Scientific Software Development in Python, AIMS Rwanda 2016
Assignment week 1

For this first assignment you are asked to create a Jupyter notebook in which you will solve the exercises. Normally you have also downloaded a notebook template to start from.

Each function should come with

- some examples
- some tests that check that the result is consistent. For example if you have programed the `gcd` function you can have the following in the cell after

```
# the code below checks the consistency of the gcd function
for a in range(50):
    for b in range(50)
        assert gcd(a, b) == gcd(b, a)
        assert gcd(a + b, b) == gcd(a, b)
        for c in range(10):
            assert gcd(c*a, c*b) == c * gcd(a, b)
```

A lot of attention will be paid to the clarity of the code. In particular

- The corrector should be able to run your cells from top to bottom without getting any error (you can try the command "Kernel → Restart and Run All" from the Jupyter menu).
- Do not put to many instruction inside a given cell. For example, no more than one function.
- give meaningful names to your variables, for example `s` for a sum and `p` for a product, `counter` for a counter in a `range`, etc
- use comments (using `#`) to explain the delicate steps of your algorithms

Exercise 1

A Pythagorean triplet is a set of three positive numbers, $x < y < z$, for which, $x^2 + y^2 = z^2$. For example, $3^2 + 4^2 = 9 + 16 = 25 = 5^2$. There exists exactly one Pythagorean triplet for which $x + y + z = 1000$. Find this triplet.

Exercise 2

Write a function `prod(l)` that returns the product of the elements in the list `l`. In case the list is empty, the function should return `1`.

For example

```
>>> prod([1, 3, 2, 3])
18
>>> prod([])
1
```

Exercise 3

Write a function `gcd(x, y)` that computes the greatest common divisor of two integers using Euclide algorithm.

Plot the graphic of the function $n \mapsto \#\{(p, q) : 1 \leq p \leq n,\ 1 \leq q \leq n,\ \gcd(p, q) = 1\}$.

Could you guess the asymptotic?

Recall that the *digits* of an integer n in base b is the sequence of numbers $(n_0, n_1, \ldots, n_d)$ in $\{0, 1, \ldots, b-1\}$ so that

$$n = \sum_{i=0}^{d} n_i b^i.$$

For example, the digits of 12 in base 10 are $(2, 1)$ and in base 2 are $(0, 1, 1)$.

Exercise 4

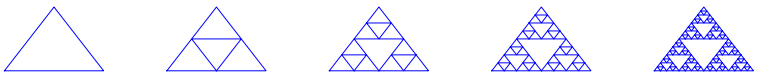
Write a function `digits(n, b)` that return the list of digits of the number `n` in base `b`.

Let $n = 123576537645123412$ written in base 10. What are its digits in base 2? In base 3?

What is the sum of the digits of 2^{100} written in base 3?

Exercise 5

Using `plot` from `matplotlib.pyplot`, build this sequence of figures



For that purpose, you might want to use a recursive function of the form

```
triangle(x0, y0, x1, y1, x2, y2, n)
```

that draws a triangle with vertices $(x_0, y_0), (x_1, y_1), (x_2, y_2)$ and if $n > 0$ calls itself with appropriate new coordinates and $n - 1$. (see worksheet 3).