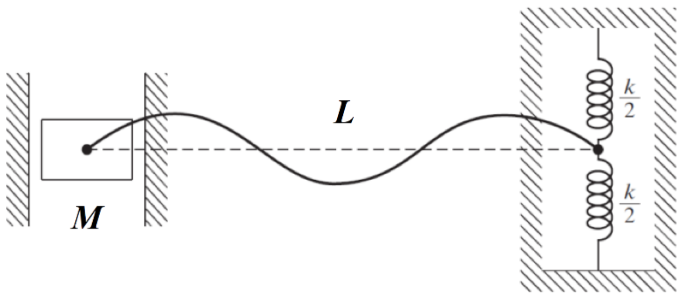
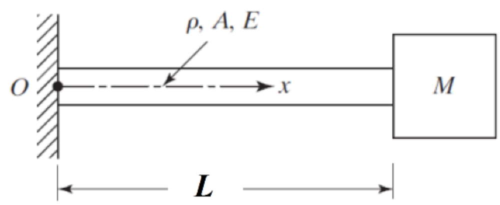
AE 543 Structural Vibrations HW #6, due 01 Dec 2025

1. A cable of length L and mass per unit length m is stretched under a tension P. One end of the cable is connected to a mass M, which can move in a frictionless slot, and the other end is fastened to a spring of stiffness k, as shown below. Derive the frequency equation for the transverse vibration of the cable.



2. A bar has one end fixed ($x = 0$) and a mass M attached at the other end ($x = L$). The rod has density ρ, cross section area A, and Young's modulus E.



a) Drive the equation of motion using Hamilton's principle. Hint: We did this in class.
b) Calculate and plot this structure first three natural frequencies as function of the ratio $\rho AL/M$.
c) Repeat (a) and (b) after adding a spring, k_a and a damper c_a at location $x = \frac{L}{2}$ to reduce the horizontal vibrations.

3. (Free vibration) A circular shaft of a helicopter power train has a uniform torsional stiffness GJ and polar mass moment of inertial I_{shaft} per unit length. The boundary at location at $x = 0$ is fixed and free at the end, $x = L$. Complete the following

(a) Derive the equation of motion and boundary conditions using Hamilton's principle

(b) Derive the exact eigenvalues and eigenfunctions shaft and plot the eigenfunctions for the first three modes.

(c) Repeat (a) and (b) after adding a round disk (in reality, it is a gear) at location $x = L$ with polar mass moment inertia I_{gear}. Hint: You have to solve and plot a transcendental equation.

4. Consider the fixed-free tapered bar, shown below and complete the following:

(a) Drive the characteristic equation of the beam. Hint: Same as the uniform beam derivation in class.
(b) Calculate the natural frequencies are given by:

(c) Show that the mode shapes are given by:

$$W_n(x) = C_{1n}\left[(\cos\beta_n x - \cosh\beta_n x) - \frac{\cos\beta_n l - \cosh\beta_n l}{\sin\beta_n l - \sinh\beta_n l}(\sin\beta_n x - \sinh\beta_n x)\right]$$

(d) Plot the first three mode shape

(e) Find the first three natural frequencies and plot their corresponding mode shapes using the Rayleigh-Ritz method and following three admissible functions:

$$\phi_1(x) = \frac{x}{L}, \qquad \phi_2(x) = \frac{x^2}{L^2}, \qquad \phi_3(x) = \frac{x^3}{L^3}$$