JOHANNES KEPLER
UNIVERSITY LINZ

Submitted by
Julian Hofer, B.Sc.

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Institute of Semiconductor and Solid State Physics

Supervisor
Prof. Dr. Armando Rastelli

Co-Supervisor
DI Christian Schimpf

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Exciting and Resolving Quantum Dot Emission with Adiabatic Rapid Passage and Fabry Perot Interferometer

Master Thesis
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Technische Physik

JOHANNES KEPLER UNIVERSITY LINZ
Altenbergerstraße 69
4040 Linz, Österreich
www.jku.at
DVR 0093696

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Abstract

This is a placeholder for the abstract. It summarizes the whole thesis to give a very short overview. Usually, this the abstract is written when the whole thesis text is finished.

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1 Introduction

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special contents, but the length of words should match the language.

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2 Quantum Dot

2.1 Processing

2.2 Properties of our dots

2.2.1 Calculate spectral range of zero-phonon line

A typical lifetime of a GaAs quantum dot is $\Delta t = 250\ ps$. According to the time-energy uncertainty relation

$$\Delta E \cdot \Delta t = \frac{h}{2\pi} \tag{2.1}$$

$$\Rightarrow \Delta E = 2.64\,\mu\text{eV} \tag{2.2}$$

The frequency uncertainty can be obtained through

$$\Delta \nu = \frac{\Delta E}{h} \tag{2.3}$$

By developing λinto a taylor series

$$\lambda = \frac{c}{\nu} \tag{2.4}$$

$$\Rightarrow \lambda(\nu) \approx \lambda(\nu_0) + \lambda'(\nu_0) \cdot (\nu - \nu_0) \tag{2.5}$$

$\Delta\lambda$ can be expressed as

$$\Delta\lambda = \lambda(\nu_0 - \Delta\nu) - \lambda(\nu_0) \tag{2.6}$$

$$= \lambda(\nu_0) - \lambda'(\nu_0) \cdot \Delta\nu - \lambda(\nu_0) \tag{2.7}$$

$$= -\lambda'(\nu_0) \cdot \Delta\nu. \tag{2.8}$$

With equation (2.4) this gives

$$\Rightarrow \Delta\lambda = \frac{c}{\nu_0^2} \cdot \Delta\nu = \frac{\lambda_0^2}{c} \cdot \Delta\nu \tag{2.9}$$

$$\approx 1.0\,\mathrm{pm} \tag{2.10}$$

Table 2.1: Parameters of GaAs quantum dots used in the laboratory of semiconductor physics department in Linz. Zero-phonon line calculates from from the theoretical limit according to the life time of the excitonic state (as can be seen in equation (**??**)) up to broader lines which are still valued enough to be measured. The phonon sideband resembles data taken from Schöll et al. [1].

Quantum dot emission	Center wavelength λ_0	Spectral range $\Delta\lambda$	Waveform
Zero-phonon line	(700 to 800) nm	(1.0 to 1.4) pm	Cauchy
Phonon sideband	0.25 nm higher than zero-phonon line	500 pm	Gauss

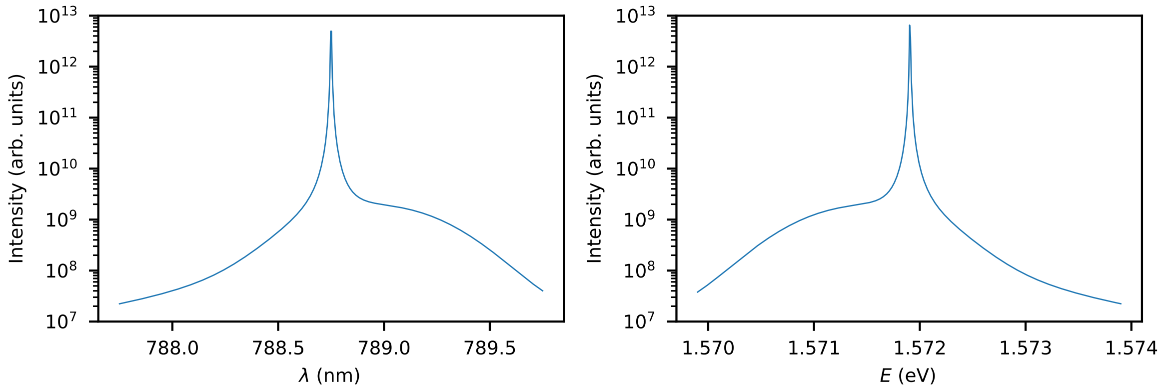

Figure 2.1: Simulated exciton emission of a GaAs quantum dot plotted dependant on the wavelength λ and the Energy E. The parameters can be found in table 2.1.

Dot-Spectra in far field is (TEM_{00}).

2.3 Adiabatic Rapid Passage

3 Chirp

Hallo [2]

4 Scanning Fabry-Pérot Interferometer

4.1 Introduction and Motivation

The Fabry-Pérot interferometer is an optical resonator developed by Charles Fabry and Alfred Pérot. An incoming light beam will only be transmitted through the resonator consisting of two semi-transparent mirrors if it fulfils the resonance condition.[3]

Resolve QD emission line.

4.2 Theory

4.2.1 Resonator losses

For the following discussion of the Fabry-Pérot interferometer, a two-mirror-resonator with the reflecting surfaces facing each other and air as medium in between is assumed. The time the light needs for one roundtrip is then given by [4]

$$t_{RT} = \frac{2l}{c} \tag{4.1}$$

where l is the geometrical length of the resonator and c is the speed of light in air.

The photon-decay time τ_c of the interferometer is then given by

$$\frac{1}{\tau_c} = -\frac{\ln(R_1 \cdot R_2)}{t_{RT}} \tag{4.2}$$

where R_1 and R_2 are the corresponding intensity reflectivities of the mirrors.

The number of photons at frequency ν inside the resonator is described by the differential rate equation

$$\frac{d}{dt}\varphi(t) = -\frac{1}{\tau_c}\varphi(t). \tag{4.3}$$

With a number φ_s of photons at $t = 0$ the integration gives

$$\varphi(t) = \varphi_s e^{-t/\tau_c} \tag{4.4}$$

4.2.2 Resonance frequencies, free spectral range and spectral line shapes

The round-trip phase shift at frequency ν is given by

$$2\phi(\nu) = 2\pi\nu t_{RT} = 2\pi\nu\frac{2l}{c} \tag{4.5}$$

where $\phi(\nu)$ is the single-pass phase shift between the mirrors.

Resonances are visible for frequencies ν at which the light interferes constructively after one round trip. Two adjacent resonance frequencies differ in their round trip phase shift by 2π. Hence, the free spectral range $\Delta\nu_{FSR}$, the frequency difference between two adjacent resonance frequencies, can be calculated from equation (4.11)

$$2\Delta\phi_{FSR} = 2\pi \tag{4.6}$$

$$\Rightarrow 2\pi\Delta\nu_{FSR}\frac{2l}{c} = 2\pi \tag{4.7}$$

$$\Rightarrow \Delta\nu_{FSR} = \frac{c}{2l} \tag{4.8}$$

According to equation (4.4) the number of photons decay with the photon-decay time τ_c. With $E_{q,s}$ representing the initial amplitude, the electric field at ν_q can is given by

$$E_q(t) = \begin{cases} E_{q,s}e^{i2\pi\nu_q t}e^{-t/(2\tau_c)} & t \geq 1 \\ 0 & t > 0 \end{cases}. \tag{4.9}$$

The Fourier transformation of the electric field can be expressed as

$$\tilde{E}_q(\nu) = \int_{-\infty}^{\infty} E_q(t)e^{-i2\pi\nu t}dt = E_q(t)\int_0^{\infty} e^{\cdot[1/(2\tau_c)+i2\pi(\nu-\nu_q)]t}dt = E_{q,s}\frac{1}{(2\tau_c)^{-1}+i2\pi(\nu-\nu_q)}. \tag{4.10}$$

The normalized spectral line shape per unit frequency is then given by

$$\tilde{\gamma}_q(\nu) = \frac{1}{\tau_c}\left|\frac{\tilde{E}_q(\nu)}{E_{q,s}}\right|^2 = \frac{1}{\tau_c}\left|\frac{1}{(2\tau_c)^{-1}+i2\pi(\nu-\nu_q)}\right|^2 = \frac{1}{\tau_c}\frac{1}{(2\tau_c)^{-2}+4\pi^2(\nu-\nu_q)^2} \tag{4.11}$$

$$= \frac{1}{\pi}\frac{1/(4\pi\tau_c)}{1/(4\pi\tau_c)^2+(\nu-\nu_q)^2} \tag{4.12}$$

with $\int \tilde{\gamma}_q(\nu)d\nu = 1$. By defining the full-width-at-half-maximum linewidth (FWHM) $\Delta\nu_c$ we get

$$\Delta\nu_c = \frac{1}{2\pi\tau_c} \Rightarrow \tilde{\gamma}_q(\nu) = \frac{1}{\pi}\frac{\Delta\nu_c/2}{(\Delta\nu_c/2)^2 + \left(\nu - \nu_q\right)^2} \tag{4.13}$$

By normalizing the Lorentzian lines so that the peak is at unity we finally obtain

$$\gamma_{q,L}(\nu) = \frac{\pi}{2}\Delta\nu_c\tilde{\gamma}_q(\nu) = \frac{(\Delta\nu_c)^2}{(\Delta\nu_c)^2 + 4\left(\nu - \nu_q\right)^2} \tag{4.14}$$

with $\gamma_{q,L}(\nu_q) = 1$.

4.2.3 Airy distribution of the Fabry-Pérot interferometer

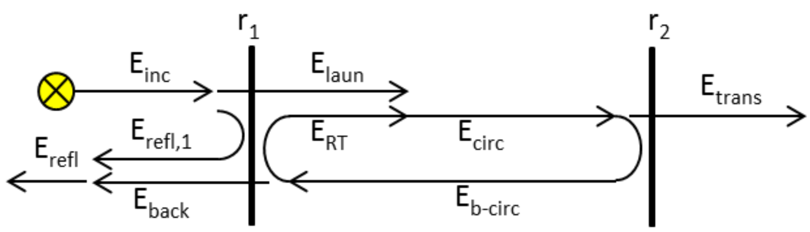

Figure 4.1: Fabry-Pérot interferometer with electric field mirror reflectivities r_1 and r_2. Indicated in this figure are the electric fields resulting from an incoming E_{inc}, the reflected field $E_{refl,1}$ and transmitted field E_{laun}. E_{circ} and $E_{circ,b}$ circulate inside the resonator, resulting in E_{RT} after one round-trip. E_{back} is the backwards transmitted field.[5]

The response of the Fabry-Pérot interferometer is calculated with the circulating-field approach [4], where a steady-state is assumed. E_{circ} is the result of E_{laun} interfering with E_{RT}. E_{laun} is the transmission of the incoming light E_{inc} and E_{RT} is E_circ after one round-trip in the resonator, i.e., after the outcoupling losses of mirror 1 and 2. Therefore, the field E_{circ} can be calculated from E_{launch} by

$$E_{circ} = E_{laun} + E_{RT} = E_{laun} + r_1 r_2 e^{-i2\phi}E_{circ} \Rightarrow \frac{E_{circ}}{E_{laun}} = \frac{1}{1 - r_1 r_2 e^{-i2\phi}} \tag{4.15}$$

where r_1 and r_2 are the electric-field reflectivities of mirror 1 and 2.

The generic Airy distribution considers only light inside the mirrors and is defined as

$$A_{circ} = \frac{I_{circ}}{I_{laun}} = \frac{|E_{circ}|^2}{|E_{laun}|^2} = \frac{1}{|1 - r_1 r_2 e^{-i2\phi}|^2} = \frac{1}{(1 - \sqrt{R_1 R_2})^2 + 4\sqrt{R_1 R_2}\sin^2(\phi)} \tag{4.16}$$

by using

$$\begin{aligned}\left|1 - r_1 r_2 e^{-i2\phi}\right|^2 &= |1 - r_1 r_2\cos(2\phi) + i r_1 r_2 \sin(2\phi)|^2 = [1 - r_1 r_2 \cos(2\phi)]^2 + r_1^2 r_2^2 \sin^2(2\phi) \\ &= 1 + R_1 R_2 - 2\sqrt{R_1 R_2}\cos(2\phi) = \left(1 - \sqrt{R_1 R_2}\right)^2 + 4\sqrt{R_1 R_2}\sin^2(\phi)\end{aligned}$$

and additionally $R_i = r_i^2$ and $\cos(2\phi) = 1 - 2\sin^2(\phi)$.

Commonly, light is sent through the Fabry-Pérot resonator. Therefore the following sections will use the Airy distribution A'_{trans}.

$$A'_{trans} = \frac{I_{trans}}{I_{inc}} = \frac{I_{circ} \cdot (1-R_2)}{I_{laun}/(1-R_1)} = (1-R_1)(1-R_2)A_{circ} = \frac{(1-R_1)(1-R_2)}{(1-\sqrt{R_1R_2})^2 + 4\sqrt{R_1R_2}\sin^2(\phi)} \quad (4.17)$$

A'_{trans} is displayed in figure 4.2 for $R_1 = R_2$. The peak value at one of its resonance frequencies calculates as follows

$$A'_{trans} = \frac{(1-R_1)(1-R_2)}{(1-\sqrt{R_1R_2})^2} \overset{R_1=R_2}{=} 1. \quad (4.18)$$

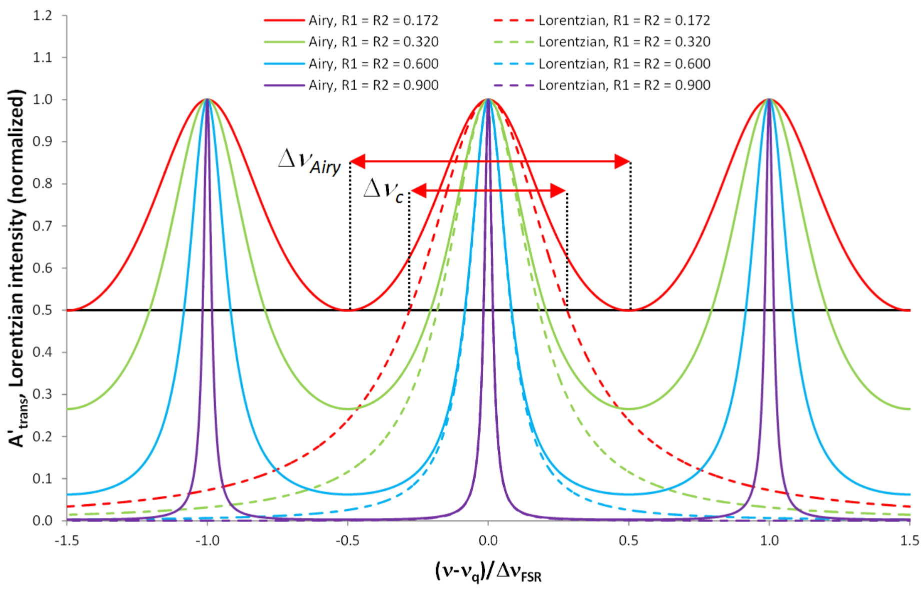

Figure 4.2: Airy distribution A'_{trans} as described in equation (4.17) compared to the Lorentzian lines $\gamma_{q,L}$ as described in equation (4.14)

4.2.4 Airy linewidth and finesse

The airy linewidth is defined as the FWHM of A'_{trans}. It can be set in relation with the free spectral range $\Delta\nu_{FSR}$ and the mirror reflectivities as follows.

A'_{trans} decreases to half of its peak value at $A'_{trans}(v_q)/2$ when the phase shift ϕ changes by the amount

$\Delta\phi$ so that the denominator of A'_{trans} in equation (4.17) is twice as big

$$\left(1-\sqrt{R_1R_2}\right)^2 = 4\sqrt{R_1R_2}\sin^2(\Delta\phi) \tag{4.19}$$

$$\Rightarrow \Delta\phi = \arcsin\left(\frac{1-\sqrt{R_1R_2}}{2\sqrt[4]{R_1R_2}}\right) \tag{4.20}$$

With equation (4.5) and (4.8), the phase shift can be expressed as

$$\phi = \frac{\pi\nu}{\Delta\nu_{FSR}} \tag{4.21}$$

$$\Rightarrow \Delta\phi = \frac{\pi(\Delta\nu_{Airy}/2)}{\Delta\nu_{FSR}}. \tag{4.22}$$

Therefore, with equation (4.20) and (4.22) the FWHM linewdith is given by

$$\Delta\nu_{Airy} = \Delta\nu_{FSR}\frac{2}{\pi}\arcsin\left(\frac{1-\sqrt{R_1R_2}}{2\sqrt[4]{R_1R_2}}\right). \tag{4.23}$$

The finesse of the Airy distribution of a Fabry-Pérot interferometer is defined as

$$F_{Airy} := \frac{\Delta\nu_{FSR}}{\Delta\nu_{Airy}} = \frac{\pi}{2}\left[\arcsin\left(\frac{1-\sqrt{R_1R_2}}{2\sqrt[4]{R_1R_2}}\right)\right]^{-1} \tag{4.24}$$

and is therefore only dependent on the mirror reflectivities R_1 and R_2.

The Airy finesse determining property when it comes to the spectral resolution of the Fabry-Pérot interferometer. This can be made visible by comparing its message with the Taylor criterion for the resolution of two adjacent peaks. The Taylor criterion proposes that two spectral lines are resolvable when the separation of the maxima is greater than the FWHM. As displayed in figure 4.3, the Airy finesse is equal to the number of Airy distributions originating from light at a certain frequencies ν_m which do not overlap at a point higher than half of their maxima. Hence, the Airy finesse describes the spectral resolution consistently with the Taylor criterion.

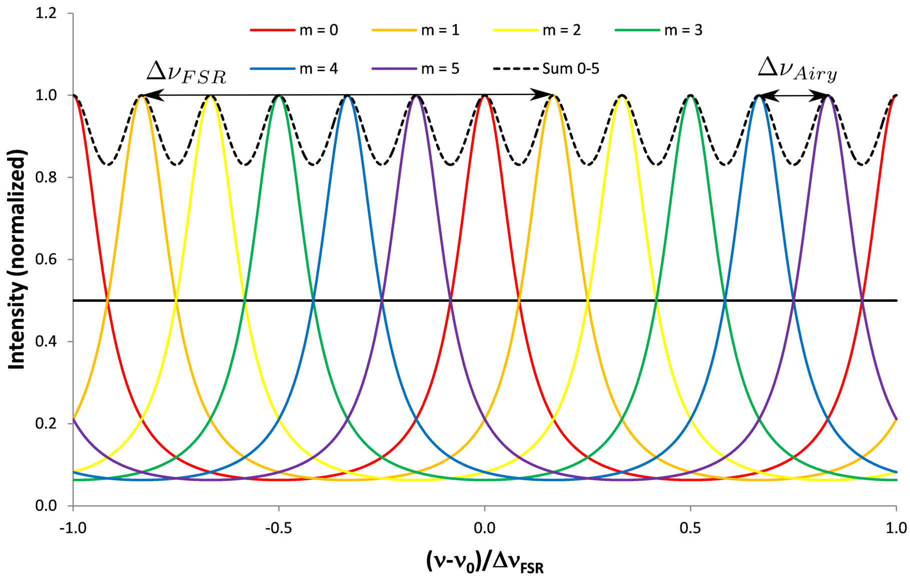

Figure 4.3: Demonstration of the physical meaning of the Airy finesse F_{Airy}. The coloured lines are Airy distributions created by light at distinct frequencies ν_m, while scanning the resonator length. When the light occurs at frequencies $\nu_m = \nu_q + m\Delta\nu_{Airy}$, the adjacent Airy distributions are separated from each other by ν_{Airy}, therefore fulfilling the Taylor criterion. Since in this example $F_{airy} = 6$ exactly six peaks fit inside the free spectral range. As can be seen in the figure the Airy finesse F_{Airy} quantifies the maximum number of peaks that can be resolved. [5]

4.2.5 Gaussian Beam

In this subsection, light beams are described by the wave picture according to Meschede [6]. They fulfil the Maxwell equations and therefore their electric field $\mathbf{E}(\mathbf{r}, t)$ the wave equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial}{\partial t^2}\right)\mathbf{E}(\mathbf{r}, t) = 0. \tag{4.25}$$

Along the propagation direction z a light beam behaves similarly to a plane wave with constant amplitude A_0 which is a known solution to the wave equation (4.25)

$$E(z, t) = A_0 e^{-i(\omega t - kz)}. \tag{4.26}$$

However, far from its source light is expected to behave like a spherical wave

$$E(\mathbf{r}, t) = A_0 \frac{e^{-i(\omega t - \mathbf{kr})}}{|\mathbf{kr}|}. \tag{4.27}$$

To get a better understanding of the propagation of light, only paraxial (near the z-axis) parts of the spherical wave are considered. Additionally, the wave is split into its longitudinal (z-axis) part and it transversal part and beams with axial symmetry are assumed, which only depend on a transversal coordinate ρ. Under these circumstances $\mathbf{kr}$ can be replaced with kr and because of $\rho << r, z$ the Fresnel approximation can be applied:

$$E(\mathbf{r}) = \frac{A(\mathbf{r})}{|\mathbf{kr}|} e^{i\mathbf{kr}} \simeq \frac{A(z,\rho)}{kz} \exp\left(i\frac{k\rho^2}{2z}\right) e^{ikz} \tag{4.28}$$

with $r = \sqrt{z^2 + \rho^2} \simeq z + \rho^2/2z$.

Equation (4.28) resembles the plain wave in equation (4.26), with the spacial phase transversal modulated by $\exp(ik\rho^2/2z)$. Another spherical wave solution can be obtained by applying the following replacement (z_0 is a real number)

$$z \to q(z) = z - iz_0. \tag{4.29}$$

Thereby, the fundamental (or TEM_{00}) Gaussian mode has been constructed

$$E(z,\rho) \simeq \frac{A_0}{kq(z)} \exp\left(i\frac{k\rho^2}{2q(z)}\right) e^{ikz}. \tag{4.30}$$

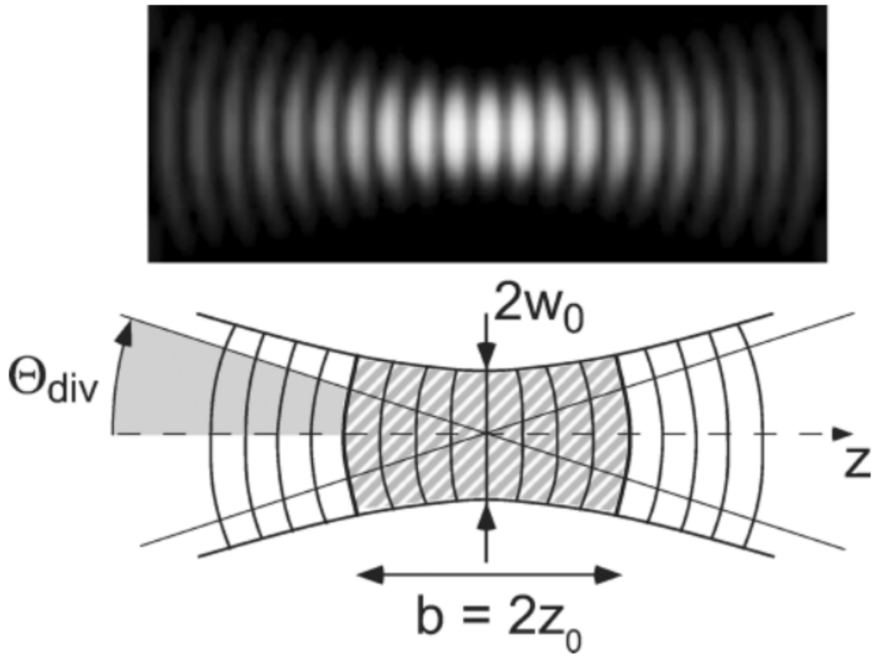

Figure 4.4: A Gaussian beam near its beam waist. Near the center they resemble plan wave fronts, while outside they converge towards spherical wave fronts. They Rayleighzone is shaded at the lower part of the figure.[6]

The electrical and magnetical fields of gauss modes are transversal to its propagation direction. These waveforms are called transversal elctrical and magnetic modes with indices (m, n). Its fundamental solution is the TEM_{00}-Mode, which is the most important one and will therefore be examined in more detail in the rest of this subsection.

To express equation (4.30) into a clearer way, the replacement $q(z) \rightarrow z - iz_0$ will be explicitly executed,

$$\frac{1}{q(z)} = \frac{z + iz_0}{z^2 + z_0^2} = \frac{1}{R(z)} + i\frac{2}{k\omega^2(z)}, \tag{4.31}$$

and new variables z_0, $R(z)$ and $\omega(z)$ are introduced. With the decomposition of the Fresnel factors into real and imaginary part, two factors can be identified: one complex phase factor, which describes the curvature of the wavefronts and a real factor, which describes the envelope of the beam. Therefore, the exponential of equation (4.30) becomes

$$\exp\left(i\frac{k\rho^2}{2q(z)}\right) \rightarrow \exp\left(i\frac{k\rho^2}{2R(z)}\right)\exp\left(-\left(\frac{\rho}{\omega(z)}\right)^2\right) \tag{4.32}$$

introduce R(z) and ω_0

4.2.6 Higher Gauss Modes

4.2.7 Mode Matching and Coupling Losses

Do the stuff according to Yariv, Yeh, and Yariv [7]

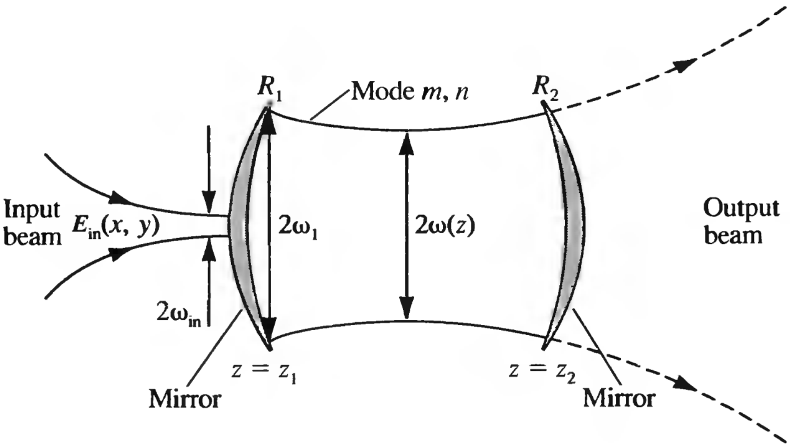

Figure 4.5:

4.2.8 Gaussian Beam Focusing

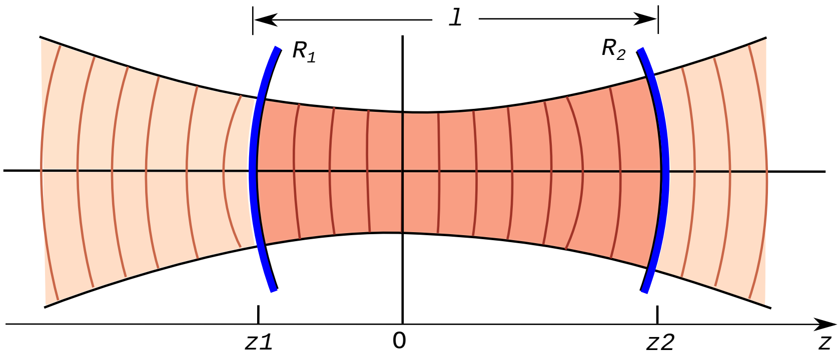

Figure 4.6:

4.2.9 Confocal Setups

4.2.10 Simulation

4.3 Setup

4.3.1 Flat mirrors

4.3.2 Concave mirrors

4.3.3 Confocal setup

4.4 Measurements and Results

Appendix

Bibliography

[1] Eva Schöll et al. "Resonance fluorescence of GaAs quantum dots with near-unity photon indistinguishability." In: *Nano Letters* 19.4 (Apr. 10, 2019), pp. 2404–2410. ISSN: 1530-6984, 1530-6992. DOI: 10.1021/acs.nanolett.8b05132. arXiv: 1901.09721. URL: http://arxiv.org/abs/1901.09721 (visited on 05/09/2019) (cit. on p. 4).

[2] Toshiyuki Hirayama and Mansoor Sheik-Bahae. "Real-time chirp diagnostic for ultrashort laser pulses." In: *Optics Letters* 27.10 (May 15, 2002), p. 860. ISSN: 0146-9592, 1539-4794. DOI: 10.1364/OL.27.000860. URL: https://www.osapublishing.org/abstract.cfm?URI=ol-27-10-860 (visited on 12/11/2018) (cit. on p. 5).

[3] Timo Kaldewey et al. "Coherent and robust high-fidelity generation of a biexciton in a quantum dot by rapid adiabatic passage." In: *Physical Review B* 95.16 (Apr. 10, 2017). ISSN: 2469-9950, 2469-9969. DOI: 10.1103/PhysRevB.95.161302. arXiv: 1701.01371. URL: http://arxiv.org/abs/1701.01371 (visited on 12/11/2018) (cit. on p. 7).

[4] Nur Ismail et al. "Fabry-Pérot resonator: spectral line shapes, generic and related Airy distributions, linewidths, finesses, and performance at low or frequency-dependent reflectivity." In: *Optics Express* 24.15 (July 25, 2016), p. 16366. ISSN: 1094-4087. DOI: 10.1364/OE.24.016366. URL: https://www.osapublishing.org/abstract.cfm?URI=oe-24-15-16366 (visited on 02/15/2019) (cit. on pp. 7, 9).

[5] *Fabry–Pérot interferometer - Wikipedia*. URL: https://en.wikipedia.org/wiki/Fabry%E2%80%93P%C3%A9rot_interferometer (visited on 05/06/2019) (cit. on pp. 9, 12).

[6] Dieter Meschede. *Optik, Licht und Laser*. 3., durchges. Aufl. Studium. OCLC: 254422596. Wiesbaden: Vieweg + Teubner, 2008. 568 pp. ISBN: 978-3-8351-0143-2 (cit. on pp. 12, 13).

[7] Amnon Yariv, Pochi Yeh, and Amnon Yariv. *Photonics: optical electronics in modern communications*. 6th ed. The Oxford series in electrical and computer engineering. OCLC: ocm58648003. New York: Oxford University Press, 2007. 836 pp. ISBN: 978-0-19-517946-0 (cit. on p. 14).