REU: Numerical optimal control

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Note 3

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0.1 Review

0.1.1 The principle of least action

The principle of stationary action – is a variational principle that, when applied to the action of a mechanical system, can be used to obtain the equations of motion for that system.

Theorem. Let $S\left[t,q,q'\right] = \int_a^b \mathcal{L}(t,q,q')dt$, the principle implies that $\delta S = 0$

1 Accelerated gradient descent implementation

The source code is here (finite iteration implementation): github.com/HoangT1215/Numerical-methods/blob/master/Accelera

2 Numerical methods for DAE

The Euler-Lagrange equation can be considered as a DAE because it has the form of $F(x,y,y')$. In this section, we will look through a number of numerical methods to solve DAE.

2.1 Quasi-Newton methods

2.1.1 Motivation

In the Newton method, it involves computing the Hessian matrix and matrix multiplication in the recursive formula. Therefore, it could be extremely computational expensive.

Based on the ideas of the Newton method, the quasi-Newton methods will use some approximation of the Hessian matrix instead of computing the Hessian matrix itself.

2.1.2 Development branches

- Improve gradient descent method using quasi-Newton methods

2.2 Euler-Lagrange application

Method:

- Introduce a fictitious time variable
- Bring in the functional derivative and define a gradient using functional derivative $q(t,\tau)$
- $\frac{dq}{d\ tau} = \frac{d^2q}{dt^2}$
- Boundary conditions: $q(0,\tau) = 0, q(1,\tau) = 1$

- Evolve the function through time. As the time approaches infinity, we will arrive at the desired function. This method is **Fictitious Time Integration Method** (FTIM)

2.3 Keywords

- Finite difference, FEM
- Numerical methods for heat equation with boundary condition
- FTIM
- Functional gradient descent

3 Task

1. Start with $q(t, tau = 0) = t^3$. Try to see how q evolves in tau assuming q satisfies the PDE with boundary conditions. (Answer: $q(t, +\infty) = t$
2. Solve $\int_0^T (q''(t) + f(q(t)))^2 dt$. What is the solution to the E-L equation? (Hint: Use differential of f)

4 Appendix

- Semi-infinite programming $https://neos - guide.org/content/semi - infinite - programming$
- $http://www.imm.dtu.dk/documents/ftp/tr99/tr08_99.pdf$
- $https://pdfs.semanticscholar.org/9be4/bde12c32488acf7e6d0a169591a5ff7bbae1.pdf$
- Solving infinite dimensional optimization problem using polynomial approximation $https://perso.uclouvain.be/franco$ $final.pdf$
- Quasi-Newton methods $https://www.youtube.com/watch?v = 2eSrCuyPscg$
- FEM $https://www.iist.ac.in/sites/default/files/people/IN08026/FEM.pdf$
- https://math.stackexchange.com/questions/2341215/numerical-implementation-of-functional-derivatives
- $http://www.techscience.com/doi/10.3970/cmes.2008.034.155.pdf$ (Original FTIM paper by Liu and Atluri)