

Performance of Hybrid-ARQ with Incremental Redundancy over Relay Channels

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Abstract—In this paper, we consider a relay network consisting of a source, a relay, and a destination. The source transmits a message to the destination using hybrid automatic repeat request (HARQ) with incremental redundancy (IR). The relay overhears the transmitted messages over the different HARQ rounds and tries to decode the data packet. In case of successful decoding at the relay, both the relay and the source cooperate to transmit the message to the destination. A maximum number M of HARQ rounds is considered. The channel realizations are independent for different HARQ rounds. We assume Rayleigh fading channels for the links source-relay, source-destination, and relay-destination. We investigate the performance of HARQ-IR over relay channel from an information theoretic perspective. Analytical expressions are derived for the information outage probability, the average number of transmissions, and the average transmission rate. We illustrate through our investigation the benefit of relaying. We also show the impact of the target outage probability ϵ and the maximum number M of HARQ rounds on the outage probability, the average number of transmissions, and the average transmission rate.

I. INTRODUCTION

In the absence of channel state information (CSI) at the transmitter the communication system capacity drops especially at low signal-to-noise ratio (SNR) [1]. It has been proven in [2]–[4] that the hybrid automatic repeat request (HARQ) technique can improve significantly the system capacity in absence of CSI at the transmitter. HARQ has received a lot of interest recently and has been included in several standards, such as High Speed Packet Access (HSPA), Long Term Evolution (LTE) [5], and WiMax [6]. An important class of HARQ protocols is HARQ with incremental redundancy (IR). For this type of HARQ, the transmitter sends new parity bits in each HARQ round. If the destination fails/succeeds to decode the message it sends a NACK/ACK message to the source. Based on the received feedback from the destination node, the source continues to send new parity bits belonging to the same data packet in case of negative feedback (i.e., NACK), or terminate the transmission of the current data packet in case of positive feedback (i.e., ACK). The receiver combines the packets received during all HARQ rounds to decode the current message. The number of HARQ rounds required to decode the message successfully at the receiver depends on the channel conditions. Once the receiver manages to decode the transmitted message or a maximum of M rounds is reached, the transmitter moves to the next data packet. Hence, even

in absence of CSI at the transmitter an implicit adaptation to the channel conditions is implemented in HARQ via the feedback mechanism. This feature is very attractive for fast varying channels where the channel coherence time is shorter than the time required to feedback the CSI from the receiver to the transmitter. Under such conditions, it is not possible to assume CSI at the transmitter and the only way to adapt the rate to the channel conditions is to use the HARQ technique.

The performance of wireless networks can be improved by the use of relay networks. Relay networks allow to improve the reliability and the multiplexing gain of communication systems [7], [8]. The performance of relay networks can be improved by the use of HARQ protocols [9]. This motivates the study of the performance of HARQ in relay networks.

In [9], the outage probability and the throughput of HARQ over relay networks has been studied. The considered relay network consists of a source, a single relay, and a destination node. The author assumes that the channel is constant during all the HARQ rounds used to transmit a data packet. Such an assumption is only valid for indoor environment where we experience low nodes mobility. A similar assumption has been considered in [10]. In that paper, the authors assume multiple relay nodes, the best relay node is first determined using a distributed algorithm. The selected relay cooperates with the source to transmit the message to the destination. The performance of the communication system is analyzed with emphasis on the outage probability and the delay-limited throughput. The long-term average throughput of HARQ for different relaying strategies has been determined in [11] using Monte Carlo simulations. The outage probability of HARQ in relay networks is studied in [12] taking into account interference from neighboring nodes.

In this paper, we investigate the performance of HARQ-IR over a relay channel. In this scenario, the source tries to send a data packet to the destination over several HARQ rounds. The relay overhears these transmissions and tries to decode the data packet. Once the relay succeeds in decoding the message it starts cooperating with the source node to transmit the message to the destination node. As opposed to [9] and [10], we consider in this paper that the channel realization are different for subsequent HARQ rounds. The channel realizations can be assumed to be constant for all HARQ rounds only in indoor environments that are characterized by low mobility. However,

for high and medium mobility environments the channel gains varies for different HARQ rounds which motivates the study of this case. For the considered relay channel, we derive closed-form expressions for the information outage probability, the average number of transmissions, and the average transmission rate.

The remainder of the paper is organized as follows. In Section II, we describe the system model and present analytical expressions of the average transmission rate and the average number of transmissions. We derive closed-form expression for the outage probability in Section III. These analytical expressions are numerically evaluated, illustrated, and analyzed in Section IV. Concluding remarks are provided in Section V.

II. SYSTEM MODEL

We consider a relay channel comprising three nodes: the source node, the relay node, and the destination node. The squared envelop of the channel gains associated with the source-destination, the source-relay, and the relay-destination links are denoted by γ_{SD} , γ_{SR} , and γ_{RD} , respectively. These squared envelop of the channel gains are independent exponentially distributed stochastic processes with mean values $\bar{\gamma}_{SD}$, $\bar{\gamma}_{SR}$, and $\bar{\gamma}_{RD}$. We assume that a data packet experiences different channel realizations in subsequent HARQ rounds. Each HARQ round corresponds to one transmission time slot. The duration of a time slot T_{slot} in modern communication systems is around one millisecond. For the LTE standard for instance, the time slot duration is 0.5 millisecond. It is well known that if the duration of the time slot is much smaller than the coherence time T_C of the channel ($T_C \gg T_{slot}$) we can consider that the channel is constant during the whole time slot duration [13], [14]. It was reported in [6] that for systems operating at a carrier frequency of 2.5 GHz and in the case that the receiver is moving with a speed of 2 km/h, 45 km/h, and 100 km/h the coherence time is equal to 200 ms, 10 ms, and 4 ms, respectively. Notice that these coherence times are 400, 20, and 8 times, respectively, larger than the time slot duration in LTE. It follows that the relationship $T_C \gg T_{slot}$ holds. Furthermore, most of the frequencies in the LTE spectrum are smaller than 2.5 GHz. It is an established fact that the coherence time of the channel increases as the carrier frequency decreases. In this way, we guarantee that the assumption $T_C \gg T_{slot}$ is valid for high, medium, and low speed values and for most frequencies in the LTE band. Accordingly, the channel realization can be assumed to be constant during one HARQ round. In order to allow decoding and feedback (ACK/NACK) to be performed at the destination, we have to separate consecutive HARQ rounds by few time slots. Consequently, we can consider the channel realizations independent for subsequent HARQ rounds. To limit the delay and avoid congestion in the network, a maximum number M of HARQ rounds is assumed.

In the HARQ-IR, b information bits are encoded into $M \times L$ symbols. We transmit L symbols in every HARQ round. If m HARQ rounds are used to transmit one data packet, this is equivalent to coding b information bits into $m \times L$ symbols.

Clearly, as m increases, the codeword becomes more robust to fading and the probability of successful decoding increases. The amount of transmitted information is implicitly adapted to the channel conditions thanks to the HARQ technique.

The relay overhears the transmission from the source. Once a successful decoding occurs at the relay, both the relay and the source start cooperating to convey the message to the destination. The source and the relay transmit simultaneously their symbols to the destination. To avoid interference the Alamouti code [15] is utilized to transmit the source and the relay symbols. An illustration of this protocol is provided in Fig. 1.

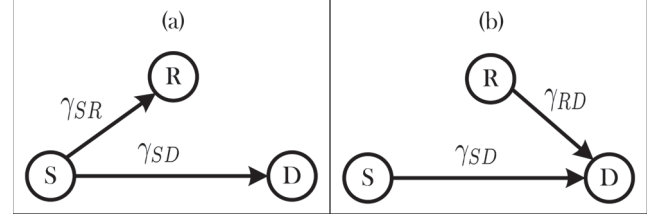


Fig. 1. Protocol illustration: (a) The source transmit the message to the destination and the relay overhears it for the first k rounds. (b) After successful decoding at the relay, the source and the relay cooperate to convey the message to the destination.

When the HARQ technique is used, the number of retransmissions of a data packet varies depending on the channel conditions. Few HARQ rounds are sufficient to decode a data packet at the receiver if the channel conditions are good, while more retransmissions are needed in the case of bad channel conditions. It follows that we need to define the average transmission rate properly. The transmission rate for the first HARQ round equals to $R_1 = b/L$. If m rounds are used to transmit successfully a data packet, the transmission rate equals to $R_m = R_1/m$. Let Q_n denote the number of HARQ rounds required for an error-free transmission of the n th data packet. The average transmission rate for N data packets can be expressed as

$$\bar{R} = \frac{Nb}{L \sum_{n=1}^N Q_n} = \frac{R_1}{\frac{1}{N} \sum_{n=1}^N Q_n} = \frac{R_1}{\bar{N}}, \quad (1)$$

with \bar{N} being the average number of transmissions per data packet.

It can be shown that the average number of transmissions \bar{N} with a maximum number M of rounds is given as [3], [4]

$$\bar{N} = 1 + \sum_{m=1}^{M-1} P_{\text{out}}(m), \quad (2)$$

where $P_{\text{out}}(m)$ stands for the probability of a decoding failure at the destination after m HARQ rounds. The closed-form expression of the outage probability $P_{\text{out}}(m)$ will be determined in the next section.

III. INFORMATION OUTAGE PROBABILITY

In our scheme, the relay tries to decode the message sent from the source. We assume that the relay successfully decode the message after k HARQ rounds. After successful decoding of the message, the relay starts cooperating with the source at round $k + 1$. The accumulated mutual information at the destination at the m th HARQ round can be expressed as

$$I_{Tot,m,k} = \begin{cases} \sum_{i=1}^k I_{SD,i} + \sum_{i=k+1}^m I_{SRD,i} & \text{if } k < m \\ \sum_{i=1}^m I_{SD,i} & \text{if } k \geq m, \end{cases} \quad (3)$$

where $I_{SD,i}$ refers to the mutual information of the channel between the source and the destination at round i , while $I_{SRD,i}$ stands for the mutual information resulting from the cooperation between the relay and the source to transmit the message to the destination at round i . These two terms can be written as

$$I_{SD,i} = \log_2(1 + \rho\gamma_{SD,i}) \quad (4)$$

$$I_{SRD,i} = \log_2(1 + \rho\gamma_{SD,i} + \rho\gamma_{RD,i}), \quad (5)$$

with $\rho = P/N_0$ is the transmit SNR. We assume that both the source and the relay transmit with the same power P .

If a capacity achieving code is utilized, the error probability is well approximated by the information outage probability [16], which motivates the study of this characteristic quantity. Let S_R denote the earliest HARQ rounds after which the relay stop listening to the current message. The information outage probability after m HARQ rounds can be expressed as [17]

$$P_{out}(m) = \sum_{k=1}^M P_{out}(m|S_R = k) \cdot P(S_R = k). \quad (6)$$

It has to be noted that if $k < m$, the relay at step m has already decoded successfully the packet sent by the source. At round m , both the relay and the source are cooperating to transmit the packet to the receiver. If $k \geq m$, the relay at round m has not yet decoded successfully the packet sent from the source. Hence, the relay cannot cooperate with the source to transmit the message to the destination. Only the source can send the message to the destination. In this way, we can expand the expression of the information outage probability in (6) as

$$P_{out}(m) = \sum_{k=1}^{m-1} P_{out}(m|k < m) \cdot P(S_R = k) + \sum_{k=m}^M P_{out}(m|k \geq m) \cdot P(S_R = k). \quad (7)$$

For $k \geq m$, there is no cooperation between the transmitter

and the relay and we can write

$$\begin{aligned} P_{out}(m|k \geq m) &= P(I_{Tot,m,k} < R_1|k \geq m) \\ &= P\left(\sum_{i=1}^m I_{SD,i} < R_1\right) \\ &= P\left(\sum_{i=1}^m \log_2(X_i) < R_1\right) \\ &= F_{V^m}(R_1), \end{aligned} \quad (8)$$

where $F_{V^m}(x)$ is the cumulative distribution function (CDF) of $V^m = \sum_{i=1}^m \log_2(X_i)$. Note that $X_i = 1 + \rho\gamma_{SD,i}$ follows a shifted exponential distribution given by

$$P_{X_i}(x) = \frac{1}{\bar{X}} \exp\left(-\frac{x-1}{\bar{X}}\right), \quad (9)$$

with $\bar{X} = \rho\bar{\gamma}_{SD,i}$. Using [18, Eq.(20)], the CDF of V^m can be derived as

$$F_{V^m}(R_1) = 1 - Y_{m+1,0}^{1,m+1}\left[\frac{2^{R_1}}{\bar{X}^m} \middle| \begin{matrix} (1, 1, 0, 1) \\ (0, 1, 0, 1), (1, 1, \frac{1}{\bar{X}}, 1), \dots, (1, 1, \frac{1}{\bar{X}}, 1) \end{matrix} \right] \quad (10)$$

where $Y_{m,n}^{p,q}[\cdot|\cdot]$ is the generalized Fox's H function introduced in [18]. It is worth mentioning that $P_{out}(m|k \geq m)$ is independent of k for $k = m, \dots, M$.

For $k < m$, the relay cooperate with the transmitter to convey the message to the destination. Under such conditions, we can express the term $P_{out}(m|k < m)$ as

$$\begin{aligned} P_{out}(m|k < m) &= P(I_{Tot,m,k} < R_1|k < m) \\ &= P\left(\sum_{i=1}^k I_{SD,i} + \sum_{i=k+1}^m I_{SRD,i} < R_1\right) \\ &= P\left(\sum_{i=1}^k \log_2(X_i) + \sum_{i=k+1}^m \log_2(Z_i) < R_1\right) = F_{W_k^m}(R_1), \end{aligned} \quad (11)$$

where $F_{W_k^m}(x)$ is the CDF of $W_k^m = \sum_{i=1}^k \log_2(X_i) + \sum_{i=k+1}^m \log_2(Z_i)$. The process $Z_i = 1 + \rho\gamma_{SD,i} + \rho\gamma_{RD,i}$. Using Laguerre series [19], the distribution of the process Z_i can be well approximated by the following gamma shifted distribution

$$P_{Z_i}(z) \approx \frac{(z-1)^{\alpha-1}}{\beta^\alpha \Gamma(\alpha)} \exp\left(-\frac{z-1}{\beta}\right), \quad (12)$$

where

$$\alpha = \frac{[E(Z_i)]^2}{\text{Var}(Z_i)} = \frac{(\bar{\gamma}_{SD} + \bar{\gamma}_{RD})^2}{\bar{\gamma}_{SD}^2 + \bar{\gamma}_{RD}^2} \quad (13)$$

$$\beta = \frac{\text{Var}(Z_i)}{E(Z_i)} = \rho \left(\frac{\bar{\gamma}_{SD}^2 + \bar{\gamma}_{RD}^2}{\bar{\gamma}_{SD} + \bar{\gamma}_{RD}} \right). \quad (14)$$

The CDF of W_k^m can be obtained using [18, Eq.(20)]. This CDF is given by (15) on the top of next page.

In order to evaluate the outage probability $P_{out}(m)$ in (7), we need to derive the expression of the term $P(S_R = k)$. For

$$F_{W_k^m}(R_1) = 1 - Y_{m+1,0}^{1,m+1} \left[\frac{2^{R_1}}{\bar{X}^k \beta^{m-k}} \middle| \begin{matrix} (1, 1, 0, 1) \\ (0, 1, 0, 1), (1, 1, \frac{1}{\bar{X}}, 1), \dots, (1, 1, \frac{1}{\bar{X}}, 1), (1, 1, \frac{1}{\beta}, \alpha), \dots, (1, 1, \frac{1}{\beta}, \alpha) \end{matrix} \right]. \quad (15)$$

$k = 1, \dots, M-1$, $S_R = k$ if the relay successfully decodes the message at the k th HARQ round. Let F_R stands for the event decoding failure at the relay, $F_R = k$ if the relay fails at round k to decode the message received from the source.

For $k = 1$, the probability $P(S_R = 1)$ reads as

$$\begin{aligned} P(S_R = 1) &= 1 - P(F_R = 1) = 1 - P(I_{SR} < R_1) \\ &= 1 - P(\log_2(1 + \rho\gamma_{SR}) < R_1) \\ &= 1 - P(\eta < 2^{R_1} - 1) = \exp\left(-\frac{2^{R_1} - 1}{\bar{\eta}}\right), \end{aligned} \quad (16)$$

where $\eta = \rho\gamma_{SR}$ and $\bar{\eta} = \rho\bar{\gamma}_{SR}$.

For $k = 2, \dots, M-1$, we can express $P(S_R = k)$ as

$$\begin{aligned} P(S_R = k) &= P(F_R = k-1) - P(F_R = k) \\ &= P\left(\sum_{i=1}^{k-1} I_{SR,i} < R_1\right) - P\left(\sum_{i=1}^k I_{SR,i} < R_1\right) \\ &= Y_{k+1,0}^{1,k+1} \left[\frac{2^{R_1}}{\bar{\eta}^k} \middle| \begin{matrix} (1, 1, 0, 1) \\ (0, 1, 0, 1), (1, 1, \frac{1}{\bar{\eta}}, 1), \dots, (1, 1, \frac{1}{\bar{\eta}}, 1) \end{matrix} \right] \\ &\quad - Y_{k,0}^{1,k} \left[\frac{2^{R_1}}{\bar{\eta}^{k-1}} \middle| \begin{matrix} (1, 1, 0, 1) \\ (0, 1, 0, 1), (1, 1, \frac{1}{\bar{\eta}}, 1), \dots, (1, 1, \frac{1}{\bar{\eta}}, 1) \end{matrix} \right]. \end{aligned} \quad (17)$$

For $k = M$, $S_R = M$ if the relay fails to decode the message after $M-1$ HARQ rounds. Thus,

$$\begin{aligned} P(S_R = M) &= P(F_R = M-1) = P\left(\sum_{i=1}^{M-1} I_{SR,i} < R_1\right) \\ &= 1 - Y_{M,0}^{1,M} \left[\frac{2^{R_1}}{\bar{\eta}^{M-1}} \middle| \begin{matrix} (1, 1, 0, 1) \\ (0, 1, 0, 1), (1, 1, \frac{1}{\bar{\eta}}, 1), \dots, (1, 1, \frac{1}{\bar{\eta}}, 1) \end{matrix} \right]. \end{aligned} \quad (18)$$

Using the above equations, we can simplify the expression of the outage probability in (7) as

$$\begin{aligned} P_{\text{out}}(m) &= \sum_{k=1}^{m-1} F_{W_k^m}(R_1) \cdot P(S_R = k) \\ &\quad + F_{V^m}(R_1) \cdot P(F_R = m-1). \end{aligned} \quad (19)$$

The expression above is valid for $m = 2, \dots, M$. For $m = 1$, the outage probability reduces to $P_{\text{out}}(1) = F_{V^1}(R_1)$.

IV. NUMERICAL RESULTS

The analytical expressions presented in the previous sections are evaluated numerically and illustrated in this section. We assume that both the source and the relay transmit with the same power P . The squared envelop of the channel gain associated with the source-destination, the source-relay, and the relay-destination links are exponentially distributed stochastic processes with mean values $\bar{\gamma}_{SD} = 0.5$, $\bar{\gamma}_{SR} = 1$, and $\bar{\gamma}_{RD} = 1$, respectively.

TABLE I
TRANSMISSION RATE R_1 FOR DIFFERENT SCENARIOS.

Scenario	Rate R_1 (bps/Hz) ($\epsilon = 0.01$)	Rate R_1 (bps/Hz) ($\epsilon = 0.1$)
HARQ without relaying $M = 2$	0.1	0.4
HARQ with relaying $M = 2$	0.25	0.55
HARQ without relaying $M = 4$	0.55	1.05
HARQ with relaying $M = 4$	1.05	1.6

The communication system is in outage if after M HARQ rounds the accumulated mutual information is less than the communication rate R_1 . Hence, the system outage probability is equal to $P_{\text{out}}(M)$. In Fig. 2, we illustrate the outage probability of HARQ-IR with and without relaying for different values of M . From this figure, it can be seen that the outage probability decreases as M increases. We observe in Fig. 2 that the gain realized thanks to relaying increases as the number of HARQ rounds M increases. The results in Fig. 2 have been obtained assuming a fixed transmission rate $R_1 = 1$ bps/Hz.

In Fig. 3, we show the dependence between the transmission rate R_1 and the outage probability $P_{\text{out}}(M)$. This latter increases as the rate R_1 increases. If we set R_1 to 2 bps/Hz and for $M = 2$, the transmission of the data packet fails with a probability of 1 either we use a relay node or not. The results shown in Fig. 3 have been obtained for an SNR of 0 dB. From this figure we have deduced the transmission rate R_1 that should be used for different scenarios such that the target outage probability ϵ is not exceeded. These rates are reported in Table. I. It is worth mentioning that these rates which are obtained for an SNR of 0 dB satisfy the target outage probability for any SNR larger than 0 dB, since the outage probability decreases with the SNR. From Table. I, we can see that if $M = 2$ and in case of HARQ-IR with relaying, the rate R_1 should not exceed 0.25 bps/Hz to achieve an outage of $\epsilon = 0.01$. If we increase M to 4, we can increase the transmission rate R_1 to 1.05 bps/Hz while keeping the same target outage probability, i.e., $\epsilon = 0.01$. We can conclude from this table that the communication rate per round R_1 can be increased if we increase the target outage probability.

The average number of transmissions is illustrated in Fig. 4 for the case of HARQ-IR with relaying. The rates R_1 have been chosen according to Table. I in order to achieve the target outage probability ϵ . From Fig. 4, we observe that the average number of transmissions drops as M decreases and as the SNR increases. More specifically, for $M = 4$ and $\epsilon = 0.1$, the average number of transmissions decreases from 3.135 to 1.34 if we increase the SNR from 0 dB to 10 dB. Additionally, for $\epsilon = 0.01$ and an SNR of 0 dB, the average number of transmissions increases from 1.3 to 2.4 if we increase the value of M from 2 to 4. We can also conclude from this figure that the average number of transmissions increases with ϵ , which

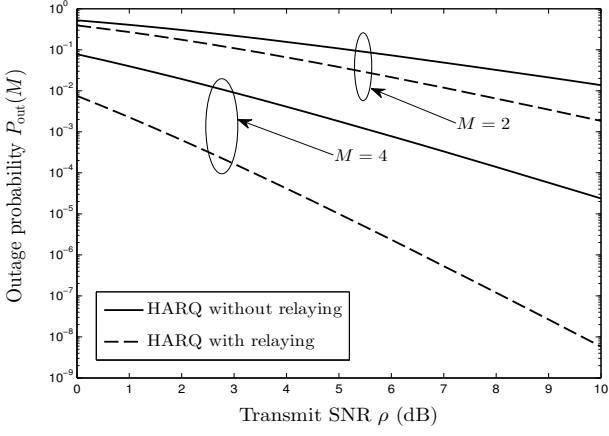


Fig. 2. System outage probability $P_{\text{out}}(M)$ of HARQ-IR with and without relaying for $R_1 = 1$ bps/Hz.

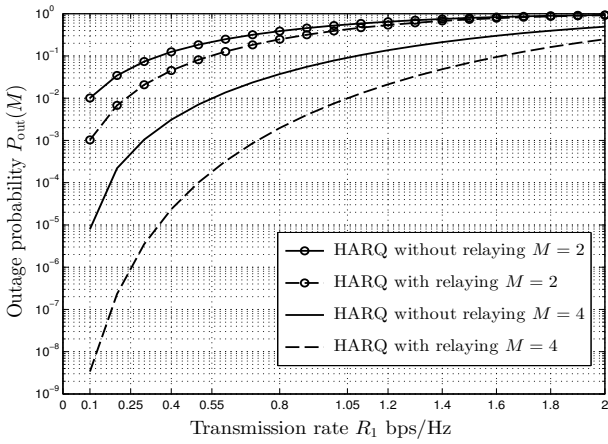


Fig. 3. System outage probability $P_{\text{out}}(M)$ versus transmission rate R_1 for a transmit SNR $\rho = 0$ dB.

can be explained as follows. An increase in the value of ϵ leads to a higher transmission rate R_1 and consequently to a larger outage probability $P_{\text{out}}(m)$ after m HARQ rounds. Hence, using the expression of \bar{N} , we can conclude that an increase in the value of ϵ results in an increase of the average number of transmissions as it can be seen in Fig. 4.

In Fig. 5, we evaluate the average transmission rate \bar{R} for the case of HARQ-IR with relaying. For a fixed target outage probability ϵ , we observe that the average transmission rate \bar{R} increases as we increase M and as the SNR increases. In particular, if we set the value of $M = 4$ and $\epsilon = 0.1$, the average transmission rate increases from 0.5 bps/Hz to 1.2 bps/Hz if we increase the SNR from 0 dB to 10 dB. Additionally, for $\epsilon = 0.01$ and an SNR of 10 dB, the average transmission rate increases from 0.24 bps/Hz to 0.9 bps/Hz if we increase the value of M from 2 to 4. Note that for high SNR, the number of retransmissions decreases thanks to the good channel conditions leading to a larger transmission rate. For a fixed value of M , the rate \bar{R} increases as we increase ϵ . We recall that $\bar{R} = R_1/\bar{N}$. If we increase ϵ the rate R_1

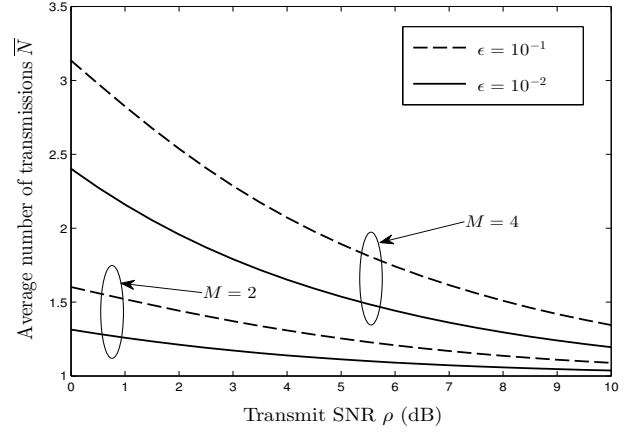


Fig. 4. Average number of transmissions of HARQ-IR with relaying for different target outage probabilities ϵ .

increases and the term \bar{N} increases. However, R_1 grows faster with ϵ compared to \bar{N} . Consequently, the average transmission rate \bar{R} increases as ϵ increases.

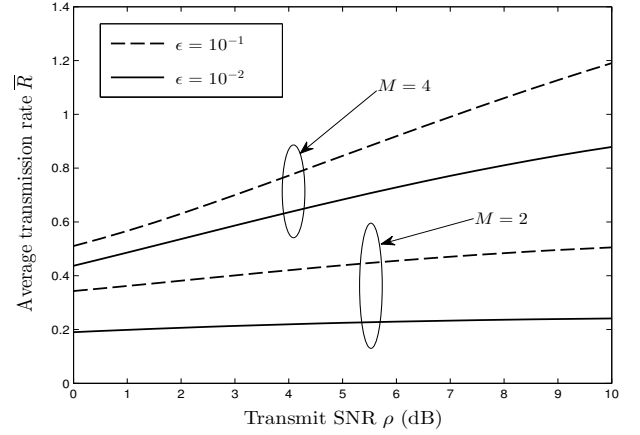


Fig. 5. Average transmission rate of HARQ-IR with relaying for different target outage probabilities ϵ .

V. CONCLUSION

In this paper, we have investigated the performance of HARQ-IR over a relay channel in case of decode and forward mode. The relay tries to decode the message from the source. Once a successful decoding occurs at the relay, it starts cooperating with the source to transmit the message to the destination. We assume that the CSI is not available at the transmitter. This occurs if the channel changes too fast, such that the CSI sent from the receiver to the transmitter is outdated compared to the actual CSI. The HARQ technique allows to deal with this problem by implicitly adapting the transmission rate according to the channel conditions. The fading channel is assumed to be varying for successive HARQ rounds. Under these conditions, we derived closed-form expressions of the outage probability, the average number of transmissions, and the average transmission rate.

Supported by our analysis, we can conclude that the use of a relay node allows to reduce the system outage probability. The gain achieved thanks to relaying increases as the maximum number M of HARQ rounds increases. The transmission rate R_1 per HARQ round should not exceed a threshold value in order to achieve a target outage probability. Moreover, if we set the target outage probability to a certain value, the average transmission rate can be increased if the maximum number M of HARQ rounds is increased. For a fixed value of M , if we increase the target outage probability ϵ , the average transmission rate increases.

Ongoing investigations focus on the performance of HARQ with code combining (CC) over a relay channel. The performance of HARQ-CC is being compared to HARQ-IR for the case of direct transmission and relay channel to illustrate the benefit of relaying.

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