$ in the index entry for $K(i)$.
- Option 3:
  - more commonly used.
  - one entry for each *distinct index field value* + *an* **extra level of indirection** to handle the multiple pointers.

- Secondary Index on non-key field: option 3



Secondary index on nonkey field

- Number of index entries?
  - Number of records in data file
  - Number of distinct index field values
- Dense or Nondense?
  - Dense/ nondense
- Search/ Insert/ Update/ Delete?

Summary of Single-level indexes

- Ordered file on indexing field?
  - Primary index
  - Clustering index
- Indexing field is Key?
  - Primary index
  - Secondary index
- Indexing field is not Key?
  - Clustering index
  - Secondary index

Summary of Single-level indexes

- Dense index?
  - Secondary index

- Nondense index?
  - Primary index
  - Clustering index
  - Secondary index

Summary of Single-level indexes

Table 18.2 Properties of Index Types

Type of Index	Number of (First-level) Index Entries	Dense or Nondense (Sparse)	Block Anchoring on the Data File
Primary	Number of blocks in data file	Nondense	Yes
Clustering	Number of distinct index field values	Nondense	Yes/no[a]
Secondary (key)	Number of records in data file	Dense	No
Secondary (nonkey)	Number of records[b] or number of distinct index field values[c]	Dense or Nondense	No

[a]Yes if every distinct value of the ordering field starts a new block; no otherwise.

[b]For option 1.

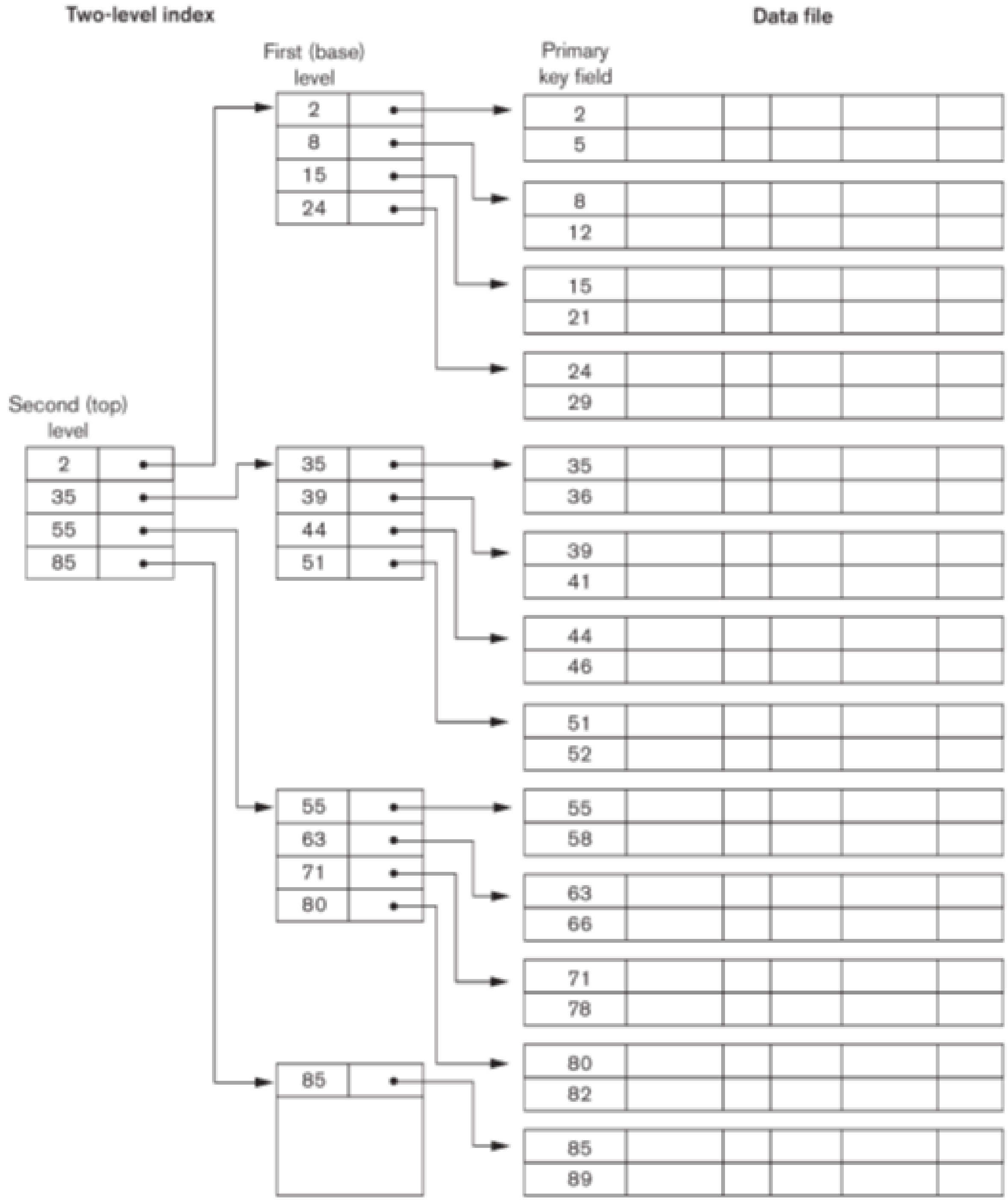
[c]For options 2 and 3.

Chapter outline

- Types of Single-level Ordered Indexes
  - Primary Indexes
  - Clustering Indexes
  - Secondary Indexes
- **Multilevel Indexes**
- Dynamic Multilevel Indexes Using B-Trees and B^+-Trees
- Indexes in Oracle

Multi-Level Indexes

- Because a single-level index is an ordered file, we can **create a primary index *to the index itself***.
  - The original index file is called the *first-level index* and the index to the index is called the *second-level index.*
- We can repeat the process, creating a third, fourth, ..., top level **until all entries of the *top level* fit in one disk block.**
- A multi-level index can be created for any type of first-level index (primary, secondary, clustering) as long as the first-level index consists of *more than one* disk block.

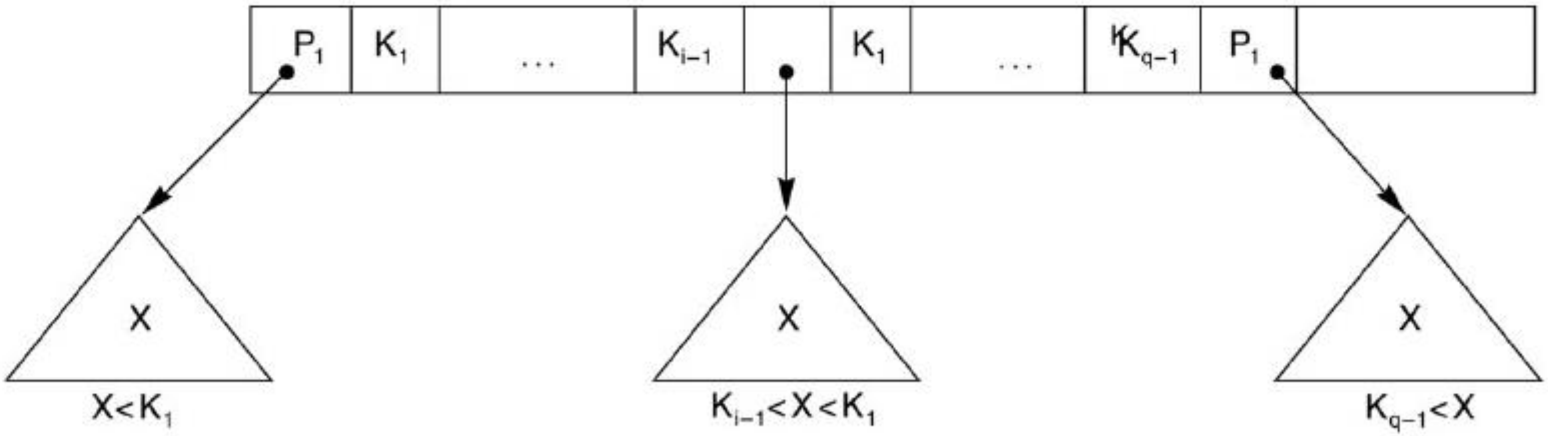


A two-level primary index resembling ISAM (Indexed Sequential Access Method) organization.

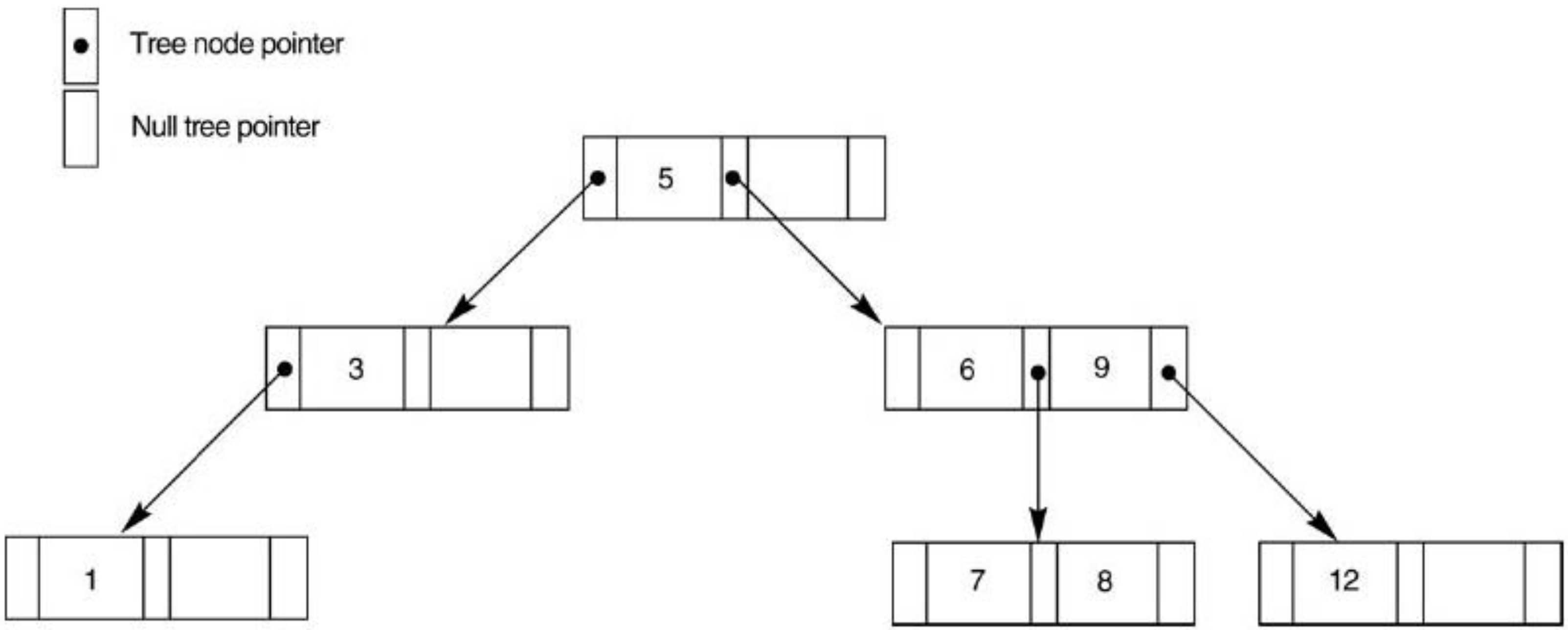
Multi-Level Indexes

- Such a multi-level index is a form of *search tree*.
- However, insertion and deletion of new index entries is a severe problem because every level of the index is an *ordered file.*

A Node in a Search Tree with Pointers to Subtrees below It



A search tree of order p = 3



Chapter outline

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- **Dynamic Multilevel Indexes Using B-Trees and B^+-Trees**
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Dynamic Multilevel Indexes Using B-Trees and B^+-Trees

- Most multi-level indexes use B-tree or B^+-tree data structures because of the insertion and deletion problem.
  - This leaves space in each tree node (disk block) to allow for new index entries
- These data structures are variations of search trees that allow efficient insertion and deletion of new search values.
- In B-Tree and B^+-Tree data structures, each node corresponds to a disk block.
- Each node is kept between half-full and completely full.

Dynamic Multilevel Indexes Using B-Trees and B^+-Trees (cont.)

- An insertion into a node that is not full is quite efficient.
  - If a node is full, the insertion causes a split into two nodes.
- Splitting may propagate to other tree levels.
- A deletion is quite efficient if a node does not become less than half full.
- If a deletion causes a node to become less than half full, it must be merged with neighboring nodes.

Difference between B-tree and B^+-tree

- In a B-Tree, pointers to data records exist at all levels of the tree.
- In a B^+-Tree, all pointers to data records exist at the leaf-level nodes.
- A B^+-Tree can have less levels (or higher capacity of search values) than the corresponding B-tree.

B-tree Structures

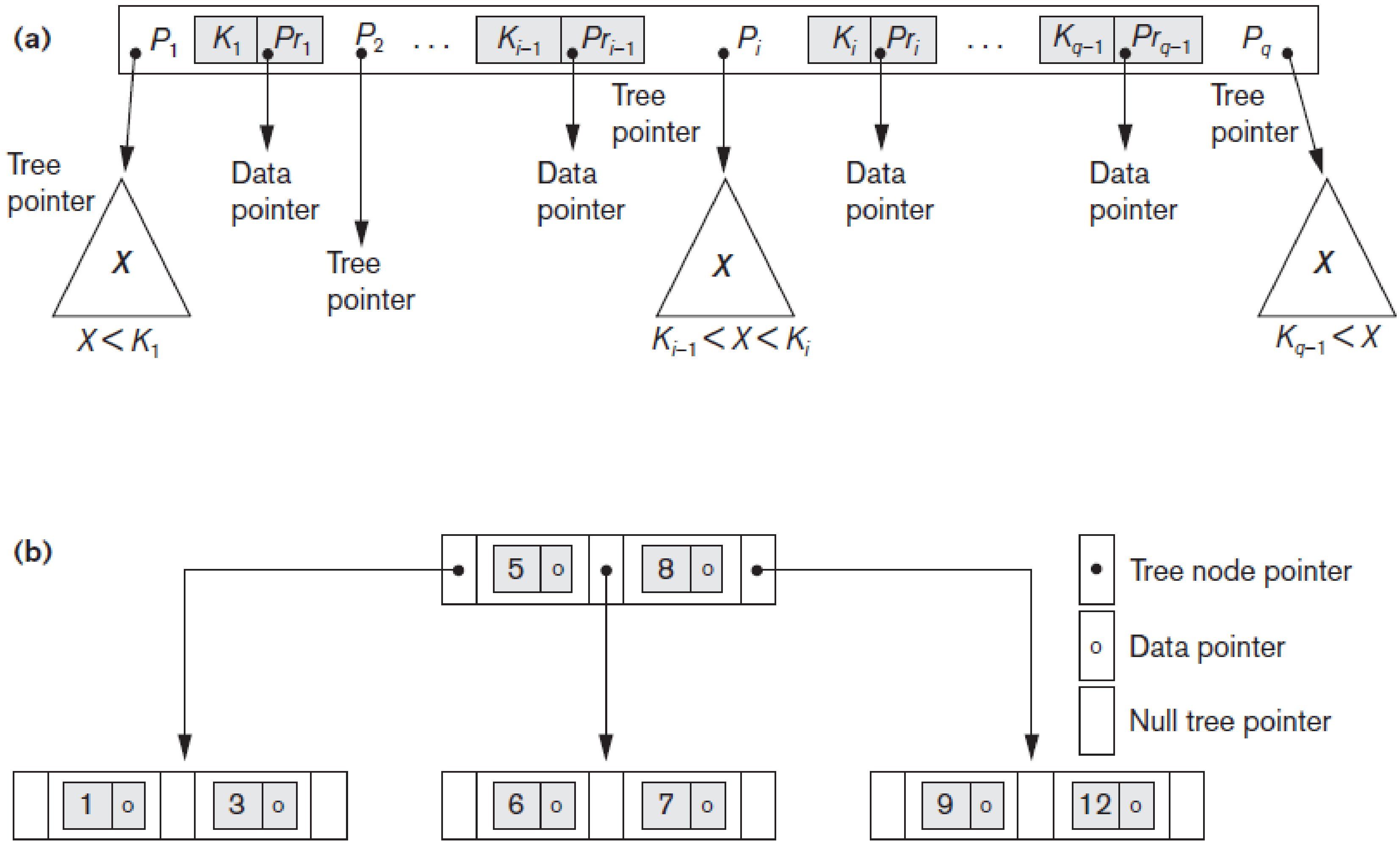


Figure 18.10
B-tree structures. (a) A node in a B-tree with $q - 1$ search values. (b) A B-tree of order $p = 3$. The values were inserted in the order 8, 5, 1, 7, 3, 12, 9, 6.

The Nodes of a B^+-Tree

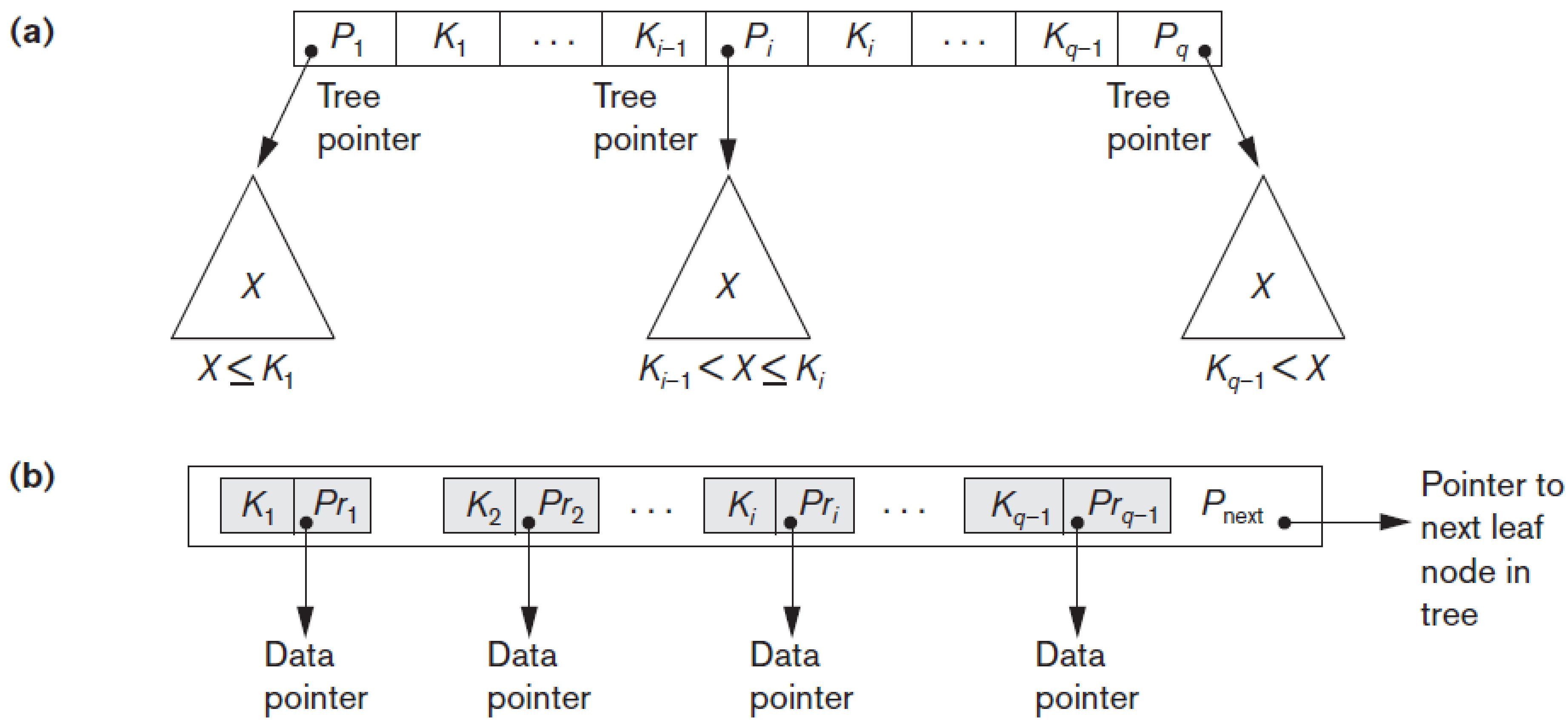


Figure 18.11
The nodes of a B^+-tree. (a) Internal node of a B^+-tree with $q - 1$ search values.
(b) Leaf node of a B^+-tree with $q - 1$ search values and $q - 1$ data pointers.

The Nodes of a B^+-Tree (cont.)

- A B^+-Tree of order p and p_{leaf}:
  - Each internal node:
    - Has at most p tree pointers.
    - Except the root, has at least $\lceil (p/2) \rceil$ tree pointer.
  - An Internal node with q pointers , q ≤ p, has q – 1 search values.
  - Each leaf node:
    - Has at most p_{leaf} data pointers.
    - has at least $\lceil (p_{leaf}/2) \rceil$

EXAMPLE 2: Suppose the search field is **V = 9 bytes** long, the disk block size is **B = 512** bytes, a record (data) pointer is **P_t = 7** bytes, and a block pointer is **P = 6** bytes. Each B-tree node can have at *most* p tree pointers, p – 1 data pointers, and p – 1 search key field values. These must fit into a single disk block if each B-tree node is to correspond to a disk block. Hence, we must have:

$$(p*P) + ((p-1)*(P_t+V)) \leq B$$

$$(p*6) + ((p-1)*(7+9)) \leq 512$$

$$(22*p) \leq 528$$

We can choose to be a large value that satisfies the above inequality, which gives p = 23 (p = 24 is not chosen because of additional information).

EXAMPLE 3: Suppose that search field of Example 2 is a non-ordering key field, and we construct a B-Tree on this field. Assume that each node of the B-tree is 69 percent full. Each node, on the average, will have:

$$p * 0.69 = 23 * 0.69$$

Or approximately 16 pointers and, hence, 15 search key field values. The average fan-out fo = 16. We can start at the root and see how many values and pointers can exist, on the average, at each subsequent level:

Level	Nodes	Index entries	Pointers
Root:	1 node	15 entries	16 pointers
Level 1:	16 nodes	240 entries	256 pointers
Level 2:	256 nodes	3840 entries	4096 pointers
Level 3:	4096 nodes	61,440 entries	

At each level, we calculated the number of entries by multiplying the total number of pointers at the previous level by 15, the average number of entries in each node. Hence, for the given block size, pointer size, and search key field size, a two-level B-tree holds 3840+240+15=**4095** entries on the average; a three-level B-tree holds **65,535** entries on the average.

23*0.69 = 15.87 ≈ 16

- **EXAMPLE 4**: Calculate the order of a B^+-tree.
- Suppose that the search key field is **V=9 bytes** long, the block size is **B=512bytes**, a record pointer **is P_r=7bytes**, and a block pointer is **P=6byte**s, as in Example 3. An internal node of the B^+-tree can have up to p tree pointers and p-1 search field values; these must fit into a single block. Hence, we have:

$$(p*P) + ((p-1)*V) \le B$$
$$\Leftrightarrow \quad (p*6) + ((p-1)*9) \le 512$$
$$\Leftrightarrow \quad 15*p \le 512$$

- We can choose p to be the largest value satisfying the above inequality, which give **p = 34**.
- This is larger than the value of 23 for the B-Tree, resulting in a larger fan-out and more entries in each internal node of a B^+-Tree than in the corresponding B-Tree.

EXAMPLE 4 (cont.)

- The leaf nodes of B+-tree will have the same number of values and pointers, except that the pointers are data pointers and a next pointer. Hence, the order p_{leaf} for the leaf nodes can be calculated as follows:

$$(p_{leaf} * (P_t+V))+P \leq B$$

$$\Leftrightarrow (p_{leaf} * (7+9))+6 \leq 512$$

$$\Leftrightarrow (16 * p_{leaf}) \leq 506$$

- If follows that each leaf node hold up to p_{leaf} = 31 key value/data pointer combinations, assuming that the data pointers are record pointers.

- **EXAMPLE 5**: Suppose that we construct a B^+-Tree on the field of Example 4. To calculate that approximate number of entries of the B^+-Tree, we assume that each node is 69 percent full. On the average, each internal node will be have 34*0.69 ≈ 23.46 or approximately 23 pointers, and hence 22 values. Each leaf node, on the average, will hold $0.69*p_{leaf}$ = 0.69*31 ≈ 21.39 or approximately 21 data record pointers. A B^+-tree will have the following average number of entries at each level:

Level	Nodes	Index entries	Pointers
Root	1 nodes	22 entries	23 pointers
Level 1	23	23*22 = 506	23^2=529 pointers
Level 2	529	529*22 = 11,638	23^3=12,167 pointers
Leaf level	12,167	12,167 *21 = 255,507	

- For the block size, pointer size, and search field size given above, a three-level B^+-tree holds up to **255,507** record pointers, on the average. Compare this to the **65,535** entries for the corresponding B-tree in Example 3.

B^+-Tree: Insert entry

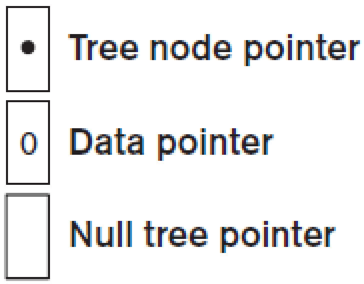
- Insert new entry at leaf node.
- If leaf node is full: overflows and must be split.
  - Create a new node.
  - The first $j = \lceil ((p_{leaf} + 1)/2) \rceil$ entries are kept in the original node.
  - The remaining entries are moved to the new node.
  - The j^{th} search value is replicated in the parent internal node in the correct sequence.
  - An extra pointer to the new node is created in the parent.

B^+-Tree: Insert entry (cont.)

- If the parent internal node is full: overflow and must be split.
  - The j^{th} ($j = \lfloor((p+1)/2)\rfloor$) search value is move to the parent.
  - The first j – 1 entries are kept.
  - The remaining entries (from j+1 to the end) is hold in a new internal node.
- This splitting can propagate all the way to create a new root node

  → new level for the B^+-tree

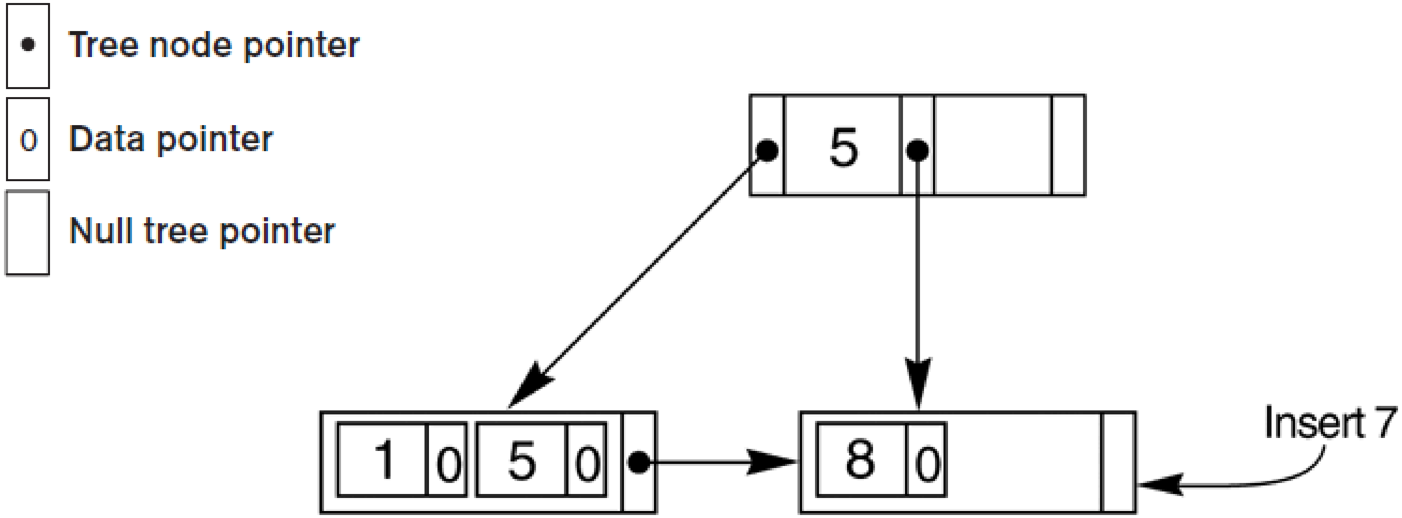
Example of insertion in B$^+$-tree



p = 3 and p_{leaf} = 2

Insertion Sequence: *8, 5,* 1, 7, 3, 12, 9, 6

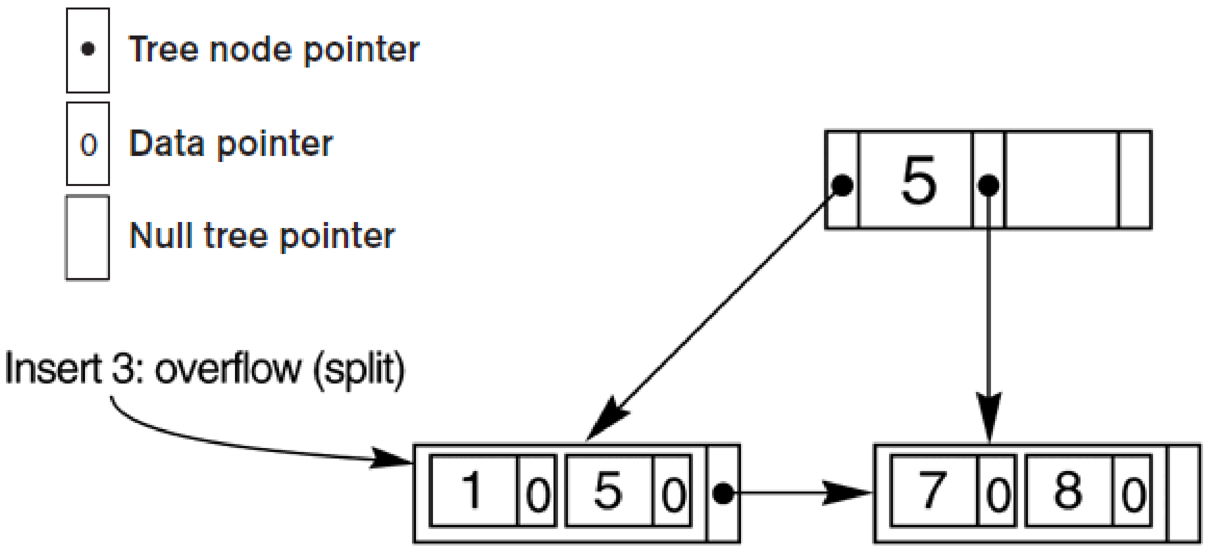
Example of insertion in B^+-tree (cont.)



p = 3 and p_{leaf} = 2

Insertion Sequence: *8, 5, 1,* 7, 3, 12, 9, 6

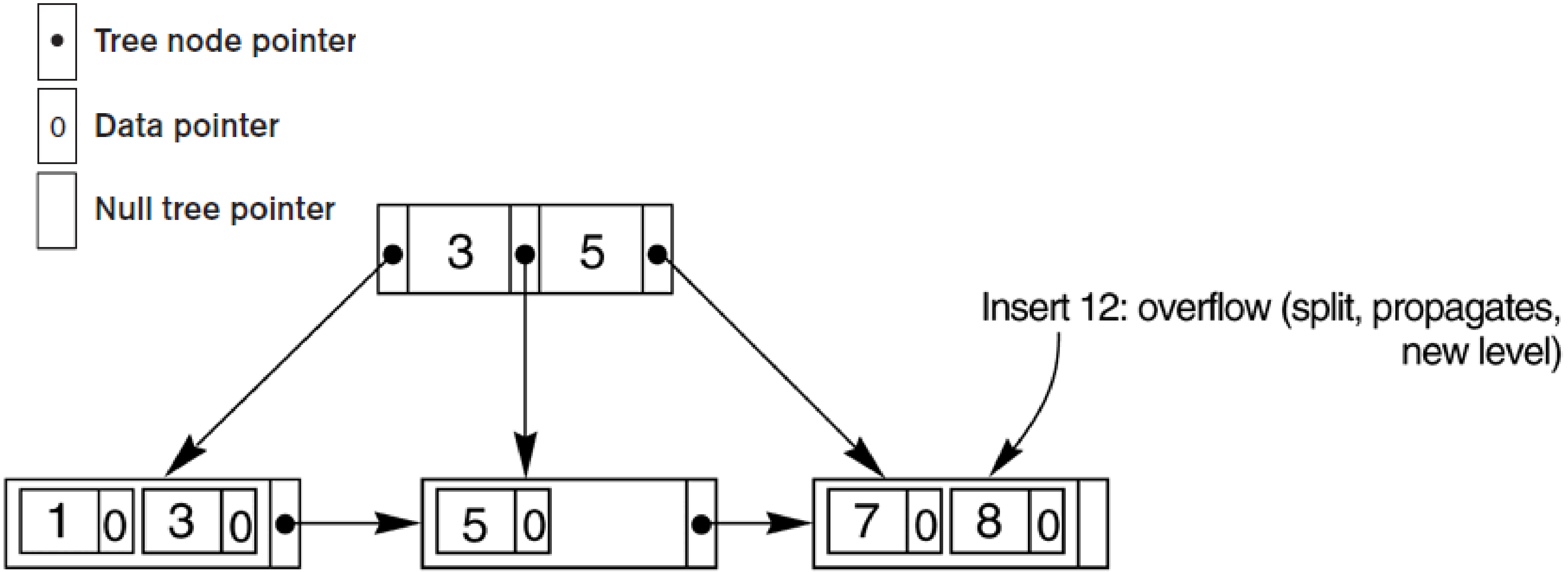
Example of insertion in B$^+$-tree (cont.)



p = 3 and p$_{leaf}$ = 2

Insertion Sequence: *8, 5, 1, 7,* 3, 12, 9, 6

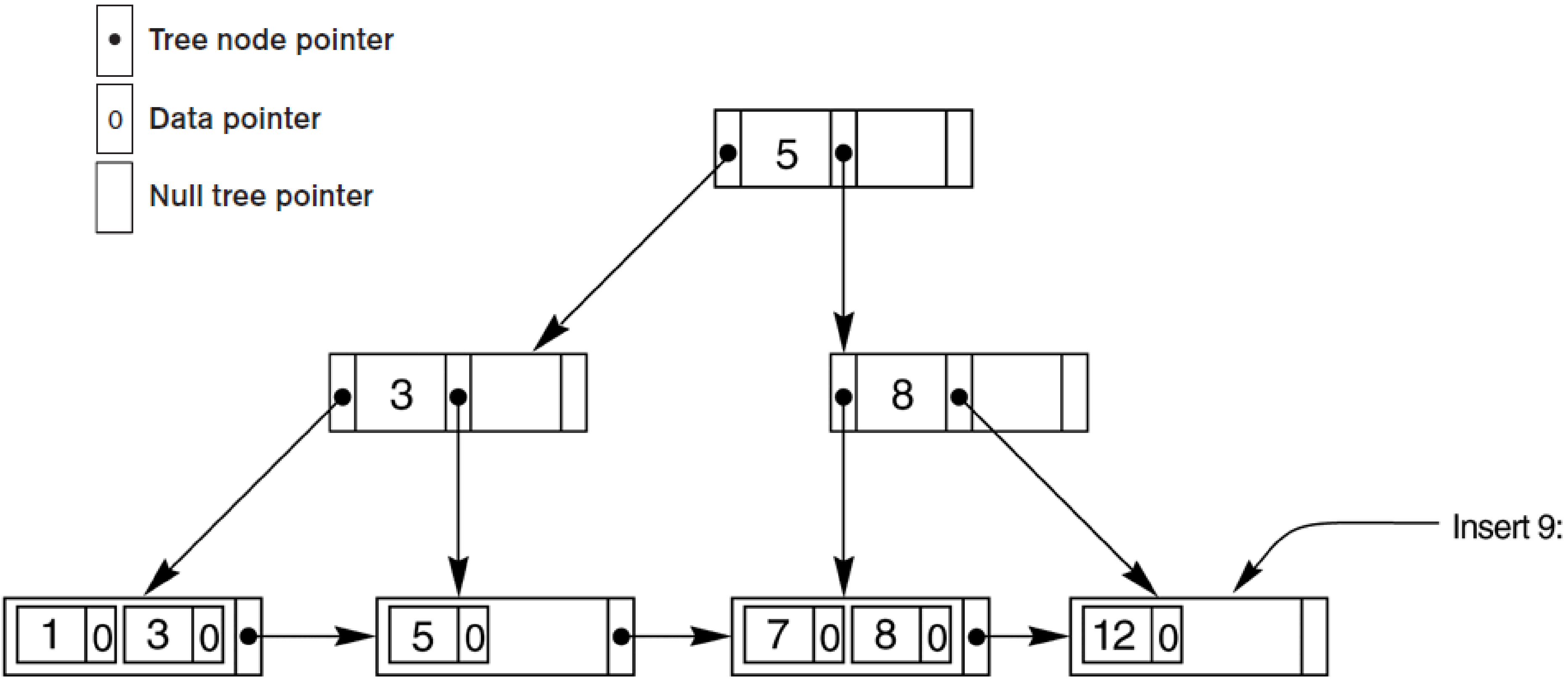
Example of insertion in B^+-tree (cont.)



p = 3 and p_{leaf} = 2

Insertion Sequence: *8, 5, 1, 7, 3,* 12, 9, 6

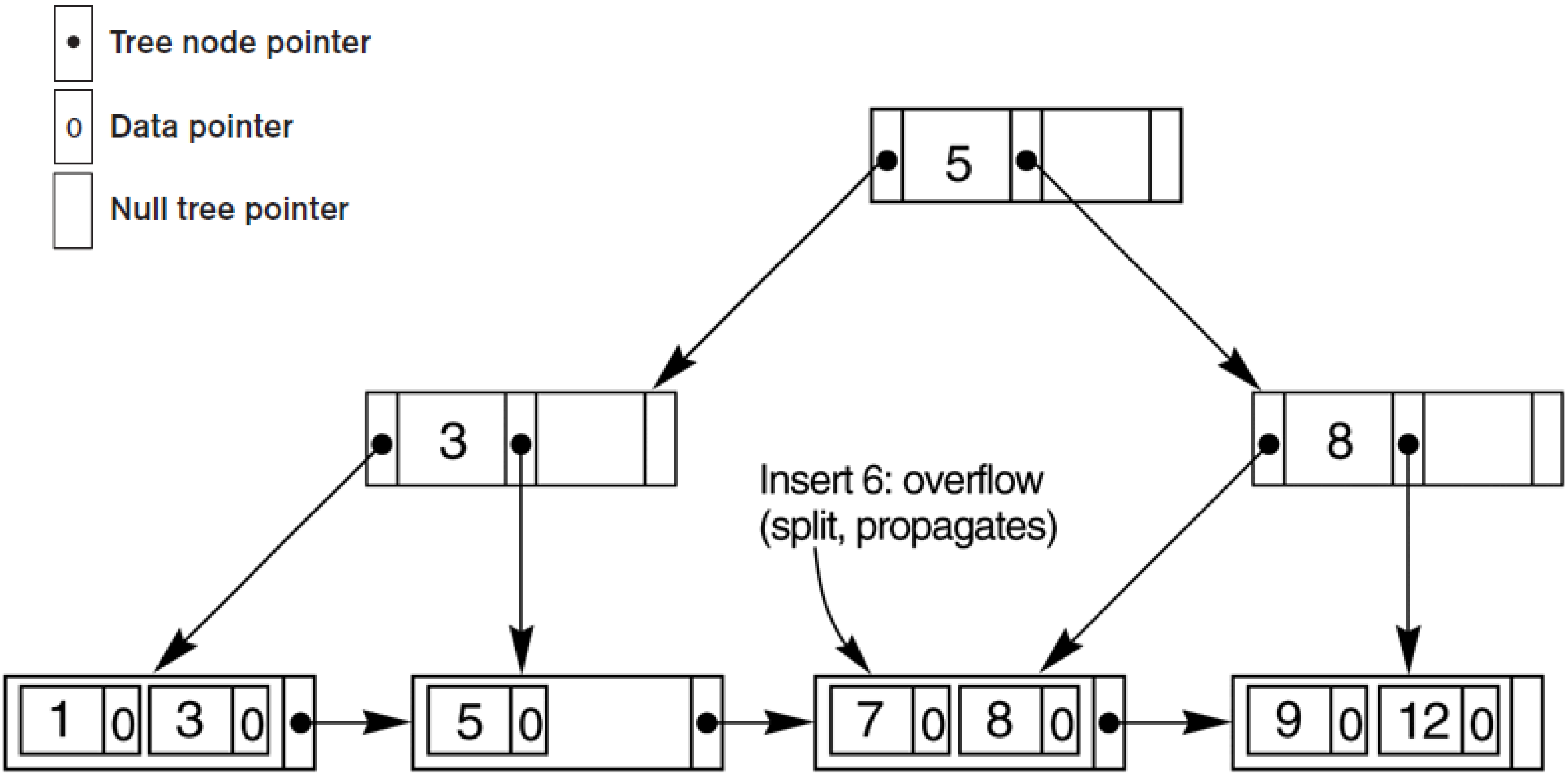
Example of insertion in B^+-tree (cont.)



p = 3 and p_{leaf} = 2

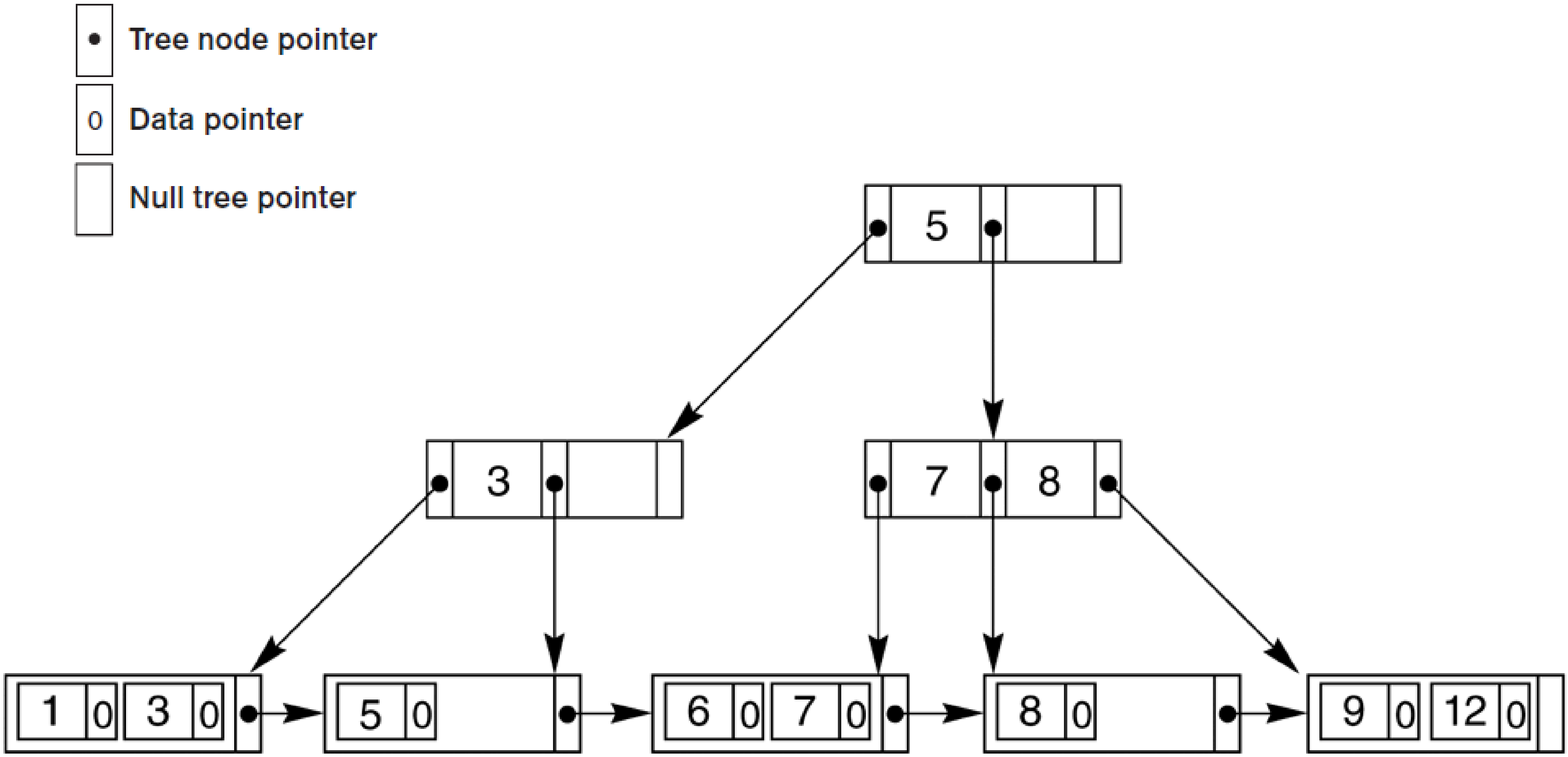
Insertion Sequence: *8, 5, 1, 7, 3, 12,* 9, 6

Example of insertion in B^+-tree (cont.)



p = 3 and p_{leaf} = 2 **Insertion Sequence:** 8, 5, 1, 7, 3, 12, 9, 6

Example of insertion in B^+-tree (cont.)



p = 3 and p_{leaf} = 2 **Insertion Sequence:** *8, 5, 1, 7, 3, 12, 9, 6*

B^+-Tree: Delete entry

- Remove the entry from the leaf node.
- If it happens to occur in an internal node:
  - Remove.
  - The value to its left in the leaf node must replace it in the internal node.
- Deletion may cause underflow in leaf node:
  - Try to find a sibling leaf node – a leaf node directly to the left or to the right of the node with underflow.
  - Redistribute the entries among the node and its siblings. (Common method: The left sibling first and the right sibling later)
  - If redistribution fails, the node is merged with its sibling.
  - If merge occurred, must delete entry (pointing to node and sibling) from parent node.

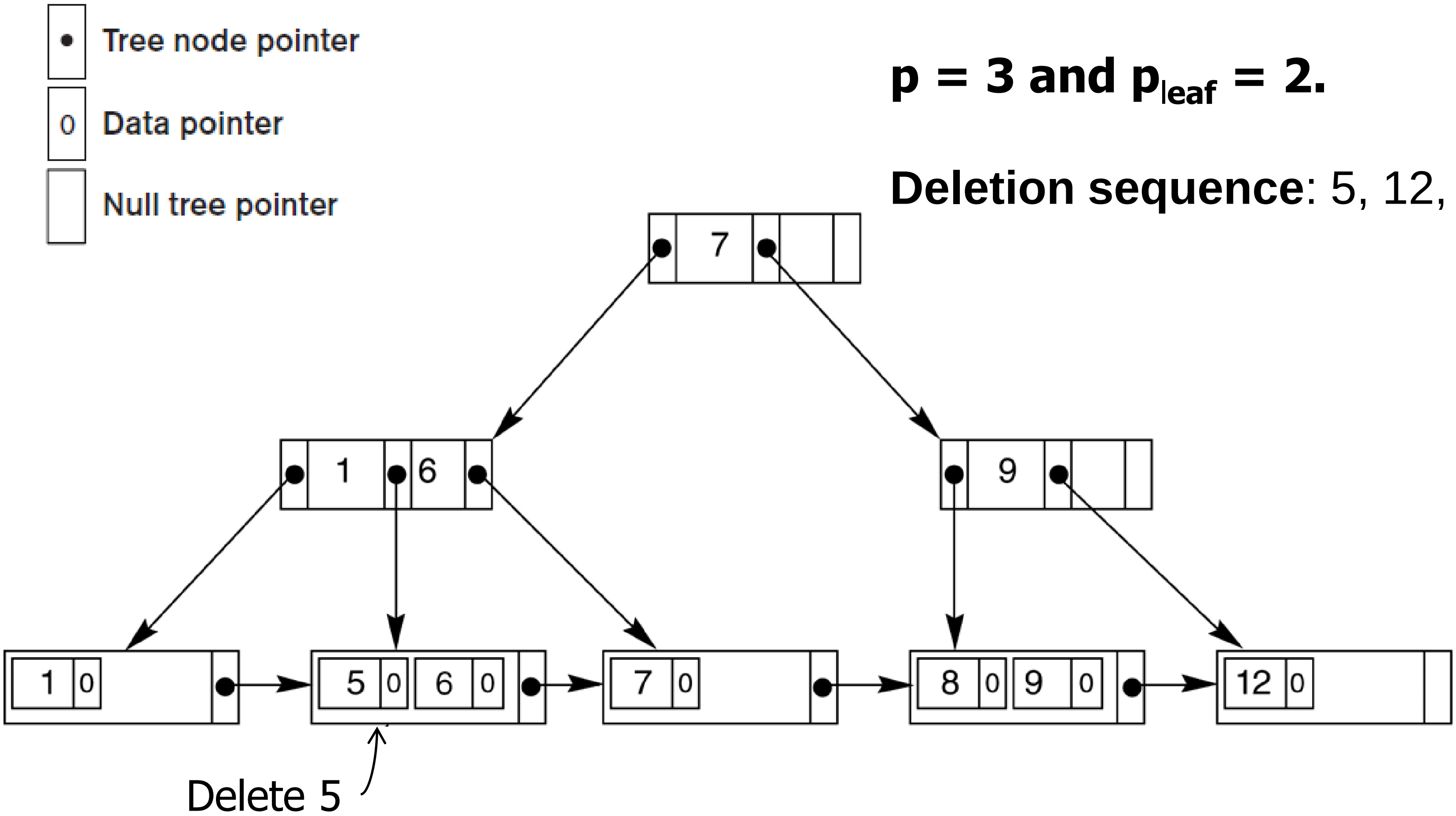
B$^+$-Tree: Delete entry (cont.)

- If an internal node is underflow:
  - Redistribute the entries among the node, its siblings and entry pointing to node and sibling of parent node .
  - If redistribution fails, the node is merged with its sibling and the entry pointing to node and sibling of parent node .
  - If merge occurred, must delete entry pointing to node and sibling from parent node.
  - If the root node is empty → the merged node becomes the new root node.
- Merge could propagate to root, reduce the tree levels.

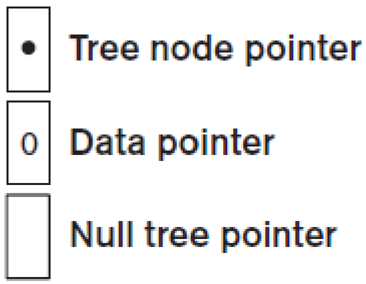
Example of deletion from B+-tree.

p = 3 and p_{leaf} = 2.

Deletion sequence: 5, 12, 9

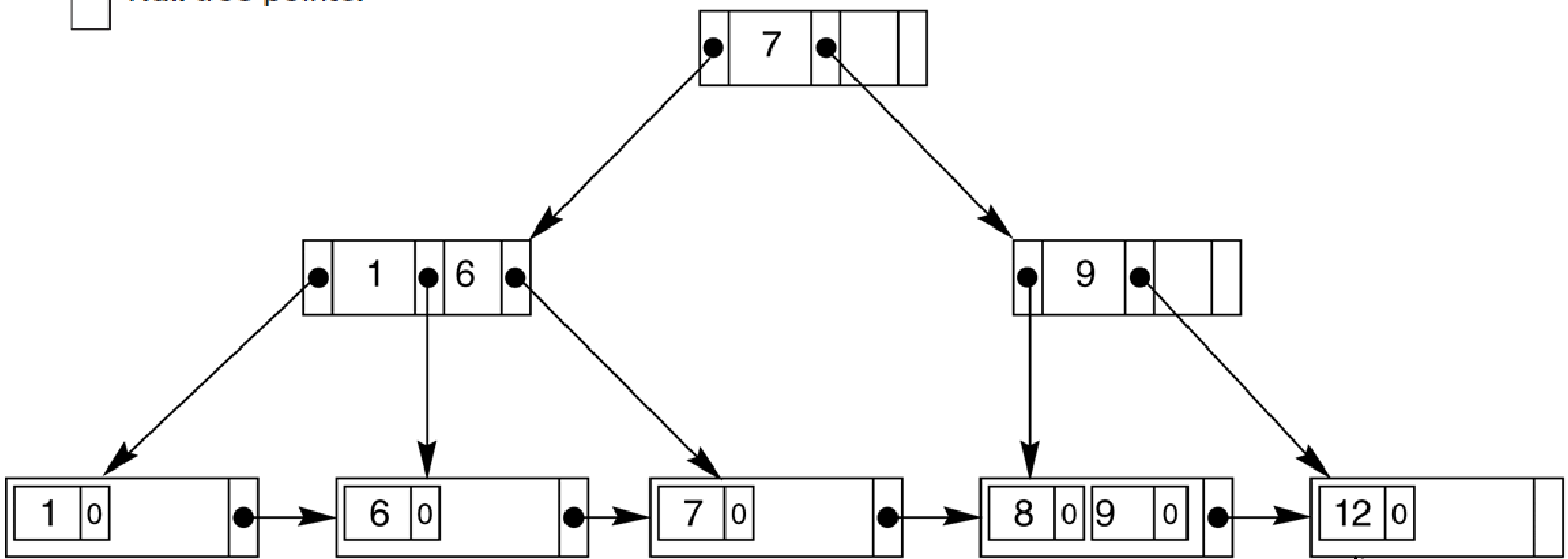


Example of deletion from B$^+$-tree (cont.)



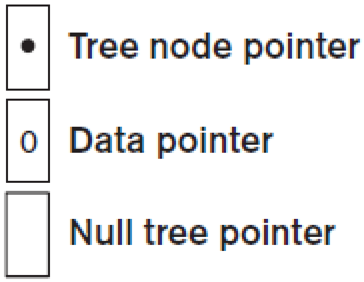
P = 3 and p$_{leaf}$ = 2.

Deletion sequence: 5, 12, 9



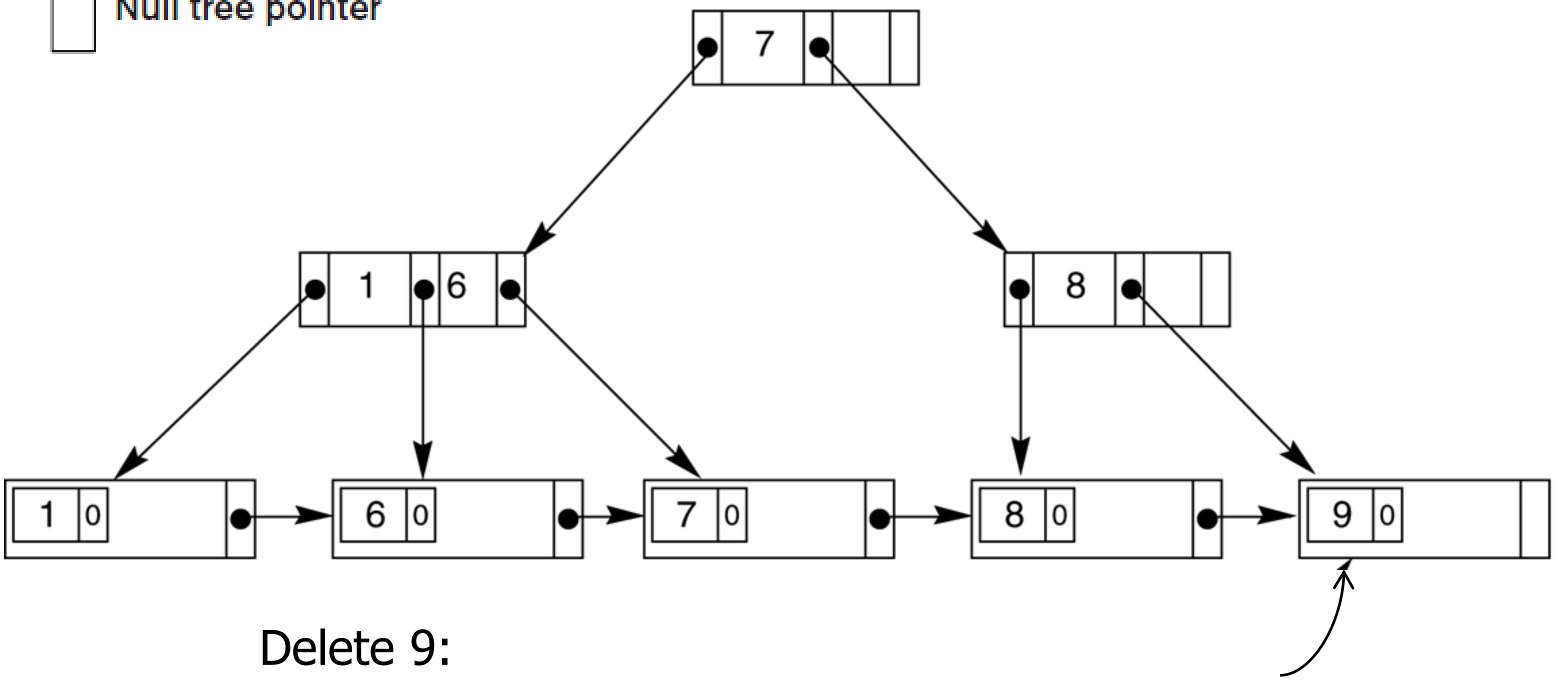
Delete 12: **underflow (redistribute)**

Example of deletion from B^+-tree (cont.)



p = 3 and p_{leaf} = 2.

Deletion sequence: *5, 12,* 9



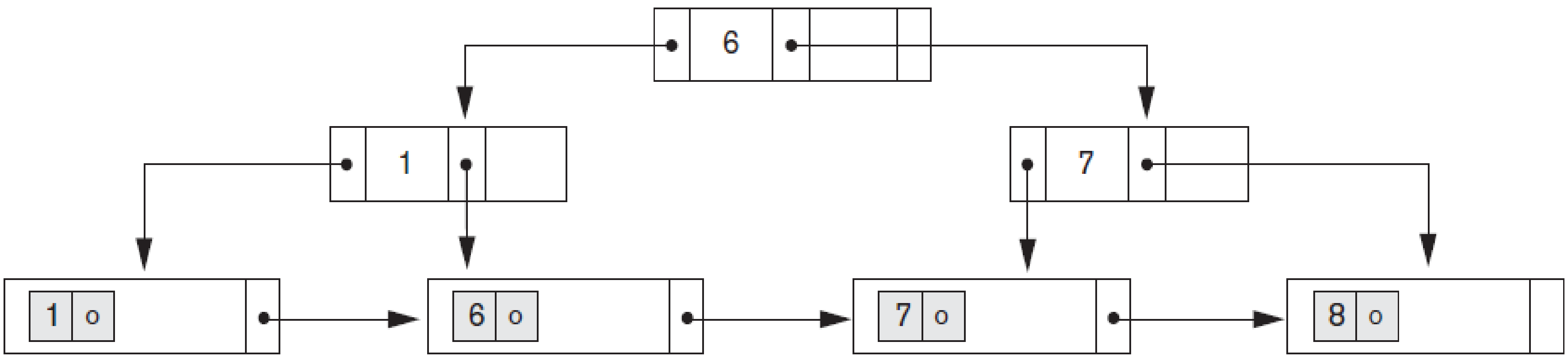
Delete 9:
Underflow (merge with left, redistribute)

Example of deletion from B+-tree (cont.)

- Tree node pointer
- o Data pointer
- Null tree pointer

p = 3 and p_{leaf} = 2.

Deletion sequence: *5, 12, 9*



Notes & Suggestions

- [1], chapter 18:
  - Index on Multiple Keys
  - Other Types of Indexes

- Search, Insertion and Deletion with B-Trees.

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  - Clustering Indexes
  - Secondary Indexes
- Multilevel Indexes
- Dynamic Multilevel Indexes Using B-Trees and B+-Trees
- **Indexes in Oracle**

Types of Indexes

- B-tree indexes: standard index type
  - *Index-organized tables:* the data is itself the index.
  - *Reverse key indexes:* the bytes of the index key are reversed. For example, 103 is stored as 301. The reversal of bytes spreads out inserts into the index over many blocks.
  - *Descending indexes:* This type of index stores data on a particular column or columns in descending order.
  - *B-tree cluster indexes:* is used to index a table cluster key. Instead of pointing to a row, the key points to the block that contains rows related to the cluster key.

Types of Indexes (cont.)

- *Bitmap and bitmap join indexes:* an index entry uses a bitmap to point to multiple rows. A bitmap join index is a bitmap index for the join of two or more tables.
- *Function-based indexes:*
  - Includes columns that are either transformed by a function, such as the UPPER function, or included in an expression.
  - B-tree or bitmap indexes can be function-based.
- *Application domain indexes:* customized index specific to an application.

Creating Indexes

- Simple create index syntax:

CREATE [UNIQUE | BITMAP] INDEX [schema.] <index_name>

ON [schema.] <table_name> (column [**ASC | DESC**] [, column [**ASC | DESC**]] ...)

[**REVERSE**];

Example of creating indexes

- **CREATE INDEX** ord_customer_ix **ON** ORDERS (customer_id);
- **CREATE INDEX** emp_name_dpt_ix ON HR.EMPLOYEES(last_name **ASC**, department_id **DESC**);
- **CREATE BITMAP INDEX** emp_gender_idx **ON** EMPLOYEES (Sex);
- **CREATE BITMAP INDEX** emp_bm_idx **ON** EMPLOYEES (JOBS.job_title) **FROM** EMPLOYEES, JOBS **WHERE** EMPLOYEES.job_id = JOBS.job_id;

Example of creating indexes (cont.)

Function-Based Indexes:

- **CREATE INDEX** emp_fname_uppercase_idx
  ON EMPLOYEES (**UPPER**(first_name));
- **SELECT** First_name, Lname
  FROM Employee **WHERE** UPPER(Lname)= “SMITH”;
- **CREATE INDEX** emp_total_sal_idx
  ON EMPLOYEES (salary + (salary * commission_pct));
- **SELECT** First_name, Lname
  FROM Employee
  WHERE ((Salary*Commission_pct) + Salary) > 15000;

Guidelines for creating indexes

- Primary and unique keys *automatically have indexes*, but you might want to create an index on a foreign key.
- Create an index on any column that the query uses to join tables.
- Create an index on any column from which you search for particular values on a regular basis.
- Create an index on columns that are commonly used in ORDER BY clauses.
- Ensure that the disk and update maintenance overhead an index introduces will not be too high.

Summary

- Types of Single-level Ordered Indexes
  - Primary Indexes
  - Clustering Indexes
  - Secondary Indexes
- Multilevel Indexes
- Dynamic Multilevel Indexes Using B-Trees and B^+-Trees
- Indexes in Oracle

Review questions

1) Define the following terms: indexing field, primary key field, clustering field, secondary key field, block anchor, dense index, and nondense (sparse) index.
2) What are the differences among primary, secondary, and clustering indexes? How do these differences affect the ways in which these indexes are implemented? Which of the indexes are dense, and which are not?
3) Why can we have at most one primary or clustering index on a file, but several secondary indexes?
4) How does multilevel indexing improve the efficiency of searching an index file?
5) What is the order p of a B-tree? Describe the structure of B-tree nodes.
6) What is the order p of a B+-tree? Describe the structure of both internal and leaf nodes of a B+-tree.
7) How does a B-tree differ from a B+-tree? Why is a B+-tree usually preferred as an access structure to a data file?