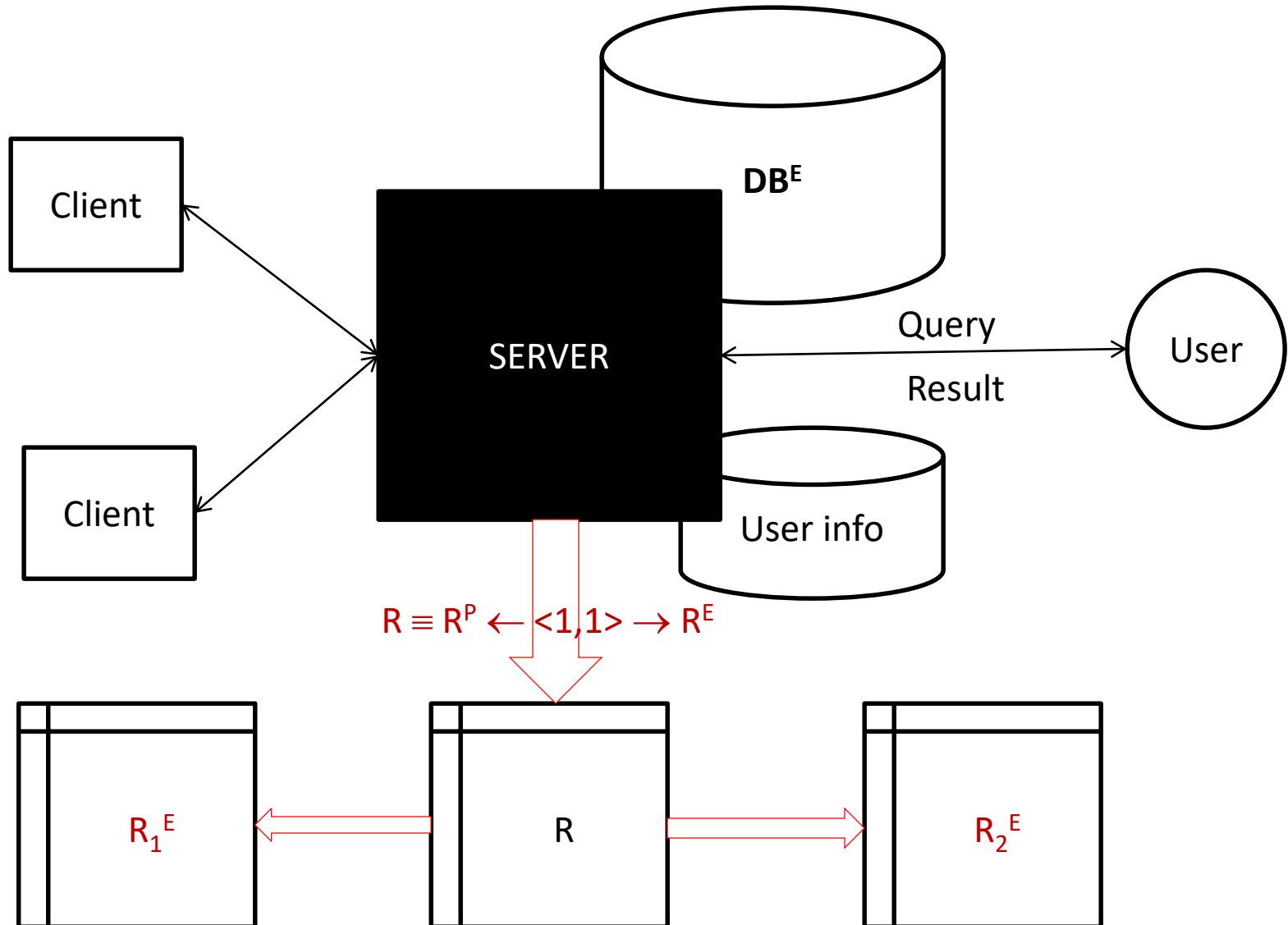


# Secure index for numeric data

## Lecture 8

# Risk model

Assume that client, user are trusted.



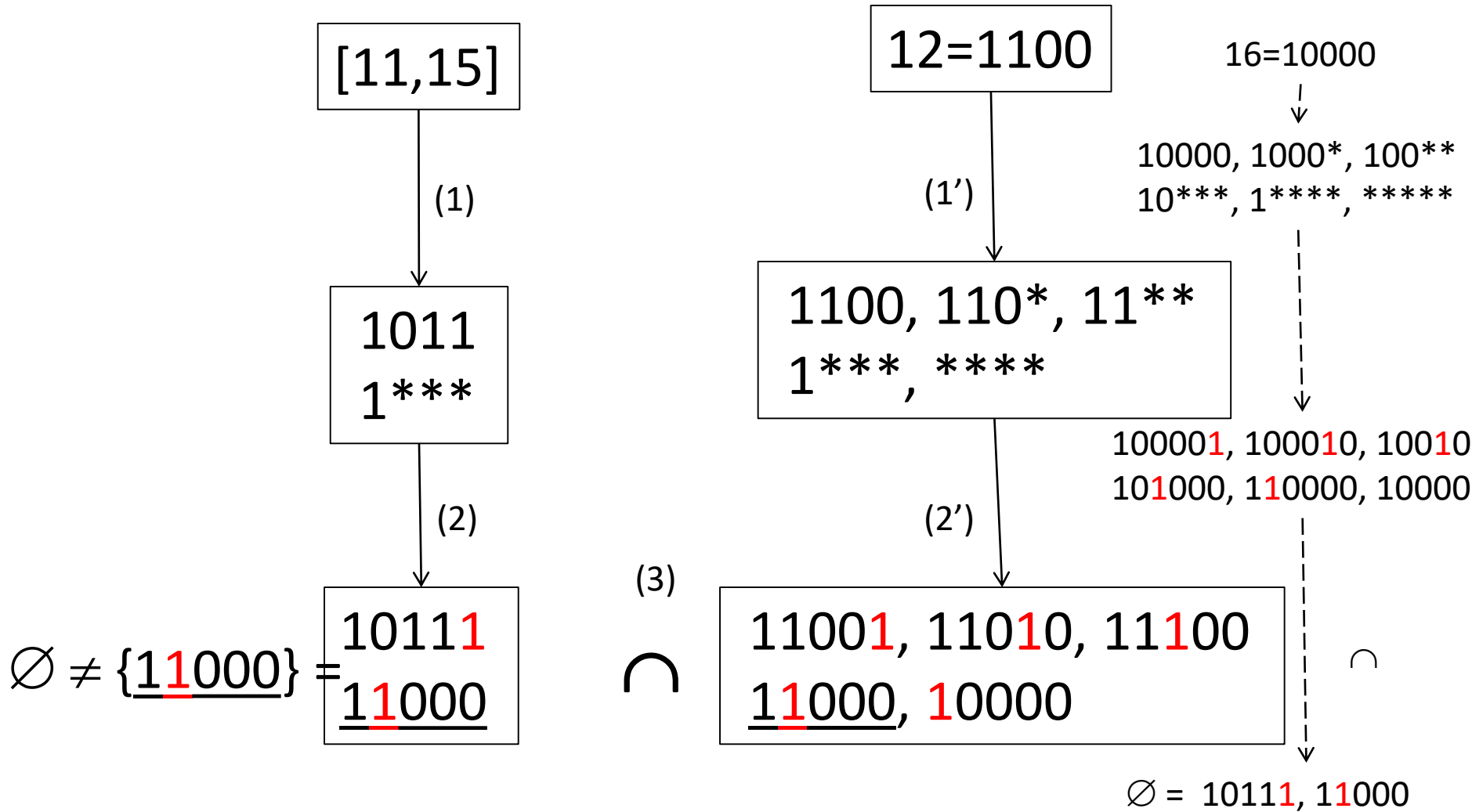
# Representing an integer

- k-prefix string: k high bits are fixed, and w-k low are free by pattern  $\{0/1\}^k \{*\}^{w-k}$ .
- k-bit string represents a set of  $2^{w-k}$  strings with the same high k bits. For example  $1^{**} = \{100, 101, 110, 111\}$ .
- Given  $x \in \mathbb{N}$ , a family of prefix for x is defined by  $F(x) = F(b_1 \dots b_w) = \{b_1 \dots b_{w-i+1}^{***}\}_i$ . For example  $F(12) = F(1100) = \{1100, 110^*, 11^{**}, 1^{***}, ^{****}\}$ .
- Given an integer x and a prefix P,  $x \in P \Leftrightarrow P \in F(x)$ .

# Representing a range

- $S([a,b])$  is the smallest set of prefix  $P_i$  such that  $\cup_i P_i = [a, b]$ . For example  $S([11,15])=\{1011,11^{**}\}$ .
- Given  $x$  and  $[a,b]$ ,  $x \in [a,b] \Leftrightarrow F(x) \cap S([a,b]) \neq \emptyset$ .
- Given prefix  $P$ .  $N(P)$  is bit string such that for all pair of prefix  $P1$  and  $P2$ ,  $P1=P2 \Leftrightarrow N(P1)=N(P2)$ .
- There are many definitions for  $N$ . This is a definition:  $N(b_1 \dots b_k^* \dots^*) = b_1 \dots b_k \mathbf{1}_{k+1} 0 \dots 0_{w+1}$ .
- Given  $x$  and  $[a,b]$ ,  $x \in [a,b] \Leftrightarrow N(F(x)) \cap N(S[a,b]) \neq \emptyset$ .

# Example: check if $12 \in [11, 15]$



# Protocols

## Submit data (t, list)

1. Sort(list):  $d_0 < d_1 < \dots < d_{n+1}$ .
2. Compute  $\{S[d_i, d_{i+1}]\}_{i=0, \dots, n}$ .
3. Compute  $\{N(S[d_i, d_{i+1}])\}_i$ .
4. Compute  $\{H(N(S[d_i, d_{i+1}]))\}_i$ .
5. Encrypt  $\{c_i = E(d_i)\}_i$ .
6. Send to server  $\{(c_i, H(N(S[d_i, d_{i+1}])))\}_i$ .

## Query $Q(t, [a, b])$ , $d_0 < a \leq b < d_{n+1}$ .

1. Compute  $F(a)$ ,  $F(b)$ .
2. Compute  $N(F(a))$ ,  $N(F(b))$ .
3. Compute  $H(N(F(a)))$ ,  $H(N(F(b)))$ .
4. Send  $\{t, H(N(F(a))), H(N(F(b)))\}$

## Query processing

(exercise)