

Euler 307: Chip Defects

k defects are randomly distributed amongst n circuit chips. Let $p(k, n)$ denote the probability that there is some chip with at least 3 defects. This problem asks us to find $p(20000, 1000000)$.

We compute the probability of the complementary event. The probability that a_i defects affect chip i for all $i \in [n]$, where $\sum_i a_i = k$, is given by the probability mass of the multinomial distribution, that is,

$$\Pr(a_1, \dots, a_n) = \frac{k!}{a_1! \dots a_n!} \frac{1}{n^k}.$$

Now, we have to find the probability that $a_i \leq 2$ for all i . This may be computed as follows:

$$\sum_{k=s+2t} \binom{n}{s} \binom{n-s}{t} \Pr(1, \dots, 1, 2, \dots, 2, 0, \dots, 0)$$

where there are s ones, t twos and $n - s - t$ zeroes. After some manipulations of the summands, this may be done without underflow issues.