

$$F = \neg (\exists y. \forall x. P(x, y) \rightarrow \forall x. \exists y. P(x, y)) \rightarrow \text{KM}$$

$$A \rightarrow B \equiv \neg A \vee B$$

$$= \neg (\neg (\exists y. \forall x. P(x, y)) \vee \forall x. \exists y. P(x, y))$$

$$= \exists y. \forall x. P(x, y) \wedge \neg (\forall x. \exists y. P(x, y))$$

De Morgan

$$= \exists y. \forall x. P(x, y) \wedge \neg \forall x. \exists y. P(x, y)$$

$$= \exists y. \forall x. P(x, y) \wedge \exists x. \neg \exists y. P(x, y)$$

$$= \text{---} \wedge \exists x. \forall y. \neg P(x, y)$$

$$= \text{---} \wedge \exists x'. \forall y'. \neg P(x', y')$$

$$= \exists y. \forall x. \exists x'. \forall y'. (P(x, y) \wedge \neg P(x', y'))$$

$$= \forall x. \exists x'. \forall y' (P(x, a) \wedge \neg P(x', y'))$$

$$= \forall x. \forall y' (P(x, a) \wedge \neg P(f(x), y'))$$

$$F = \neg (\exists y. \forall x. P(x, y) \rightarrow \forall x. \exists y. P(x, y))$$

$$= \neg (\neg (\exists y. \forall x. P(x, y)) \vee \forall x. \exists y. P(x, y))$$

$$= \exists y. \forall x. P(x, y) \wedge \neg \forall x. \exists y. P(x, y)$$

$$= \exists y. \forall x. P(x, y) \wedge \exists x'. \forall y'. \neg P(x', y')$$

$$= \exists y. \forall x. \exists x'. \forall y'. (P(x, y) \wedge \neg P(x', y'))$$

$$\forall x. \exists x'. \forall y' (P(x, a) \wedge \neg P(x', y'))$$

$$\forall x. \forall y' (P(x, a) \wedge \neg P(f(x), y'))$$

$$\{P(x, a)\} \wedge \neg P(f(x), y')$$