CS111 - Structure: Mathematical and Computational Models

Hoang Nguyen
Class 2.1 Dimensional analysis

September 16, 2017

1 Theory

Dimensional analysis is a method by which we deduce information about a phenomenon from the single premise that the phenomenon can be described by a dimensionally correct equation among certain variables.

Theorem A: Dimensionally consistent theorem. All of the terms in the equation must be dimensionally consistent

1.1 Methods of dimensional analysis

Dimensional analysis is a process to ensure dimensional consistency. It ensures that we used the right dimensions for the problem being modeled. The steps for dimensional analysis as follows:

- 1. Check the dimensions of all derived quantities to see that they are properly represented in terms of the chosen primary quantities and their dimensions.
- 2. Identify the proper dimensionless groups of variables.

1.2 Identify dimensionless groups

- **Basic method**: Formulate equation so that units cancel out in the equation.
- **Buckingham π theorem**:

Theorem B: Buckingham π theorem. A dimensionally homogeneous equation involving n variables in m primary or fundamental dimensions can be reduced to a single relationship among n m independent dimensionless products.

Proof. Given a system of n dimensional variables (with physical dimensions) in k fundamental (basis) dimensions, write the dimensional matrix M, whose rows are the fundamental dimensions and whose columns are the dimensions of the variables: the (i, j)th entry is the power of the i-th fundamental dimension in the j-th variable.

A dimensionless variable is a quantity with fundamental dimensions raised to the zeroth power (the zero vector of the vector space over the fundamental dimensions), which is equivalent to the kernel of this matrix. By the ranknullity theorem, a system of n vectors (matrix columns) in k linearly independent dimensions (the rank of the matrix is the number of fundamental dimensions) leaves a nullity, p, satisfying (p = n k), where the nullity is the number of extraneous dimensions which may be chosen to be dimensionless. □

1.3 Importance of dimensional analysis

- Helps identify the necessary variables and fundamental dimensions.
- Helps reach the desired function of the modeling problem.
- Dimensional analysis developed as an attempt to perform extended, costly experiments in a more organized, more efficient fashion.

2 Class notes

Modeling reality: The diagram demonstrates the relationship between modeling and reality. It also shows that these two worlds don't overlap, implying there are phenomena that cannot be properly modeled and modeling can happen outside the real world. Inside the modeling world, math modeling is just a small subset inside a larger range of modeling methods.

Dimensionally independent quantities are the quantities that form the base dimensions. Base dimensions build up the problem space and they cannot be represent by a combination of other existing variables/vectors. In linear algebra language, dimensionally independent quantities are equivalent to linearly independent rows of a matrix, and the number of such rows of matrix is called the rank of the matrix.

Discussion 1: Dimensionally homogeneous can be understood as the consistency of units among the terms of the equation. In the class discussion, we gave the example of the distance equation. Acceleration is similar to gravity (if we consider gravity as the acceleration of the free falling speed)

Activity 1: Use scaling arguments to solve idealized problems In the first problem, we are given the question of how many dimensionless quantities are presented in a right triangle.

Solution. From Buckingham π theorem, we have: there are 4 primary variables (the lengths of triangle sides and angle ϕ), and two base dimensions (sides and angles). Thus, in any dimensionally homogeneous equation involving triangle properties, we can present in exactly $4 - 2 = 2$ dimensionless products. □

We then apply dimensional analysis in proving Pythagoras theorem.

Activity 2: Apply dimensional analysis to solve fluid problems From the Buckingham π theorem, we have there are 5 primary variables and 3 main dimensions (distance, time, weight), thus there are 2 dimensionless quantities, which are shown in Precious' group notes.

In the debrief, I learn that dimensional analysis can help us identify the necessary variables for modeling. Furthermore, dimensional analysis ensure the dimensional homogenuity of the equation, which is vital for later calculations and mathematical manipulations.