

Submitted by
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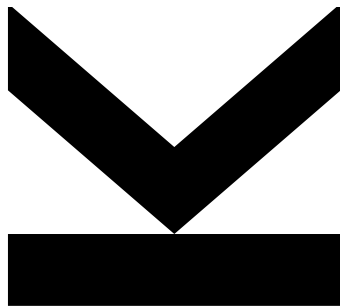
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Physics**

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month year

Exciting and Resolving Quantum Dot Emission with Adiabatic Rapid Passage and Fabry Perot Interferometer



Master Thesis
to obtain the academic degree of
Diplom-Ingenieur
in the Master's Program
Technische Physik

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Abstract

This is a placeholder for the abstract. It summarizes the whole thesis to give a very short overview. Usually, this the abstract is written when the whole thesis text is finished.

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1 Introduction

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And after the second paragraph follows the third paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special contents, but the length of words should match the language.

After this fourth paragraph, we start a new paragraph sequence. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and

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2 Quantum Dot

2.1 Processing

2.2 Properties of our dots

Table 2.1: My caption		
Quantum dot emission	Energy	Frequency
Center	(1.38 to 2.07) eV	$(3.33 \text{ to } 5.00) \times 10^{14} \text{ Hz}$
Spectral range	(100 to 500) μeV	$(24.20 \text{ to } 120.90) \times 10^9 \text{ Hz}$

2.3 Adiabatic Rapid Passage

3 Chirp

Hallo [1]

4 Scanning Fabry-Pérot Interferometer

4.1 Introduction and Motivation

The Fabry-Pérot interferometer is an optical resonator developed by Charles Fabry and Alfred Pérot. An incoming light beam will only be transmitted through the resonator consisting of two semi-transparent mirrors if it fulfils the resonance condition.[2]

Resolve QD emission line.

4.2 Theory

4.2.1 Resonator losses

For the following discussion of the Fabry-Pérot interferometer, a two-mirror-resonator with the reflecting surfaces facing each other and air as medium in between is assumed. The time the light needs for one roundtrip is then given by [3]

$$t_{RT} = \frac{2l}{c} \quad (4.1)$$

where l is the geometrical length of the resonator and c is the speed of light in air.

The photon-decay time τ_c of the interferometer is then given by

$$\frac{1}{\tau_c} = -\frac{\ln(R_1 \cdot R_2)}{t_{RT}} \quad (4.2)$$

where R_1 and R_2 are the corresponding intensity reflectivities of the mirrors.

The number of photons at frequency ν inside the resonator is described by the differential rate equation

$$\frac{d}{dt}\varphi(t) = -\frac{1}{\tau_c}\varphi(t). \quad (4.3)$$

With a number φ_s of photons at $t = 0$ the integration gives

$$\varphi(t) = \varphi_s e^{-t/\tau_c} \quad (4.4)$$

4.2.2 Resonance frequencies, free spectral range and spectral line shapes

The round-trip phase shift at frequency ν is given by

$$2\phi(\nu) = 2\pi\nu t_{RT} = 2\pi\nu \frac{2l}{c} \quad (4.5)$$

where $\phi(\nu)$ is the single-pass phase shift between the mirrors.

Resonances are visible for frequencies ν at which the light interferes constructively after one round trip. Two adjacent resonance frequencies differ in their round trip phase shift by 2π . Hence, the free spectral range $\Delta\nu_{FSR}$, the frequency difference between two adjacent resonance frequencies, can be calculated from equation (4.11)

$$2\Delta\phi_{FSR} = 2\pi \quad (4.6)$$

$$\Rightarrow 2\pi\Delta\nu_{FSR} \frac{2l}{c} = 2\pi \quad (4.7)$$

$$\Rightarrow \Delta\nu_{FSR} = \frac{c}{2l} \quad (4.8)$$

According to equation (4.4) the number of photons decay with the photon-decay time τ_c . With $E_{q,s}$ representing the initial amplitude, the electric field at ν_q can be given by

$$E_q(t) = \begin{cases} E_{q,s} e^{i2\pi\nu_q t} e^{-t/(2\tau_c)} & t \geq 0 \\ 0 & t < 0 \end{cases}. \quad (4.9)$$

The Fourier transformation of the electric field can be expressed as

$$\tilde{E}_q(\nu) = \int_{-\infty}^{\infty} E_q(t) e^{-i2\pi\nu t} dt = E_q(t) \int_0^{\infty} e^{[1/(2\tau_c) + i2\pi(\nu - \nu_q)]t} dt = E_{q,s} \frac{1}{(2\tau_c)^{-1} + i2\pi(\nu - \nu_q)}. \quad (4.10)$$

The normalized spectral line shape per unit frequency is then given by

$$\tilde{\gamma}_q(\nu) = \frac{1}{\tau_c} \left| \frac{\tilde{E}_q(\nu)}{E_{q,s}} \right|^2 = \frac{1}{\tau_c} \left| \frac{1}{(2\tau_c)^{-1} + i2\pi(\nu - \nu_q)} \right|^2 = \frac{1}{\tau_c} \frac{1}{(2\tau_c)^{-2} + 4\pi^2(\nu - \nu_q)^2} \quad (4.11)$$

$$= \frac{1}{\pi} \frac{1/(4\pi\tau_c)}{1/(4\pi\tau_c)^2 + (\nu - \nu_q)^2} \quad (4.12)$$

with $\int \tilde{\gamma}_q(\nu) d\nu = 1$. By defining the full-width-at-half-maximum linewidth (FWHM) $\Delta\nu_c$ we get

$$\Delta\nu_c = \frac{1}{2\pi\tau_c} \Rightarrow \tilde{\gamma}_q(\nu) = \frac{1}{\pi} \frac{\Delta\nu_c/2}{(\Delta\nu_c/2)^2 + (\nu - \nu_q)^2} \quad (4.13)$$

By normalizing the Lorentzian lines so that the peak is at unity we finally obtain

$$\gamma_{q,L}(\nu) = \frac{\pi}{2} \Delta\nu_c \tilde{\gamma}_q(\nu) = \frac{(\Delta\nu_c)^2}{(\Delta\nu_c)^2 + 4(\nu - \nu_q)^2} \quad (4.14)$$

with $\gamma_{q,L}(\nu_q) = 1$.

4.2.3 Airy distribution of the Fabry-Pérot interferometer

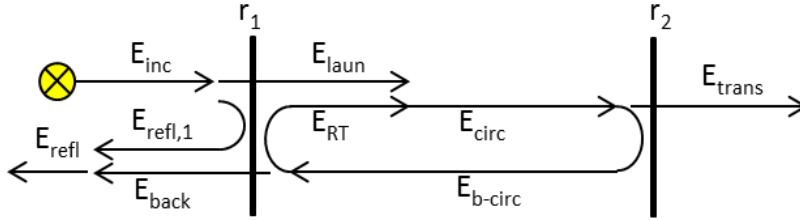


Figure 4.1: Fabry-Pérot interferometer with electric field mirror reflectivities r_1 and r_2 . Indicated in this figure are the electric fields resulting from an incoming E_{inc} , the reflected field $E_{refl,1}$ and transmitted field E_{laun} . E_{circ} and $E_{circ,b}$ circulate inside the resonator, resulting in E_{RT} after one round-trip. E_{back} is the backwards transmitted field.[4]

The response of the Fabry-Pérot interferometer is calculated with the circulating-field approach [3], where a steady-state is assumed. E_{circ} is the result of E_{laun} interfering with E_{RT} . E_{laun} is the transmission of the incoming light E_{inc} and E_{RT} is E_{circ} after one round-trip in the resonator, i.e., after the outcoupling losses of mirror 1 and 2. Therefore, the field E_{circ} can be calculated from E_{laun} by

$$E_{circ} = E_{laun} + E_{RT} = E_{laun} + r_1 r_2 e^{-i2\phi} E_{circ} \Rightarrow \frac{E_{circ}}{E_{laun}} = \frac{1}{1 - r_1 r_2 e^{-i2\phi}} \quad (4.15)$$

where r_1 and r_2 are the electric-field reflectivities of mirror 1 and 2.

The generic Airy distribution considers only light inside the mirrors and is defined as

$$A_{circ} = \frac{I_{circ}}{I_{laun}} = \frac{|E_{circ}|^2}{|E_{laun}|^2} = \frac{1}{|1 - r_1 r_2 e^{-i2\phi}|^2} = \frac{1}{(1 - \sqrt{R_1 R_2})^2 + 4\sqrt{R_1 R_2} \sin^2(\phi)} \quad (4.16)$$

by using

$$\begin{aligned} |1 - r_1 r_2 e^{-i2\phi}|^2 &= |1 - r_1 r_2 \cos(2\phi) + i r_1 r_2 \sin(2\phi)|^2 = [1 - r_1 r_2 \cos(2\phi)]^2 + r_1^2 r_2^2 \sin^2(2\phi) \\ &= 1 + R_1 R_2 - 2\sqrt{R_1 R_2} \cos(2\phi) = (1 - \sqrt{R_1 R_2})^2 + 4\sqrt{R_1 R_2} \sin^2(\phi) \end{aligned}$$

and additionally $R_i = r_i^2$ and $\cos(2\phi) = 1 - 2\sin^2(\phi)$.

Commonly, light is sent through the Fabry-Pérot resonator. Therefore the following sections will use the Airy distribution A'_{trans} .

$$A'_{trans} = \frac{I_{trans}}{I_{inc}} = \frac{I_{circ} \cdot (1 - R_2)}{I_{laun} / (1 - R_1)} = (1 - R_1)(1 - R_2) A_{circ} = \frac{(1 - R_1)(1 - R_2)}{(1 - \sqrt{R_1 R_2})^2 + 4\sqrt{R_1 R_2} \sin^2(\phi)} \quad (4.17)$$

A'_{trans} is displayed in figure 4.2 for $R_1 = R_2$. The peak value at one of its resonance frequencies calculates as follows

$$A'_{trans} = \frac{(1 - R_1)(1 - R_2)}{(1 - \sqrt{R_1 R_2})^2} \Big|_{R_1=R_2} 1. \quad (4.18)$$

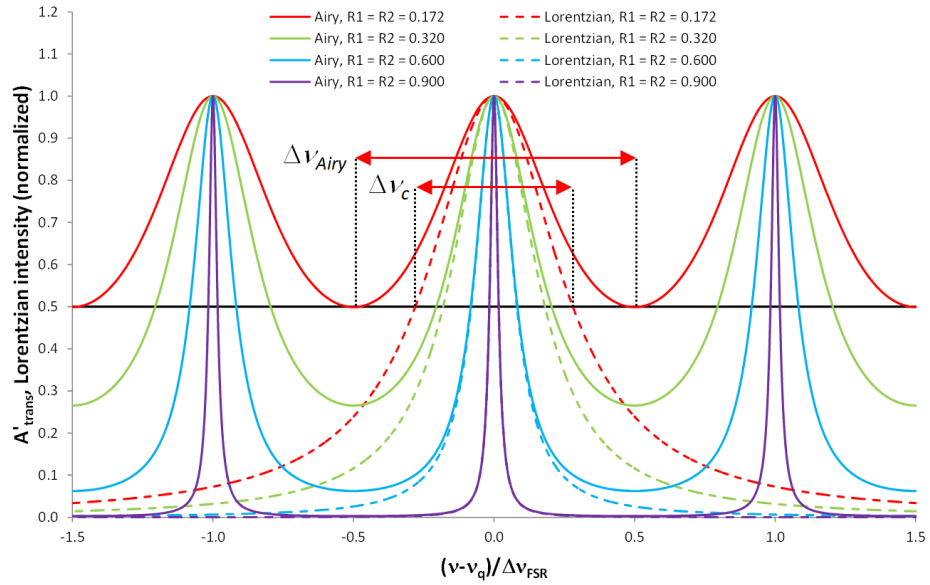


Figure 4.2: Airy distribution A'_{trans} as described in equation (4.17) compared to the Lorentzian lines $\gamma_{q,L}$ as described in equation (4.14)

4.2.4 Airy linewidth and finesse

The airy linewidth is defined as the FWHM of A'_{trans} . It can be set in relation with the free spectral range Δv_{FSR} and the mirror reflectivities as follows.

A'_{trans} decreases to half of its peak value at $A'_{trans}(v_q)/2$ when the phase shift ϕ changes by the amount

$\Delta\phi$ so that the denominator of A'_{trans} in equation (4.17) is twice as big

$$\left(1 - \sqrt{R_1 R_2}\right)^2 = 4\sqrt{R_1 R_2} \sin^2(\Delta\phi) \quad (4.19)$$

$$\Rightarrow \Delta\phi = \arcsin\left(\frac{1 - \sqrt{R_1 R_2}}{2\sqrt[4]{R_1 R_2}}\right) \quad (4.20)$$

With equation (4.5) and (4.8), the phase shift can be expressed as

$$\phi = \frac{\pi\nu}{\Delta\nu_{FSR}} \quad (4.21)$$

$$\Rightarrow \Delta\phi = \frac{\pi(\Delta\nu_{Airy}/2)}{\Delta\nu_{FSR}}. \quad (4.22)$$

Therefore, with equation (4.20) and (4.22) the FWHM linewidth is given by

$$\Delta\nu_{Airy} = \Delta\nu_{FSR} \frac{2}{\pi} \arcsin\left(\frac{1 - \sqrt{R_1 R_2}}{2\sqrt[4]{R_1 R_2}}\right). \quad (4.23)$$

The finesse of the Airy distribution of a Fabry-Pérot interferometer is defined as

$$F_{Airy} := \frac{\Delta\nu_{FSR}}{\Delta\nu_{Airy}} = \frac{\pi}{2} \left[\arcsin\left(\frac{1 - \sqrt{R_1 R_2}}{2\sqrt[4]{R_1 R_2}}\right) \right]^{-1} \quad (4.24)$$

and is therefore only dependent on the mirror reflectivities R_1 and R_2 .

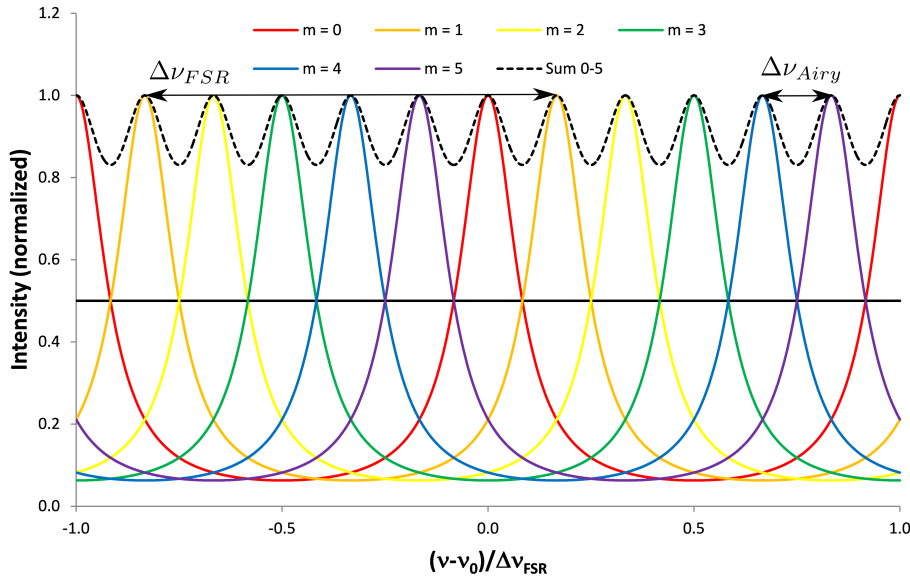


Figure 4.3: Demonstration of the physical meaning of the Airy finesse F_{Airy} . The coloured lines are Airy distributions created by light of frequencies $\nu_m = \nu_q + m\Delta\nu_{Airy}$. Since in this example $F_{Airy} = 6$ exactly six peaks fit inside the free spectral range according to the Taylor criterion. [4]

The Airy finesse determining property when it comes to the spectral resolution of the Fabry-Pérot interferometer. This can be made visible by comparing its message with the Taylor criterion for the resolution of two adjacent peaks. The Taylor criterion proposes that two spectral lines are resolvable when the separation of the maxima is greater than the FWHM. As displayed in figure 4.3, the Airy finesse is equal to the number of Airy distributions originating from light at a certain frequencies ν_m which do not overlap at a point higher than half of their maxima. Hence, the Airy finesse describes the spectral resolution consistently with the Taylor criterion.

4.2.5 Gaussian Beam

Dot-Spectra in far field is (TEM₀₀).

4.2.6 Simulation

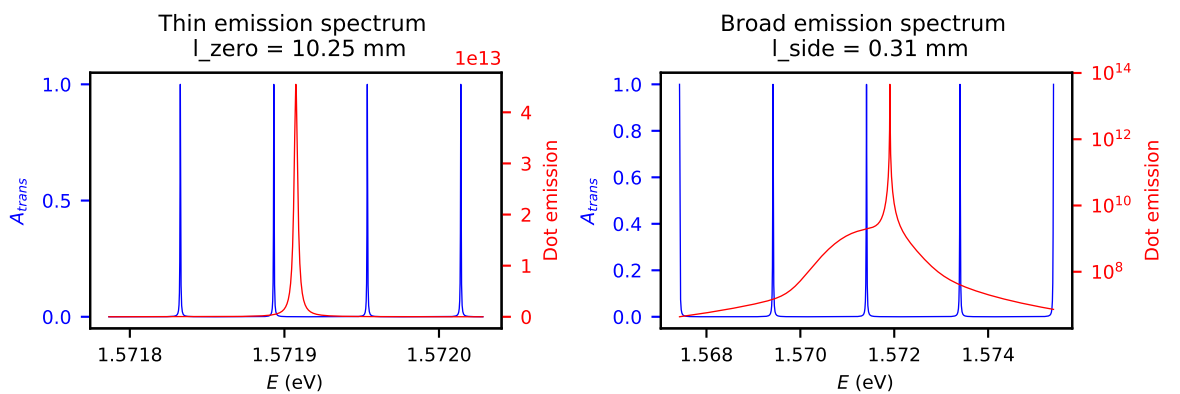


Figure 4.4:

4.3 Setup

4.3.1 Flat mirrors

4.3.2 Concave mirrors

4.3.3 Confocal setup

4.4 Measurements and Results

Appendix

Bibliography

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- [2] Timo Kaldewey et al. "Coherent and robust high-fidelity generation of a biexciton in a quantum dot by rapid adiabatic passage." In: *Physical Review B* 95.16 (Apr. 10, 2017). ISSN: 2469-9950, 2469-9969. DOI: 10.1103/PhysRevB.95.161302. arXiv: 1701.01371. URL: <http://arxiv.org/abs/1701.01371> (visited on 12/11/2018) (cit. on p. 7).
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- [4] *Fabry-Pérot interferometer* - Wikipedia. URL: https://en.wikipedia.org/wiki/Fabry%E2%80%93P%C3%A9rot_interferometer (visited on 05/06/2019) (cit. on pp. 9, 11).