Group project - Nonlinear model

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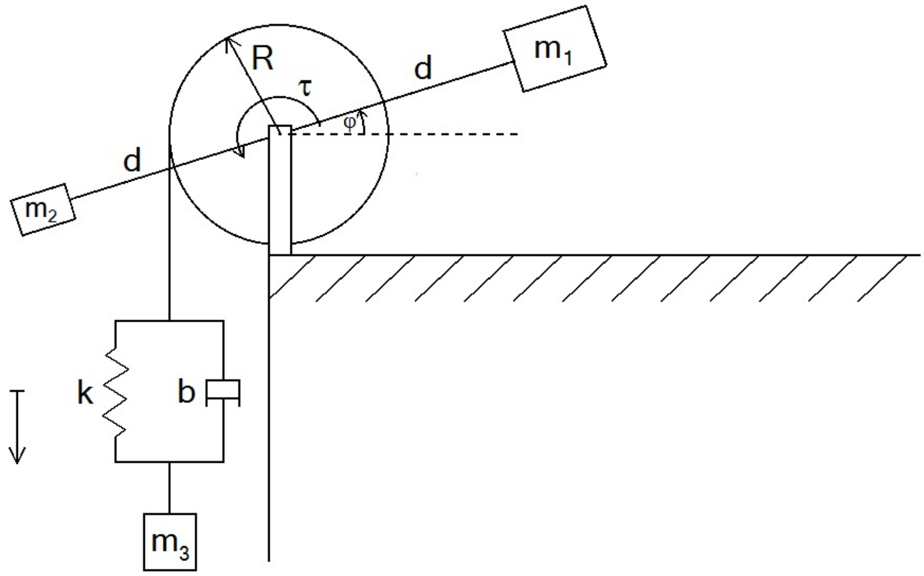
Consider the following mechanical system



We assume now that the wheel doesn't have a mass, that m_1 has a moment of inertia of $J_1 = c_1 m_1$ and that m_2 has $J_2 = c_2 m_2$ with respect to the wheel's hub. Both masses are modelled as dots with distance d to the hub. Furthermore we are assuming that the force of the damper is given by $F_b = b \cdot \Delta z$. Using Newton's laws we obtain the equations

$$\begin{aligned} m_3 \ddot{z} &= m_3 g + k(\varphi R - z) + b(\dot{\varphi} R - \dot{z}) \\ (J_1 + J_2)\ddot{\varphi} &= \tau + gd\cos(\varphi)(m_2 - m_1) + k(z - \varphi R) + b(\dot{z} - \dot{\varphi} R) \end{aligned}$$

By definig the states $x_1 = z$, $x_2 = \dot{z}$, $x_3 = \varphi$ and $x_4 = \dot{\varphi}$, the input $u = \tau$ and the output $y = \dot{\varphi}$ we find the following differential equations

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= g + \frac{k}{m_3}(x_3 R - x_1) + \frac{b}{m_3}(x_4 R - x_2) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{1}{c_1 m_1 + c_2 m_2}\left(u + gd\cos(x_3)(m_2 - m_1) + k(x_1 - x_3 R) + b(x_2 - x_4 R)\right) \end{aligned}$$

$$y = x_4$$

In a next step we transform our continuous time system into a discrete time system. We use a simple Euler step for this task

$$\dot{x} \approx \frac{x_{k+1} - x_k}{T}$$

where T is the sampling period. The discrete system's equations now look as follows

$$\begin{aligned}
x_{1,k+1} &= x_{1,k} + T \cdot x_{2,k} \\
x_{2,k+1} &= x_{2,k} + T \cdot \left(g + \frac{k}{m_3}(x_{3,k}R - x_{1,k}) + \frac{b}{m_3}(x_{4,k}R - x_{2,k}) \right) \\
x_{3,k+1} &= x_{3,k} + T \cdot x_{4,k} \\
x_{4,k+1} &= x_{4,k} + T \cdot \left(\frac{1}{c_1 m_1 + c_2 m_2} \left(u_k + gd\cos(x_{3,k})(m_2 - m_1) + k(x_{1,k} - x_{3,k}R) + b(x_{2,k} - x_{4,k}R)\right) \right) \\
y_k &= x_{4,k}
\end{aligned}$$

Since we are dealing with nonideal effects, caused by model uncertainties or measurement errors, we add process noise w_k and measurement noise v_k to our model. We will get equations of the form

$$\begin{aligned}
x_{k+1} &= f(x_k, u_k, w_k) \\
y_k &= h(x_k, v_k)
\end{aligned}$$

Note that x_{k+1} as well as w_k and v_k are column vectors. In our nonlinear model, w_k and v_k are realizations of white Gaussian noise processes

$$\begin{aligned}
E[w_k w_n] &= W_k Q_k W_k^T \delta[k-n] \\
E[v_k v_n] &= V_k R_k V_k^T \delta[k-n]
\end{aligned}$$

where $W_k = \frac{\partial f}{\partial w_k}(x_{k-1}, u_{k-1}, 0)$ and $V_k = \frac{\partial h}{\partial v_k}(x_k, 0)$. Note that Q_k and R_k could change every timestep k. Possible values for them are

$$Q_k = \begin{bmatrix} 10^{-6} & 0 & 0 & 0 \\ 0 & 10^{-6} & 0 & 0 \\ 0 & 0 & 10^{-6} & 0 \\ 0 & 0 & 0 & 10^{-6} \end{bmatrix}, \qquad R_k = 10^{-2}$$

We have a full description of our nonlinear system and are now able to implement an extended Kalman filter by choosing reasonable values for the system's parameters.