

Aufgabe 1

$$\min z = -10x_1 + 57x_2 + 9x_3 + 24x_4$$

$$\text{U.d.N. } \frac{1}{2}x_1 - \frac{11}{2}x_2 - \frac{5}{2}x_3 + 9x_4 + x_5 = 0$$

$$\frac{1}{2}x_1 - \frac{3}{2}x_2 - \frac{1}{2}x_3 + x_4 + x_6 = 0$$

$$x_1 + x_7 = 1$$

$$x_1, \dots, x_7 \geq 0$$

$$C^T = (-10, 57, 9, 24)$$

Tableau Tabelle

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	R
I	-z	-10	57	9	24	0	0	0	0
II	x_5	$\boxed{1/2}$	$-11/2$	$-5/2$	9	1	0	0	0
III	x_6	$1/2$	$-3/2$	$-1/2$	1	0	1	0	0
IV	x_7	1	0	0	0	0	0	1	1

Kleinste Wert = -10 $\Rightarrow x_1$ tritt in Basis ein

Quotient Kriterium $\min \left\{ \frac{0}{1/2}; \frac{0}{1/2}; \frac{1}{1} \right\} = \frac{0}{1/2} \Rightarrow x_5$ tritt aus

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	R
I + 10 II \rightarrow	-z	0	-53	-41	204	20	0	0	0
	x_1	1	-11	-5	18	2	0	0	0
III - $\frac{1}{2}$ II	x_6	0	$\boxed{4}$	2	-8	-1	1	0	0
IV - II	x_7	0	11	5	-18	-2	0	1	1

Kleinste Wert = -53 $\Rightarrow x_2$ tritt in Basis ein

Quotient Kriterium $\min \left\{ \frac{0}{4}; \frac{1}{11} \right\} = \frac{0}{4} \Rightarrow x_6$ tritt aus

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	R
I + 53 III	-z	0	0	$\boxed{-29/2}$	98	$27/4$	$53/4$	0	0
II + 11 III	x_1	1	0	$\boxed{1/2}$	-4	$-3/4$	$11/4$	0	0
	x_2	0	1	$1/2$	-2	$-1/4$	$1/4$	0	0
IV - 11 III	x_7	0	0	$-1/2$	4	$3/4$	$-11/4$	1	1

Kleinste Wert = $-29/2$ $\Rightarrow x_3$ tritt in Basis ein

Quotient Kriterium $\min \left\{ \frac{0}{1/2}; \frac{0}{1/2} \right\} = \frac{0}{1/2} \Rightarrow x_1$ tritt aus

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	R
$I + 29/2 II$	$-z$	29	0	0	-18	-15	93	0	0
	x_3	2	0	1	-8	$-3/2$	$11/2$	0	0
$III - \frac{1}{2} II$	x_2	-1	1	0	2	$1/2$	$-5/2$	0	0
$IV + \frac{1}{2} II$	x_7	1	0	0	0	0	0	1	1

Kleinste Wert = -18 $\Rightarrow x_4$ tritt in Basis ein
 Quotient Kriterium $\min \left\{ \frac{0}{2} \right\} = 0 \Rightarrow x_2$ tritt aus

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	R
$I + 18 III$	$-z$	20	9	0	0	$-21/2$	$141/2$	0	0
$II + 8 III$	x_3	-2	4	1	0	$1/2$	$-9/2$	0	0
	x_4	$-1/2$	$1/2$	0	1	$1/4$	$-5/4$	0	0
	x_7	1	0	0	0	0	0	1	1

Kleinste Wert = $-21/2 \Rightarrow x_5$ tritt in Basis ein
 Quotient Kriterium $\min \left\{ \frac{0}{1/2}, \frac{0}{1/4} \right\} = 0 \Rightarrow x_1$ tritt aus

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	R
$I + \frac{21}{2} II$	$-z$	-22	93	21	0	0	-24	0	0
	x_5	-4	8	2	0	1	-9	0	0
$III - 1/4 II$	x_4	$1/2$	$-3/2$	$-1/2$	1	0	1	0	0
	x_7	1	0	0	0	0	0	1	1

Kleinste Wert = -24 $\Rightarrow x_6$ tritt in Basis ein
 Quotient Kriterium $\min \left\{ \frac{0}{1} \right\} = 0 \Rightarrow x_4$ tritt aus

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	R
$I + 24 III$	$-z$	-10	57	9	24	0	0	0	0
$II + 9 III$	x_5	$1/2$	$-11/2$	$-5/2$	9	1	0	0	0
	x_4	$1/2$	$-3/2$	$-1/2$	1	0	1	0	0
	x_7	1	0	0	0	0	0	1	1

\Rightarrow Zyklus
 - durchläuft \textcircled{B}

Aufgabe 2: Band's rule

$$\begin{aligned} \min z &= -10x_1 + 57x_2 + 9x_3 + 24x_4 \\ \text{U.d.N. } \frac{1}{2}x_1 - \frac{11}{2}x_2 - \frac{5}{2}x_3 + 9x_4 + x_5 &= 0 \\ \frac{1}{2}x_1 - \frac{3}{2}x_2 - \frac{1}{2}x_3 + x_4 + x_6 &= 0 \\ x_1 + x_7 &= 1 \\ x_1, \dots, x_7 &\geq 0 \end{aligned}$$

Tableau Tabelle

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	R
I	-z	-10	57	9	24	0	0	0	0
II	x_5	$\boxed{1/2}$	$-11/2$	$-5/2$	9	1	0	0	0
III	x_6	$1/2$	$-3/2$	$-1/2$	1	0	1	0	0
IV	x_7	1	0	0	0	0	0	1	1

I + 10 II →

III - $\frac{1}{2}$ II

IV - II

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	R
	-z	0	-53	-41	204	20	0	0	0
	x_1	1	-11	-5	18	2	0	0	0
	x_6	0	$\boxed{4}$	2	-8	-1	1	0	0
	x_7	0	11	5	-18	-2	0	1	1

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	R
	-z	0	0	-29/2	98	27/4	53/4	0	0
	x_1	1	0	$\boxed{1/2}$	-4	-3/4	1/4	0	0
	x_2	0	1	1/2	-2	-1/4	1/4	0	0
	x_7	0	0	-1/2	4	3/4	-1/4	1	1

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	R
	-z	29	0	0	-18	-15	93	0	0
	x_3	2	0	1	-8	-3/2	11/2	0	0
	x_2	-1	1	0	$\boxed{2}$	1/2	-5/2	0	0
	x_7	1	0	0	0	0	0	1	1

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	R
	-z	20	9	0	0	-21/2	141/2	0	0
	x_3	-2	4	1	0	$\boxed{1/2}$	-3/2	0	0
	x_4	-1/2	1/2	0	1	1/4	-5/4	0	0
	x_7	1	0	0	0	0	0	1	1

$$I + \frac{21}{2} II$$

$$III - 1/4 II$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	R
- z	-22	93	21	0	0	-24	0	0
x_5	-4	8	2	0	1	-9	0	0
x_4	$\boxed{1/2}$	$-3/2$	$-1/2$	1	0	1	0	0
x_7	1	0	0	0	0	0	1	1



hier in der 3. Spalte
habt ihr müsste -2, -1
stehen statt andersrum

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	R
- z	0	27	-1	44	0	20	0	0
x_5	0	-4	-1	8	1	-1	0	0
x_1	1	-3	-2	2	0	2	0	0
x_7	0	3	$\boxed{1}$	-2	0	-2	1	1



richtig weitergerechnet

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	R
- z	0	30	0	42	0	18	1	1
x_5	0	2	0	4	1	-5	2	2
x_1	1	0	0	0	0	0	1	1
x_3	0	3	1	-2	0	-2	1	1



Optimale Basis = $\{x_5, x_1, x_3\}$ mit $z = -1$

4,5/5

Optimallösung $\hat{x} = \{1, 0, 1, 0, 2\}$ als Vektor schreiben

Aufgabe 3

$$a_1 x_1 + a_2 x_2 + \dots + a_n x_n = b \quad a \in \mathbb{R}^n$$

$$\text{basis } \{i_1\} \text{ mit Startbasis } B = a_{i_1} \in \mathbb{R}^{n \times n} \text{ mit } a \neq 0, B^{-1} = \frac{1}{a_{i_1}}$$

$$\text{ZBL } x_b = B^{-1} \cdot b = \frac{1}{a_{i_1}} \cdot b$$

$$y^T = c_{i_1} \cdot \frac{1}{a_{i_1}} \quad \hat{c}^1 = \underbrace{(c_1, \dots, c_{i_1-1}, c_{i_1}, c_{i_1+1}, \dots, c_n)}_c - c_{i_1} \frac{1}{a_{i_1}} \underbrace{(a_1, \dots, a_{i_1-1}, a_{i_1}, a_{i_1+1}, \dots, a_n)}_a$$

$$\text{Sei } \hat{c}_{i_2}^1 < 0 \quad (\text{sonst optimal}) \quad \text{mit } \hat{A}_{i_2} = B^{-1} A_2 = \frac{1}{a_{i_1}} \cdot a_{i_2} \geq 0$$