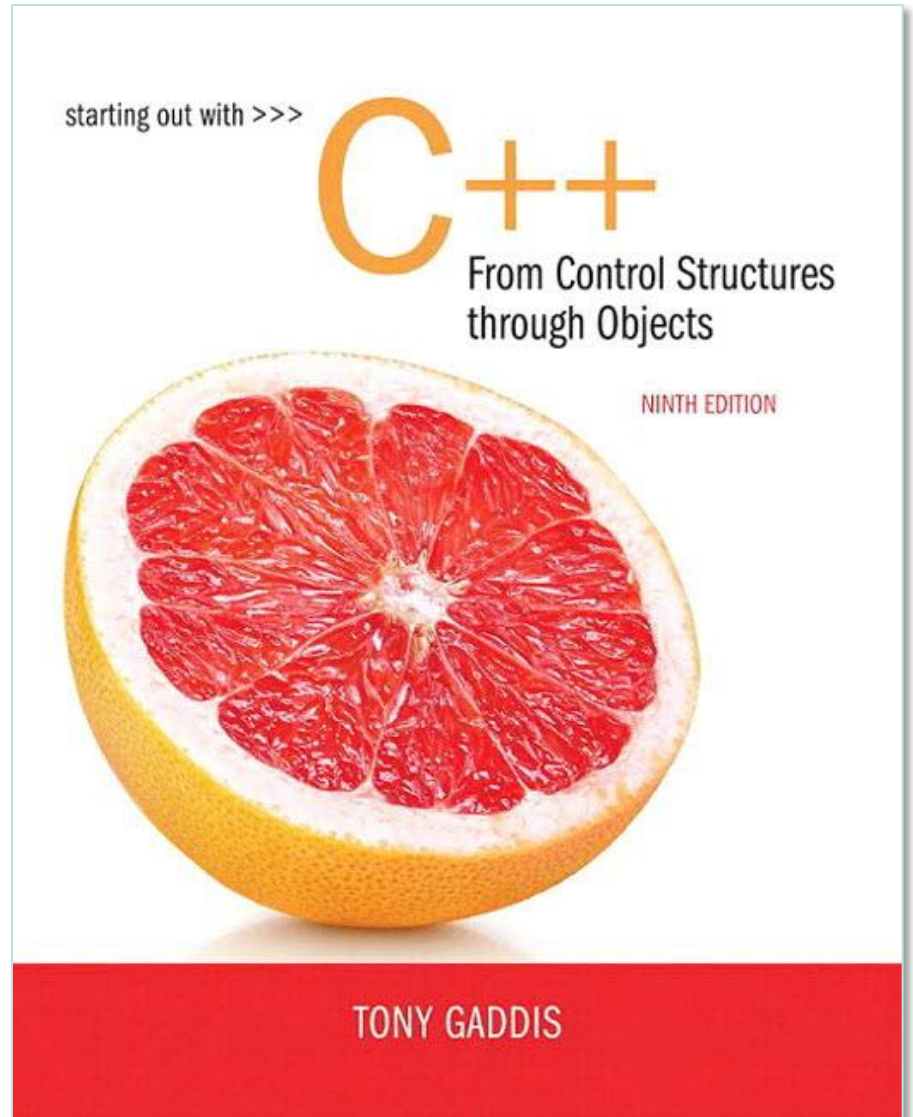


Chapter 20:

Recursion



Introduction to Recursion

- A recursive function contains a call to itself:

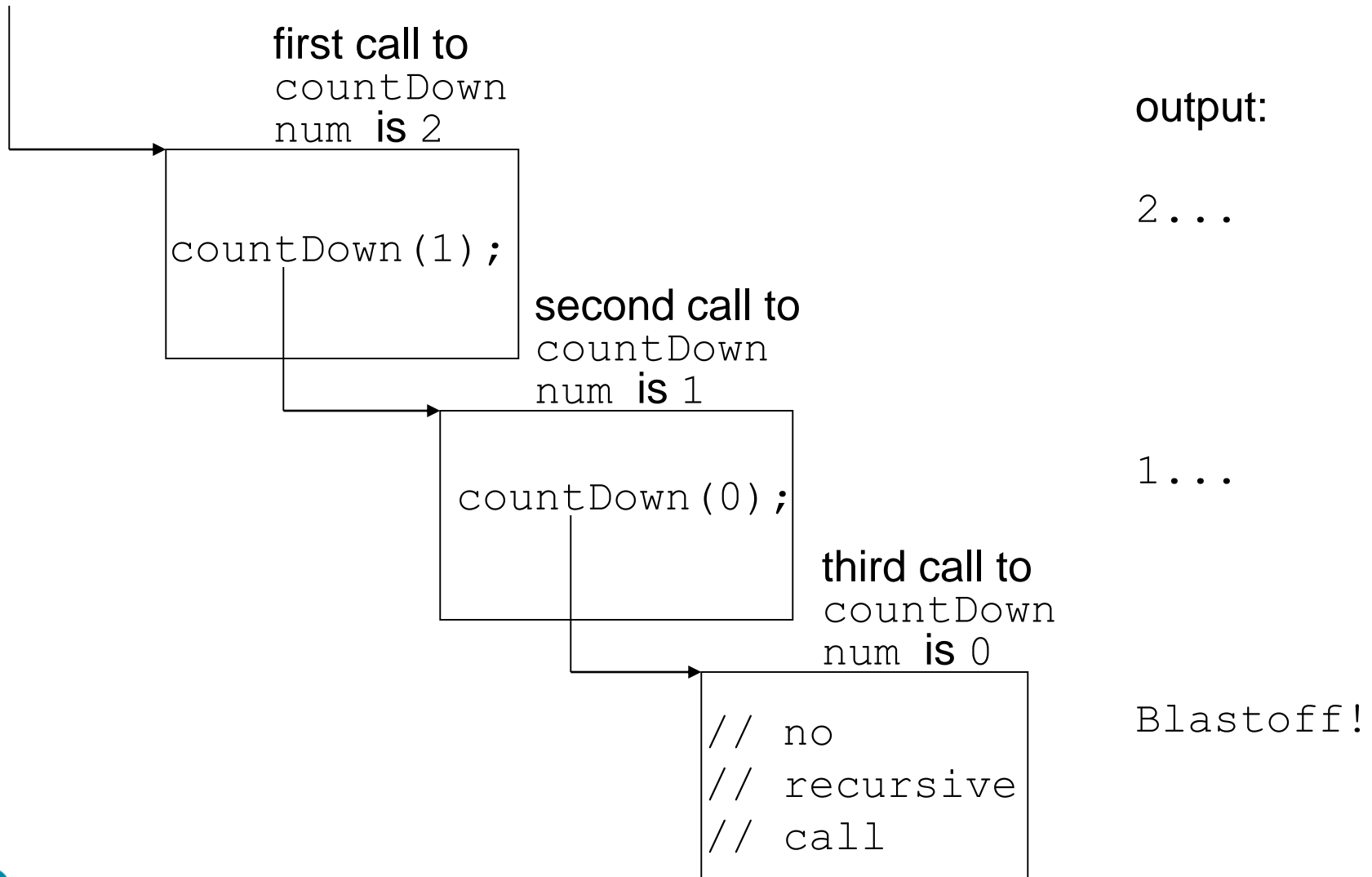
```
void countDown(int num)
{
    if (num == 0)
        cout << "Blastoff!";
    else
    {
        cout << num << "... \n";
        countDown(num-1); // recursive
    }                     // call
}
```

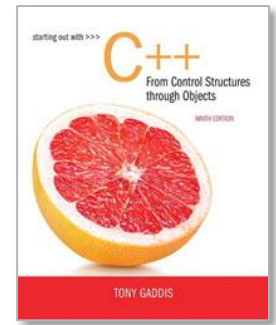
What Happens When Called?

If a program contains a line like `countDown(2);`

1. `countDown(2)` generates the output `2...`, then it **calls** `countDown(1)`
2. `countDown(1)` generates the output `1...`, then it **calls** `countDown(0)`
3. `countDown(0)` generates the output `Blastoff!`, then **returns to** `countDown(1)`
4. `countDown(1)` **returns to** `countDown(2)`
5. `countDown(2)` **returns to the calling function**

What Happens When Called?





20.2

Solving Problems with Recursion

Recursive Functions - Purpose

- Recursive functions are used to reduce a complex problem to a simpler-to-solve problem.
- The simpler-to-solve problem is known as the base case
- Recursive calls stop when the base case is reached

Stopping the Recursion

- A recursive function must always include a test to determine if another recursive call should be made, or if the recursion should stop with this call
- In the sample program, the test is:

```
if (num == 0)
```

Stopping the Recursion

```
void countDown(int num)
{
    if (num == 0) // test
        cout << "Blastoff!";
    else
    {
        cout << num << "...\\n";
        countDown(num-1); // recursive
    } // call
}
```


Stopping the Recursion

- Recursion uses a process of breaking a problem down into smaller problems until the problem can be solved
- In the `countDown` function, a different value is passed to the function each time it is called
- Eventually, the parameter reaches the value in the test, and the recursion stops

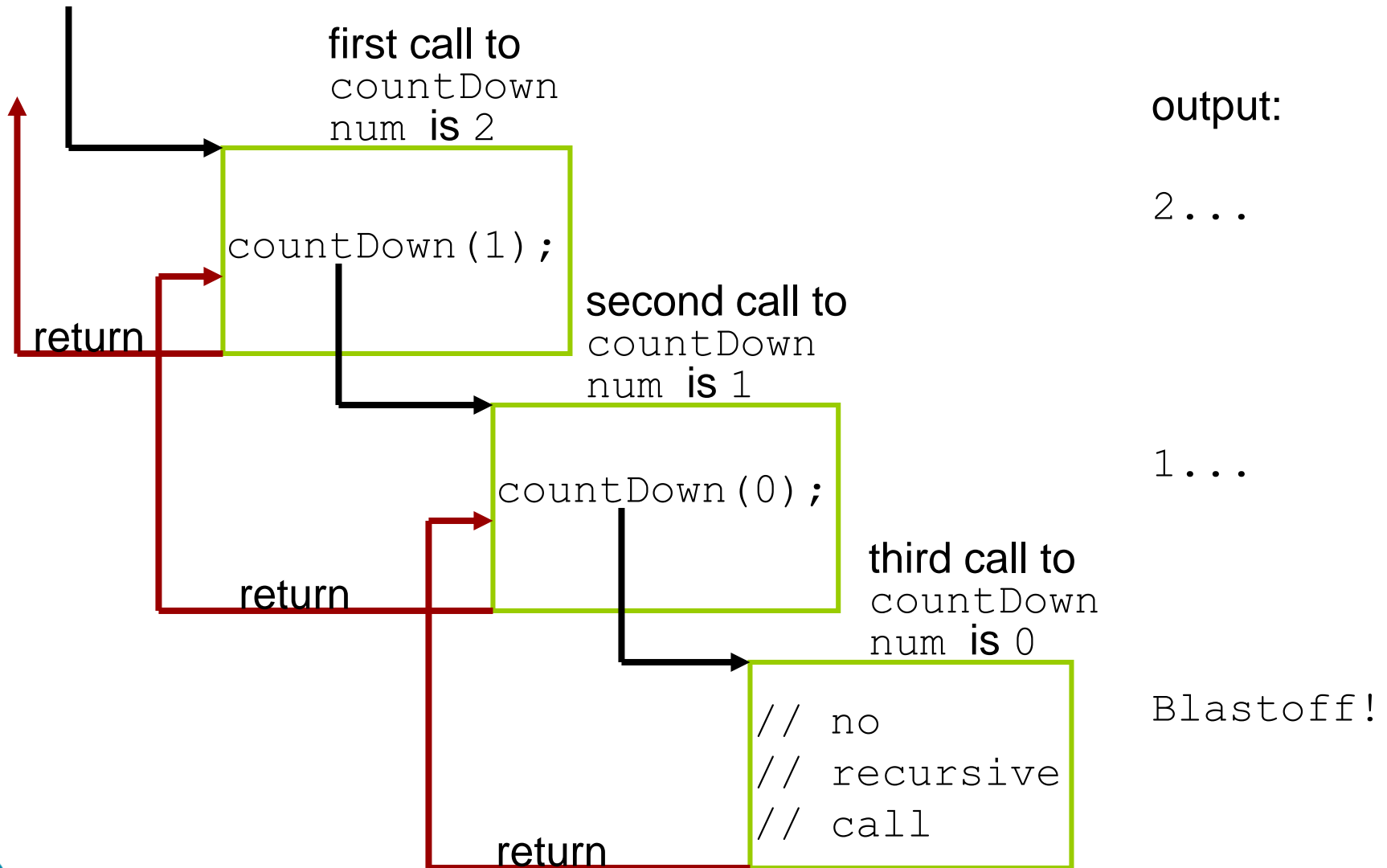
Stopping the Recursion

```
void countDown(int num)
{
    if (num == 0)
        cout << "Blastoff!";
    else
    {
        cout << num << "... \n";
        countDown(num-1) ; // note that the value
    }                       // passed to recursive
    // calls decreases by
    // one for each call
}
```

What Happens When Called?

- Each time a recursive function is called, a new copy of the function runs, with new instances of parameters and local variables created
- As each copy finishes executing, it returns to the copy of the function that called it
- When the initial copy finishes executing, it returns to the part of the program that made the initial call to the function

What Happens When Called?



Types of Recursion

● Direct

- a function calls itself

● Indirect

- function A calls function B, and function B calls function A
- function A calls function B, which calls ..., which calls function A

The Recursive Factorial Function

- The factorial function:

$$n! = n * (n-1) * (n-2) * \dots * 3 * 2 * 1 \text{ if } n > 0$$

$$n! = 1 \text{ if } n = 0$$

- Can compute factorial of n if the factorial of $(n-1)$ is known:

$$n! = n * (n-1)!$$

- $n = 0$ is the base case

The Recursive Factorial Function

```
int factorial (int num)
{
    if (num > 0)
        return num * factorial(num - 1);
    else
        return 1;
}
```

Program 20-3

```
1  // This program demonstrates a recursive function to
2  // calculate the factorial of a number.
3  #include <iostream>
4  using namespace std;
5
6  // Function prototype
7  int factorial(int);
8
9  int main()
10 {
11     int number;
12
```

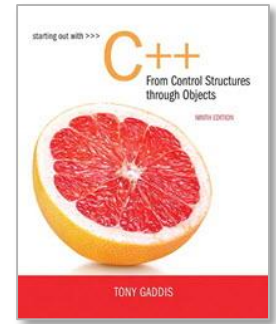
(program continues)

Program 20-3 (Continued)

```
13      // Get a number from the user.
14      cout << "Enter an integer value and I will display\n";
15      cout << "its factorial: ";
16      cin >> number;
17
18      // Display the factorial of the number.
19      cout << "The factorial of " << number << " is ";
20      cout << factorial(number) << endl;
21      return 0;
22  }
23
24  //*****
25  // Definition of factorial. A recursive function to calculate *
26  // the factorial of the parameter n. *
27  //*****
28
29  int factorial(int n)
30  {
31      if (n == 0)
32          return 1;                // Base case
33      else
34          return n * factorial(n - 1); // Recursive case
35  }
```

Program Output with Example Input Shown in Bold

Enter an integer value and I will display
its factorial: **4**
The factorial of 4 is 24



20.3

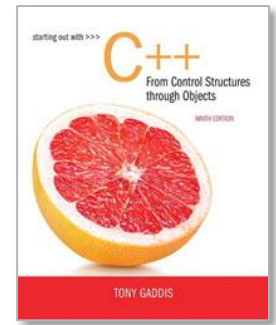
The Recursive gcd Function

The Recursive gcd Function

- Greatest common divisor (gcd) is the largest factor that two integers have in common
- Computed using Euclid's algorithm:
 $\text{gcd}(x, y) = y$ if y divides x evenly
 $\text{gcd}(x, y) = \text{gcd}(y, x \% y)$ otherwise
- $\text{gcd}(x, y) = y$ is the base case

The Recursive gcd Function

```
int gcd(int x, int y)
{
    if (x % y == 0)
        return y;
    else
        return gcd(y, x % y);
}
```



20.4

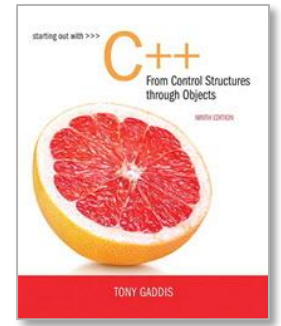
Solving Recursively Defined Problems

Solving Recursively Defined Problems

- The natural definition of some problems leads to a recursive solution
- Example: Fibonacci numbers:
0, 1, 1, 2, 3, 5, 8, 13, 21, ...
- After the starting 0, 1, each number is the sum of the two preceding numbers
- Recursive solution:
$$\text{fib}(n) = \text{fib}(n - 1) + \text{fib}(n - 2);$$
- Base cases: $n \leq 0, n == 1$

Solving Recursively Defined Problems

```
int fib(int n)
{
    if (n <= 0)
        return 0;
    else if (n == 1)
        return 1;
    else
        return fib(n - 1) + fib(n - 2);
}
```



20.5

Recursive Linked List Operations

Recursive Linked List Operations

- Recursive functions can be members of a linked list class
- Some applications:
 - Compute the size of (number of nodes in) a list
 - Traverse the list in reverse order

Counting the Nodes in a Linked List

- Uses a pointer to visit each node
- Algorithm:
 - pointer starts at head of list
 - If pointer is null pointer, return 0 (base case)
else, return 1 + number of nodes in the list pointed to by current node
- See the `NumberList` class in Chapter 19

The countNodes function, a private member function

```
173 int NumberList::countNodes(ListNode *nodePtr) const
174 {
175     if (nodePtr != nullptr)
176         return 1 + countNodes(nodePtr->next);
177     else
178         return 0;
179 }
```

- The countNodes function is executed by the public numNodes function:

```
int numNodes() const
{ return countNodes(head); }
```

Contents of a List in Reverse Order

Algorithm:

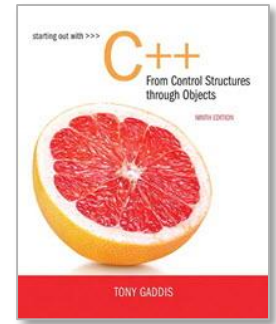
- pointer starts at head of list
- If the pointer is null pointer, return (base case)
- If the pointer is not null pointer, advance to next node
- Upon returning from recursive call, display contents of current node

The showReverse function, a private member function

```
187 void NumberList::showReverse(ListNode *nodePtr) const
188 {
189     if (nodePtr != nullptr)
190     {
191         showReverse(nodePtr->next);
192         cout << nodePtr->value << " ";
193     }
194 }
```

- The showReverse function is executed by the public displayBackwards function:

```
void displayBackwards() const
{ showReverse(head); }
```



20.6

A Recursive Binary Search Function

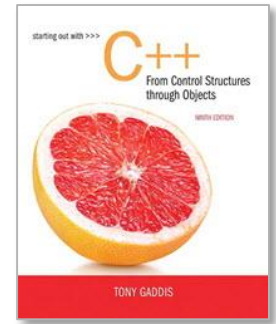
A Recursive Binary Search Function

- Binary search algorithm can easily be written to use recursion
- Base cases: desired value is found, or no more array elements to search
- Algorithm (array in ascending order):
 - If middle element of array segment is desired value, then done
 - Else, if the middle element is too large, repeat binary search in first half of array segment
 - Else, if the middle element is too small, repeat binary search on the second half of array segment

A Recursive Binary Search Function (Continued)

```
int binarySearch(int array[], int first, int last, int value)
{
    int middle;    // Mid point of search

    if (first > last)
        return -1;
    middle = (first + last) / 2;
    if (array[middle] == value)
        return middle;
    if (array[middle] < value)
        return binarySearch(array, middle+1, last, value);
    else
        return binarySearch(array, first, middle-1, value);
}
```

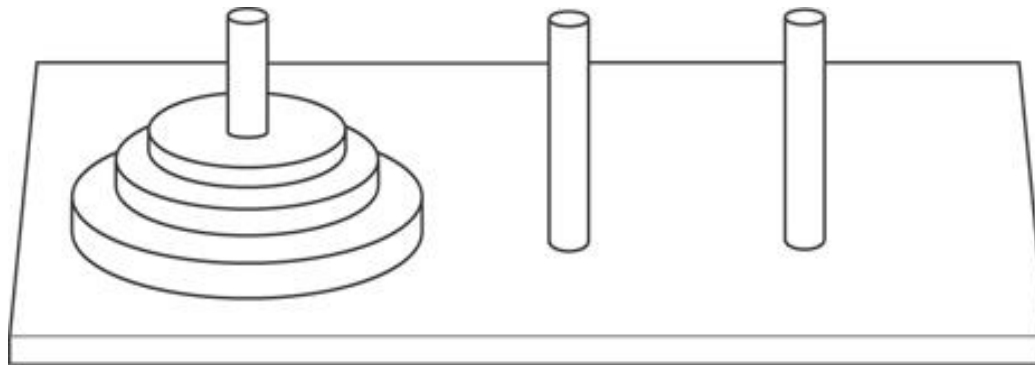



20.7

The Towers of Hanoi

The Towers of Hanoi

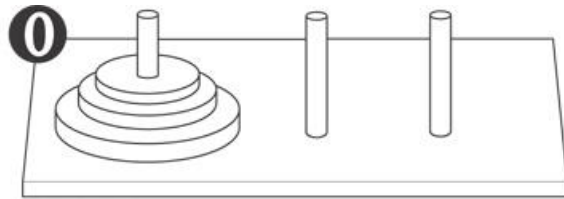
- The Towers of Hanoi is a mathematical game that is often used to demonstrate the power of recursion.
- The game uses three pegs and a set of discs, stacked on one of the pegs.



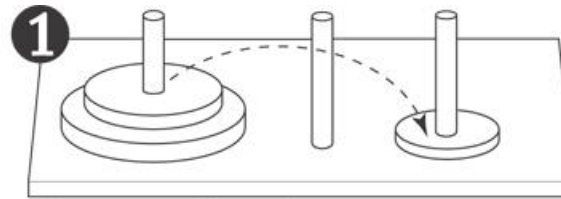
The Towers of Hanoi

- The object of the game is to move the discs from the first peg to the third peg. Here are the rules:
 - Only one disc may be moved at a time.
 - A disc cannot be placed on top of a smaller disc.
 - All discs must be stored on a peg except while being moved.

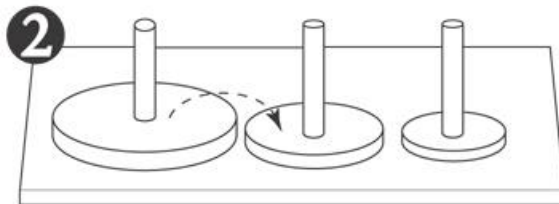
Moving Three Discs



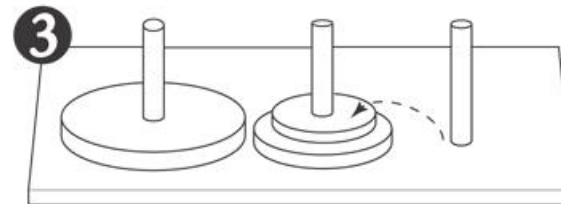
Original setup.



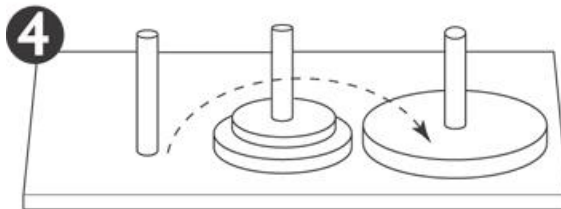
First move: Move disc 1 to peg 3.



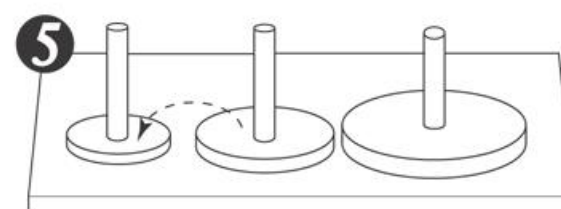
Second move: Move disc 2 to peg 2.



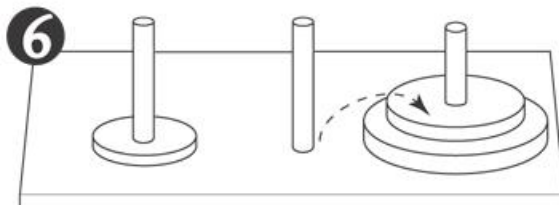
Third move: Move disc 1 to peg 2.



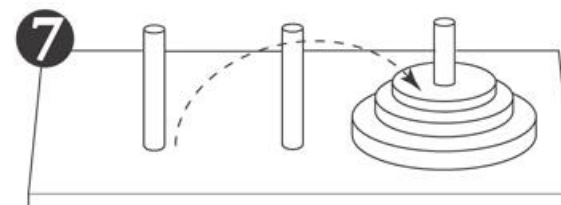
Fourth move: Move disc 3 to peg 3.



Fifth move: Move disc 1 to peg 1.



Sixth move: Move disc 2 to peg 3.



Seventh move: Move disc 1 to peg 3.

The Towers of Hanoi

- The following statement describes the overall solution to the problem:
 - *Move n discs from peg 1 to peg 3 using peg 2 as a temporary peg.*

The Towers of Hanoi

Algorithm

- *To move n discs from peg A to peg C, using peg B as a temporary peg:*

If $n > 0$ Then

Move $n - 1$ discs from peg A to peg B, using peg C as a temporary peg.

Move the remaining disc from the peg A to peg C.

Move $n - 1$ discs from peg B to peg C, using peg A as a temporary peg.

End If

Program 20-10

```
1 // This program displays a solution to the Towers of
2 // Hanoi game.
3 #include <iostream>
4 using namespace std;
5
6 // Function prototype
7 void moveDiscs(int, int, int, int);
8
9 int main()
10 {
11     const int NUM_DISCS = 3; // Number of discs to move
12     const int FROM_PEG = 1;  // Initial "from" peg
13     const int TO_PEG = 3;    // Initial "to" peg
14     const int TEMP_PEG = 2;  // Initial "temp" peg
15
16     // Play the game.
17     moveDiscs(NUM_DISCS, FROM_PEG, TO_PEG, TEMP_PEG);
18     cout << "All the pegs are moved!\n";
```

(program continues)

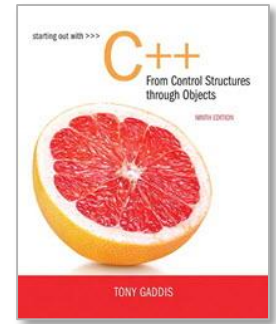
Program 20-10 *(continued)*

```
19         return 0;
20     }
21
22     //*****
23     // The moveDiscs function displays a disc move in *
24     // the Towers of Hanoi game.                      *
25     // The parameters are:                             *
26     // num: The number of discs to move.               *
27     // fromPeg: The peg to move from.                  *
28     // toPeg: The peg to move to.                     *
29     // tempPeg: The temporary peg.                    *
30     //*****
31
32     void moveDiscs(int num, int fromPeg, int toPeg, int tempPeg)
33     {
34         if (num > 0)
35         {
36             moveDiscs(num - 1, fromPeg, tempPeg, toPeg);
37             cout << "Move a disc from peg " << fromPeg
38                  << " to peg " << toPeg << endl;
39             moveDiscs(num - 1, tempPeg, toPeg, fromPeg);
40         }
41     }
```


Program 20-10 (Continued)

Program Output

```
Move a disc from peg 1 to peg 3
Move a disc from peg 1 to peg 2
Move a disc from peg 3 to peg 2
Move a disc from peg 1 to peg 3
Move a disc from peg 2 to peg 1
Move a disc from peg 2 to peg 3
Move a disc from peg 1 to peg 3
All the pegs are moved!
```

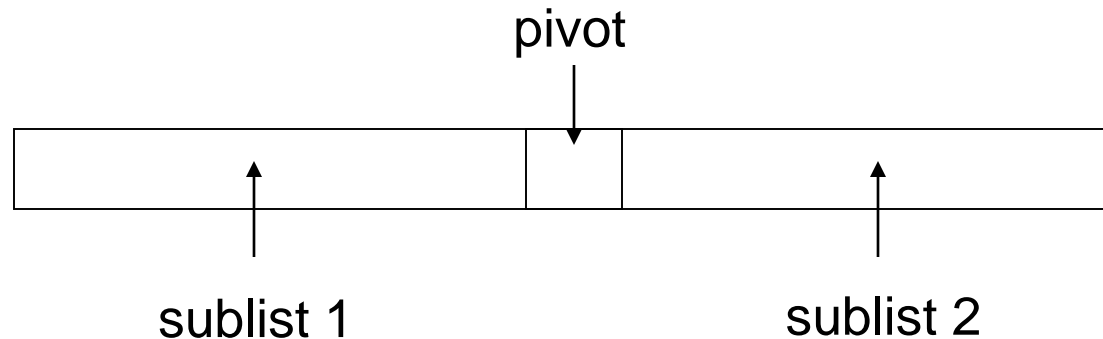


20.8

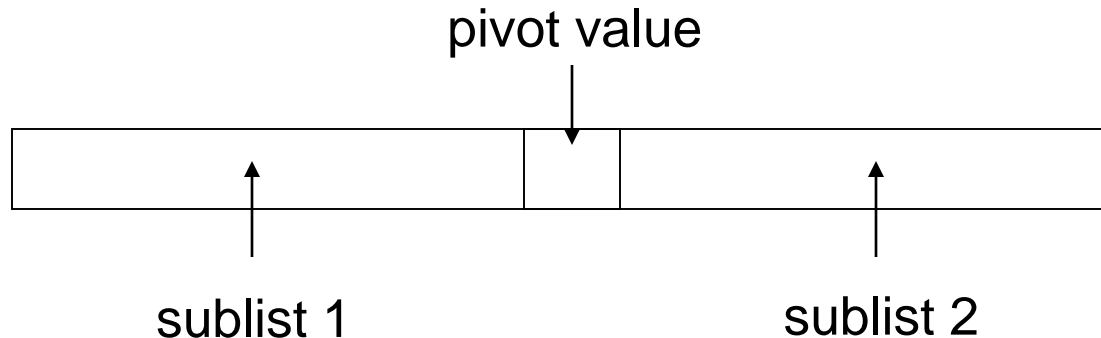
The QuickSort Algorithm

The QuickSort Algorithm

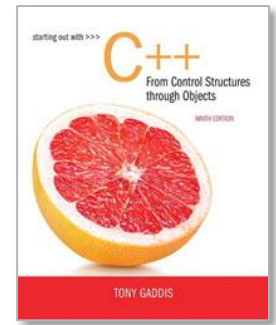
- Recursive algorithm that can sort an array or a linear linked list
- Determines an element/node to use as pivot value:



The QuickSort Algorithm



- Once pivot value is determined, values are shifted so that elements in sublist1 are $<$ pivot and elements in sublist2 are $>$ pivot
- Algorithm then sorts sublist1 and sublist2
- Base case: sublist has size 1



20.9

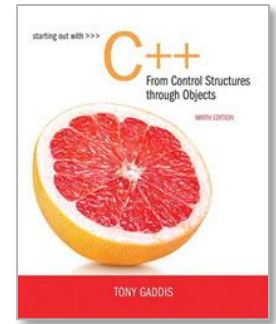
Exhaustive and Enumeration Algorithms

Exhaustive and Enumeration Algorithms

- Exhaustive algorithm: search a set of combinations to find an optimal one

Example: change for a certain amount of money that uses the fewest coins

- Uses the generation of all possible combinations when determining the optimal one.



20.10

Recursion vs. Iteration

Recursion vs. Iteration

- Benefits (+), disadvantages(-) for recursion:
 - + Models certain algorithms most accurately
 - + Results in shorter, simpler functions
 - May not execute very efficiently
- Benefits (+), disadvantages(-) for iteration:
 - + Executes more efficiently than recursion
 - Often is harder to code or understand