Group project - Nonlinear model

Daniel Gilgen, David Lehnen, Matthias Roggo, Fabio Marti

April 21, 2012

Consider the following mechanical system

We assume now that the wheel doesn't have a mass and that mass m_1 has a moment of inertia of $J_1 = c_1 m_1$ and mass m_2 has $J_2 = c_2 m_2$ with respect to the wheel's hub. Furthermore we are assuming that the force of the damper is given by $F_b = b \cdot \Delta z$. Using Newton's laws we obtain the equations

$$\begin{aligned} m_3 \ddot{z} &= m_3 g + k(\varphi R - z) + b(\dot{\varphi} R - \dot{z}) \\ (J_1 + J_2)\ddot{\varphi} &= \tau + gd\cos(\varphi)(m_2 - m_1) + k(z - \varphi R) + b(\dot{z} - \dot{\varphi} R) \end{aligned}$$

By definig the states $x_1 = z$, $x_2 = \dot{z}$, $x_3 = \varphi$ and $x_4 = \dot{\varphi}$, the input $u = \tau$ and the output $y = \dot{\varphi}$ we find the following differential equations

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= g + \frac{k}{m_3}(x_3 R - x_1) + \frac{b}{m_3}(x_4 R - x_2) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{1}{c_1 m_1 + c_2 m_2}\left(u + gd\cos(x_3)(m_2 - m_1) + k(x_1 - x_3 R) + b(x_2 - x_4 R)\right) \\ y &= x_4 \end{aligned}$$

In a next step we transform our continuous time system into a discrete time system. We use a simple Euler step for this task

$$\dot{x} \approx \frac{x_{k+1} - x_k}{T}$$

where T is the sampling period. The discrete system's equations now look as follows

$$
\begin{aligned}
x_{1,k+1} &= x_{1,k} + T \cdot x_{2,k} \\
x_{2,k+1} &= x_{2,k} + T \cdot \left(g + \frac{k}{m_3}(x_{3,k}R - x_{1,k}) + \frac{b}{m_3}(x_{4,k}R - x_{2,k}) \right) \\
x_{3,k+1} &= x_{3,k} + T \cdot x_{4,k} \\
x_{4,k+1} &= x_{4,k} + T \cdot \left(\frac{1}{c_1 m_1 + c_2 m_2} \left(\tau + gd\cos(x_{3,k})(m_2 - m_1) + k(x_{1,k} - x_{3,k}R) + b(x_{2,k} - x_{4,k}R) \right) \right) \\
y_k &= x_{4,k}
\end{aligned}
$$