

mappe1_SOK-2009

Assignment 6

Du får i oppgave å lage et spill til kasino, Hver gang spillerne får et tall x på to terninger, i.e. $T_2 = x$ tjener spilleren en bonus (B) på 10000 kroner.

1)

En formel for forventet premie B om det bare er et utfall som gir premie er $E[B] = p(B)B$; kan du sette opp denne formelen dersom to eller N utfall gir premie?

To find the expected bonus $E[x]$ when there are multiple outcomes that yield a bonus, we can use this formula:

$$E[B] = \sum_{i=1}^n p(B_i)B_i$$

Here we are summing all outcomes of the probability of receiving the bonus $p(B)B$.

So if we create an example game where we roll two dices, and we have the win condition of double six.

With 2d6 total outcome is $6 * 6 = 36$, so the outcome of doubler 6 is $\frac{1}{36}$ each. And lets use the bonus (B) on 10,000 KR

The expected bonus would then be:

$$E[B] = \frac{1}{36} * 10,000 + \frac{1}{36} * 10,000 = 555.5555.... \approx 555.56$$

This means if a player played enough games, they will win on average 555.56 kr each game.

2)

Hva er minste spillet kan koste samtidig som kasino ikke taper penger. (Kasinoet har en kjempe stor egenkapital og kan låne så mye de vil i banken).

Here we would do it similar to what we did in the example, but since double N is the victory condition instead of 1/36 possible outcome now there are 6/36 or 1/6.

$$E[B] = \frac{1}{6} * 10,000 + \frac{1}{6} * 10,000 = 3333.3333... \approx 3333.34$$

So if a player can expect to win 3333.34 on each game over a large enough games, the casino can expect to lose on average 3333.4 on each game played over time.

So a minimum price would be approximate 3333.34 kr

3)

Kan du sette opp en B for få et eller flere tall, en pris for deltakelse D for å spille slik at spillerene vinner med sannsynlighet:

A. 1/36

B. 2/36

C. 1/2

B = Bonus

D = Buy-in

So here we can adjust the formula we have used, since D will cover some amount of B , we can change it to:

$$E[B] = p(B)B - D$$

Here we know the buy in D needs to be above 0 to have any effect and lowering the risk for the casino, as well we know it needs to be below the expected bonus when $D = 0$ so that players can gain profit.

So let's first solve for D , if $p(B) = \frac{1}{36}$.

$$D = \frac{1}{36} * 10,000 = 277.7777.... \approx 277.78$$

So we know that D needs to be somewhere between 0 and 277.78 or $0 \leq D \leq 277.78$. But since the question ask us to just set a number for B and D .

I have already set $B = 10,000$, and I can set $D = 150$ to soften the risk for the casino. So with this the casino can lower there price with D , and go below $E[B]$ without lose.

And we can continue doing the same for each game:

Here if $p(B) = \frac{2}{36}$.

$$D = \frac{2}{36} * 10,000 = 555.5555.... \approx 555.56$$

So we need something where $0 \leq D \leq 555.56$. Anything between that range.

Lets just sett $B = 10,000$ and $D = 250$

And since the probability is 50/50 here, lets change the B if $p(B) = \frac{1}{2}$, to $B = 100$

$$D = \frac{1}{2} * 100 = 50$$

And the same here, D needs to be between 0 and 50.

With $B = 100$ and $D = 25$

4.1)

En gruppe kunder har tapt myke penger og er lei seg. De klager og sier det er urettferdig at kasinoet ditt tar mer betalt for å spille en forventet utbetaling. Hvordan forklarer du dette til kundene?

Expected pay-out is what the casino can expect to lose over a infinite amount of players, so if they didn't take more at best they would go in 0 assuming everyone works for free and cost of running the casino is 0.

4.2)

Eieren av kasinoet er lei seg fordi banken tar høyere rente av lånet hans enn det banken betaler i innskudsrente. Hvordan forklarer du til eieren at banken må ta rente.

Similar to a casino, a bank is gambling when lending money. So similar to what the casino does, it needs a "House-Edge". So they need to be able to cover expected loss from Interest Rate and other

5)

En gruppe på 5 studenter $i \in 1, 2, 3, 4, 5$ går på kasino. Vi skriver poengsummen i statistikkfaget deres på skalaen $p_i \in [1, 100]$ og gevinsten på kasino $[-100, 100]$. Data er som følger, s_i, p_i : 10, -50, 78, -100, 98, 100, 60, 50, 30, 25.

Kan du sett opp gjennomsnitt og median for p_i og s_i . Er det noe type tall i utvalget? Sett også opp varians, standardavvik for s_i, p_i og korrelasjon og kovarians mellom s_i, p_i

Mean of s_i and p_i

$$s = \frac{10 + 78 + 98 + 60 + 30}{5} = 55.5$$

$$p = \frac{-50 + (-100) + 100 + 50 + 30 + 25}{5} = 5$$

Median:

$$s = (10, 30, 60, 78, 98) = 60$$

$$p = (-100, -50, 25, 50, 100) = 25$$

Variance

$$Var(s) = \frac{\sum_{i=1}^n (s_i - \bar{s})^2}{n - 1} = 1263.2$$

$$Var(p) = \frac{\sum_{i=1}^n (p_i - \bar{p})^2}{n - 1} = 6375$$

Standard Deviation

$$\sigma_s = \sqrt{Var(s)} = 35.541$$

$$\sigma_p = \sqrt{Var(p)} = 79.843$$

Covariance

$$COV_{s,p} = \frac{\sum_{i=1}^n (s_i - \bar{s})(p_i - \bar{p})}{n - 1} = 967.5$$

Correlation

$$r = \frac{\sum_{i=1}^n (s_i - \bar{s})(p_i - \bar{p})}{\sqrt{\sum_{i=1}^n (s_i - \bar{s})^2 \sum_{i=1}^n (p_i - \bar{p})^2}} = 0.34$$
