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Robocup: Cooperative estimation and prediction of player and ball movement

Group Project

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Abstract

Hier kommt das Abstract ...

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3.1 Linear Kalman Filter

There exists a recursive Kalman filter algorithm for discrete time systems.[1] This ongoing Kalman filter cycle can be divided into two groups of equations:

1. Time update equations

$$\begin{aligned} \hat{x}_k^- &= A\hat{x}_{k-1} + Bu_{k-1} \\ P_k^- &= AP_{k-1}A^T + Q \end{aligned} \tag{1}$$

2. Measurement update equations

$$\begin{aligned} K_k &= P_k^- H^T (HP_k^- H^T + R)^{-1} \\ \hat{x}_k &= A\hat{x}_{k-1}^- + K_k(z_k - H\hat{x}_k^-) \\ P_k &= (I - K_k H)P_k^- \end{aligned} \tag{2}$$

To test this algorithm and to learn something about applying the Kalman filter on linear system we created an example. A detailed description of the following example can be found in appendix A.1.
We consider a linear, timeinvariant model, given by the following circuit diagram in Figure 1. First we added process noise and measurement noise

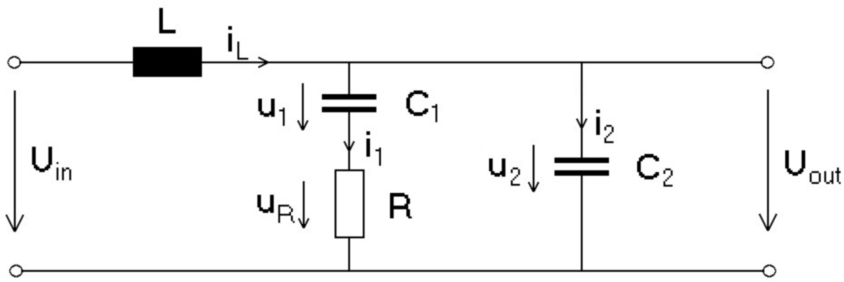

Figure 1: Example: Linear electrical circuit.

to the system output. The aim is to get an estimation of the noisy output. Therefore we applied the Kalman filter algorithm based on the equations 1. In the figure 2, above we can see the input signal and the ideal measurement of the output signal. That ideal measurement includes process noise, which can obviously not be filtered by the Kalman filtering algorithm. Below one can see the noisy measurement on the left side and the filtered output on the right side.

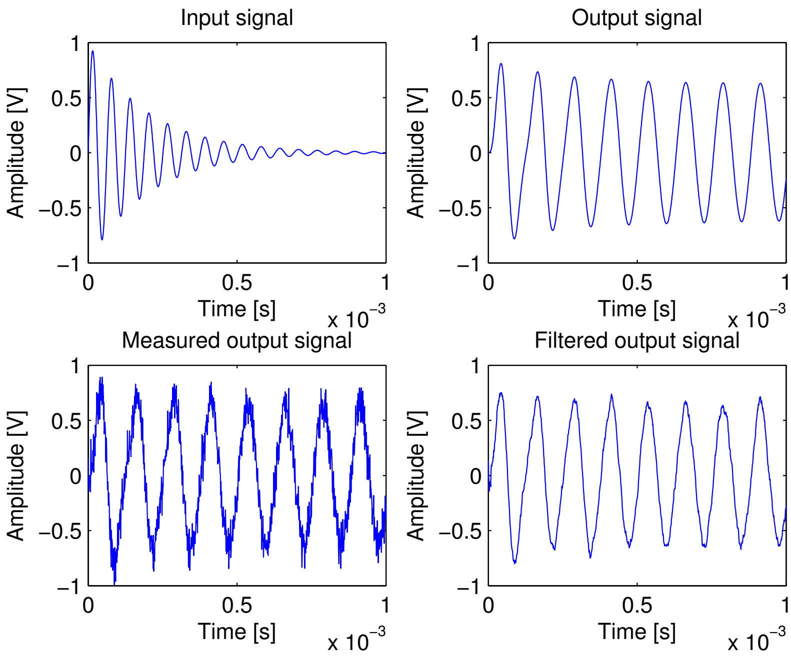

Figure 2: Example: Input signal, output signal, noisy output, and filtered ouptut.

3.2 Extended Kalman Filter (EKF)

4 Estimation

4.1 Estimate of the Ball

4.2 Estimate of the Robots

A Examples

A.1 Kalman Filtering of Linear System

Group project - Linear model

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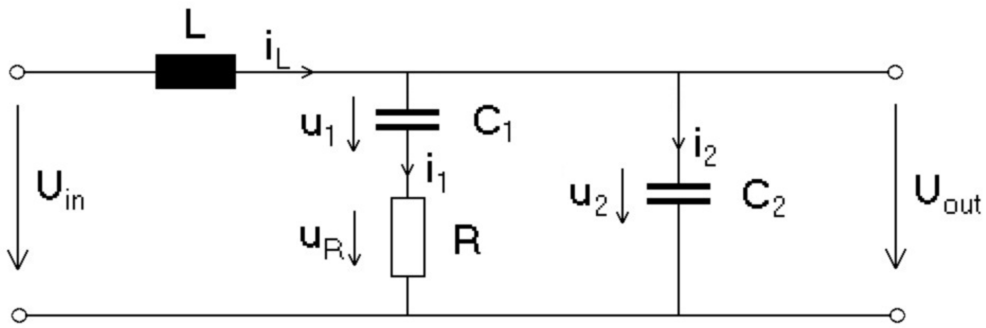
We are considering a linear, timeinvariant model, given by the following circuit diagram



This is obviously a system of order 3 with the states $x_1 = i_L$, $x_2 = u_1$ and $x_3 = u_2$, the input $u = U_{\text{in}}$ and the output $y = U_{\text{out}}$. The system's equations are given by

$$L \cdot \frac{di_L}{dt} = u_L = U_{\text{in}} - u_2$$

$$\Longrightarrow \qquad \frac{di_L}{dt} = \frac{U_{\text{in}}}{L} - \frac{u_2}{L}$$

$$C_1 \cdot \frac{du_1}{dt} = i_1 = u_R \cdot R = (u_2 - u_1) \cdot R$$

$$\Longrightarrow \qquad \frac{du_1}{dt} = \frac{R}{C_1} \cdot u_2 - \frac{R}{C_1} \cdot u_1$$

$$C_2 \cdot \frac{du_2}{dt} = i_2 = i_L - i_1 = i_L - (u_2 - u_1) \cdot R$$

$$\Longrightarrow \qquad \frac{du_2}{dt} = \frac{i_L}{C_2} - \frac{R}{C_2} \cdot u_2 + \frac{R}{C_2} \cdot u_1$$

This leads to the following state space representation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1}{L} \\ 0 & -\frac{R}{C_1} & \frac{R}{C_1} \\ \frac{1}{C_2} & \frac{R}{C_2} & -\frac{R}{C_2} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

What we have now is a continuous time state space model of the form

$$\dot{x}(t) = \bar{A}x(t) + \bar{B}u(t)$$
$$y(t) = \bar{C}x(t)$$

Now we can assume, that all electrical devices are not ideal or that there are external disturbances like fluctuations in temperature or air moisture, such that their behaviour is not ideal. These factors will lead to process noise $w(t)$. Furthermore we will measure the output voltage U_{out} with a voltmeter, which will not measure the values exactly or which has an inappropriate resolution. This will add some measurement noise $v(t)$ to our model. Our new state space representation will be

$$\dot{x}(t) = \bar{A}x(t) + \bar{B}u(t) + w(t)$$
$$y(t) = \bar{C}x(t) + v(t)$$

with

$$E[w(t)w(\tau)] = Q_e\delta(t-\tau)$$
$$E[v(t)v(\tau)] = R_e\delta(t-\tau)$$

so $w(t)$ and $v(t)$ are white Gaussian noise processes. Since we assume that our model is quite reliable and there are only few external disturbances, the process noise will be much smaller that the measurement noise. Reasonable choices for Q_e and R_e could be

$$Q_e = \begin{bmatrix} 10^{-4} & 0 & 0 \\ 0 & 10^{-4} & 0 \\ 0 & 0 & 10^{-4} \end{bmatrix}, \qquad R_e = 10^{-1}$$

The last step is to convert this system into a discrete time linear system. The state space representation for such a system is given by

$$x_{k+1} = Ax_k + Bu_k + w_k$$
$$y_k = Cx_k + v_k$$

The matrices Q_e and R_e will stay the same. All other matrices are given by

$$A = e^{\bar{A}T}, \qquad B = \int_0^T e^{\bar{A}(T-\tau)}\bar{B}\, d\tau, \qquad C = \bar{C}$$

where T represents the sampling rate of our discrete time model. By choosing appropriate values for L, C_1, C_2 and R, we finally have enough information to build a Kalman filter for this linear system.

References

[1] G. Welch and G. Bishop, “An Introduction to the Kalman Filter,” UNC-Chapel Hill, 2006.

[2] RoboCup Technical Committee, “RoboCup Standard Platform League (Nao) Rule Book,” RoboCup, 2011.