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Optimal Online Path Planning for Approach and Landing Guidance

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A method for solving finite-horizon optimal control of nonlinear systems has been developed in this paper and used for an online path planning problem. The new controller synthesis is motivated by the state-dependent Riccati equation (SDRE) technique that was developed for solving regulator and tracking problems. However, finite-time problems need to meet specified boundary conditions for the associated Riccati equations. A closed form solution, called the Finite-SDRE, is obtained for the time-varying Riccati equations by using certain approximations. The application in this paper is the closed loop guidance of a reusable launch vehicle (RLV) during its approach and landing (A&L) phase in such a way to land the RLV at a fixed downrange with the least possible vertical velocity and flight path angle. Simulation studies show that the controller provides an excellent performance in terms of meeting the objectives and is quite robust to initial conditions.

Nomenclature

C_D	= *drag coefficient*
C_{D_0}	= *zero-lift drag coefficient*
C_L	= *lift coefficient*
C_{L0}	= *zero-angle-of-attack lift coefficient*
D	= *drag force, lb*
g	= *Earth's gravitational acceleration, 32.174* ft/sec^2
H	= *scale height, 8.5* km
h	= *altitude,* ft
K_I	= *lift-induced drag coefficient parameter*
L	= *lift force, lb*
m	= *reusable launch vehicle mass,* $slugs$
$\bar{q}$	= *dynamic pressure* lb/ft^2
Q	= *state penalizing matrix*
R	= *control penalizing matrix*
S	= *final state penalizing matrix*
S_a	= *aerodynamic reference area* ft^2
t	= *time,* sec
u	= *control vector*
V	= *velocity magnitude,* ft/sec
x	= *state vector*
X	= *downrange,* ft
X_f	= *fixed final downrange,* ft
α	= *angle of attack,* deg
γ	= *flight-path angle,* deg
ρ	= *air density,* $slugs/ft^3$
ρ_0	= *sea-level air density, 0.0027* $slugs/ft^3$

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I. Introduction

Extreme conditions experienced by a reentry vehicle injects uncertainty in its flight path and demands that the guidance techniques for succeeding phases be robust to variable conditions. Consequently, there needs to be developed techniques that can improve the safety and reliability of reusable launch vehicles (RLVs). In particular, guidance technologies are desired that can accommodate for aero-surface failures, poor vehicle performance, or dispersions from the desired trajectory,[1]. These types of technologies are particularly critical during the approach and landing (A&L) reentry phase. In this paper, a novel method is presented that can handle dispersions from the desired trajectory. It should be noted, however, that the aero-surface failures are not compensated by this technique.

For the space shuttle,[2] the A&L phase is the final phase of the reentry process, beginning at the end of the Terminal Area Energy Management (TAEM) phase at an altitude of roughly 10,000 ft and ending with touchdown on the runway. The goal of this flight phase is for the RLV to land at a desired runway with a near-zero vertical velocity. The vertical velocity at touchdown is desired to be below 5 ft/s, but velocities up to 9 ft/s are generally still considered acceptable for the space shuttle,[2]. One of the concerns with guidance during the A&L phase is that there is the possibility that crucial control surfaces on the RLV may be damaged during the previous reentry phases, leading to a loss of controllability,[1]. A second concern which is addressed by this paper is that at the start of the A&L phase the RLV may be at a point that is significantly different from the desired or pre-planned trajectory. This concern is very real since the current space shuttle guidance methods during the A&L phase rely on the shuttle to follow a predetermined trajectory,[2]. It is important then to develop new A&L guidance technologies that do not require predetermined reference trajectories to be implemented, but instead would be capable of obtaining feasible trajectories on-line that meet the desired landing conditions.

There have been many previous papers which solve the A&L guidance problem while attempting to minimize the amount of off-line information required. Schierman et al. in Ref. 3-5 use an Optimum-Path-To-Go (OPTG) approach for obtaining feasible A&L trajectories in the presence of major control surface errors. In this approach, a large database of neighboring optimal trajectories is first generated off-line. While integrating the trajectory on-line, at each point the states are observed and a trajectory is reconfigured to follow the particular off-line trajectory that leads to the best performance. Later, in Ref. 6 and 7, Schierman et al. use their OPTG approach to account for control surface errors between the atmospheric entry and terminal area energy management phases. In Ref. 7, the authors use scenarios where alternative landing locations are considered. Their numerical results show that the use of the OPTG trajectory reconfiguration approach leads to mission success under different stressed conditions.

Bollino et al.,[8] propose a real-time optimal control approach for generating reentry trajectories through a pseudospectral method under different wind conditions. This approach is not limited to a specific phase. The authors generate on-line trajectories from the point of atmospheric entry to a “flight corridor” of approximately 2000 ft altitude. The wind gusts used range from very small levels to those present in a category 5 hurricane. The authors present results which show that their method can successfully develop feasible trajectories even in the presence of severe wind gusts.

Fahroo et al.,[9] use a Legendre pseudospectral method to develop an approach for trajectory reshaping during approach and landing. The authors first present a technique for predicting flight envelope constraints, and then use it with the Legendre pseudospectral approach to determine optimal feasible trajectories in the event of a failure. Numerical simulations presented with “nominal” conditions and with a rudder stuck fault are used to demonstrate the efficacy of the technique shown.

Kluever in Ref. [10] developed an A&L guidance method based on trajectory planning. Kluever’s method involved computing a desirable reference trajectory on-line that brings the RLV from its initial state to a desired landing condition by piecing together several flight segments. An iterative method determined the quasi-equilibrium glide slope for a constant dynamic pressure, and then a backward trajectory propagation method adjusted the flight path angle such that dynamic pressure was matched at the pull-up altitude. Results were presented which showed that the trajectory-planning method is able to generate feasible reference trajectories very quickly, even with the addition of variations in wind and vehicle drag. Kluever later applied his trajectory planning approach for the case of a RLV with damaged control surfaces,[11]. The “damaged control surfaces” scenario was modeled by placing a limit on the normal acceleration capabilities. With this approach, a small but constant normal acceleration is applied which continuously rotates the flight-path angle upward. The results showed that the generated trajectories satisfy the condition of a near-zero vertical velocity at touchdown, with the desired touchdown position being missed by 300 ft.

Guidance problems fall into the category of finite-time/horizon problems in the optimal control literature. Finite-horizon optimal control of nonlinear systems is a difficult and challenging problem. The resulting equations from optimal control theory form a two point boundary value problem (TPBVP) where initial states are given in the beginning of the horizon and the final costates are given at the terminal time,[12].

One method of solving the resulted TPBVP is through numerical iterative methods. The results, however, are open loop and each set of initial conditions needs a whole new iteration. Hence, online applicability of the iterative methods is severely limited. In this paper a novel approach is developed for the purpose of solving finite-horizon optimal control of nonlinear input affine systems and its performance is tested on the approach and landing guidance of an RLV. It uses an State Dependent Ricatti Equation (SDRE), [16] approach along with some modifications to solve the associated state dependent *differential* Riccati equation, however, this is a major step since the time-varying nature of the Ricatti equation needs to be considered due to the finite-time nature of guidance problems. The real time computational burden of the controller mainly includes solving one algebraic Riccati equation (ARE) and two matrix inversions in real-time in each time step during the implementation as compared with one ARE equation solved online in each time step in the SDRE approach.

The rest of the paper is organized as follows: in section II the approach and landing dynamics is modeled and the problem is formulated in the finite-horizon optimal control form. Section III presents the theory of the novel proposed finite-horizon optimal controller and section IV contains the simulation results. Finally, the conclusion is given in section V.

II. System Modeling

In this section the dynamic model of the landing of an RLV is presented. Assuming zero cross range to the runway, the approach and landing phase of an RLV in a single vertical plane can be modeled as follows,[18].

$$\dot{V} = -\frac{D}{m} - g\sin\gamma \tag{1}$$

$$\dot{\gamma} = \frac{L}{mV} - \frac{g}{V}\cos\gamma \tag{2}$$

$$\dot{h} = V\sin\gamma \tag{3}$$

$$\dot{X} = V\cos\gamma \tag{4}$$

where

$$L = \bar{q} S_a C_L \tag{5}$$

$$D = \bar{q} S_a C_D \tag{6}$$

$$\bar{q} = \frac{1}{2}\rho V^2 \tag{7}$$

Assuming an exponential air density profile results in

$$\rho = \rho_0 \exp(-h/H) \tag{8}$$

The lift and drag coefficient are modeled as

$$C_L = C_{L_0}\alpha \tag{9}$$

$$C_D = C_{D_0} + K_I C_L^2 \tag{10}$$

The equations (1) to (4) represent a state space with states being $V,\ \gamma,\ h$ and X and the control being α. This system is non-affine in the control because it contains the square of the control in the equation for C_D in (10). In order to convert the non-affine system into an input-affine form, i.e. linear in the control, the state vector is augmented with α, and its derivative is considered as the new control,[19]. That is,

$$\dot{\alpha} = u \tag{11}$$

Now, α is a new state of the systems and the control to the augmented state space equation is u which appears in an affine form.

The objective of the problem in this paper is to optimally land the RLV with minimum final vertical velocity and flight path angle at a given downrange location.

Note that in practical situations, one is usually interested in an optimal landing in a predetermined and fixed *downrange*, not in a fixed *time*. Therefore, the independent variable of the above system needs to be changed from t to X, the downrange, for convenience. In order to do that, divide (1) to (3) and (11) by (4). Denoting the derivative of a variable with respect to X by the prime notation, one has

$$\mathrm{V}' \equiv dV/dX = \frac{1}{V\cos\gamma}\left(-\frac{D}{m} - g\sin\gamma\right) \tag{12}$$

$$\gamma' \equiv d\gamma/dX = \frac{1}{V\cos\gamma}\left(\frac{L}{mV} - \frac{g}{V}\cos\gamma\right) \tag{13}$$

$$h' \equiv dh/dX = \tan\gamma \tag{14}$$

$$\alpha' \equiv d\alpha/dX = \frac{1}{V\cos\gamma}u \tag{15}$$

The time becomes an incidental variable given by

$$t' \equiv \frac{dt}{dX} = \frac{1}{\dot{X}} = \frac{1}{V\cos\gamma} \tag{16}$$

The state vector can then be formed as

$$x = [\,V,\ \gamma,\ h,\ t,\ \alpha]^T \tag{17}$$

where superscript T denotes the transpose operation. The cost function to be minimized is assumed quadratic as

$$J = \frac{1}{2}x^T(X_f)Sx(X_f) + \frac{1}{2}\int_{X_0}^{X_f}(x^TQx + u^TRu)dX \tag{18}$$

where S, Q, and R are final states, states and control penalizing matrices, respectively, and X_f denotes the fixed downrange in which the RLV is supposed to land on the runway. The guidance problem based on minimizing the cost function (18) is a finite-horizon optimal control problem for a nonlinear input-affine system. The solution in a feedback form will be obtained through a novice approach. Development of this approach is described in the next section.

III. Introducing Finite-SDRE: A New Finite-Horizon Nonlinear Optimal Controller

A. Problem statement

The state equation of nonlinear input-affine systems is given as

$$\dot{x}(t) = f(x) + B(x)u \tag{19}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ are the state and control vectors, respectively, and n and m are the order of the system and number of inputs, respectively. $f(x)$ and $B(x)$ are proportionately dimensioned vector and matrix based on the dynamics of the system. For simplicity in the notation, instead of $x(t)$, the notation x has been used in some places throughout this paper.

In finite-horizon optimal control, one in interested in finding the optimal control history $u^*(t)$ for $t_0 \le t < t_f$ which minimizes a cost function, like the following

$$J = \frac{1}{2}x^T(t_f)Sx(t_f) + \frac{1}{2}\int_{t_0}^{t_f}(x^TQx + u^TRu)dt \tag{20}$$

The approach developed here for solving finite-horizon optimal nonlinear problems is inspired by the State Dependent Riccati Equation (SDRE) method developed in Ref. 16. In SDRE, one rewrites the state space equation (19) in the below linear like form

$$\dot{x}(t) = A(x)x(t) + B(x)u(t) \tag{21}$$

where $A(x)x(t) = f(x)$, and solves the following algebraic Riccati equation (ARE) with the state dependent coefficients at each time step during the real-time implementation to end up with $P(x)$.

$$P(x)A(x) + A^T(x)P(x) - P(x)B(x)R^{-1}B^T(x)P(x) + Q = 0 \tag{22}$$

The solution, $P(x)$, is then used for calculating the optimal costate vector λ^* in this manner

$$\lambda^*(t) = P(x)x(t) \tag{23}$$

Hence, at each time step the optimal costate vector can be calculated which is used for calculating the (near) optimal control through the below equation

$$u^*(t) = -R^{-1}B^T(x)\lambda^*(t) \tag{24}$$

Interested readers are referred to Ref. 16 and for more information about forming the linear like state equation (21) from (19) one may refer to Ref. 20.

Note that the SDRE is a solution to the *infinite-horizon* optimal control problem, however, the problem to be solved here, belongs to the *finite-horizon* category. In the finite-horizon case, the feedback solution is time-dependent and a differential equation, rather than an algebraic one, needs to be solved to calculate the control. Following the method of solving the differential Riccati equation (DRE) for linear systems, the below state dependent DRE is suggested to be solved for the near optimal finite-horizon solution.

$$P(x,t)A(x) + A^T(x)P(x,t) - P(x,t)B(x)R^{-1}B^T(x)P(x,t) + Q = -\dot{P}(x,t) \tag{25}$$

The final condition is given by

$$P\left(x,t_f\right) = S \tag{26}$$

In order for (26) to be satisfied at the boundary, the typical SDRE is not applicable here. Note that since the states at future times are not known ahead, one cannot calculate the state dependent coefficients to integrate (25) backward from t_f to t and end up with the unknown at time t, i.e. $P(x,t)$.

To remedy the problem, an approximate analytical approach is developed in this paper. The basis for this approach uses the closed form solution obtained by using a transformation with the solution to the ARE that converts the original nonlinear Ricatti equation to a Lyapunov equation,[21,22]. Details are presented in the next section.

B. Solution process for linear quadratic problems

Consider the constant-coefficient differential Ricatti equation (DRE)

$$P(t)A + A^T P(t) - P(t)BR^{-1}B^{\mathrm{T}}P(t) + Q = -\dot{P}(t) \tag{27}$$

$$P\left(t_f\right) = S \tag{28}$$

where $P(t)$, is the Ricatti solution associated with the linear dynamics and quadratic cost given below.

$$\dot{x} = Ax + Bu \tag{29}$$

$$J = \frac{1}{2}x^T\left(t_f\right)Sx\left(t_f\right) + \frac{1}{2}\int_{t_0}^{t_f}\left(x^T Qx + u^T Ru\right)dt \tag{30}$$

The corresponding ARE for (27) with the unknown denoted by P_{ss} is

$$P_{ss}A + A^{\mathrm{T}}P_{ss} - P_{ss}BR^{-1}B^{\mathrm{T}}P_{ss} + Q = 0 \tag{31}$$

Following Ref. 22, subtracting (31) from (27) results in

$$(P(t) - P_{ss})A + A^{\mathrm{T}}(P(t) - P_{ss}) - P(t)BR^{-1}B^{\mathrm{T}}P(t) + P_{ss}BR^{-1}B^{\mathrm{T}}P_{ss} = -\dot{P}(t) \tag{32}$$

Using the change of variable of $K(t) \equiv (P(t) - P_{ss})^{-1}$, (32) leads to

$$\dot{K}(t) = A_{cl}K(t) + K(t)A_{cl}^{T} - BR^{-1}B^{\mathrm{T}} \tag{33}$$

where $A_{cl} \equiv A - BR^{-1}B^{\mathrm{T}}P_{ss}$.

Interestingly, (33) is a differential Lyapunov equation (DLE), which is a linear equation to be solved with the following boundary condition

$$K(t_f) = (S - P_{ss})^{-1} \tag{34}$$

Moreover, it can be seen by substitution, that the solution to (33) with final condition (34) is given as,[23]

$$K(t) = e^{A_{cl}(t-t_f)}\left(K(t_f) - E\right)e^{A_{cl}^{T}(t-t_f)} + E \tag{35}$$

where E is the solution to the following algebraic Lyapunov equation (ALE) which can simply be solved.

$$A_{cl}E + EA_{cl}^{T} = BR^{-1}B^{\mathrm{T}} \tag{36}$$

Hence, instead of solving DRE (27) through backward integration, one can solve (31), (36) and (35), the desired solution will be

$$P(t) = K^{-1}(t) + P_{ss} \tag{37}$$

In Ref. 24 the authors have suggested to use the negative definite solution of ARE (31) instead of the positive definite one for P_{ss} in order to avoid probable singularity of $P(t) - P_{ss}$ in case of time-dependent solution of DRE converging to the steady state solution for $t \ll t_f$. In such scenario, one needs to replace A in (31) with $-A$, solve the equation and flip the sign of the solution.

C. Solving State Dependent DRE for optimal control of nonlinear systems

Inspired by the idea of linear like re-formulation of the nonlinear system and solving the resulted ARE with state dependent coefficient in the well known SDRE method, the authors suggest solving the state dependent differential Riccati equation given in (25) through the method discussed in the previous subsection without needing backward integration. In this process, called Finite-SDRE, one needs to solve the following SDRE and ALE at each time step

$$P_{ss}(x)A(x) + A^{T}(x)P_{ss}(x) - P_{ss}(x)B(x)R^{-1}B^{T}(x)P_{ss}(x) + Q = 0 \tag{38}$$

$$A_{cl}(x)E(x) + E(x)A_{cl}^{T}(x) = B(x)R^{-1}B^{T}(x) \tag{39}$$

where $A_{cl}(x) = A(x) - B(x)R^{-1}B^{T}(x)P_{ss}(x)$. The next step is calculating

$$K(x,t) = e^{A_{cl}(x)(t-t_f)}\left(K(x,t_f) - E(x)\right)e^{A_{cl}^{T}(x)(t-t_f)} + E(x) \tag{40}$$

where

$$K(x, t_f) = (S - P_{ss}(x))^{-1} \tag{41}$$

Finally the solution will be

$$P(x, t) = K^{-1}(x, t) + P_{ss}(x) \tag{42}$$

Equation (42) gives the desired approximate solution to (25), which is to be used for finite-horizon (near) optimal control of nonlinear input-affine systems. The control can then be calculated in the feedback form as

$$u^*(t) = -R^{-1}B^T(x)P(x, t)x(t) \tag{43}$$

The computational effort needed to be done in real-time at each time step to end up with the closed loop solution mainly includes a SDRE solving and two matrix inversions. Note that in this approach, in order to have (40) as the solution to the corresponding state dependent DLE, the state values are assumed constant over the entire time at each control calculation. However, the numerical results show that with a high frequency control calculation, i.e. short time steps, this approximation is justifiable and the result is excellent.

IV. Numerical Analysis

For simulating the proposed controller on the landing path planning problem, the following values have been selected for the RLV parameters: $C_{L_0} = 2.3$, $C_{D_0} = 0.0975$, $K_I = 0.1819$, $S/m = 0.912\ ft^2/slug$. The weight matrices are selected as: $R = 1$, $Q = diag(0\ \ 0.05\ \ 0.01\ \ 10^{-6}\ \ 1)$, and $S = diag(0\ \ 10^6\ \ 10^3\ \ 0\ \ 0)$. In matrix Q, the last element corresponds to α, and to force the controller to give an admissible angles of attacks history, a relatively higher value is assigned to that. The rest of the elements of Q are selected through trial and error to end up with a good performance.

Since the goal of the problem is to land at some small vertical velocity and flight path angle, the second and the third states which are γ and h are penalized with high values to the corresponding elements of S. The initial condition for the simulation is selected as initial height of 10,000 ft, initial velocity of 300 ft/sec, initial γ and α of -30 and 10 deg, respectively. The problem is to land the RLV at a fixed downrange of 20,000 ft.

Simulation were carried out with a sampling interval of 20 ft. Note that the independent variable is downrange X, hence the sampling interval has the unit of ft. In this simulation the negative definite solution of (38) has been used for $P_{ss}(x)$ in solving the state dependent DRE (25) to avoid the singularity problem explained earlier.

Sample flight path profile and the flight path angle history are shown in Fig. 1 versus the downrange. The histories of the horizontal velocity, the vertical velocity, the angle of attack and the applied control are shown in Fig. 2 and 3. As can be seen in Fig. 1, the controller has been nicely able to force the γ to go to zero at the touchdown moment resulting in a close to zero vertical velocity at the moment shown, as in Fig. 2. The final value of γ in this simulation turns out to be around $-0.01\ deg$ and the touchdown vertical velocity is $0.06\ ft/sec$ which are quite small. This smooth touchdown can be seen in the zoomed view of the path profile at the terminal moment in Fig. 4. As seen through Fig. 3 the angle of attack is small enough for the theory to be acceptable for actual implementations.

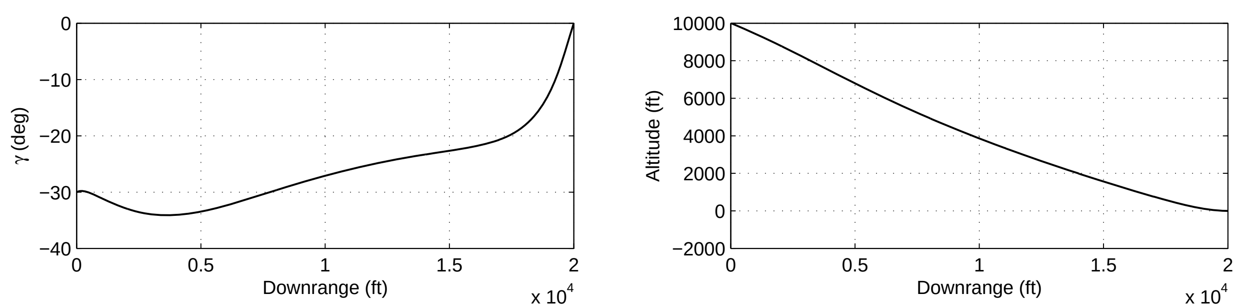

Figure 1. Altitude and flight path angle histories versus downrange

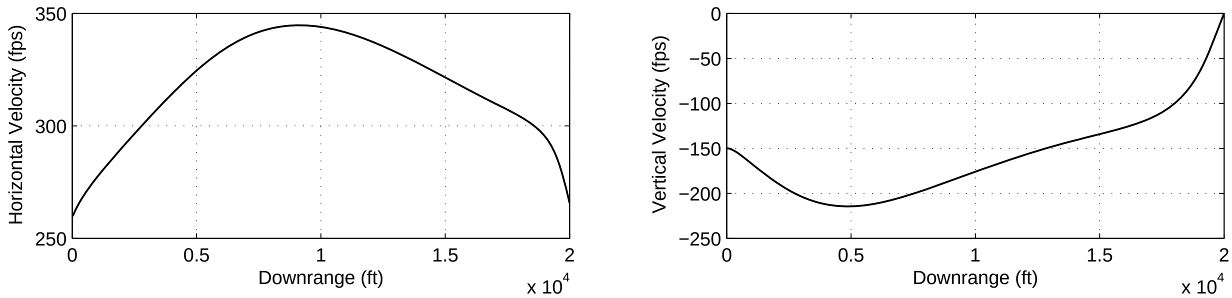

Figure 2. Histories of horizontal and vertical velocities

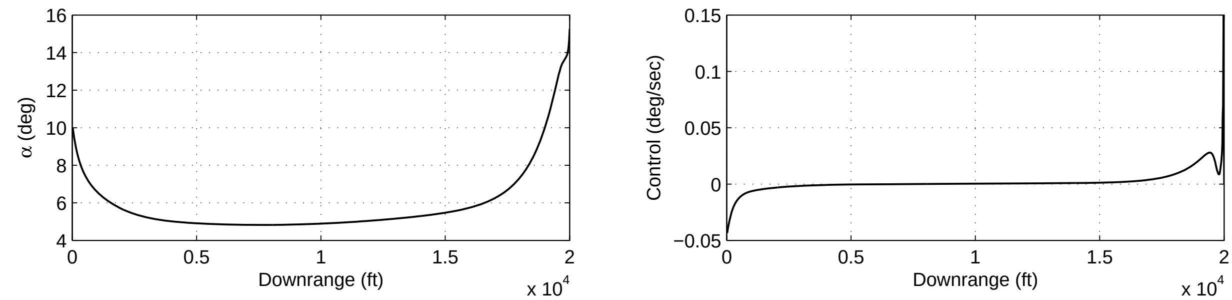

Figure 3. Applied control history and resulted angle of attack trajectory

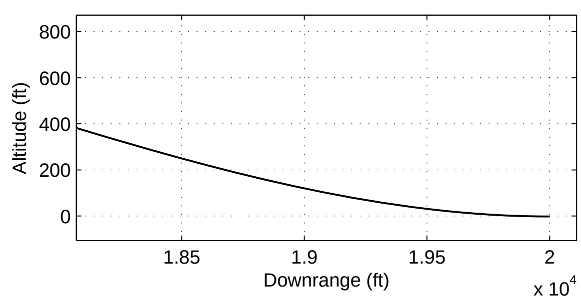

Figure 4. A zoomed view of the landing path at the touchdown moment

To show the robustness of the proposed controller to different initial conditions some other initial conditions are also selected and simulated and the results are presented and analyzed below.

Different Initial Altitudes

Three different initial altitudes of 7,000, 12,000 and 17,000 ft are simulated with other initial conditions being the same as selected earlier and the path profile along with the history of the flight path angles are depicted in Fig. 5. As can be seen, the controller has been able to force γ to converge to close to zero at the touchdown moment for all of the different initial altitudes.

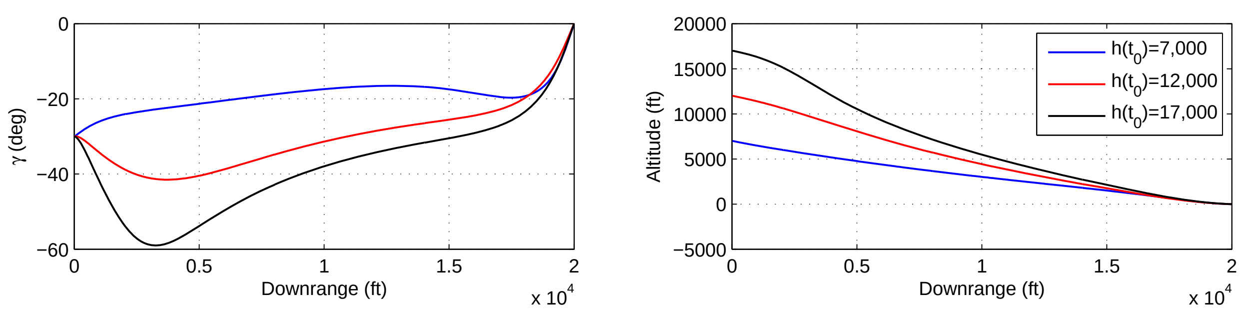

Figure 5. Altitude and flight path angle histories versus downrange for different initial altitude

Different Initial Flight Path Angles

The results of the simulations of three different initial flight path angles of -50, -30, and -10 deg are depicted in 6. Other initial conditions are the same as in the initial simulation. In all these cases, the controller achieves the desired objectives as can be seen from the flight path angle and the altitude histories.

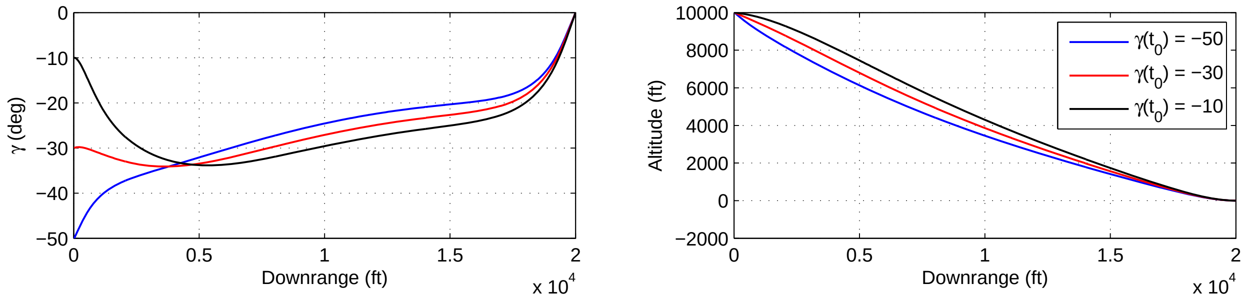

Figure 6. Altitude and flight path angle histories versus downrange for different initial γ

Different Initial Velocities

Different initial velocities of 200, 350, and 500 ft/sec were assumed and the corresponding simulation results are presented in Fig. 7 and Fig. 8. Again, the robustness of the controller to different initial velocities is depicted through very small final flight path angle and vertical velocity at the touchdown.

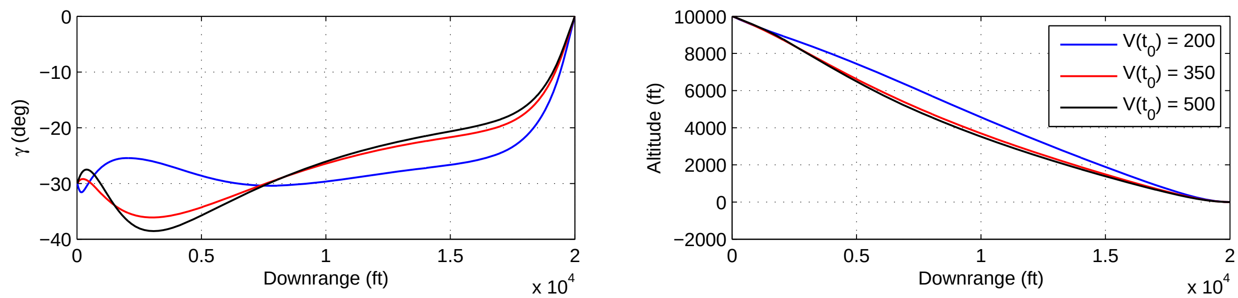

Figure 7. Altitude and flight path angle histories versus downrange for different initial velocities

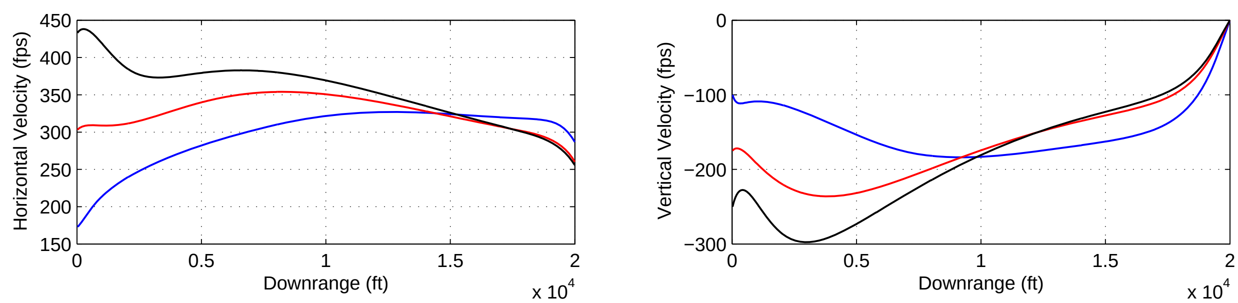

Figure 8. Vertical and horizontal velocities' histories versus downrange for different initial velocities

Different Initial Angles of Attack

As the angle of attack is a crucial flight variable, the performance of the proposed controller to different initial angles of attack of 0, 10, and 20 deg were tested and the results are given in Fig. 9 and Fig. 10. Interestingly, looking at Fig. 10, it can be seen that the controller very quickly changes different initial angle of attacks to similar histories soon into the engagement. The touchdown behavior is shown to be very good for these simulations as well.

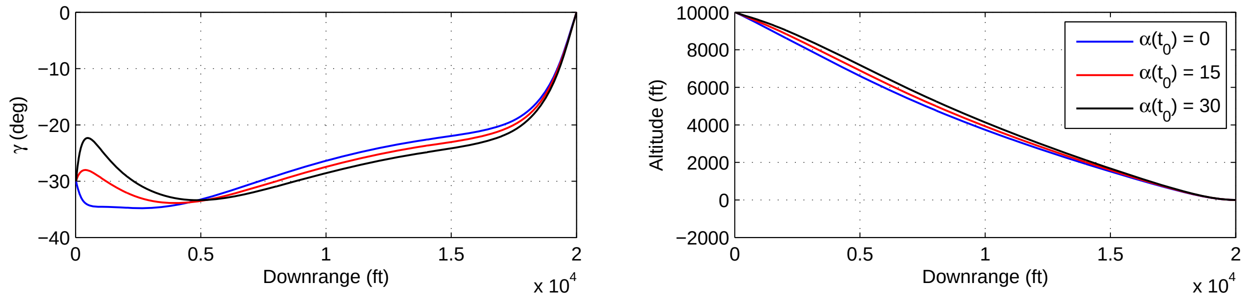

Figure 9. Altitude and flight path angle histories versus downrange for different initial α

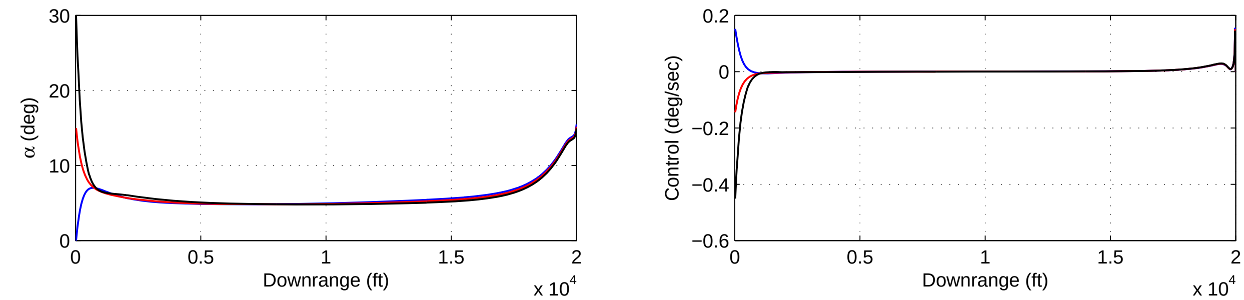

Figure 10. Angle of attack and the control histories versus downrange for different initial α

Different Downrange

Finally, to test the dependency of the solution to the fixed horizon, three simulations were done for three different horizons of 15000, 20000, and 25000 ft and the results are shown in Fig. 11. The excellent capability of the controller can be seen through the close to zero flight path angle at the touchdown at different fixed horizons. Comparing this controller with other methods, the accuracy of the results of this controller is very exciting.

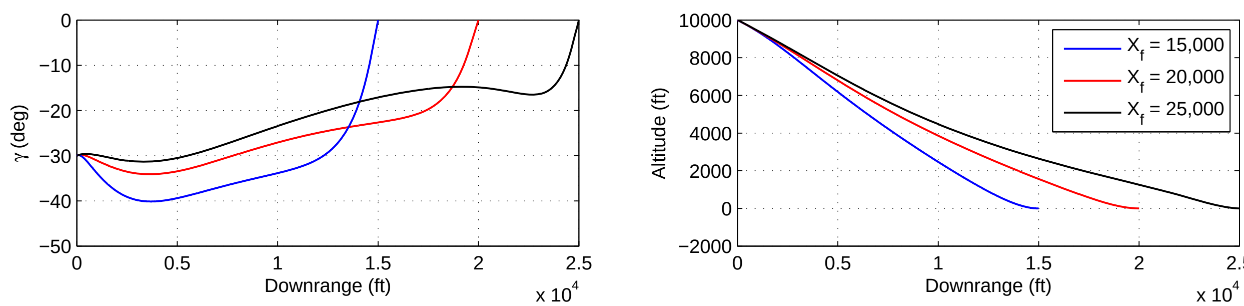

Figure 11. Altitude and flight path angle histories versus downrange for different horizons

V. Conclusions

A new state dependent differential Riccati equation controller for solving finite-horizon optimal control problem has been developed, called the Finite-SDRE. Its application to an A&L guidance problem shows a lot of promise. Although only a guidance problem has been used as an example, this approach is applicable to any input-affine nonlinear finite-time optimal control problem that has a quadratic cost function. The computational burden of the Finite-SDRE is as small as solving an ARE along with performing two matrix inversions which in return for calculating a non-iterative closed-form solution in real-time, makes it very rewarding for different finite-horizon applications.

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