RECURSION

Bùi Tiến Lên

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Contents

Recursion

Introduction to Recursion?

Concept 1

Recursion occurs when a function calls itself.

- Recursion can be **direct** (when the function calls itself) or **indirect** (when the function calls some other function that then calls the first function).
- Recursion can also be **single** (when the function calls itself once) or **multiple** (when the function calls itself multiple times).

Recursive Algorithm

When designing a recursive algorithm, We must identify

- The **base case**, which is the part of the problem that we can solve without recursion.
- The **recursive case**, or the part of the problem that we use recursion to solve.

Factorial

The factorial of a number n is written $n!$ and pronounced “N factorial.”

- The **base case**: if $n = 0$ then $factorial(0) = 1$
- The **recursive case**: if $n > 0$ then $factorial(n) = factorial(n-1) \times n$

```
int factorial(int n) {
  if(n==0)
    return 1;
  else
    return n * factorial(n-1);
}
```

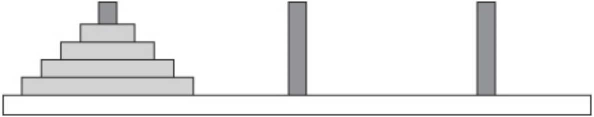
Fibonacci Numbers

- The Fibonacci numbers are defined by these equations:
  - $f_0 = 1$ and $f_1 = 1$
  - $f_n = f_{n-1} + f_{n-2}$ for $n > 1$

```
int fibonacci(int n) {
  if(n <= 1)
    return 1;
  else
    return fibonacci(n-1) + fibonacci(n-2);
}
```

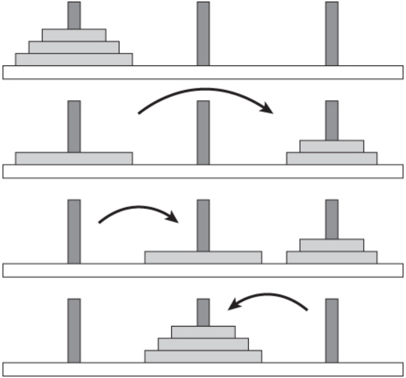
Tower of Hanoi

- The Tower of Hanoi puzzle has three pegs.
- One peg holds a stack of disks of different sizes, ordered from smallest to largest.
- You cannot place a disk on top of another disk that has a smaller radius.
- **The goal**: move disks from one peg to another without placing a disk on top of a smaller disk.



Tower of Hanoi (cont.)

- To move n disks, recursively move the upper $n-1$ disks to the temporary peg.
- Then move the remaining disk to the destination peg.
- Finally, move the $n-1$ upper disks from the temporary peg to the destination peg.



Tower of Hanoi (cont.)

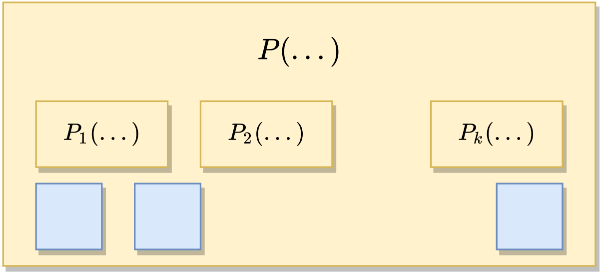
```c
void TowerOfHanoi(int n, char A, char B, char C)  {
  if(n==1)
    printf("Di chuyen dia tren cung tu %d den %d\n", A, C);
  else {
    TowerOfHanoi(n-1, A, C, B);
    printf("Di chuyen dia tren cung tu %d den %d\n", A, C);
    TowerOfHanoi(n-1, B, A, C);
  }
}
```

Divide-and-Conquer

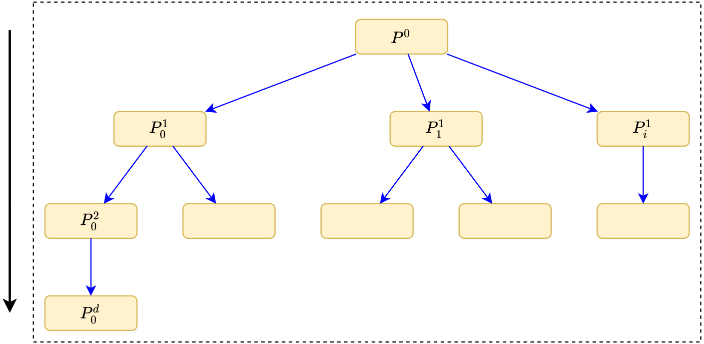
Divide-and-Conquer

- The **divide-and-conquer** approach employs this same strategy on an instance of a problem.
- It divides an instance of a problem into two or more smaller instances.
- The smaller instances are usually instances of the original problem, solves them recursively
  - If solutions to the smaller instances can be obtained readily, the solution to the original instance can be obtained by combining these solutions.
  - If the smaller instances are still too large to be solved readily, they can be divided into still smaller instances.
- The divide-and-conquer approach is a **top-down** approach.

Divide-and-Conquer (cont.)



Divide-and-Conquer (cont.)



Binary Search

Locates a key x in a sorted (nondecreasing order) array $\boldsymbol{a}$

- If x equals the middle item, quit. Otherwise:
  1. **Divide** the array into two subarrays about half as large. If x is smaller than the middle item, choose the left subarray. If x is larger than the middle item, choose the right subarray.
  2. **Conquer** (solve) the subarray by determining whether x is in that subarray. Unless the subarray is sufficiently small, use recursion to do this.
  3. **Obtain** the solution to the array from the solution to the subarray.

Binary Search (cont.)

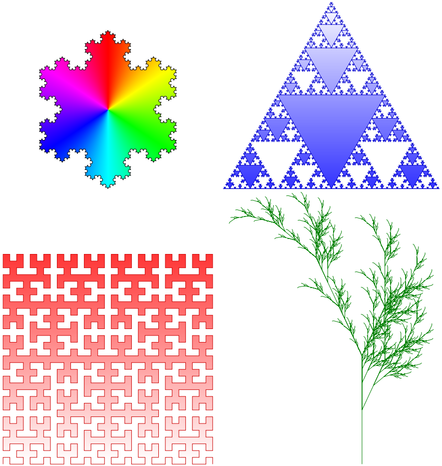
```
int binarySearch(int a[], int l, int r, int x) {
  int m = (l+r)/2;
  if(l>r) return -1;
  if(a[m]==x)
    return m;
  if(a[m]>x)
    return binarySearch(a, l, m-1, x);
  if(a[m]<x)
    return binarySearch(a, m+1, r, x);
}
```

Fractal

Fractal

Concept 2

Fractal is a *self-similar* geometry



Koch Curves

- A curve in which pieces of the small self-similar curve resemble the curve as a whole.

Base case:

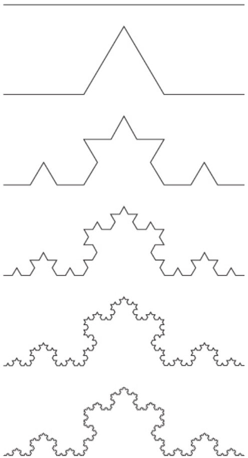
- start with an *initiator*, a curve that determines the basic shape.

Recursive case:

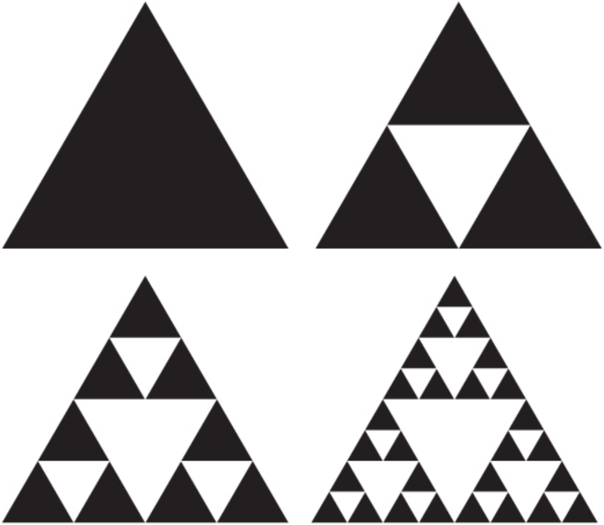
- At each level of recursion, some or all of the *initiator* is replaced by a suitably scaled, rotated, and translated version of a curve called a *generator*.

Koch Curves (cont.)

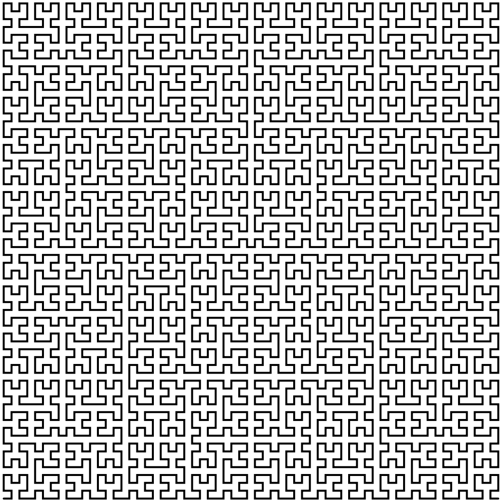
The Koch curve with levels of recursion 0 through 5 produces these shapes.



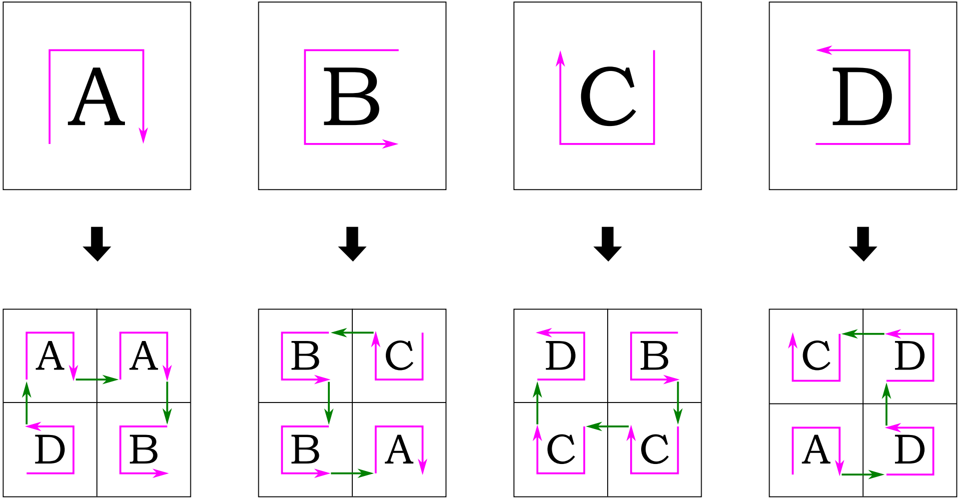
Sierpinski triangle



Hilbert Curves



Hilbert Curves



Workshop

Quiz

1. What is the recursion?

. . . .

. . . .

. . . .

Exercises

- Write a program that solves the Tower of Hanoi puzzle and then displays the moves by graphically drawing disks moving between the pegs

References

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