# DeepSequent

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## Contents

# 1 Introduction

DeepSequent is a program for proving validity of QBF formulas via sequence calculus.

It supports several AI methods which will be discussed in more detail in further sections.

## 1.1 Terminology

- **Proof tree -** Tree graph with the assigned problem at its root. For every applicable action, there is an edge connecting the root to the state of the proof after applying this action. Deeper nodes are connected with their successors in the same way. Basically the game tree for the sequence calculus.
- **Proof state -** Every node of the proof tree.
- **Leaf state -** A proof state whose successors have not been yet derived.
- **Action -** Application of a rule which will, given a proof state, derive one or more successor sates.

# 2 General project structure

## 2.1 Files included

- **solver.py**: Main file. Calls the parser to create an object representation of the formula and then solves it with the corresponding method.
- **formula_parser.py**: Contains function *parse* which takes a file directory as its argument and returns its object representation. The parsing process is described in more detail later.
- **connectives.py**: Contains operator and literal classes.
- **rules.py**: Contains rules for deriving new proof states.
- **axioms.py**: Contains rules for ending the proofs and thus successfully closing the branches.

## 2.2 Building blocks

This subsection provides an exhaustive list of objects and classes with which DeepSequent operates.

- State: Corresponds to a single *proof state*. Contains a list of formulas of which the last one is the goal. The definition of this class can be found in *Implementation/rules.py*.
- Literal: This object will be renamed to 'Variable' in the near future because that's what it is. It contains the value which identifies the corresponding variable. The class also has a static attribute *used_tokens* which comprises all variables present in the current problem. Class is defined in *Implementation/connectives.py*.

### 2.2.1 Connectives

The following classes can be found in *Implementation/connectives.py*.

- Universal quantifier: Has attribute *variables* which is the list of all variables bounded by corresponding quantifier instance and also attribute *successor* containing the link for its uppermost successor.
- Existential quantifier: Has attribute *variables* which is the list of all variables bounded by corresponding quantifier instance and also attribute *successor* containing the link for its uppermost successor.
- And: Simple object with a single property *operands* containing list of its operands. And with an empty operand list is used as $\top$.
- Or: Simple object with a single property *operands* containing list of its operands. Or with an empty operand list is used as $\bot$.

- Xor: Simple object with a single property *operands* containing list of its operands.
- Not: Simple object with a single property *operand* containing a link to its operands.

### 2.2.2 Rules

The following classes can be found in *Implementation/rules.py*. Those classes have only static methods because there was no use in creating their instances.

All rules operate only on the uppermost connectives of state's formulas.

- Negation rules
  - Eliminate double negation: This rule takes a formula and removes all negations from its top if there is even number of them. Otherwise, it leaves it negated with a single negation.
  - Negate goal: This rule is applicable only if the goal is not $\perp$. It sets the goal to $\perp$, negates the previous one and moves it among assumptions.
  - Negate a sequent: This rule is applicable only if the goal is $\perp$. It sets the goal to be a negation of a condition formula and removes this formula from the assumption.
- De Morgan's laws: Those are well known and are implemented in the most standard way so they hopefully do not need any special explanation.
  - And
  - Or
  - Universal
  - Existential
- Quantifier replacement rules: These rules find all possible truth value assignments of variables bounded by the given quantifier. For each of those assignments replace variables in the successor formula by corresponding truth constants and add the result among operands of a corresponding connective.
  - Universal: Corresponding connective is *And*.
  - Existential: Corresponding connective is *Or*.
- Assumption rules
  - And: Splits formula into its operands.
  - Or: For each operand creates a separate branch where this operand replaces the original *Or* formula. All branches must be proven.

- Xor: For each operand creates a separate branch where this operand replaces the original *Xor* formula. Other operands are negated and added to the assumption too. All branches must be proven.
- Universal: Instantiates. Removes the quantifier and replaces the variables bounded by it by other variables used in the other formulas of the proof.
- Existential: Skolemnizies. Removes the quantifier and replaces the variables bounded by it with newly introduced skolem variables.

- Goal rules
  - And: For each operand creates a separate branch where this operand replaces the original *And* formula. All branches must be proven.
  - Or: For each operand creates a separate branch where this operand replaces the original *Or* formula. This operation is indeterministic so proving one branch is enough.
  - Xor: Sets $\perp$ as the goal and creates two branches, each with a formula corresponding to *Xor* negation. All branches must be proven.
  - Universal: Skolemnizies. Removes the quantifier and replaces the variables bounded by it with newly introduced skolem variables.
  - Existential: Instantiates. Removes the quantifier and replaces the variables bounded by it by other variables used in the other formulas of the proof.

#### 2.2.3 Axioms

Those are also well-known and hopefully do not need any special commentary.

- Goal in assumption.
- Contradiction in assumption.

### 2.3 Parsing .qcir files

The parsing procedure is as follows:

1. Check that the given file is in *qcir* format.
2. Read the prenex: Store first quantifier in *Formula* variable.
3. Each time a quantifier is read, set it as a successor of the downer most *Formula* successor.
4. When *output(...)* is read, switch to post-prenex mode and save the output variable

5. Each time a new line is read, replace those of its variables that are stored as placeholders for more elaborate formulas by those formulas. Store the result together with its placeholder.

6. When the output formula is met, set it as a successor of the downer most *Formula* successor.

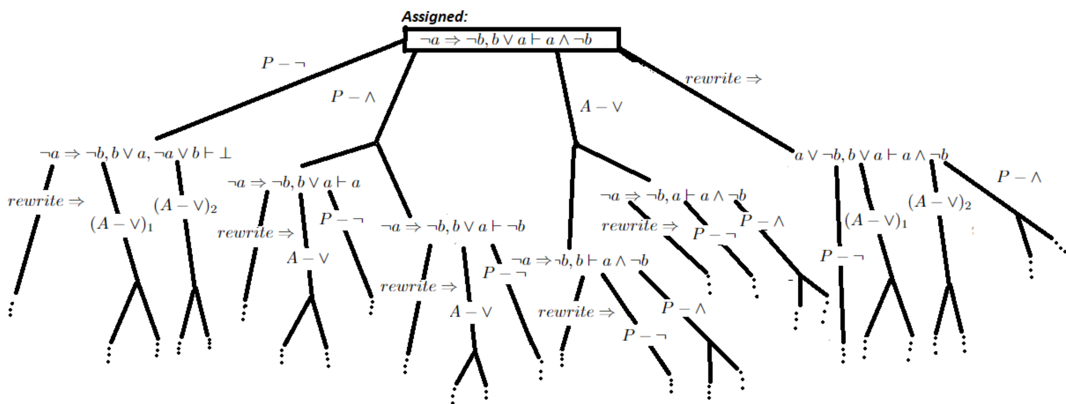
7. Create a new *State*, where the assumption is $\neg$*Formula* and goal is $\bot$.

# 3 Brute force

## 3.1 Basic idea

For each leaf state, derive and store all of its successors. This approach could be complete in theory, but because of physical limitations, it cannot be implemented in both a sound and complete way.

Figure 1: Figure and code for search-space of brute-force search solving




```python
# Algorithm: Brute force search

# Data:Graph node
# Return: proof found ? True : False
def brute_force(node):
    for action in node.possibleActions():
        succesor_state = node.applyAction(action)
        if succesor_state.fitAxioms():
            return True
        else:
            succesor_result = brute_force(succesor_state)
            if succesor_result: return True
    return False
```

## 3.2 Actual implementation

This subsection describes the logic of *brute_force* function which is recursively called to get a proof of the validity of a formula.

1. Check whether the given state fits the axioms. If it does, return the fulfilled axiom.
2. If not, save the state to the transposition table together with an empty string indicating that proof for this state has not been yet found.
3. Get all actions, that can be applied on the uppermost formulas of the state. Also, do some heuristics to reduce a search space a bit (e.g. by removing *Negate sequent* if there are only literals among the state's formulas).
4. For each action, get a list of formulas on which it can be applied and get the states derived by applying it.
5. If those states are already saved in the transposition table, get their proofs from there. Otherwise, apply *brute_force* to them.
6. If proofs were found for all derived states (or at least one in case of indeterministic rules applications), add the current state and action to the obtained proof. Save the proof to the transposition table and return it.
7. If no of the operations resulted in a proof or if no applicable action was found or if a recursion error occurred, return an empty string indicating that proof was not found (empty strings have false boolean values in Python, which makes checking, that proof was found, very convenient).

## 4 References

[1] Martina Seidl Charles Jordan Will Klieber. *Non-CNF QBF Solving with QCIR*. URL: `https://fmv.jku.at/papers/JKS-BNP.pdf`.

[2] Uwe Egly. *On Sequent Systems and Resolution for QBFs*. URL: `https://publik.tuwien.ac.at/files/PubDat_214419.pdf`.

[3] Lars Hupel Tobias Nipkow. *Logics Exercise*. URL: `https://www21.in.tum.de/teaching/logik/SS18/sol03.pdf`.