REU: Numerical optimal control

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Note 5

June 21, 2018

0.1 Review

0.1.1 Chain rule notation

$\delta S = \int_0^T \delta[q''^2]dt = \int_0^T 2q''\delta q'' dt$ (We can put the delta notation inside the integral)
Taking integration by parts, we have: $\int_0^T 2q''\delta q'' dt = \int_0^T -2q'''\delta q' dt = \int_0^T 2q'''' dt$

0.1.2 Chain rule for vector

$d(||x||_2^2) = d(x^T x) = (dx)^T x + x^T dx = \sum (dx)_i x_i + \sum x_i dx_i$

Example

Given $S[q] = \frac{1}{2}\int_0^T ||q' - f(q)||_2^2 dt$, we have:

$$\delta S = \int_0^T [q' - f(q)]^T \delta(q' - f(q))dt$$

$$= \int_0^T [q' - f(q)]^T \delta q' - [q' - f(q)]^T \delta f(q))dt$$

$$= \int_0^1 -[q' - f(q)]'^T \delta q - [q' - f(q)]^T f'(q)\delta q$$

$$= \int_0^1 [-q'' + f'(q)q']^T \delta q - [q' - f(q)]^T f'(q)\delta q$$

$$= \int_0^1 [-q''^T + q'^T f'(q)^T - q'^T f'(q) + f(q)^T f'(q)]\delta q = 0$$

From the fundamental lemma, we have:

$$-q''^T + q'^T f'(q)^T - q'^T f'(q) + f(q)^T f'(q) = 0$$

Taking transpose and using $(AB)^T = B^T A^T$, we have:

$$\frac{\delta S}{\delta q} = -q'' + [f'(q) - f'(q)^T]q' + f'(q)^T f(q) = 0$$

If $f(q) = \nabla U(q)$ for $U : \mathbb{R}^n \to \mathbb{R}$ then $f' = (f')^T$

0.1.3 Fictitious time method

Given $q(\tau, t)$

Step 1: t discretization

$$\delta_\tau q = \delta_t^2 q$$

$$\frac{\delta q}{\delta \tau} = \frac{q(i+1,j) - 2q(i,j) + q(i-1,j)}{\delta^2}$$

Step 2: τ discretization

$$\frac{q(i,j+1) - q(i,j)}{h} = \frac{q(i+1,j) - 2q(i,j) + q(i-1,j)}{\delta^2}$$

$$\Leftrightarrow q(i,j+1) = q(i,j) + h \times \frac{q(i+1,j) - 2q(i,j) + q(i-1,j)}{\delta^2}$$

We can proceed in two ways: explicit and implicit. For explicit method, we use finite difference to simulate the function. For implicit method, we generate n equations and solve it.

1 Task

Task 1

Consider $q' = q + u$, minimize $\int_0^T u^2 dt$ subject to $q(0) = 0, q(1) = 1$

Task 2

Consider two dimensional dynamics satisfy $x' = u_1, y' = u_2 - g$. Minimize $\int_0^1 (u_1^2 + u_2^2) dt$ such that $x(0) = 0, y(0) = 0, x(1) = 2, y(1) = 1$

Task 3

Simulate:

$$\delta_\tau q = \delta_t^2 q$$

Given

$$q(\tau, 0) = 0, q(\tau, 1) = 1, q(0, t) = \sin t$$

2 Appendix

Keywords

Exact differential

Changelog

- 16/06/2018 First version
- 21/06/2018 Fix the wrong notation and correct the proof for the control problem with arbitrary cost function