Case Study 1: Modeling T-33 Wing Vibrations

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1 Learning objectives

- **Connecting solid mechanics to vibration fundamentals:** This will involve determining the structural stiffness of a simplified wing structure using its geometric and material properties.
- **Conducting Systematic structural vibration Analysis:** The assignment will guide you through the typical procedure used in structural vibration analysis. You will gain an understanding of the importance of mathematically idealizing complex systems into simplified single-degree-of-freedom (SDoF) and multi-degree-of-freedom (MDoF) models prior to employing advanced finite element method (FEM) tools.
- **Realizing the advantages of energy methods over Newtonian mechanics:** You will use both Lagrangian and Newtonian mechanics to derive the equations of motion for a wing strucutre as an MDoF system. This will deepen your appreciation of Lagrangian analytical power and versatility.

1.1 Why Use Jupyter Notebook?

Jupyter Notebook serves as both a coding and reporting platform, enabling a seamless combination of computation, documentation, and visualization. It encourages clear commentary and explanation of your code in a structured, report-like format.

2 Expectations:

The project will be submitted as a Jupyter Notebook product. Throughout your notebook, briefly describe your approach, explain the underlying physics of your results. Your submission should reflect both sound technical reasoning and clarity in presentation. Figures without proparly lable axes, grids, title and legends will not be accepted. You will recieve 25 points for completing the assignment tasks on time. You will recieve an extra point for each figure from your model beyond the assignment requirment upto five points. However, you must explain why you provided the figure with an interpretation of your engineering findings.

3 Background

The wing of a typical fighter jet, such as the Lockheed T-33 Shooting Star or T-Bird, consists of a lightweight structural airframe with attachment mechanisms at the tip for carrying a mission-related payload or an additional fuel tank attached to the wing tip, denoted as m_p. In this case study, you will perform a dynamic analysis of this aircraft wing using various idealized lumped-parameter modeling approaches. For modeling purposes, the wing may be reasonably approximated as a cantilevered closed-rectagular-cell beam (hollow beam) with a concentrated tip mass. In some

tasks you will be asked to assume that the distributed structural mass of the beam, m_w is negligible relative to the tip mass, which is given as m_p = 100 kg. Each wing extends 5.0 m from the fuselage.

Idealizing the airfoil closed-rectagular-cell, assume the chord and height remain constant along the span, with values of 0.70 m and 0.25 m, respectively. Accordingly, the beam's external cross-sectional area is 0.70×0.25 m ^{2}. A uniform wall thickness of 0.010 m is assumed for the box-beam structure. The wing is fabricated from aluminum alloy 6061-T6. For the purposes of this assignment, the following material properties for aluminum 6061-T6 are obtained from matweb.com: Young's modulus, E=69 GPa, density ρ=2700 kg/m ^{2}, and Poisson's ratio ν=0.33.

4 Approach 0.1: Preliminary analysis (3 points)

(d) Estimate the effective tip displacements and stiffness due to vertical, F_z, axial, F_x, and tortional, F_θ, loads at the tip. Assume the loads are static and m_w is negligible.

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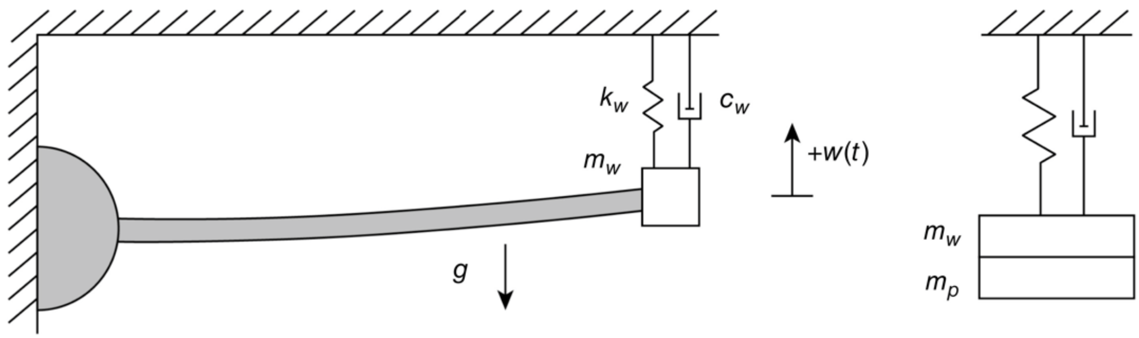
(d) Estimate the undamped natural frequencies for extension, bending and torsion of this idealized wing using the information provided above with and without m_w. Provide units in Hz and rad/s. Can w_w neglected?

Complete the above tasks under this section.

5 Approach 1.0: Free and impulse vibrations using SDoF modeling. (7 points)

Model the wing as an SDOF mass-spring-damper system. This time include the wing's mass in the analysis, m_w. Don't plug in any numbers. The wing stiffness and modal damping ratio are k_w, and ζ_w, respectively. In steady level flight, the wing will typically deflect vertically due to the aerodynamic forces and its own weight.



(e) Estimate the effective tip vertical deflection, $w(t)$, for $t \geq 0$, after m_p is released at $t = 0$. Assuming the aerodynamics forces do not change. Provide the damped and undamped resonance frequencies. Hint this is a free vibration problem.

(e) After releasing the payload, the aircraft encounters a gust wind at time $t = 0^+$. Idealize this change in the aerodynamic loads as an impulse force, F_i. Estimate the effective tip vertical deflection, $w(t)$, for $t \geq 0$.

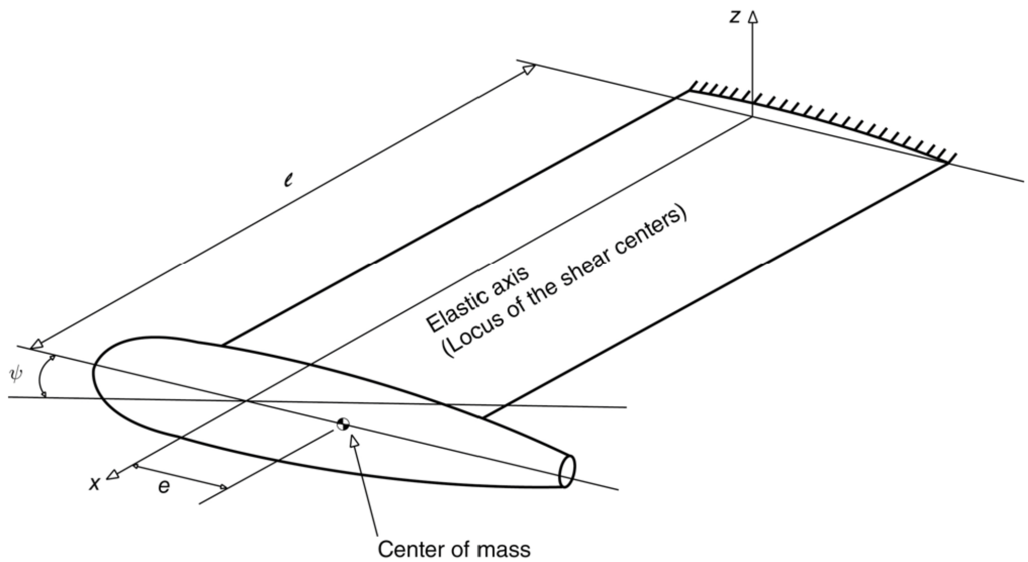
(e) Plot the deflections in both cases (a and b) as a function of time, t. Assume ζ_w is 1

(e) Provide the phase plane plots (velocity vs. displacement). What do you infer from the phase plane plots that we can see in task c.

Complete the above tasks under this section.

6 Approach 2.0: Equation of motion as an MDoF system (7 points)

In this approach, you will not assume that the payload m_p is a point mass. Accordingly, the contribution of m_p radius of gyration, κ, to the dynamics must be



considered. The center of m_p is located at e distance from elastic axis X. Assuming the wing experiences only vertical bending and twisting deformation and neglecting its mass, m_w, formulate the equations of motion using:

(a) Newtonian mechanics, and

(b) Lagrange's equations. Ignore damping for a and b

Complete the above tasks under this section.

7 Approach 3: Modal analysis of the wing as an MDoF system (8 points)

In this approach the wing deflection is modeled as a segmented beam into %n% masses. . Considering only vertical bending of the wing, formulate the equations of motion for n=5. Assume m_p is a point mass.

(a) Estimate the mode shapes with and without m_p.

(b) Estimate the flextural natural frequencies with and without m_p.

(c) Compare your findings with Approach 01.