Date :
No:

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Problem 1.6

- A sample of <u>10 marbles</u> draw <u>independently</u> from a bin of red and green marbles.

The probability of a red marble is μ

[a] only one sample

The probability of <u>no</u> red marbles $P(\nu = 0)$:

$P(\nu = 0) = (1 - \mu)^{10}$

$\mu = 0.05$ $\quad$ $P(\nu = 0) = 0.5987369$

$\mu = 0.5$ $\quad$ $P(\nu = 0) = 9.7656 \times 10^{-4}$

$\mu = 0.8$ $\quad$ $P(\nu = 0) = 1.024 \times 10^{-7}$

$\boxed{b}$ 1000 independent samples

The probability that at least one of the samples has $\nu = 0$:

P(at least one sample has $\nu = 0$) =
$1 - P$(none of the samples has $\nu = 0$) =
$1 - P$(all of the samples has $\nu > 0$)

P(all of the samples has $\nu > 0$) =

$$\underbrace{\prod_{i=1}^{1000}}_{\text{independent samples}} [1 - P(\nu = 0)] = \prod_{i=1}^{1000} [1 - (1 - \mu)^{10}] = $$

$$[1 - (1 - \mu)^{10}]^{1000}$$

P(at least one sample has $\nu = 0$) =

$$1 - [1 - (1 - \mu)^{10}]^{1000}$$

$\mu = 0.05$ $P = 1$
$\mu = 0.5$ $P = 0.623576$
$\mu = 0.8$ $P = 1.0239476 \times 10^{-4}$

[e] 1000000 independent samples

same as b

$P(\text{at least one sample has } \nu = 0) =$

$1 - [1 - (1-\mu)^{10}]^{1000000}$

$\mu = 0.05$ $\quad P = 1$

$\mu = 0.5$ $\quad P = 1$

$\mu = 0.8$ $\quad P = 0.09733159$

No:

problem 2.5

prove by induction that $\sum_{i=0}^{D} \binom{N}{i} \leq N^{D}+1$

hence $m_H(N) \leq N^{dvc}+1$

* prove by induction that $\sum_{i=0}^{D} \binom{N}{i} \leq N^{D}+1$

Base Case:

$N=1$

$D=0 \rightarrow \sum_{i=0}^{D} \binom{N}{i} = \binom{1}{0} = 1 \leq 1^{0}+1 \leq N^{D}+1$

$D=1 \rightarrow \sum_{i=0}^{D} \binom{N}{i} = \binom{1}{0}+\binom{1}{1} = 2 \leq 1^{1}+1 \leq N^{D}+1$

the inequality holds for the base case

Assume that it holds for N or less

Assume $\sum_{i=0}^{D} \binom{N}{i} \leq N^{D}+1$

Does it hold for $N+1$?

$$\sum_{i=0}^{D} \binom{N+1}{i} \leq (N+1)^{D}+1$$

$\because \binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} \qquad 0 \leq k \leq n \quad , n \geq 1$

$$\sum_{i=0}^{D} \binom{N+1}{i} = \sum_{i=0}^{D} \left[\binom{N}{i} + \binom{N}{i-1} \right]$$

$$= \sum_{i=0}^{D} \binom{N}{i} + \sum_{j=0}^{D-1} \binom{N}{j}$$

$$\leq (N^D + 1) + (N^{D-1} + 1)$$

$$= (N^D + N^{D-1} + 1) + 1$$

$$\leq (N+1)^D + 1$$

$\downarrow$

$$(N+1)^D = \sum_{K=0}^{D} \binom{D}{K} N^K = N^D + DN^{D-1} + \dots + 1$$

So $N^D + N^{D-1} + 1 \leq (N+1)^D$

but this holds for $D > 1$

$D = 0 \longrightarrow \sum_{i=0}^{D} \binom{N}{i} = \binom{N}{0} = 1 \leq N^0 + 1$

$D = 1 \longrightarrow \sum_{i=0}^{D} \binom{N}{i} = \binom{N}{0} + \binom{N}{1} = 1 + N \leq N^1 + 1$

So $\sum_{i=0}^{D} \binom{N}{i} \leq N^D + 1$ is proved

* To prove $m_H(N) \leq N^{d_{vc}} + 1$

using Theorem $m_H(N) \leq \sum_{i=0}^{d_{vc}} \binom{N}{i}$

and $\sum_{i=0}^{D} \binom{N}{i} \leq N^D + 1$

$\sum_{i=0}^{d_{vc}} \binom{N}{i} \leq N^{d_{vc}} + 1 \longrightarrow 1$

$m_H(N) \leq \sum_{i=0}^{d_{vc}} \binom{N}{i} \longrightarrow 2$

$m_H(N) \leq \sum_{i=0}^{d_{vc}} \binom{N}{i} \leq N^{d_{vc}} + 1$

So $m_H(N) \leq N^{d_{vc}} + 1$ ✓