A particle-based elastic wave simulation method in 3D arbitrary anisotropic media using GPU

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Introduction

Forward modeling of the behavior for the geological materials is a fundamental issue in seismology. In terms of elastic wave simulation, numerical methods such as the finite element methods, the finite difference methods are widely used to give approximate solutions to wave equation. O'Brien & Bean (2004a,2004b) bring up a particle based elastic lattice method for 3D elastic wave simulation with topography and use this method to study volcano earthquake. Xia et al. (2017) shows the implementation of the several lattice spring models in wave simulation. As for anisotropic case, difficulty lies in building the right anisotropic lattice model. To overcome this challenge, Hu and Jia (2016) bring up the dynamic lattice method where a 2-dimensional anisotropic lattice model is designed for elastic wave simulation in TTI media. Latter Hu and Jia (2018) establish the theory of high-order DLM to suppress spatial dispersion during wave simulation. The major purpose of this article is to expand the application of the DLM to elastic wave modeling in 3D complex anisotropic media. In this article, a 3D body-centered lattice model used for elastic wave simulation in 3D anisotropic media is introduced. We show the particle interaction patterns in this particular lattice model and demonstrate the general relation between the lattice model and the anisotropic media. We also use GPU to accelerate the calculation of the wave propagation in 3D model. In the numerical example, we carry out elastic wave simulations in 3D arbitrary anisotropic media.

3D anisotropic lattice model

To simulate 3D arbitrary anisotropic media, we employ a body-centered lattice model shown in figure 1. In the basic lattice unit, the central particle is connected with its 18 neighboring particles through lattice bonds. As figure 2 shows, these bonds work like linear springs while the angles between these bonds are regarded as angular springs. The coefficients k_{ij} is used to describe the stiffness of the linear spring between particles i and j while the coefficient b_{ijk} represents the stiffness of the angular spring connecting particles i,j and k with the particle i as the pivot. In the case of anisotropy, the angular springs exist in the *xy-*,*xz-*,*yz-* planes (e.g., figures 3a) as well as four hexagonal planes(e.g., figures 3b). The balanced angles between the angular springs of *xy-*,*xz-*,*yz-* planes are $\frac{\pi}{2}$ while in the four hexagonal planes the balanced angles between the angular springs are $\frac{\pi}{3}$. The elastic energy in the lattice unit is given by

$$E=\frac{1}{2}\sum_{i}k_{0i}(|\mathbf{r}_{0i}|-|\mathbf{R}_{0i}|)^2+\frac{1}{2}\sum_{j,k}b_{j0k}(\cos\theta_{j0k}-\cos\theta^0{}_{j0k})^2 \qquad (1)$$

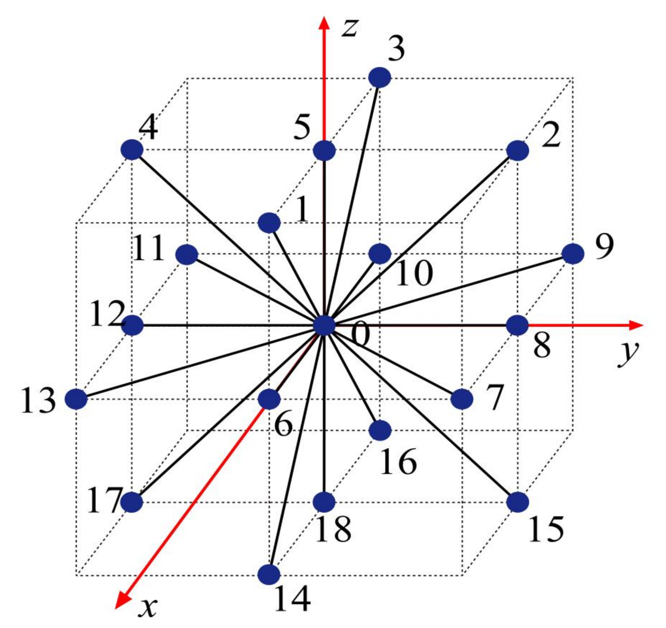

Figure 1: One particle lattice unit for the 3D lattice model. The central particle is connected with its 18 neighboring particles through lattice bonds.

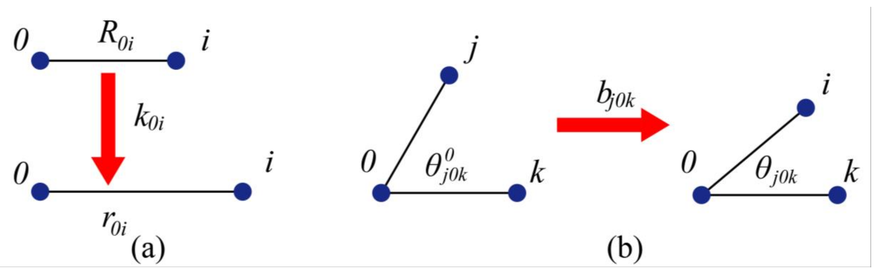

Figure 2. Stretching and bending of the bonds. (a) linear springs before and after distortion (b) angular springs before and after distortion. Red arrows denote the distortion of the particles.

The first sum of the equation is taken over half of the linear springs in the lattice unit to avoid double counting of the linear spring. The second sums over all angular springs that use particle *0* as their pivots and the particle *j*, *k* are adjacent particles. In our body-centered lattice model, the total elastic energy counts the three *xy-*, *xz-*, *yz-* planes and the four hexagonal planes. Differentiating the elastic energy with respect to the particle displacements yields the interaction forces on the corresponding particles. Therefore, the interaction force on the central particle *0* is given by

$$\mathbf{F}_0=\frac{\partial E}{\partial \mathbf{u}_0}=\sum_i -k_{0i}\left(\frac{(\mathbf{u}_i-\mathbf{u}_0)\cdot\mathbf{R}_{0i}}{|\mathbf{R}_{0i}|^2}\right)\cdot\mathbf{R}_{0i}-\sum_{j,k}b_{j0k}\left[(\mathbf{u}_j-\mathbf{u}_0)\cdot\mathbf{s}_{kj}+(\mathbf{u}_k-\mathbf{u}_0)\cdot\mathbf{s}_{jk}\right](\mathbf{s}_{kj}+\mathbf{s}_{jk}) \qquad (2)$$

And for non-central particle *i*, the interaction force takes the form as

$$\mathbf{F}_i=\frac{\partial E}{\partial \mathbf{u}_i}=k_{0i}\left(\frac{(\mathbf{u}_i-\mathbf{u}_0)\cdot\mathbf{R}_{0i}}{|\mathbf{R}_{0i}|^2}\right)\cdot\mathbf{R}_{0i}+\sum_{k}b_{i0k}\left[(\mathbf{u}_i-\mathbf{u}_0)\cdot\mathbf{s}_{ki}+(\mathbf{u}_k-\mathbf{u}_0)\cdot\mathbf{s}_{ik}\right]\cdot\mathbf{s}_{ki} \qquad (3)$$

Another critical part of this dynamic lattice method is to find its connection with the anisotropic medium. Considering the symmetry between the stiffness coefficients in the lattice unit, we reduce the independent stiffness coefficients into k_{01}, k_{02}, k_{03}, k_{04}, k_{07}, k_{09} and b^1_{h1}, b^2_{h1}, b^3_{h1}, b^1_{h2}, b^2_{h2}, b^3_{h2}, b^1_{h3}, b^2_{h3}, b^3_{h3}, b^1_{h4}, b^2_{h4} ,b^3_{h4} . Comparing the elastic energy density in the lattice unit with the strain energy in the anisotropic continuum, we can get the relation between the stiffness coefficients and the elasticity tensor of the arbitrary anisotropic medium. Therefore, the connection between the stiffness coefficients and the elasticity tensor lies in

$$E=V_{eff}\varphi \qquad (4)$$

The effective volume of this body-centered elastic model held by each lattice unit is $V_{eff}=a^3$.

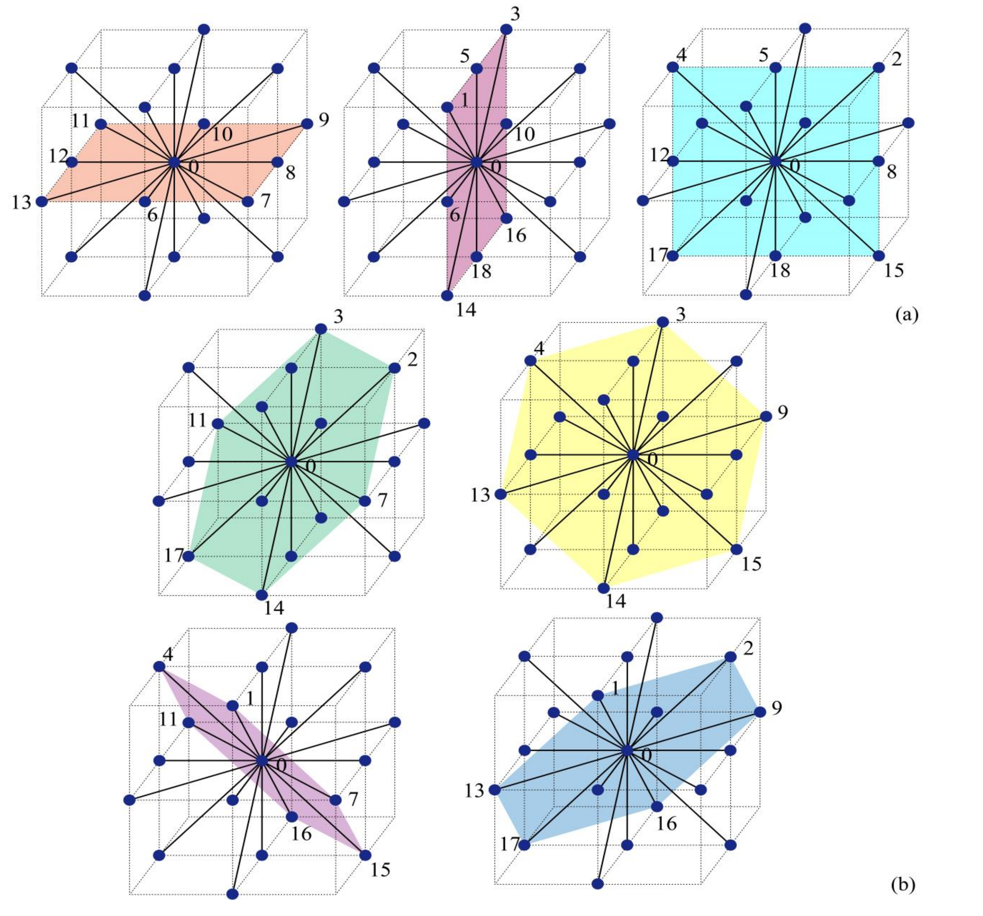

Figure 3. Planes of the body-centered lattice model. (a) *xy-*, *xz-*, *yz-* planes with 8 particles on each plane (b) four hexagonal planes with 6 particles on each plane.

Then we can get the linear transformation between the independent stiffness coefficients and the elasticity tensor of the anisotropic medium. According to equation 2, we can get the interaction force on the central particle *0*. And then we use GPU(GeForce RTX 3090) to accelerate the calculation of force by transforming the calculated stiffness coefficients from CPU device to GPU device and then do the computing of the forces on GPU device. Once the forces of the particles are given, the displacement of the particles are calculated by the Verlet accumulation.

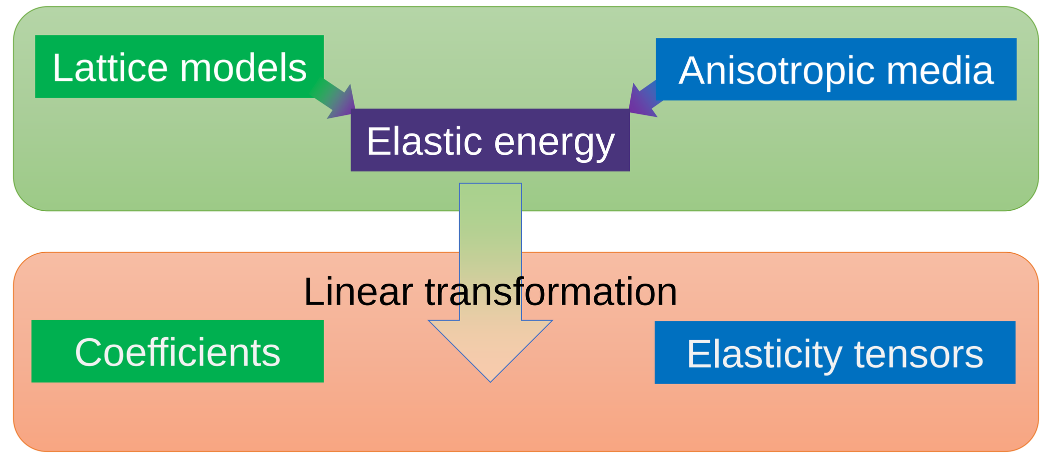

Figure 4. The diagram shows how we find the connection between the stiffness coefficients and the elasticity tensors of the anisotropic media with the elastic energy as the bridge.

Numerical examples

A complex lattice model is used to simulate the elastic wave propagation in anisotropic media. The elastic properties of the anisotropic medium are chosen according to the standard model given by Igel et al. (1995) and we use the velocity and density of the SEG overthrust model to replace the C_{ij} of the following elasticity tensor with the following equation

$$e_{ijknm}=v_{ijk}*v_{ijk}*\rho_{ijk}*C_{nm}/C_{33}$$

$$C_{nm}=\begin{pmatrix}10.0 & 3.5 & 2.5 & -5.0 & 0.1 & 0.3\\ 3.5 & 8.0 & 1.5 & 0.2 & -0.1 & -0.15\\ 2.5 & 1.5 & 6.0 & 1.0 & 0.4 & 0.24\\ -5.0 & 0.2 & 1.0 & 5.0 & 0.35 & 0.525\\ 0.1 & -0.1 & 0.4 & 0.35 & 4.0 & -1.0\\ 0.3 & -0.15 & 0.24 & 0.525 & -1.0 & 3.0\end{pmatrix} \qquad (5)$$

We use the Ricker wavelet with 20 Hz dominant frequency as the source time function. The lattice spacing for this numerical test is 5 m and the time stepping is 0.2ms. We place a single force at the center of the test area. And in this complex lattice model, we need to store $18\times9\times Nx\times Ny\times Nz$ coefficients on the GPU device. So the computation efficiency can only be increased by 8 times then running on the CPU device due to device limitation.

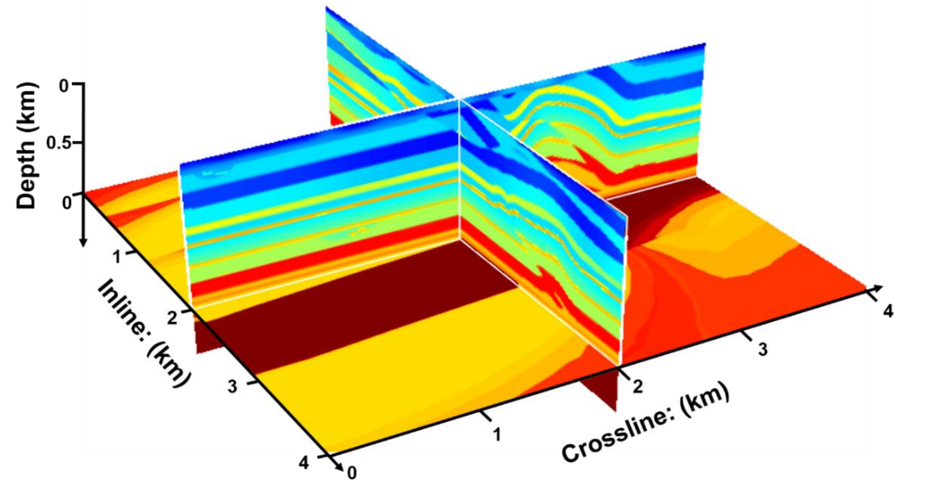

Figure 5. The velocity we used in the anisotropic model.

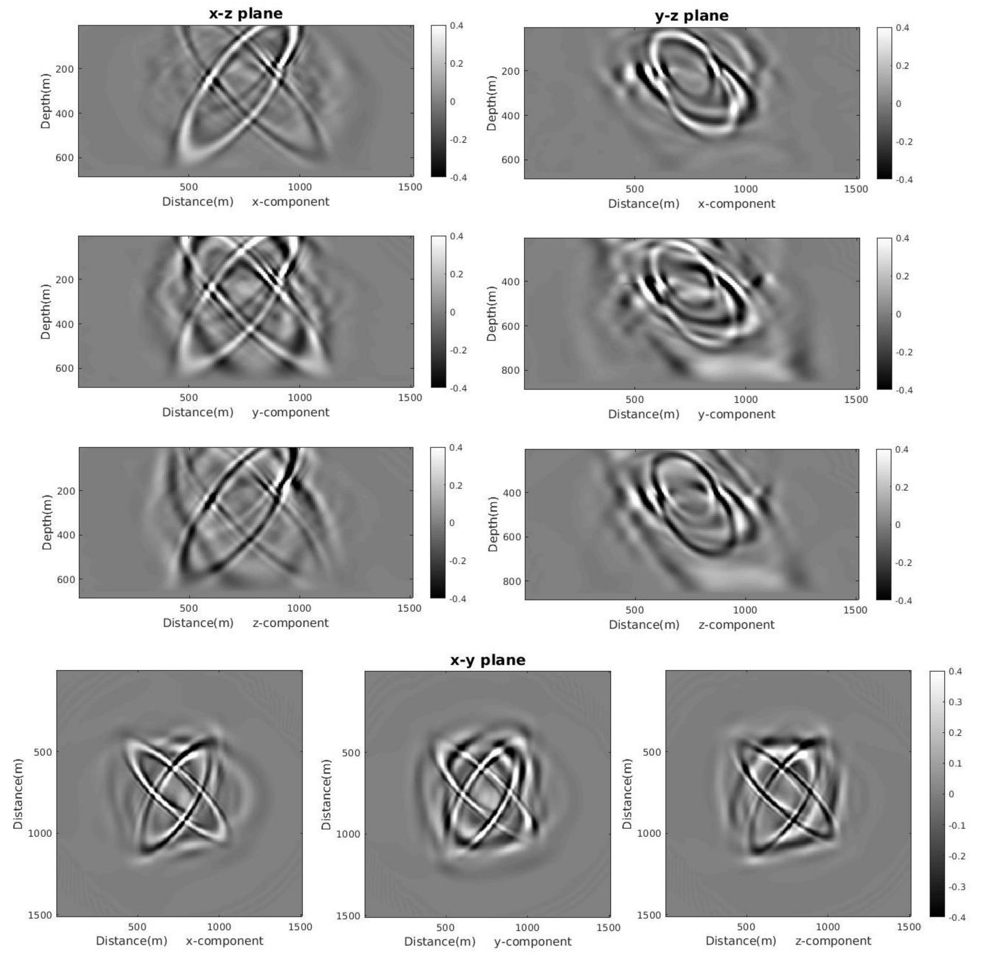

Figure 6. The snapshots simulated by DLM in the anisotropic medium at 0.16s of *x-*,*y-* and *z-*component in the three planes.

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