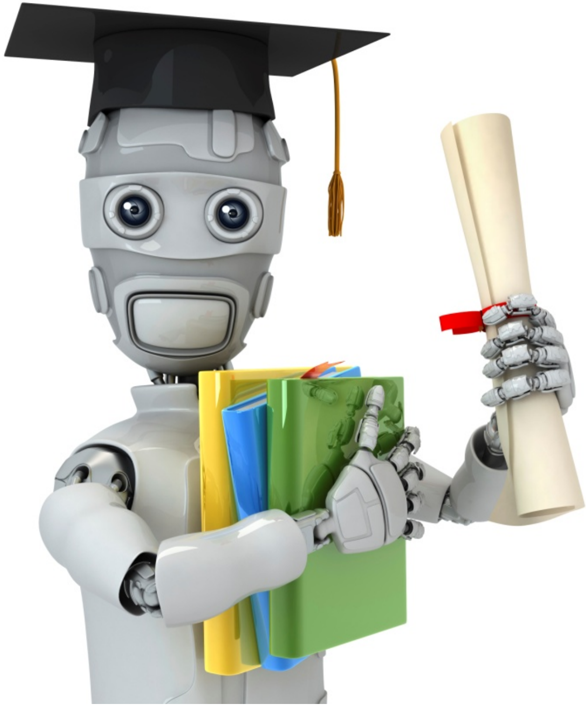
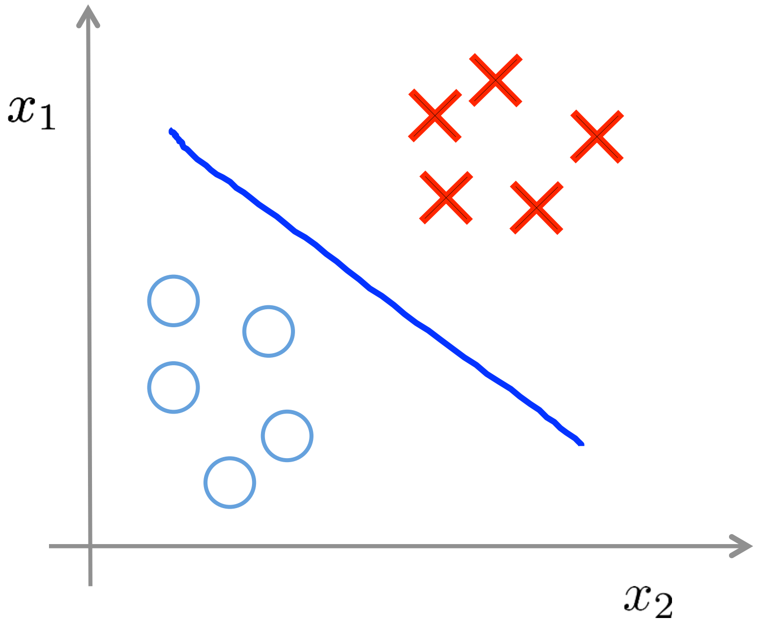


Machine Learning

Clustering

Unsupervised learning introduction

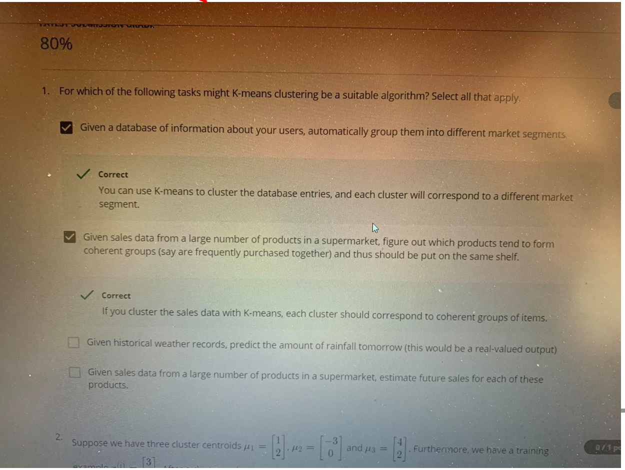
Supervised learning



Training set: $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), (x^{(3)}, y^{(3)}), \ldots, (x^{(m)}, y^{(m)})\}$

Unsupervised learning

Unsupervised learning **application**



80%

1. For which of the following tasks might K-means clustering be a suitable algorithm? Select all that apply.

- [x] Given a database of information about your users, automatically group them into different market segments.

  ✓ Correct
  You can use K-means to cluster the database entries, and each cluster will correspond to a different market segment.

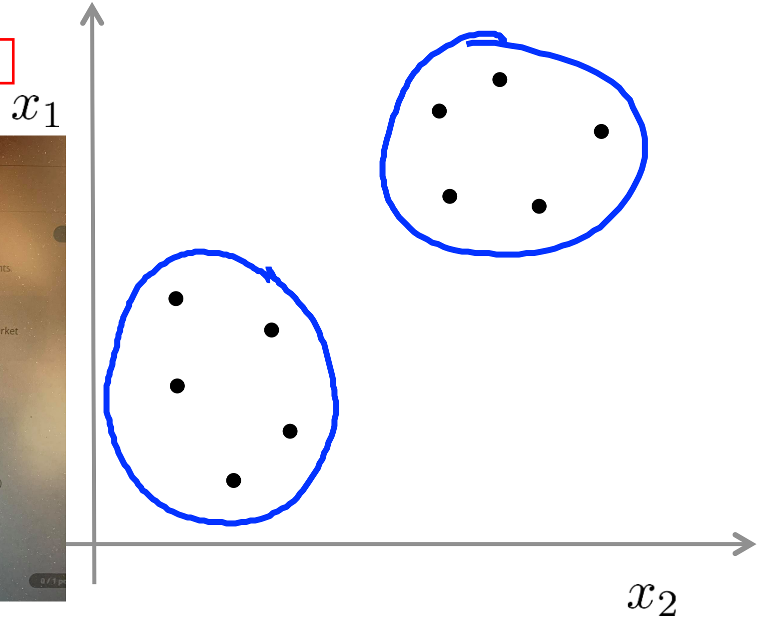
- [x] Given sales data from a large number of products in a supermarket, figure out which products tend to form coherent groups (say are frequently purchased together) and thus should be put on the same shelf.

  ✓ Correct
  If you cluster the sales data with K-means, each cluster should correspond to coherent groups of items.

- [ ] Given historical weather records, predict the amount of rainfall tomorrow (this would be a real-valued output)

- [ ] Given sales data from a large number of products in a supermarket, estimate future sales for each of these products.

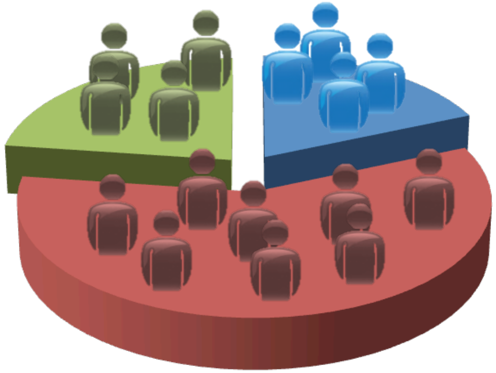
2. Suppose we have three cluster centroids $\mu_1 = \begin{bmatrix}1\\2\end{bmatrix}$, $\mu_2 = \begin{bmatrix}-3\\0\end{bmatrix}$ and $\mu_3 = \begin{bmatrix}4\\2\end{bmatrix}$. Furthermore, we have a training



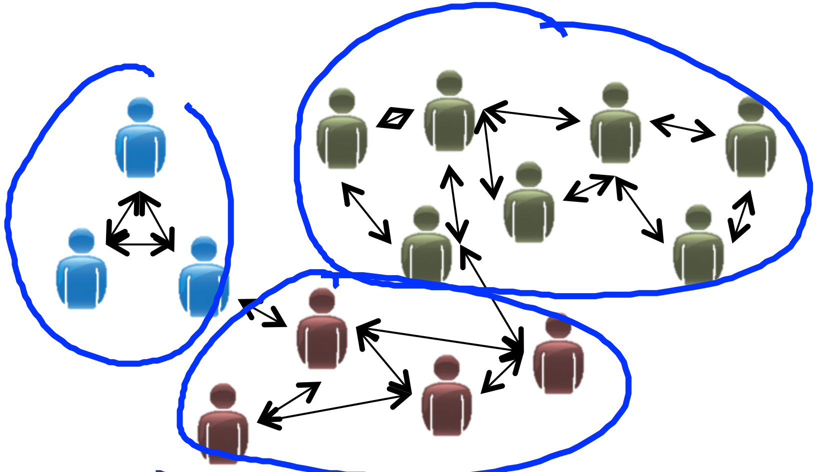
Clustering algorithm

Training set: $\{\underline{x^{(1)}}, \underline{x^{(2)}}, x^{(3)}, \ldots, \underline{x^{(m)}}\} \leftarrow$

Applications of clustering



Market segmentation



Social network analysis

Organize computing clusters

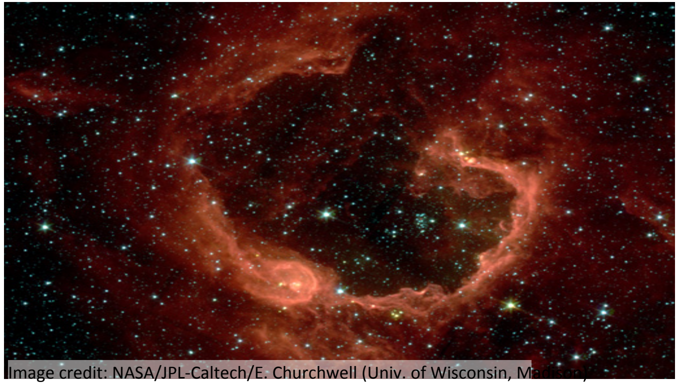
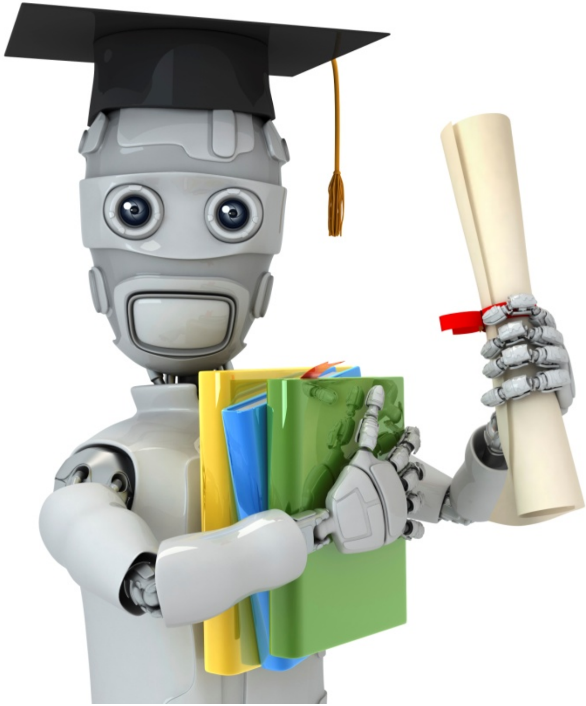


Image credit: NASA/JPL-Caltech/E. Churchwell (Univ. of Wisconsin, Madison)

Astronomical data analysis

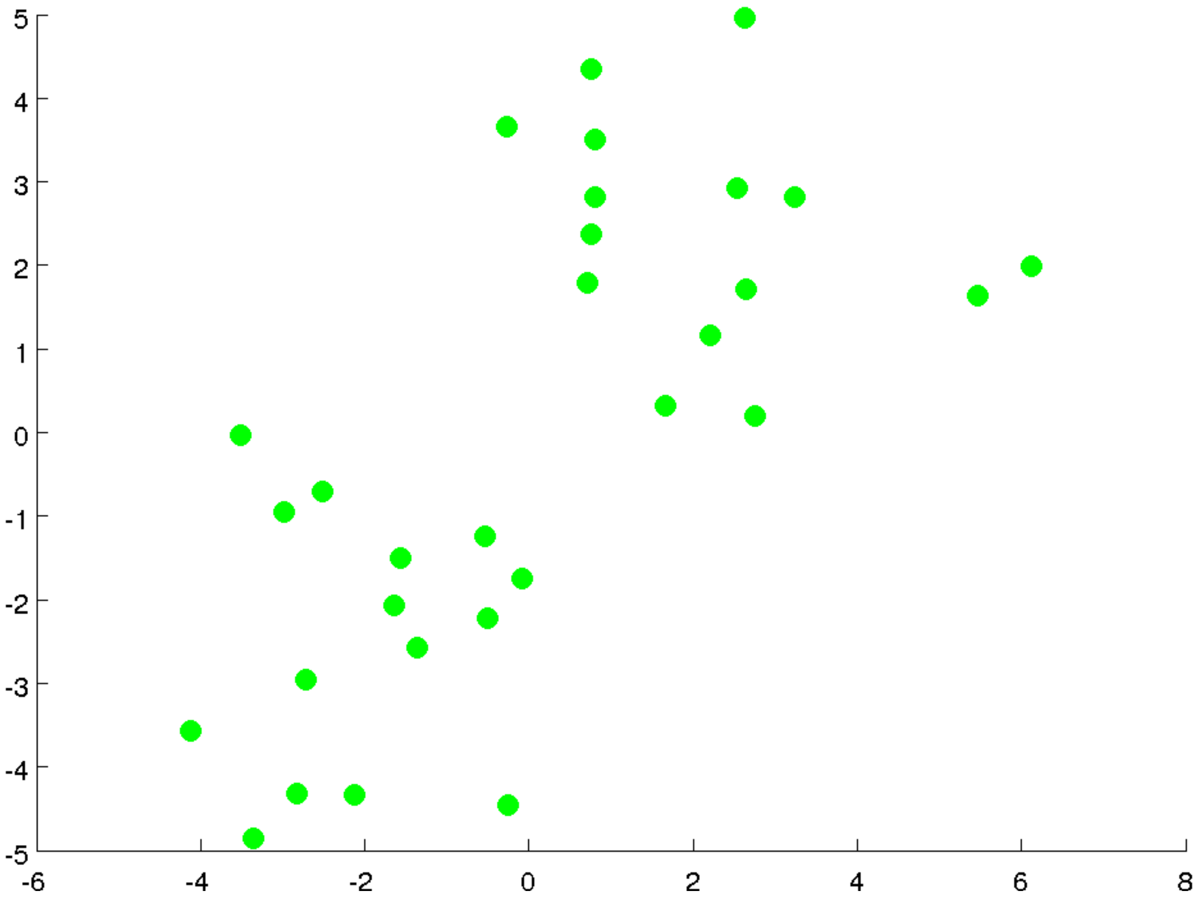
Andrew Ng



Machine Learning

Clustering

K-means algorithm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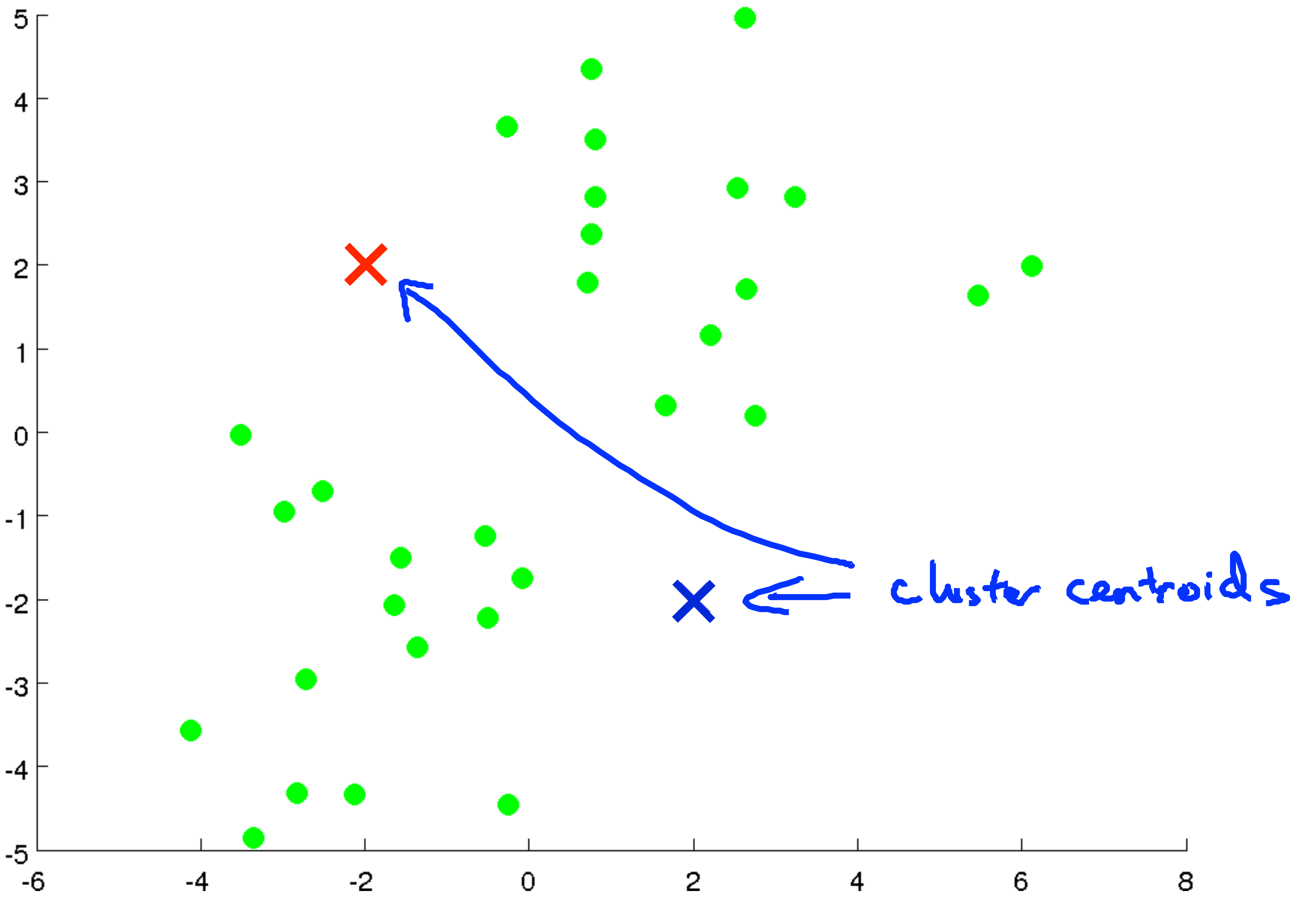

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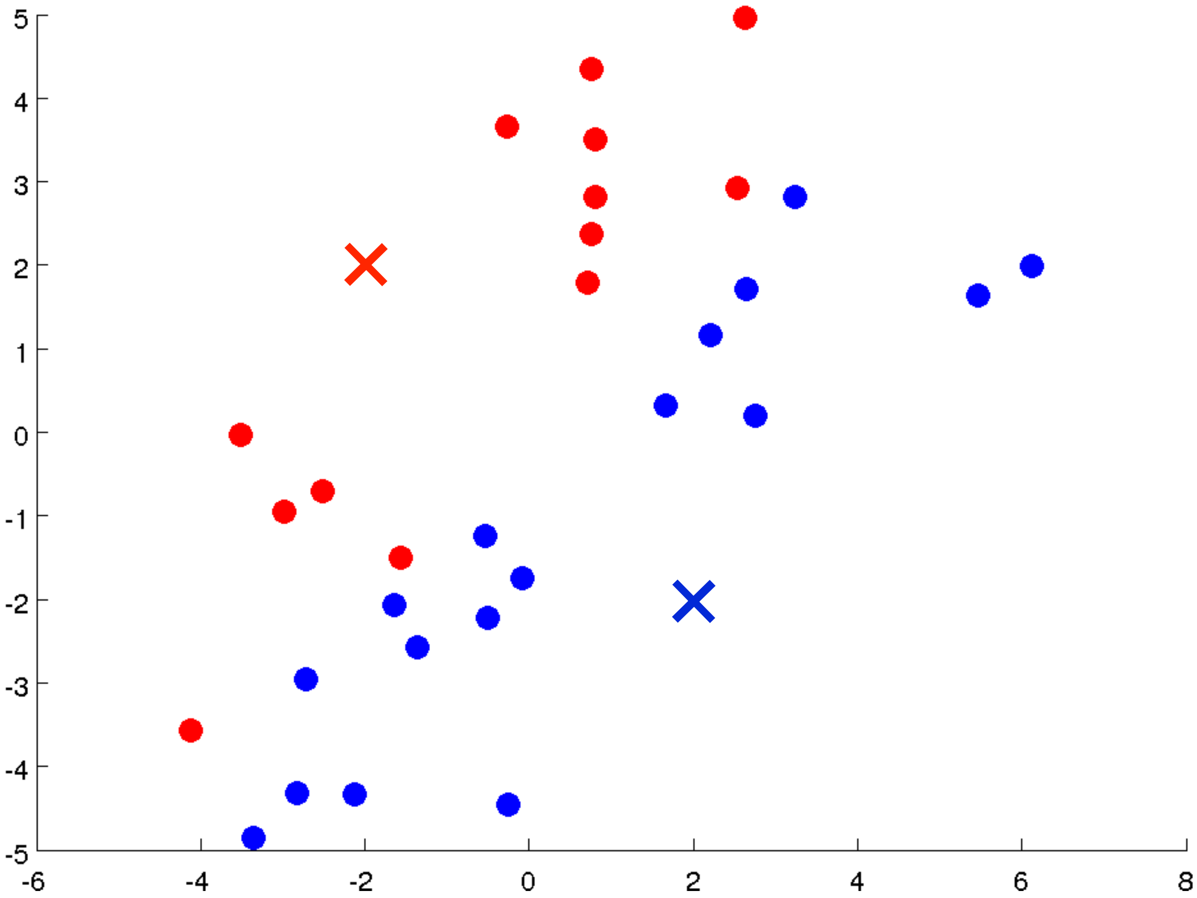
cluster centroids



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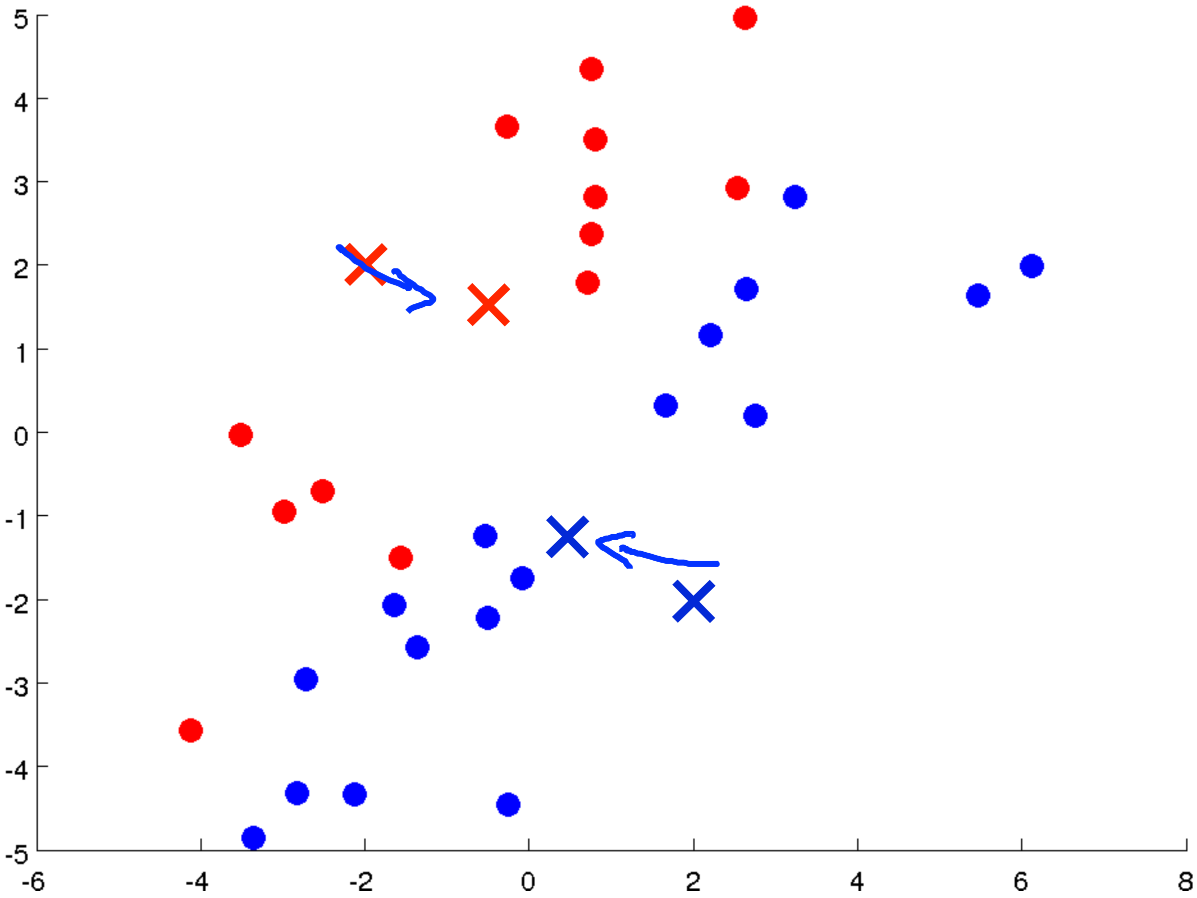

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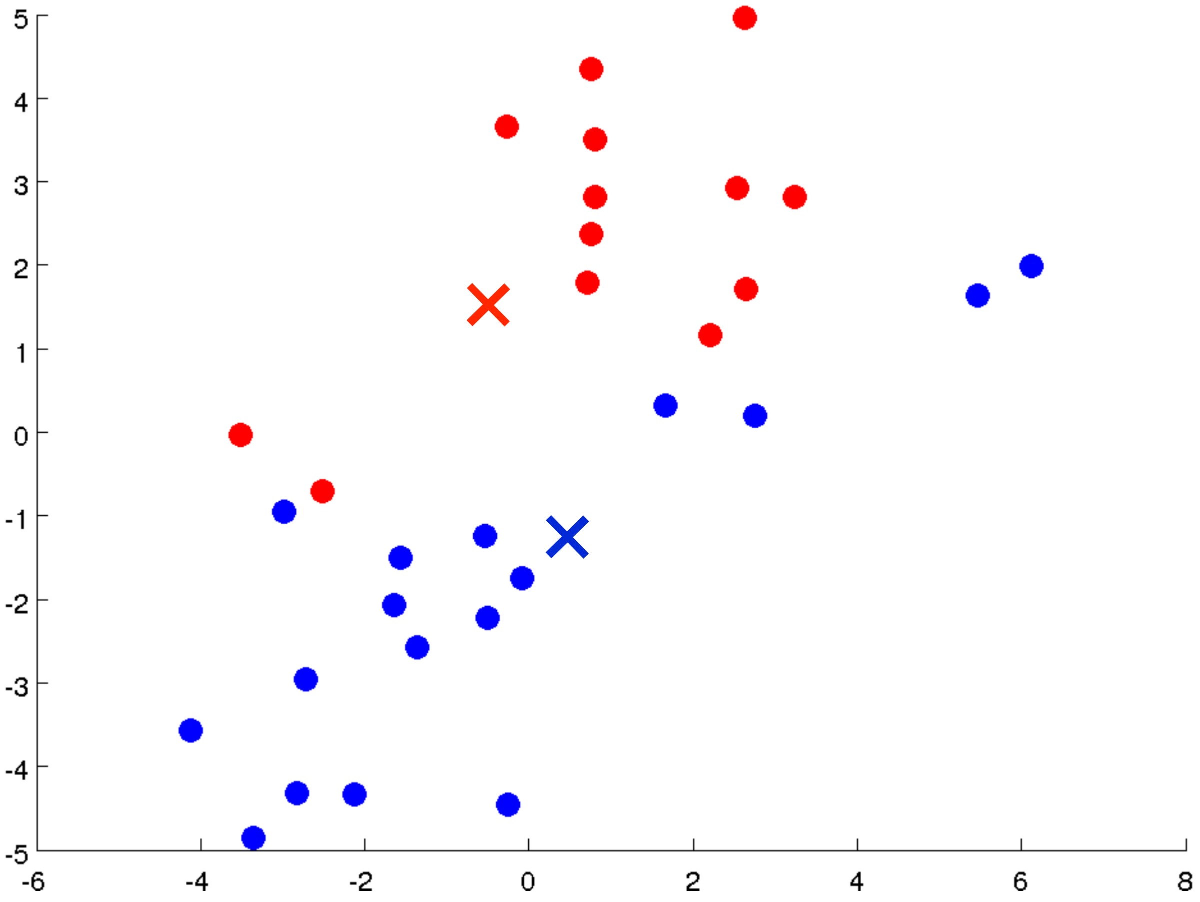

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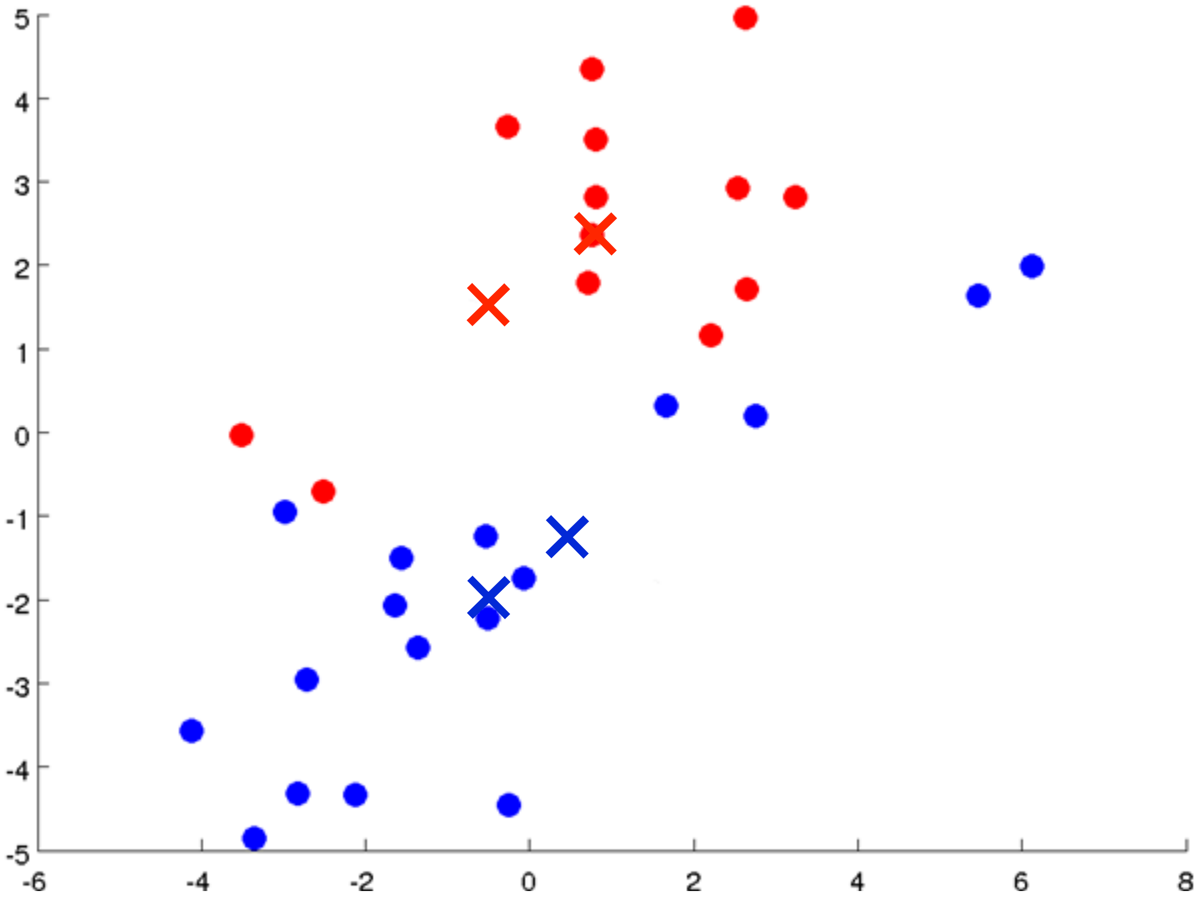

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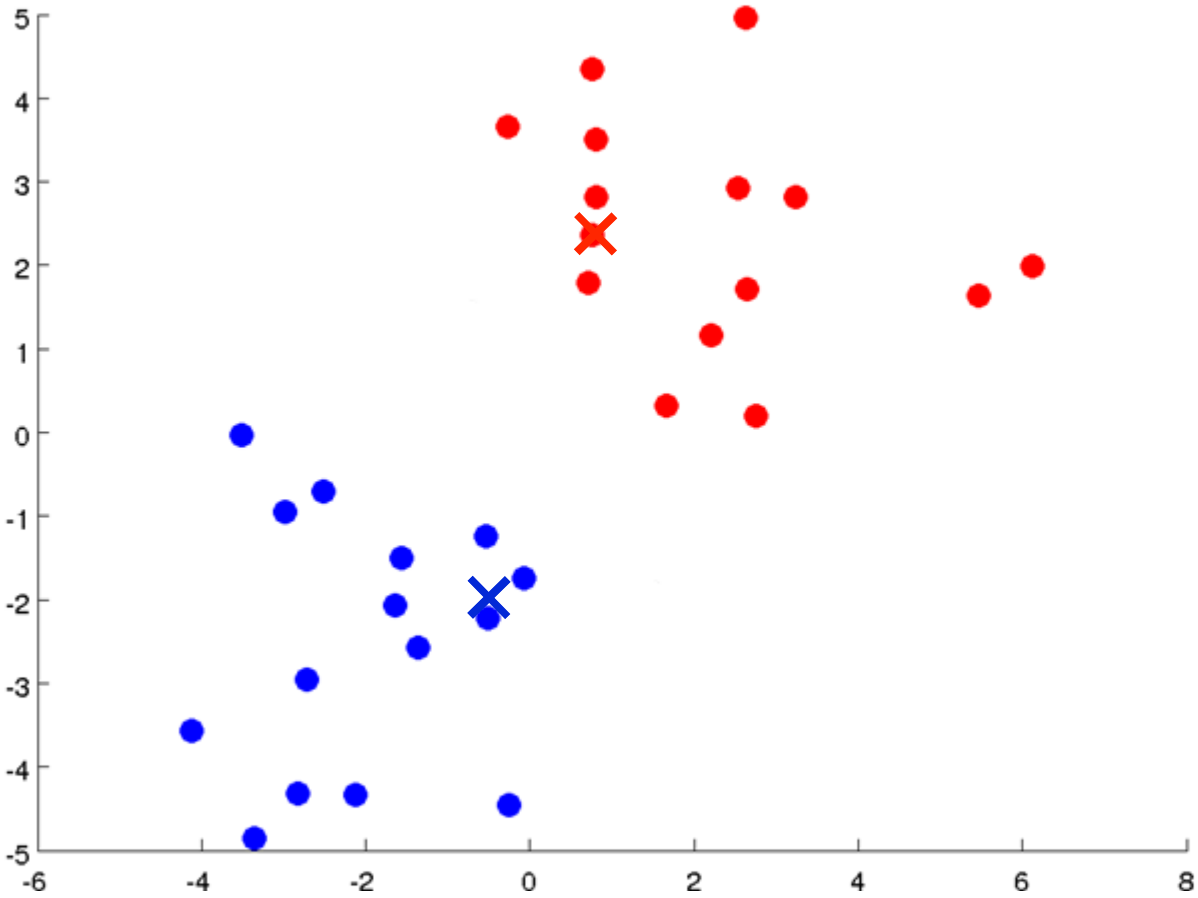
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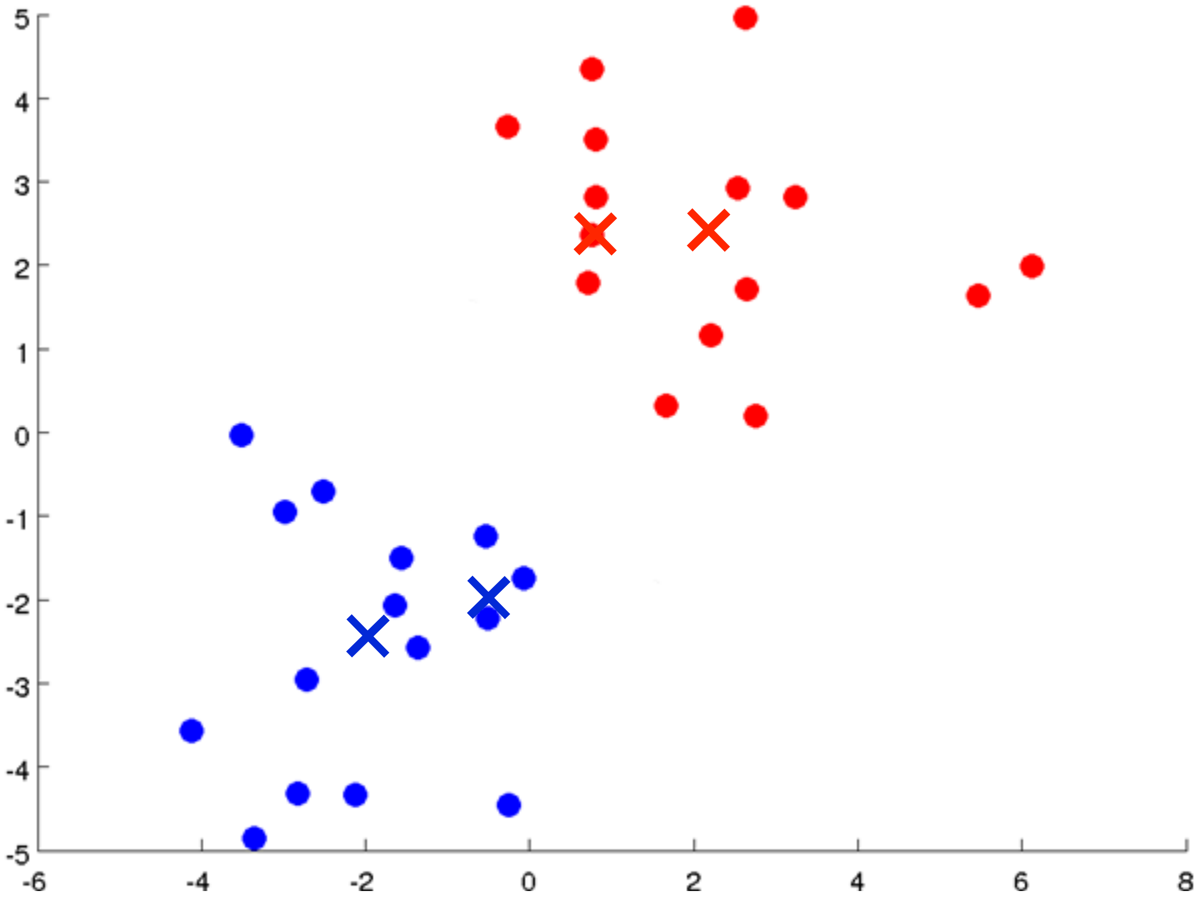

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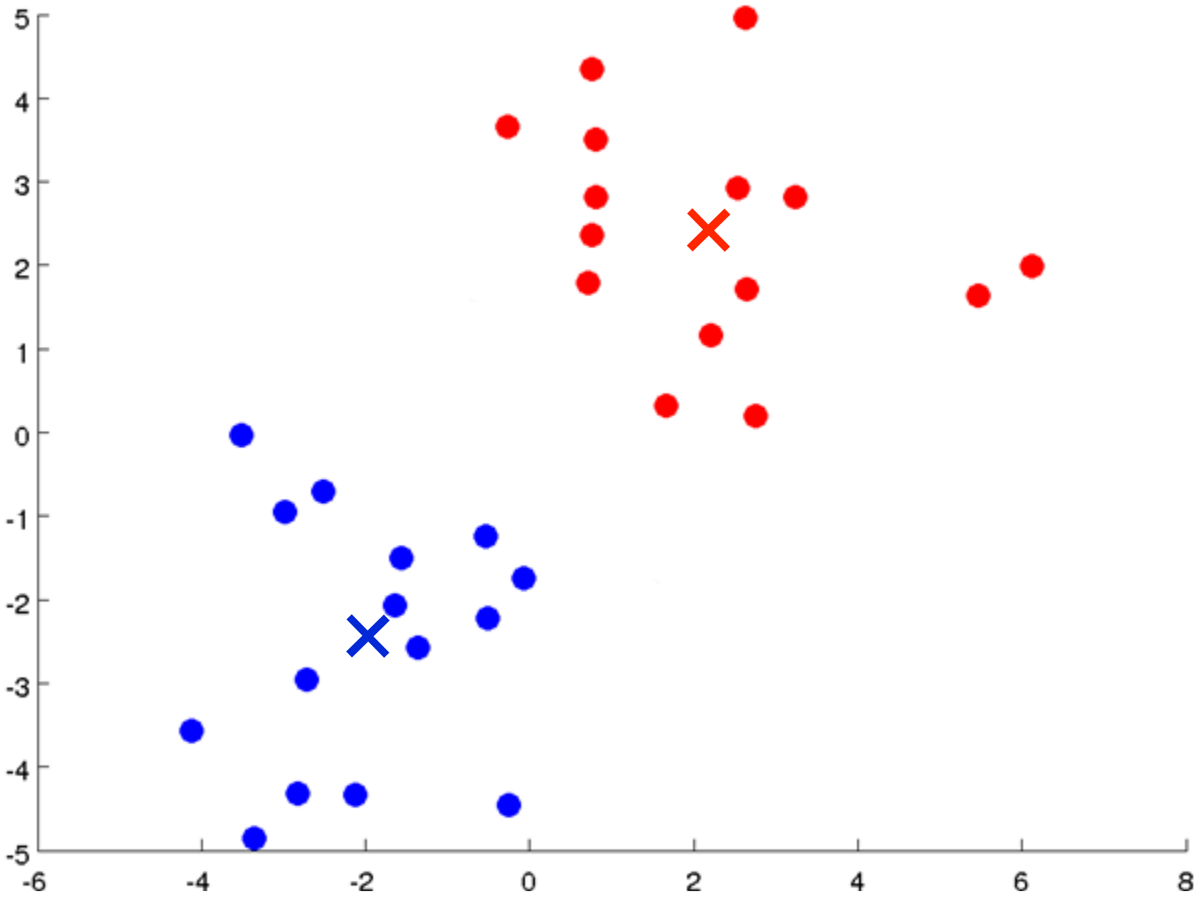

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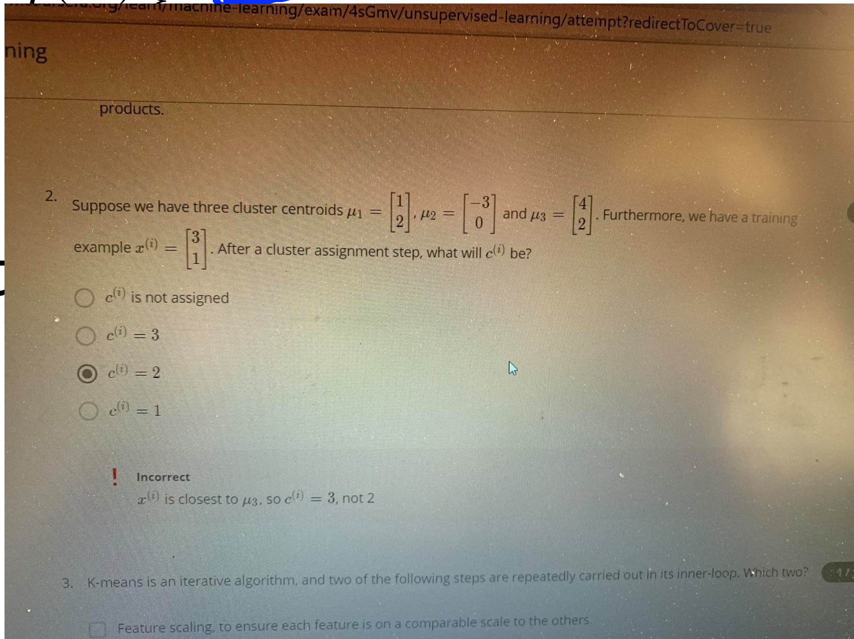
K-means algorithm

Input:

- K (number of clusters)
- Training set $\{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}$

Way to ***determine c(i)***

$x^{(i)} \in \mathbb{R}^n$ (drop $x_0 = 1$ convent



K-means algorithm

μ_1 μ_2

Randomly initialize K cluster centroids $\mu_1, \mu_2, \ldots, \mu_K \in \mathbb{R}^n$

Repeat {

Cluster assignment step

for i = 1 to m

$c^{(i)}$:= index (from 1 to K) of cluster centroid closest to $x^{(i)}$

$\min_k \|x^{(i)} - \mu_k\|^2 \rightarrow c^{(i)}$

Move centroid

for k = 1 to K

μ_k := average (mean) of points assigned to cluster k

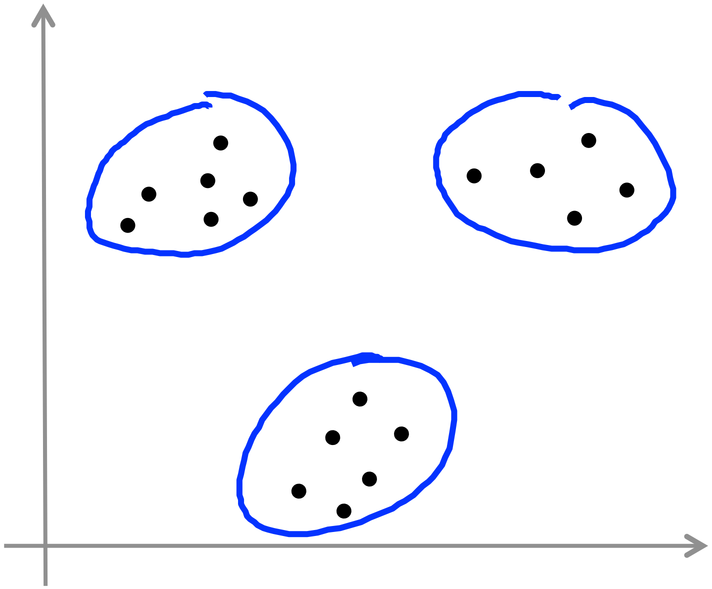
$x^{(1)}, x^{(5)}, x^{(6)}, x^{(10)} \rightarrow c^{(1)}=2, c^{(5)}=2, c^{(6)}=2, c^{(10)}=2$

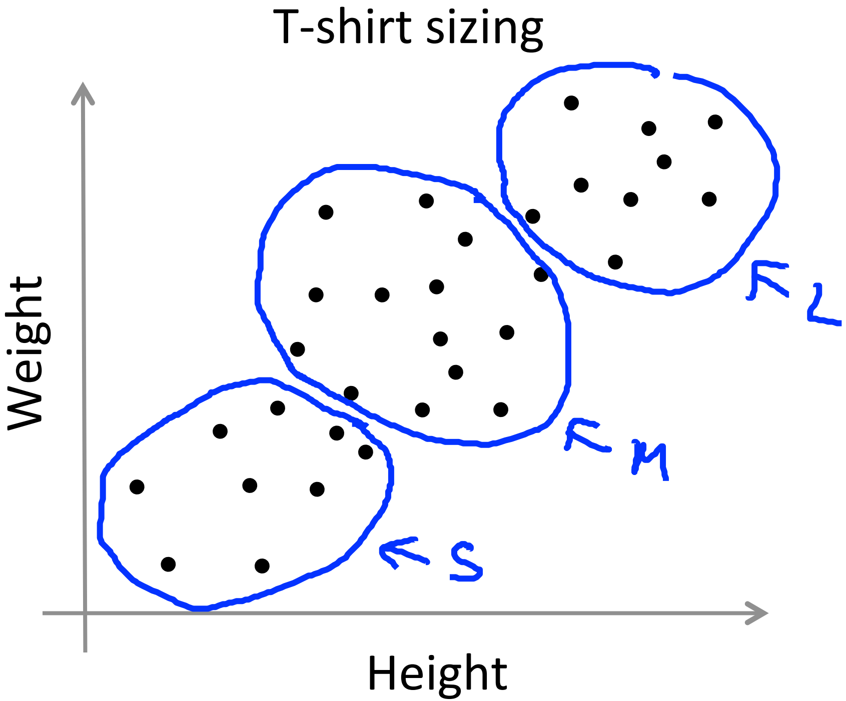
$$\mu_2 = \frac{1}{4}\left[x^{(1)} + x^{(5)} + x^{(6)} + x^{(10)}\right] \in \mathbb{R}^n$$

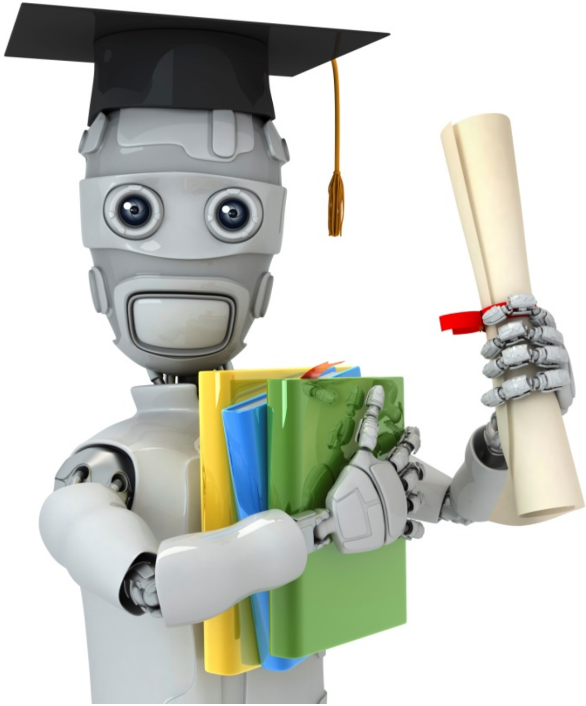
}

K-means for non-separated clusters

S, M, L







Machine Learning

Clustering

Optimization objective

K-means optimization objective

$c^{(i)}$ = index of cluster (1,2,…,K) to which example $x^{(i)}$ is currently assigned

μ_k = cluster centroid k ($\mu_k \in \mathbb{R}^n$) K $k \in \{1,2,\ldots,K\}$

$\mu_{c^{(i)}}$ = cluster centroid of cluster to which example $x^{(i)}$ has been assigned

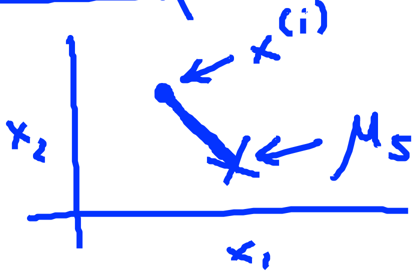
$x^{(i)} \rightarrow 5$ $c^{(i)} = 5$ $\mu_{c^{(i)}} = \mu_5$

Optimization objective:

$$J(c^{(1)},\ldots,c^{(m)},\mu_1,\ldots,\mu_K) = \frac{1}{m}\sum_{i=1}^{m} \boxed{||x^{(i)} - \mu_{c^{(i)}}||^2}$$

$$\min_{\substack{c^{(1)},\ldots,c^{(m)},\\ \mu_1,\ldots,\mu_K}} J(c^{(1)},\ldots,c^{(m)},\mu_1,\ldots,\mu_K)$$

Distortion



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K-means algorithm

Randomly initialize K cluster centroids $\mu_1, \mu_2, \ldots, \mu_K \in \mathbb{R}^n$

Cluster assignment step

Minimize $J(\ldots)$ wrt $\boxed{c^{(1)}, c^{(2)}, \ldots, c^{(m)}}$ ←

(holding $\mu_1, \ldots, \mu_K$ fixed)

Repeat {

for i = 1 to m

$c^{(i)}$:= index (from 1 to K) of cluster centroid closest to $x^{(i)}$

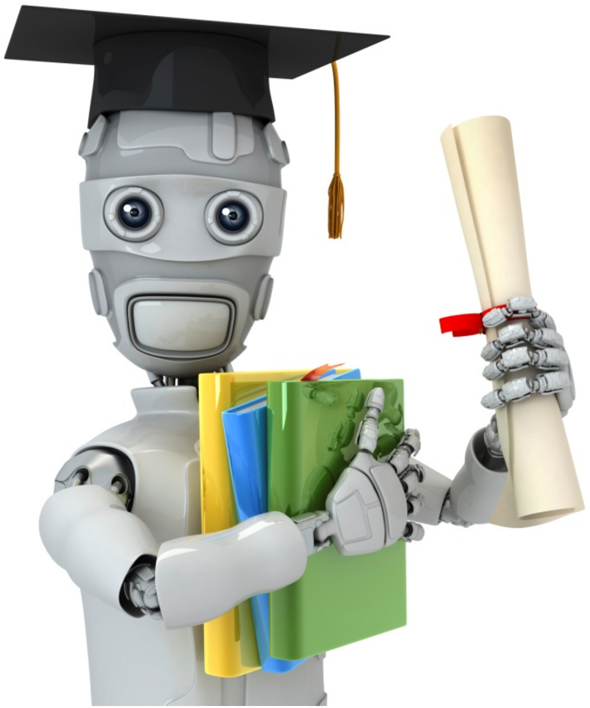
move centroid

for k = 1 to K

μ_k := average (mean) of points assigned to cluster k

}

minimize $J(\ldots)$ wrt $\boxed{\mu_1, \ldots, \mu_K}$



Machine Learning

Clustering

Random initialization

K-means algorithm

Randomly initialize K cluster centroids $\mu_1, \mu_2, \ldots, \mu_K \in \mathbb{R}^n$

Repeat {

for i = 1 to m

$c^{(i)}$:= index (from 1 to K) of cluster centroid closest to $x^{(i)}$

for k = 1 to K

μ_k := average (mean) of points assigned to cluster k

}

Random initialization

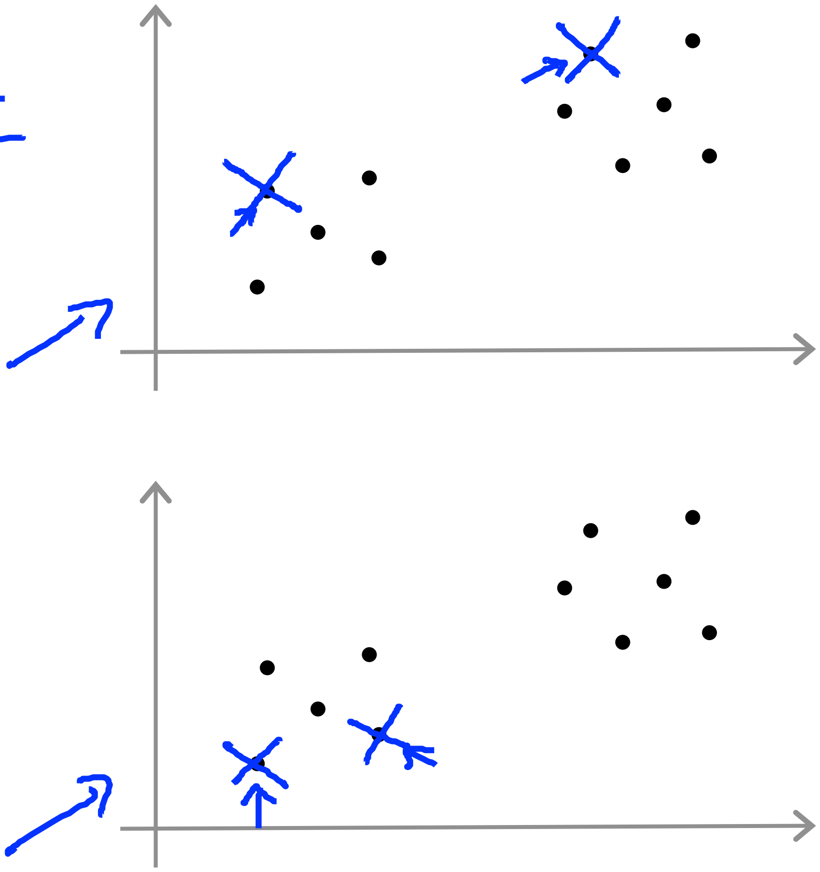
Should have $K < m$

K=2

Randomly pick K training examples.

Set $\mu_1, \ldots, \mu_K$ equal to these K examples.

$\mu_1 = x^{(i)}$
$\mu_2 = x^{(j)}$
$\vdots$



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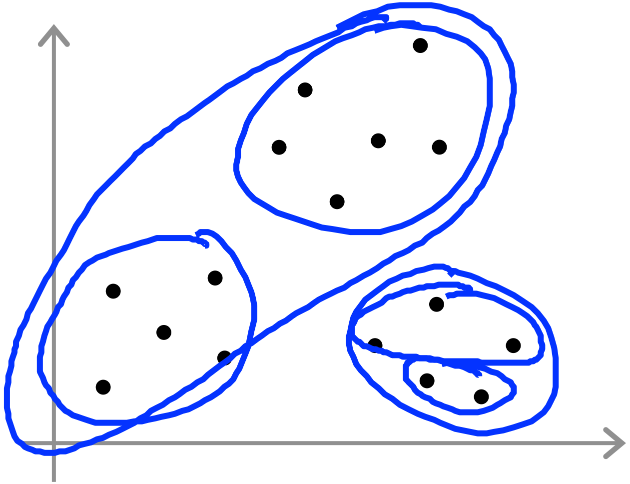
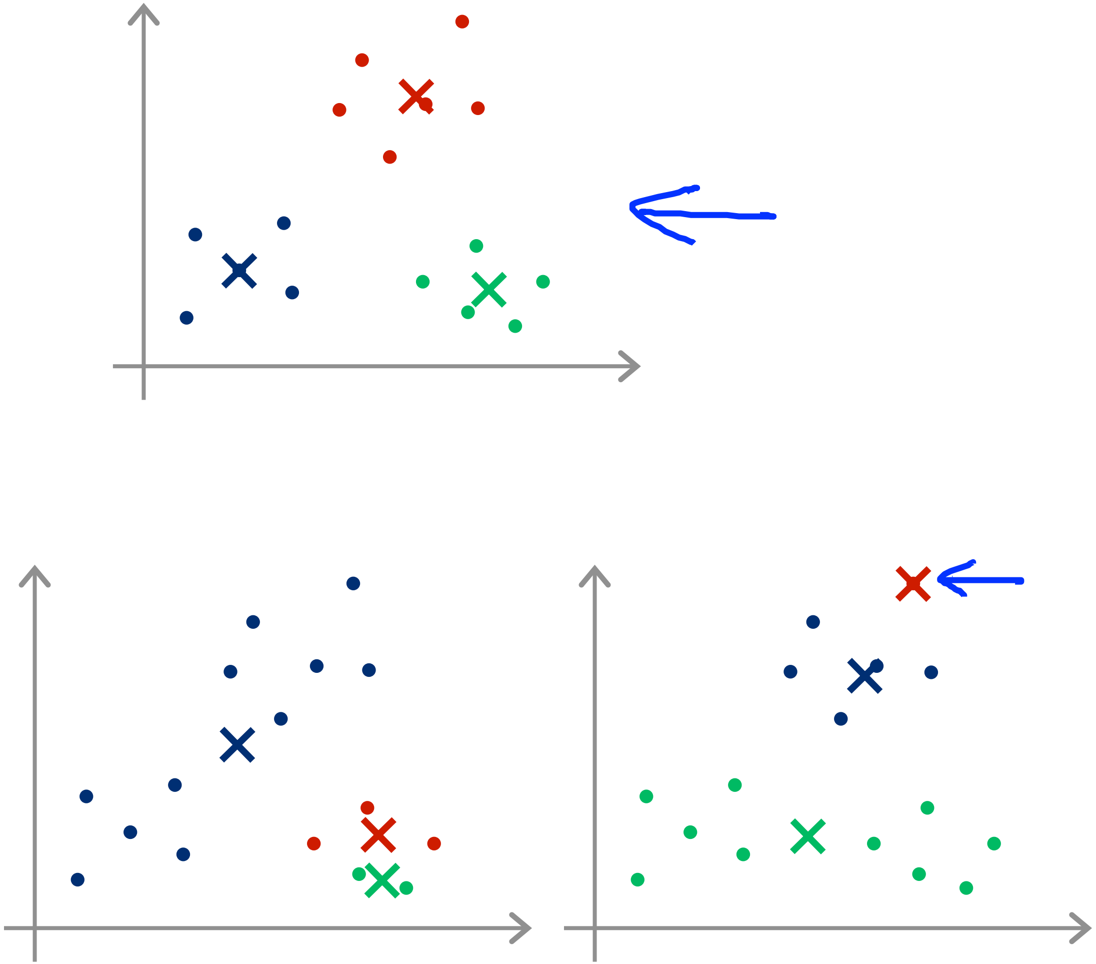
Local optima



$J(c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_K)$



Random initialization

For i = 1 to 100 {

For coding

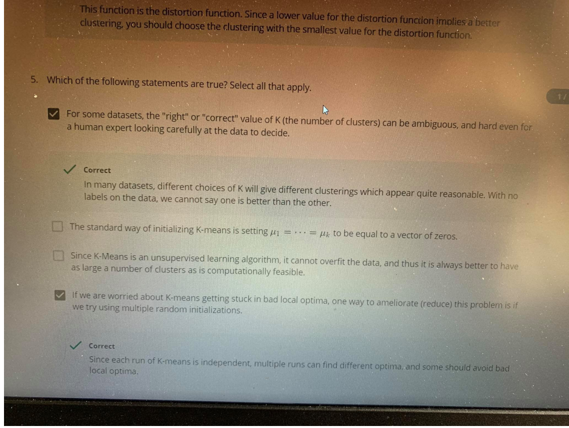
Randomly initialize K-means.

Run K-means. Get $c^{(1)}, \ldots, c^{(m)}, \mu_1, \ldots, \mu_K$.

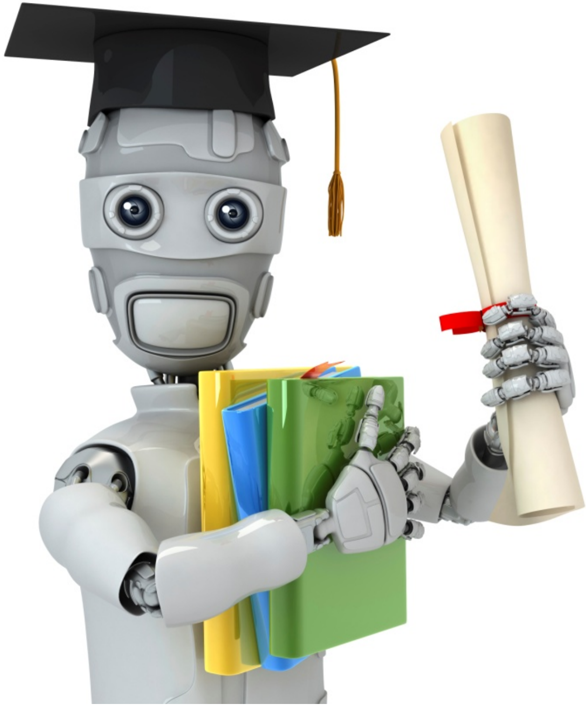
Compute cost function (distortion)

$$J(c^{(1)}, \ldots, c^{(m)}, \mu_1, \ldots, \mu_K)$$

}



Pick clustering that gave lowest cost $J(c^{(1)}, \ldots, c^{(m)}, \mu_1, \ldots, \mu_K)$

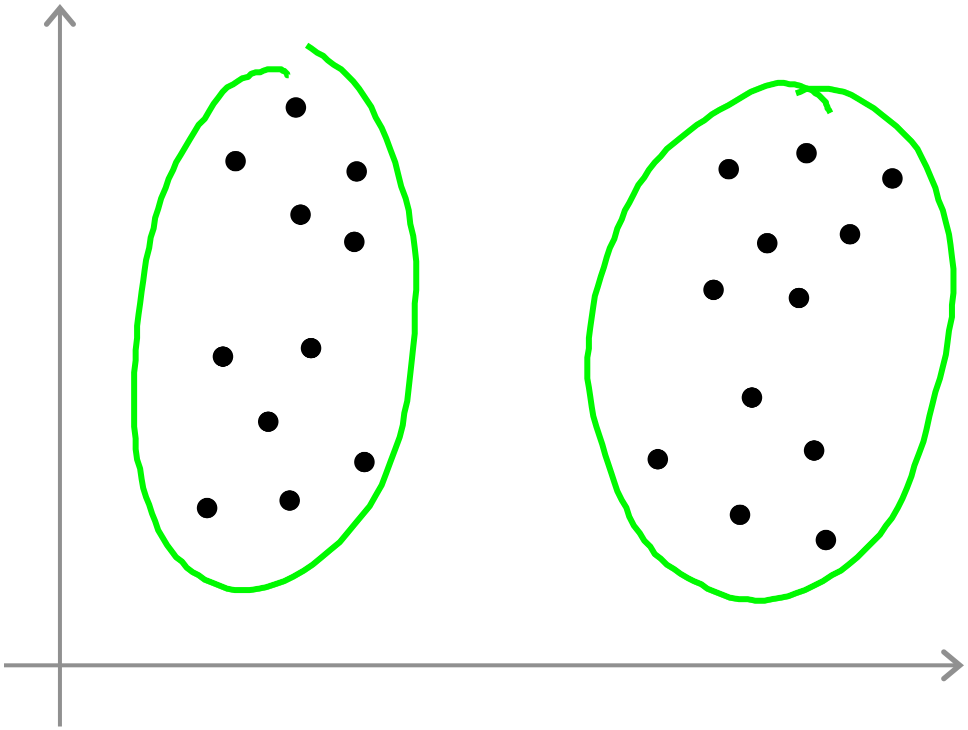


Machine Learning

Clustering

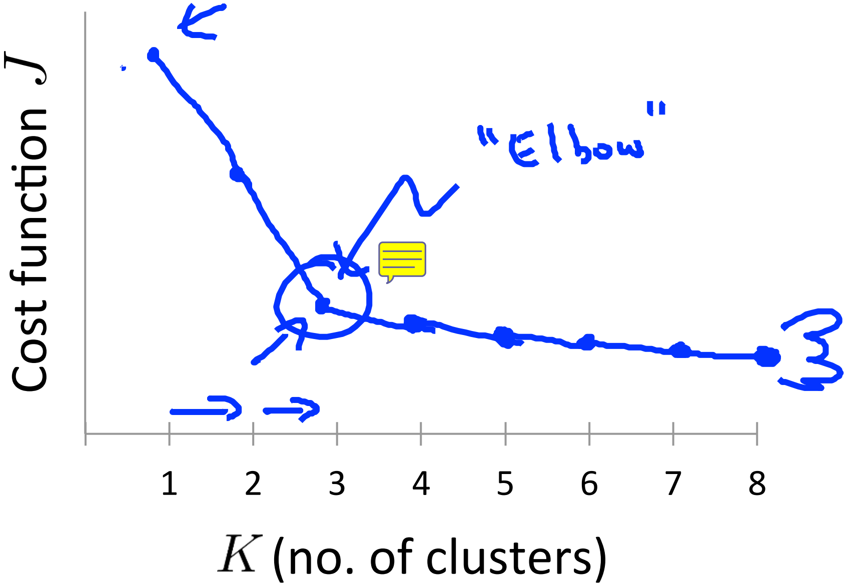
Choosing the number of clusters

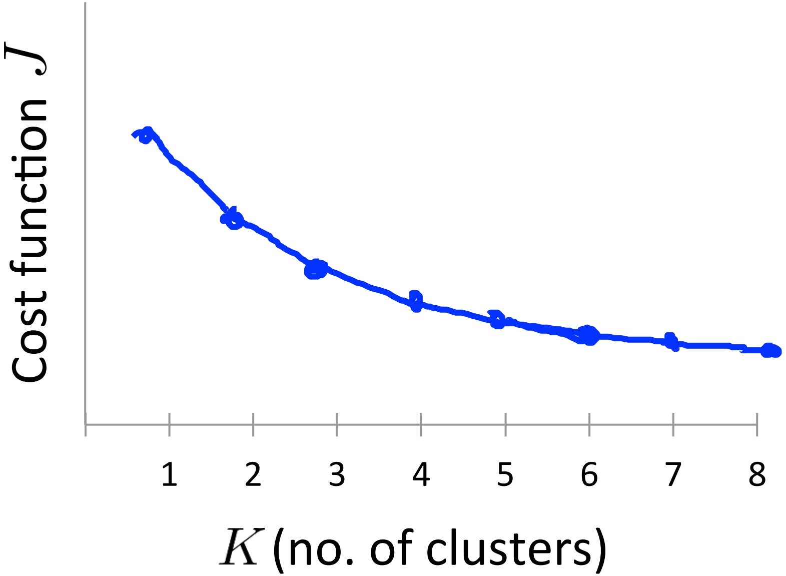
What is the right value of K?



Choosing the value of K

Elbow method:





Choosing the value of K

Sometimes, you're running K-means to get clusters to use for some later/downstream purpose. Evaluate K-means based on a metric for how well it performs for that later purpose.

K=3 S, M, L

K=5 XS, S, M, L, XL

E.g.