REU: Numerical optimal control

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Note 7

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1 Implicit method of simulation

In the implicit method, we have the following equations:

$$\delta_\tau q = \delta_t^2 q$$

$$\Leftrightarrow \frac{q(t,\tau+h)-q(t,\tau)}{h} = \frac{q(t+\delta,\tau+h)-2q(t,\tau+h)+q(t-\delta,\tau+h)}{\delta^2}$$

$$\Leftrightarrow q(t,\tau) = q(t,\tau+h) - \frac{h}{\delta^2}\left(q(t+\delta,\tau+h)-2q(t,\tau+h)+q(t-\delta,\tau+h)\right)$$

If we map $t \to i, \tau \to j$, we have:

$$q(i,j) = -\frac{h}{\delta^2}q(i+1,j+1) + \left(1-\frac{2h}{\delta^2}\right)q(i,j) - \frac{h}{\delta^2}q(i-1,j+1)$$

For $i = n-1$, we have:

$$q(i+1,j+1) = 1$$

$$\Rightarrow q(i,j) + \frac{h}{\delta^2} = \left(1-\frac{2h}{\delta^2}\right)q(i,j) - \frac{h}{\delta^2}q(i-1,j+1)$$

Implementation

Matlab code can be found here: `https://github.com/HoangT1215/Numerical-methods/blob/master/implicit_method.m`

2 Appendix

Keywords

- Tridiagonal linear system

References

- https://stackoverflow.com/questions/29583026/implementing-gradient-descent-algorithm-in-matlab

Questions

- How to perform differential with matrix entries
- Implement accelerated gradient descent with fictitious time method