Study program: Computer Science, VGU
Algebra

Exercise Sheet: Logic

1. Let consider the interpretation v where $v(p) = F, v(q) = T, v(r) = T$. Does v satisfy the following propositional formulas?

   (a) $(p \Rightarrow \neg q) \vee \neg(r \wedge q)$

   (b) $(\neg p \vee \neg q) \Rightarrow (p \vee \neg r)$

   (c) $\neg(\neg p \Rightarrow \neg q) \wedge r$

   (d) $\neg(\neg p \Rightarrow q \wedge \neg r)$

2. Construct a truth table for the formula $\neg p \wedge (p \Rightarrow q)$.

3. Show that $(p \Rightarrow q) \vee (q \Rightarrow p)$ is a tautology.

4. Construct a truth table for $(p \Rightarrow q) \wedge (q \Rightarrow r)$.

5. Write down the negation of the following statements, simplifying so that only simple statements are negated.

   (a) $p \vee \neg q$

   (b) $(p \wedge q) \Rightarrow r$

6. Construct a truth table for $(\neg A \vee B) \Rightarrow (\neg C \wedge D)$.

7. Use a truth table to determine the validity of the argument:

   If Tweety is a bat then Tweety can fly. Tweety is not a bat. Therefore, Tweety can not fly.

8. Use a truth table to determine the validity of the argument:

   If Joe studies hard, he will make the dean's list. Joe made the Dean's list. Therefore, Joe studied hard.

9. Use a truth table to determine the validity of the argument:

   If Joe can afford it, he will ask Sally to go out on a date. If Bill pays Joe the money he owes him then Joe can afford to go out on a date. Joe does not ask Sally to go out on a date. Therefore, Bill did not pay Joe the money he owes him.

10. Use a truth table to determine the validity of the argument:

    If you have high blood pressure then you are at risk for having a stroke. Lynn has high blood pressure. Therefore, Lynn is at risk for having a stroke.

11. Prove the following statements by direct proof

    (a) The product of two odd number integers is odd.

    (b) The product of two invertible matrices is invertible.

12. Prove that following statements by contraposition.

    (a) If a product of two positive real numbers is greater than 100, then at least one of the number is greater than 10.

    (b) There is no greatest even integer.

13. Use the truth table to determine if $p \wedge \neg q \rightarrow p \wedge q$ is a logical consequence of the formular $\neg p$.

14. Check if each of the following statements is satisfiable or not

    (a) If new messages are not queued, then they will be sent to the message buffer.

    (b) $(\neg p \vee q) \wedge (q \rightarrow \neg r \wedge \neg p) \wedge (p \vee r)$

15. Express the negation of these propositions using quanti-fiers, and then express the negation in English.

    (a) Some old dogs can learn new tricks.

    (b) No rabbit knows Algebra.

    (c) Every bird can fly.

    (d) There is no dog that can talk.

    (e) There is no one in this class who knows German and Spanish.