

Fabio Marti, Matthias Roggo, David Lehen, Daniel Gilgen

# **Robocup: Cooperative estimation and prediction of player and ball movement**

Group Project

Department:

IfA – Automatic Control Laboratory, ETH Zürich

Supervising Professor:

Prof. Dr. John Lygeros, ETH Zürich

Supervisor:

M.S., Ph.D. Student Sean Summer, ETH Zürich

Zürich, March 2012

## *ABSTRACT*

### **Abstract**

Hier kommt das Abstract ...

# Contents

<b>1. Introduction</b>	<b>5</b>
<b>2. Simulation</b>	<b>6</b>
2.1. Playing Field, Robots and Ball . . . . .	6
2.2. Random Simulation . . . . .	6
<b>3. Kalman Filtering</b>	<b>7</b>
3.1. Linear Kalman Filter . . . . .	7
3.2. Extended Kalman Filter (EKF) . . . . .	8
<b>4. Estimation</b>	<b>10</b>
4.1. Estimate of the Ball . . . . .	10
4.2. Estimate of the Robots . . . . .	10
<b>A. Examples</b>	<b>11</b>
A.1. Kalman Filtering of Linear System . . . . .	11
A.2. Kalman Filtering of Nonlinear System . . . . .	14
Literatur . . . . .	15

# List of Figures

3.1. Example: Linear electrical circuit. . . . .	7
3.2. Example: Input signal, output signal, noisy output, and filtered output. .	8
3.3. Example: Input signal, output signal, noisy output, and filtered output. .	9

# 1. Introduction

## 2. Simulation

### 2.1. Playing Field, Robots and Ball

### 2.2. Random Simulation

## 3. Kalman Filtering

### 3.1. Linear Kalman Filter

There exists a recursive Kalman filter algorithm for discrete time systems.[1] This ongoing Kalman filter cycle can be divided into two groups of equations

1. Time update equations

$$\begin{aligned}\hat{x}_k^- &= A\hat{x}_{k-1} + Bu_{k-1} \\ P_k^- &= AP_{k-1}A^T + Q\end{aligned}\tag{3.1}$$

2. Measurement update equations

$$\begin{aligned}K_k &= P_k^- H^T (H P_k^- H^T + R)^{-1} \\ \hat{x}_k &= \hat{x}_{k-1}^- + K_k(z_k - H\hat{x}_{k-1}^-) \\ P_k &= (I - K_k H)P_k^-\end{aligned}\tag{3.2}$$

To test this algorithm and to learn something about applying the Kalman filter on linear systems we created an example. A detailed description of the following example can be found in appendix A.1.

We consider a linear, timeinvariant model, given by the following circuit diagram in figure 3.1. First we added process noise and measurement noise to the system output.

```
(-3,-1)(9,5) (-2,4)A (-2,0)B (8,4)C0 (8,0)D (3,4)A2 (3,2)M (3,0)B2 (6,4)C2 (6,0)D2
[intensitylabel= $i_L$ ,labeloffset=-0.2](A)(A2)L [tensionlabel= $U_1$ ,tensionlabeloffset=-
1.2,tensionoffset=-0.8,intensitylabel= $i_1$ ,dipoleconvention=generator](A2)(M)C1
[tensionlabel= $U_R$ ,tensionoffset=0.8,labeloffset=-0.5](B2)(M)R
[tensionlabel= $U_2$ ,tensionlabeloffset=-1.2,tensionoffset=-
0.8,intensitylabel= $i_2$ ,dipoleconvention=generator](C2)(D2)C2 (B)(D) (A2)(C0)
[fillstyle=solid](A)0.075 [fillstyle=solid](B)0.075 [fillstyle=solid](C0)0.075
[fillstyle=solid](D)0.075 (A)(B) $U_{in}$  (C0)(D) $U_{out}$ 
```

Figure 3.1.: Example: Linear electrical circuit.

The aim is to get an estimation of the noisy output. Therefore we applied the Kalman filter algorithm based on the equations 3.1. In the figure 3.2, above we can see the input signal and the ideal measurement of the output signal. That ideal measurement includes process noise, which can obviously not be filtered by the Kalman filtering algorithm. Below one can see the noisy measurement on the left side and the filtered output on the right side.

### 3. Kalman Filtering

Figure 3.2.: Example: Input signal, output signal, noisy output, and filtered output.

## 3.2. Extended Kalman Filter (EKF)

In the section above we discussed the usage of a Kalman filter on linear systems. Most systems, including the motion equations of our robots, however are nonlinear. Nevertheless linearizing our system around the current estimate still makes our Kalman filter useful and leads to the concept of the extended Kalman filter [1]. The recursive equations are quite similar to those of the linear Kalman filter

1. Time update equations

$$\begin{aligned}\hat{x}_k^- &= f(\hat{x}_{k-1}, u_{k-1}, 0) \\ P_k^- &= A_k P_{k-1} A_k^T + W_k Q_{k-1} W_k^T\end{aligned}\tag{3.3}$$

2. Measurement update equations

$$\begin{aligned}K_k &= P_k^- H_k^T (H_k P_k^- H_k^T + V_k R_k V_k^T)^{-1} \\ \hat{x}_k &= \hat{x}_{k-1}^- + K_k (z_k - h(\hat{x}_{k-1}^-, 0)) \\ P_k &= (I - K_k H_k) P_k^-\end{aligned}\tag{3.4}$$

The function  $f(\hat{x}_k, u_k, w_k)$  contains the system's nonlinear dynamics where  $w_k$  represents the current process noise and  $h(\hat{x}_k, v_k)$  denotes the nonlinear state-to-output relationship with the measurement noise  $v_k$ . Furthermore the matrices  $A_k$ ,  $H_k$ ,  $W_k$  and  $V_k$  are linearizations, i.e. partial derivatives, of their respective functions at time step  $k$

$$\begin{aligned}A_{i,j} &= \frac{\partial f_i}{\partial x_j}(\hat{x}_{k-1}, u_{k-1}, 0) \\ H_{i,j} &= \frac{\partial h_i}{\partial x_j}(\hat{x}_{k-1}, u_{k-1}, 0) \\ W_{i,j} &= \frac{\partial f_i}{\partial w_j}(\tilde{x}_k, 0) \\ V_{i,j} &= \frac{\partial h_i}{\partial v_j}(\tilde{x}_k, 0)\end{aligned}\tag{3.5}$$

We dropped the fact that all matrices should have a subscript  $k$  and that they are allowed be different at each time step. Again, before applying the theory to our simulation, we created a generic example of a nonlinear system. Further details can be looked up in section A.2. We did essentially the same as we did for the linear system: We simulated the ideal system, then added process and measurement noise and in the last



### 3. Kalman Filtering

Figure 3.3.: Example: Input signal, output signal, noisy output, and filtered output.

step checked whether we could get rid of our artificially added measurement noise. The results for a sample run on MATLAB are shown in figure 3.3 below

The two pictures above show the sinusoidal input on the left and the ideal output, i.e. without process and measurement noise, on the right. Thereunder we can see the plots of the noisy signal and the signal after it has been filtered by the extended Kalman filter. We can conclude that the extended Kalman filter too provides an acceptable performance if our measurement noise is not too big.

## 4. Estimation

### 4.1. Estimate of the Ball

### 4.2. Estimate of the Robots

## A. Examples

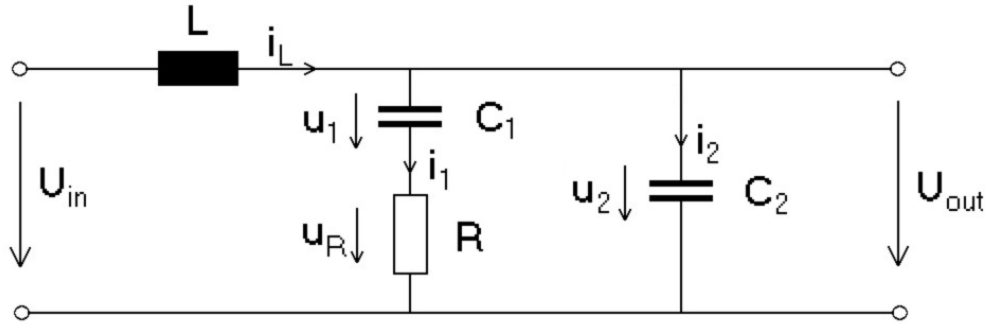
### A.1. Kalman Filtering of Linear System

# Group project - Linear model

Daniel Gilgen, David Lehen, Matthias Roggo, Fabio Marti

March 12, 2012

We are considering a linear, timeinvariant model, given by the following circuit diagram



This is obviously a system of order 3 with the states  $x_1 = i_L$ ,  $x_2 = u_1$  and  $x_3 = u_2$ , the input  $u = U_{\text{in}}$  and the output  $y = U_{\text{out}}$ . The system's equations are given by

$$L \cdot \frac{di_L}{dt} = u_L = U_{\text{in}} - u_2$$

$$\Rightarrow \frac{di_L}{dt} = \frac{U_{\text{in}}}{L} - \frac{u_2}{L}$$

$$C_1 \cdot \frac{du_1}{dt} = i_1 = u_R \cdot R = (u_2 - u_1) \cdot R$$

$$\Rightarrow \frac{du_1}{dt} = \frac{R}{C_1} \cdot u_2 - \frac{R}{C_1} \cdot u_1$$

$$C_2 \cdot \frac{du_2}{dt} = i_2 = i_L - i_1 = i_L - (u_2 - u_1) \cdot R$$

$$\Rightarrow \frac{du_2}{dt} = \frac{i_L}{C_2} - \frac{R}{C_2} \cdot u_2 + \frac{R}{C_2} \cdot u_1$$

This leads to the following state space representation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1}{L} \\ 0 & -\frac{R}{C_1} & \frac{R}{C_1} \\ \frac{1}{C_2} & \frac{R}{C_2} & -\frac{R}{C_2} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

What we have now is a continuous time state space model of the form

$$\begin{aligned}\dot{x}(t) &= \bar{A}x(t) + \bar{B}u(t) \\ y(t) &= \bar{C}x(t)\end{aligned}$$

Now we can assume, that all electrical devices are not ideal or that there are external disturbances like fluctuations in temperature or air moisture, such that their behaviour is not ideal. These factors will lead to process noise  $w(t)$ . Furthermore we will measure the output voltage  $U_{\text{out}}$  with a voltmeter, which will not measure the values exactly or which has an inappropriate resolution. This will add some measurement noise  $v(t)$  to our model. Our new state space representation will be

$$\begin{aligned}\dot{x}(t) &= \bar{A}x(t) + \bar{B}u(t) + w(t) \\ y(t) &= \bar{C}x(t) + v(t)\end{aligned}$$

with

$$\begin{aligned}E[w(t)w(\tau)] &= Q_e \delta(t - \tau) \\ E[v(t)v(\tau)] &= R_e \delta(t - \tau)\end{aligned}$$

so  $w(t)$  and  $v(t)$  are white Gaussian noise processes. Since we assume that our model is quite reliable and there are only few external disturbances, the process noise will be much smaller than the measurement noise. Reasonable choices for  $Q_e$  and  $R_e$  could be

$$Q_e = \begin{bmatrix} 10^{-4} & 0 & 0 \\ 0 & 10^{-4} & 0 \\ 0 & 0 & 10^{-4} \end{bmatrix}, \quad R_e = 10^{-1}$$

The last step is to convert this system into a discrete time linear system. The state space representation for such a system is given by

$$\begin{aligned}x_{k+1} &= Ax_k + Bu_k + w_k \\ y_k &= Cx_k + v_k\end{aligned}$$

The matrices  $Q_e$  and  $R_e$  will stay the same. All other matrices are given by

$$A = e^{\bar{A}T}, \quad B = \int_0^T e^{\bar{A}(T-\tau)} \bar{B} d\tau, \quad C = \bar{C}$$

where  $T$  represents the sampling rate of our discrete time model. By choosing appropriate values for  $L$ ,  $C_1$ ,  $C_2$  and  $R$ , we finally have enough information to build a Kalman filter for this linear system.

## *A. Examples*

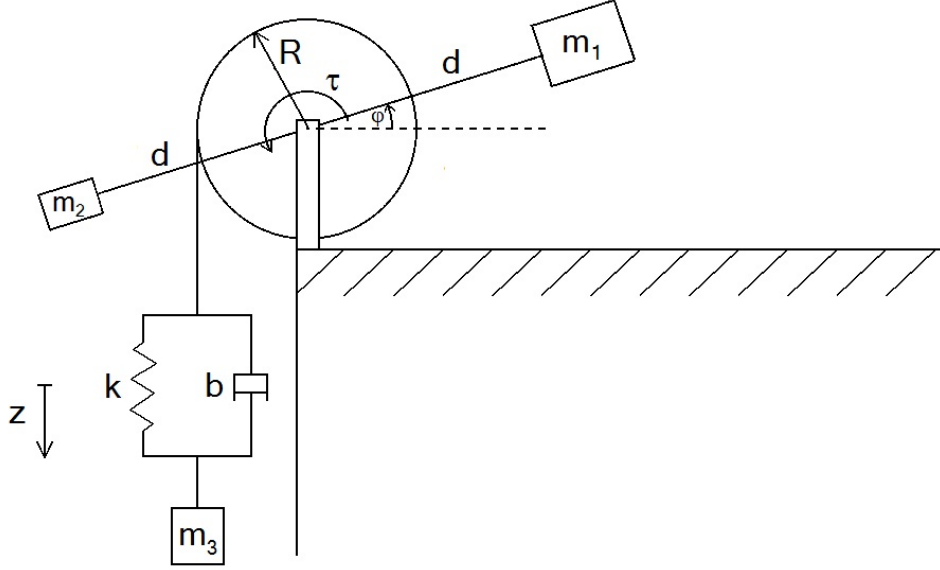
### **A.2. Kalman Filtering of Nonlinear System**

# Group project - Nonlinear model

Daniel Gilgen, David Lehnert, Matthias Roggo, Fabio Marti

April 22, 2012

Consider the following mechanical system



We assume now that the wheel doesn't have a mass, that  $m_1$  has a moment of inertia of  $J_1 = c_1 m_1$  and that  $m_2$  has  $J_2 = c_2 m_2$  with respect to the wheel's hub. Both masses are modelled as dots with distance  $d$  to the hub. Furthermore we are assuming that the force of the damper is given by  $F_b = b \cdot \Delta z$ . Using Newton's laws we obtain the equations

$$m_3 \ddot{z} = m_3 g + k(\varphi R - z) + b(\dot{\varphi} R - \dot{z})$$

$$(J_1 + J_2) \ddot{\varphi} = \tau + g d \cos(\varphi)(m_2 - m_1) + k(z - \varphi R) + b(\dot{z} - \dot{\varphi} R)$$

By defining the states  $x_1 = z$ ,  $x_2 = \dot{z}$ ,  $x_3 = \varphi$  and  $x_4 = \dot{\varphi}$ , the input  $u = \tau$  and the output  $y = \dot{\varphi}$  we find the following differential equations

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = g + \frac{k}{m_3}(x_3 R - x_1) + \frac{b}{m_3}(x_4 R - x_2)$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{1}{c_1 m_1 + c_2 m_2} (u + g d \cos(x_3)(m_2 - m_1) + k(x_1 - x_3 R) + b(x_2 - x_4 R))$$

$$y = x_4$$

In a next step we transform our continuous time system into a discrete time system. We use a simple Euler step for this task

$$\dot{x} \approx \frac{x_{k+1} - x_k}{T}$$

where  $T$  is the sampling period. The discrete system's equations now look as follows

$$x_{1,k+1} = x_{1,k} + T \cdot x_{2,k}$$

$$x_{2,k+1} = x_{2,k} + T \cdot \left( g + \frac{k}{m_3}(x_{3,k}R - x_{1,k}) + \frac{b}{m_3}(x_{4,k}R - x_{2,k}) \right)$$

$$x_{3,k+1} = x_{3,k} + T \cdot x_{4,k}$$

$$x_{4,k+1} = x_{4,k} + T \cdot \left( \frac{1}{c_1 m_1 + c_2 m_2} (u_k + g d \cos(x_{3,k})(m_2 - m_1) + k(x_{1,k} - x_{3,k}R) + b(x_{2,k} - x_{4,k}R)) \right)$$

$$y_k = x_{4,k}$$

Since we are dealing with nonideal effects, caused by model uncertainties or measurement errors, we add process noise  $w_k$  and measurement noise  $v_k$  to our model. We will get equations of the form

$$x_{k+1} = f(x_k, u_k, w_k)$$

$$y_k = h(x_k, v_k)$$

Note that  $x_{k+1}$  as well as  $w_k$  and  $v_k$  are column vectors. In our nonlinear model,  $w_k$  and  $v_k$  are realizations of white Gaussian noise processes

$$E[w_k w_n^T] = W_k Q_k W_k^T \delta[k - n]$$

$$E[v_k v_n^T] = V_k R_k V_k^T \delta[k - n]$$

where  $W_k = \frac{\partial f}{\partial w_k}(x_{k-1}, u_{k-1}, 0)$  and  $V_k = \frac{\partial h}{\partial v_k}(x_k, 0)$ . Note that  $Q_k$  and  $R_k$  could change every timestep  $k$ . Possible values for them are

$$Q_k = \begin{bmatrix} 10^{-6} & 0 & 0 & 0 \\ 0 & 10^{-6} & 0 & 0 \\ 0 & 0 & 10^{-6} & 0 \\ 0 & 0 & 0 & 10^{-6} \end{bmatrix}, \quad R_k = 10^{-2}$$

We have a full description of our nonlinear system and are now able to implement an extended Kalman filter by choosing reasonable values for the system's parameters.



# Bibliography

- [1] G. Welch and G. Bishop, “An Introduction to the Kalman Filter,” UNC-Chapel Hill, 2006.
- [2] RoboCup Technical Committee, “RoboCup Standard Platform League (Nao) Rule Book,” RoboCup, 2011.