



Frequency control and stability requirements on hydro power plants

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October 21, 2019



Outline



Problem

Initial tests using time domain vector fitting Paper I

Development of a simple test system Paper II

Theoretical validation of the PMU approach Paper III

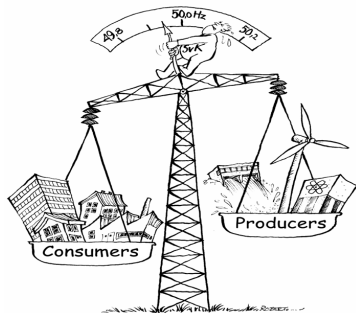
Comparison of a PMU-based approach and the draft requirements approach using tests from two of Statkraft's power plants Paper IV

An extension of Paper IV, with more discussion, simulation comparisons and more simulation validations Paper V

Checking the requirements using measurements from the control system of a hydro power plant Paper VI

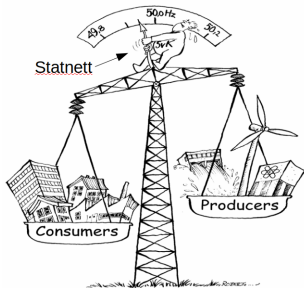
Load and production balancing

- The power system frequency measures the power balance.



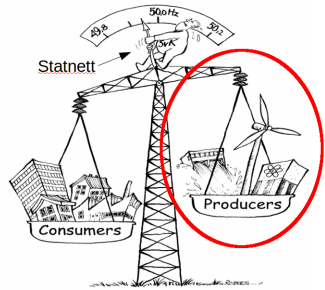
Load and production balancing

- The power system frequency measures the power balance.
- It is the responsibility of Statnett to control the frequency.



Load and production balancing

- The power system frequency measures the power balance.
- It is the responsibility of Statnett to control the frequency.
- However, it is the power plant owners who can control the frequency.



Buying frequency control

- Statnett pays all power plant owners to provide frequency control.

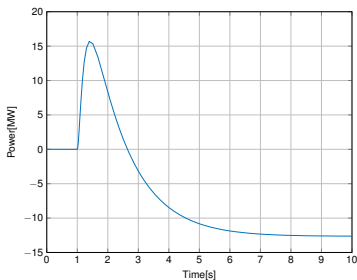


Figure: Frequency control response to step change in frequency

Buying frequency control

- Statnett pays all power plant owners to provide frequency control.
- However, they don't provide the same quality of service.

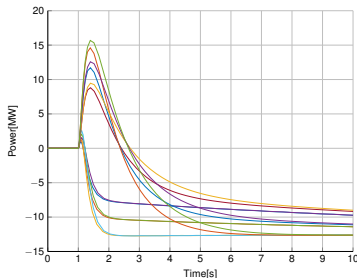


Figure: Frequency control response to step change in frequency

Buying frequency control

- Statnett pays all power plant owners to provide frequency control.
- However, they don't provide the same quality of service.
- Renewable energy sources such as wind and solar don't contribute.

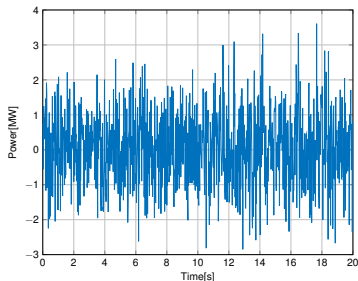


Figure: Frequency control response to step change in frequency

Future of frequency control



- Power plants have to pass tests to get paid to provide frequency control.
- Only those who pass the tests get paid for the service.

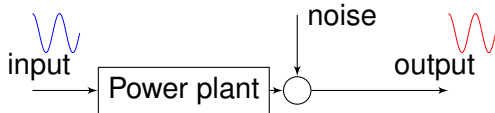


Figure: Test of power plant

Tests proposed by the industry

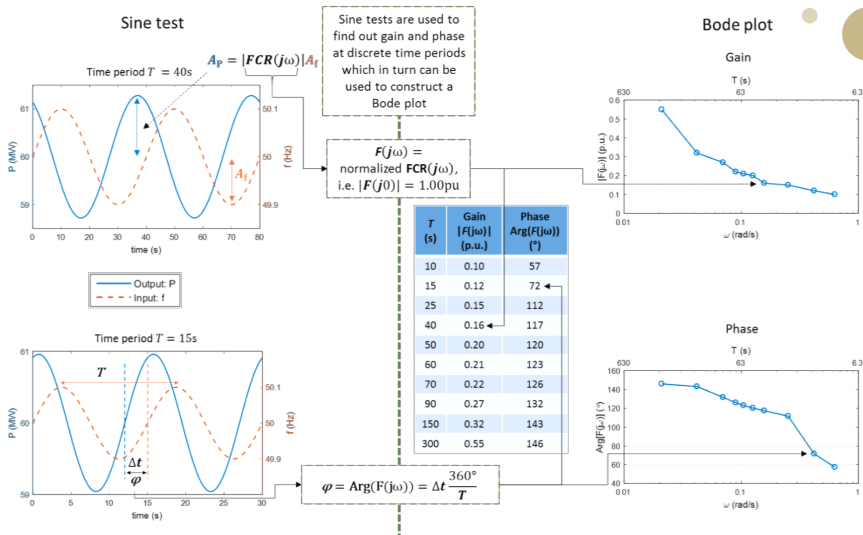
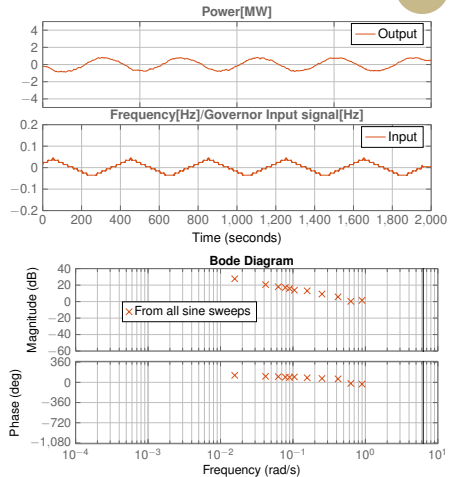


Figure: Testing procedure [source:ENTSO-E]

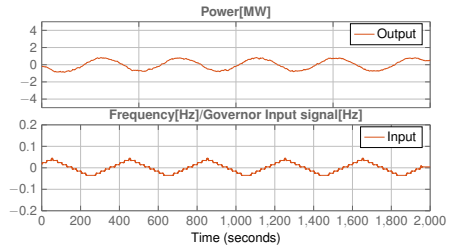
Example from real tests

- The power plant needs to be disconnected



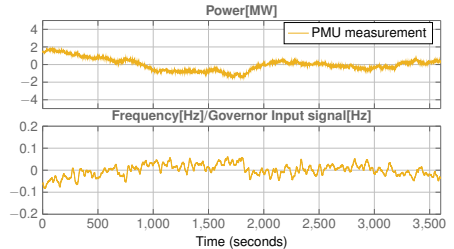
Example from real tests

- The power plant needs to be disconnected
- Takes up to 20 hours.



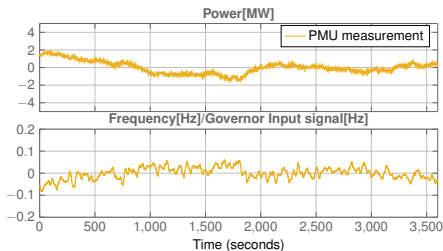
Motivation

- The power system is never really in steady state.



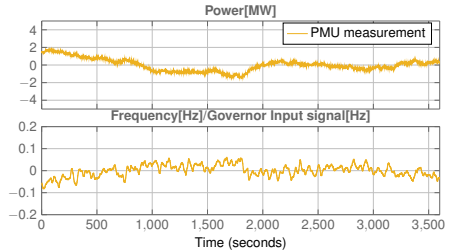
Motivation

- The power system is never really in steady state.
- Can the power plant dynamics be identified from normal operation measurements?



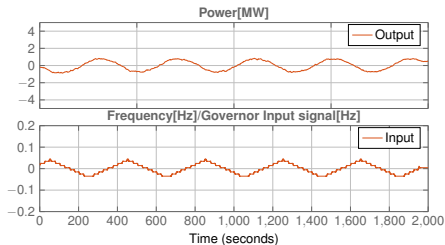
Research questions

- Can power plant dynamics be identified using a PMU?



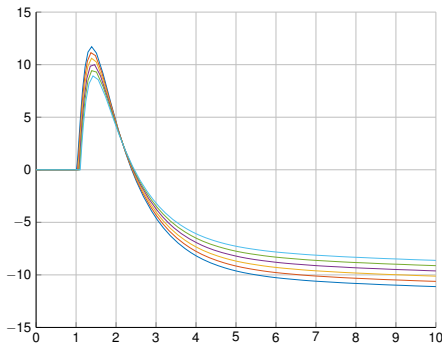
Research questions

- Can power plant dynamics be identified using a PMU?
- Can power plant dynamics be identified using control system measurements without disturbing the operation of the plant?



Research questions

- Can power plant dynamics be identified using a PMU?
- Can power plant dynamics be identified using control system measurements without disturbing the operation of the plant?
- What is the effect of nonlinearities on the identification?



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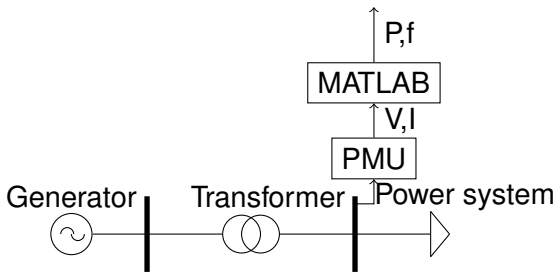
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Background

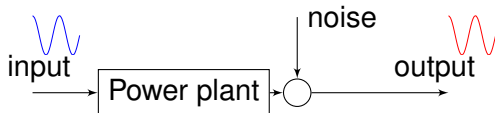
- Idea from¹ can the power plant dynamics be identified using PMUs



¹Dinh Thuc Duong et al. "Estimation of Hydro Turbine-Governor's Transfer Function from PMU Measurements". In: *IEEE PES General Meeting*. Boston: IEEE, July 2016

Background

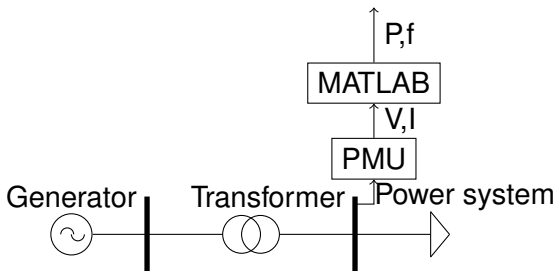
- Idea from¹ can the power plant dynamics be identified using PMUs
- Uses the same input and output measurements as in the requirements:
 - Input: Power system frequency.
 - Output: Electric power.



¹Dinh Thuc Duong et al. "Estimation of Hydro Turbine-Governor's Transfer Function from PMU Measurements". In: [IEEE PES General Meeting. Boston: IEEE, July 2016](#)

Methodology

- Collect several datasets from PMUs.
- Calculate power and frequency from the measurements.
- Identify dynamics using vector fitting.
- Compare models.



Vector fitting basics



$$Y(s) = H(s) \cdot U(s) \quad (1)$$

- Vector fitting fits a transfer function to measured input and output data

Vector fitting basics

- Vector fitting fits a transfer function to measured input and output data
- It assumes the system to have the following structure.

$$Y(s) = H(s) \cdot U(s) \quad (1)$$

$$H(s) = d + \sum_{i=1}^{n_p} \frac{r_i}{s - p_i} \quad (2)$$

Vector fitting basics



$$Y(s) = H(s) \cdot U(s) \quad (1)$$

- Vector fitting fits a transfer function to measured input and output data
- It assumes the system to have the following structure.
- In time domain it is.

$$H(s) = d + \sum_{i=1}^{n_p} \frac{r_i}{s - p_i} \quad (2)$$

$$y(t) \approx \tilde{d}x(t) + \sum_{i=1}^{n_p} \tilde{r}_i x_i - \sum_{i=1}^{n_p} \tilde{k}_i y_i \quad (3)$$

$$x_i = \int_0^t e^{\tilde{p}_i(t-\tau)} x_i(\tau) d\tau \quad (4)$$

$$y_i = \int_0^t e^{\tilde{p}_i(t-\tau)} y_i(\tau) d\tau \quad (5)$$

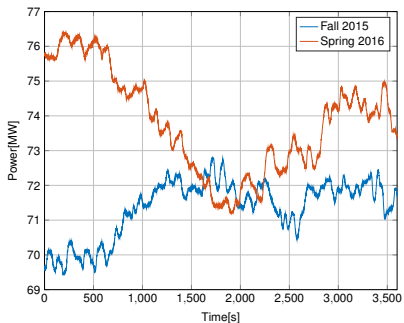
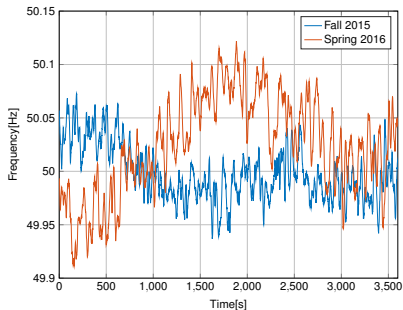
Vector fitting basics ctd.



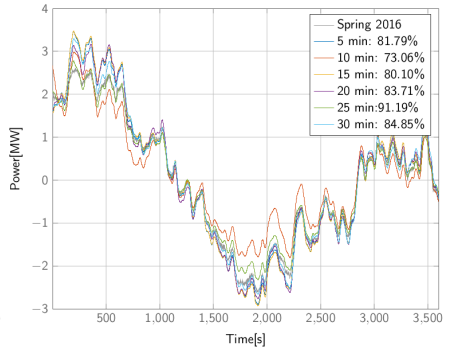
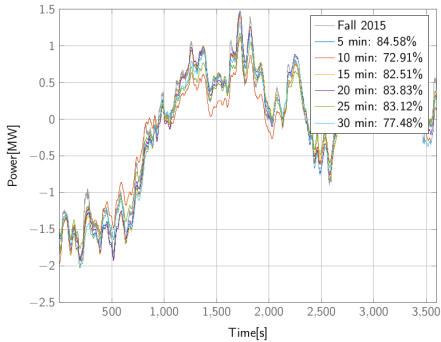
— Find \tilde{d} , \tilde{r}_i and \tilde{k}_i to minimize:

$$y(t) - (\tilde{d}x(t) + \sum_{i=1}^{n_p} \tilde{r}_i x_i - \sum_{i=1}^{n_p} \tilde{k}_i y_i) \quad (6)$$

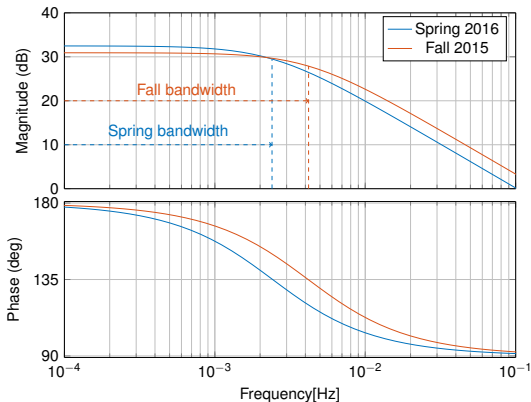
Cross validation using distant data sets



Cross validation using distant data sets



Estimated droop and bandwidth

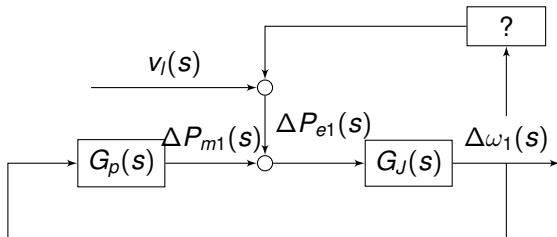


Dataset	Droop[%]	Bandwidth[mHz]
Fall 2015	10	4.16
Spring 2016	8	2.41

Shortcoming with the paper

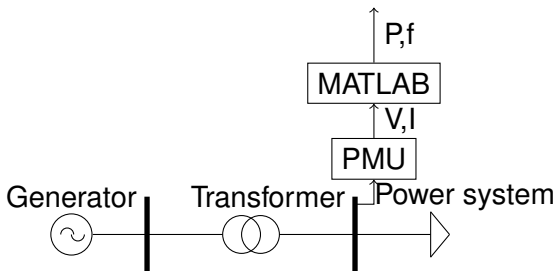


- No theoretical validation of the results.
- No simulation validation of the results.



Main contributions to the research questions

- Promising results for 19 datasets.



Main contributions to the research questions



- Promising results for 19 datasets.
- Developed code for interfacing with the PMU data.

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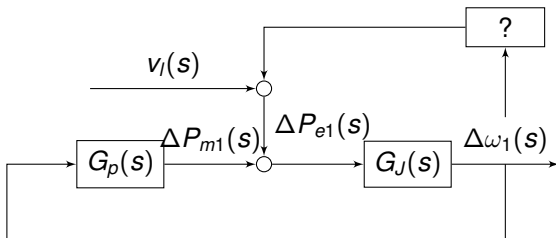
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Motivation

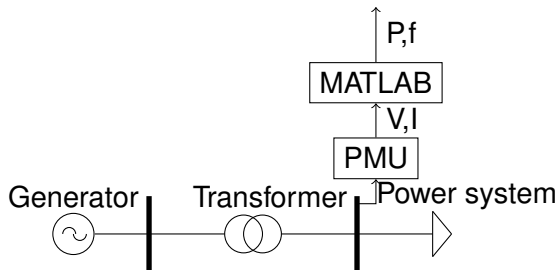


- Explain the problem to my co-supervisor.
- Create a model for analysing the identifiability of hydro power plant dynamics.



What do we need to model?

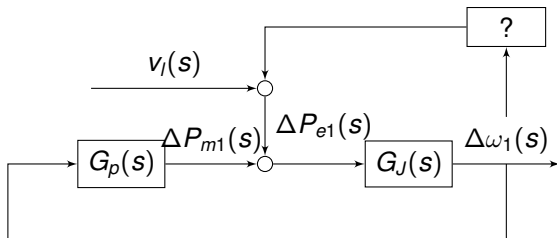
— From the PMU we get



What do we need to model?

— From the PMU we get

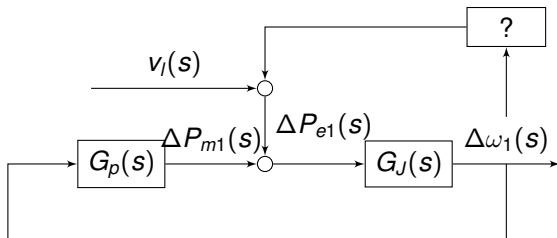
- Power: $\Delta P_{e1}(s)$.



What do we need to model?

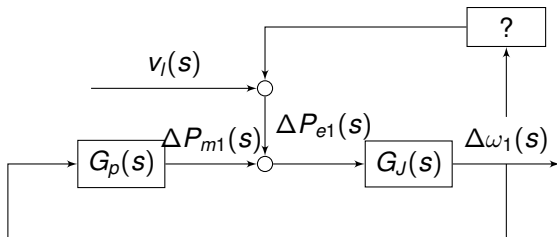
— From the PMU we get

- Power: $\Delta P_{e1}(s)$.
- Frequency: $\Delta f(s)$.



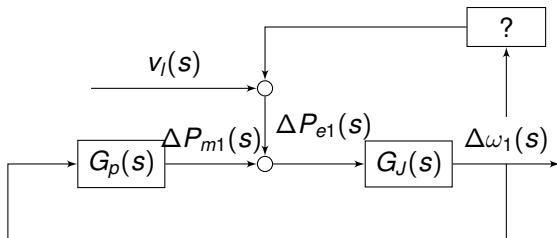
What do we need to model?

- From the PMU we get
 - Power: $\Delta P_{e1}(s)$.
 - Frequency: $\Delta f(s)$.
- We need to model how $\Delta P_{e1}(s)$ and $\Delta f(s)$ is related through the power system.



What do we need to model?

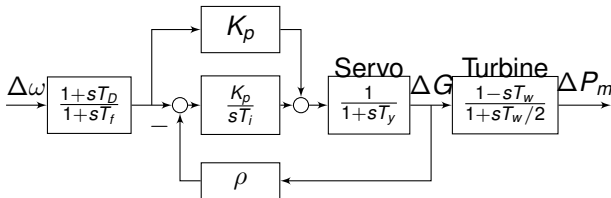
- From the PMU we get
 - Power: $\Delta P_{e1}(s)$.
 - Frequency: $\Delta f(s)$.
- We need to model how $\Delta P_{e1}(s)$ and $\Delta f(s)$ is related through the power system.
- We also need to model the power plant consisting of $G_p(s)$ and $G_J(s)$.



Power plant model

- Model for $G_p(s)$
- Model for $G_J(s)$

$$G_J(s) = \frac{1}{2Hs + K_d} \quad (7)$$



Power system model

- The frequency and power system angle is related.

$$\Delta\theta(s) = \frac{2\pi f_s}{s} f(s) \quad (8)$$

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Power system model



- The frequency and power system angle is related.
- The angle and power is related.
- On matrix form.

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$$\mathbf{P} = \mathbf{Y}\theta \quad (10)$$

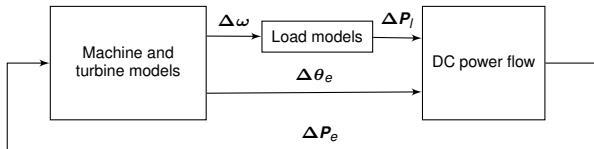
Power system model

- The frequency and power system angle is related.
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- On matrix form.
- In software

$$\Delta\theta(s) = \frac{2\pi f_s}{s} f(s) \quad (8)$$

$$P_k \approx \sum_{m \in \Omega_k} x_{km}^{-1} \theta_{km} \quad (9)$$

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Test system

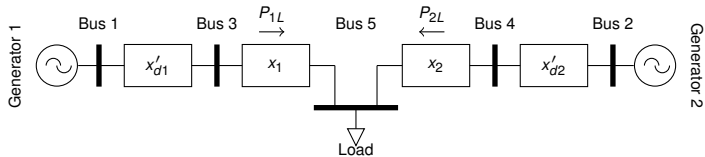
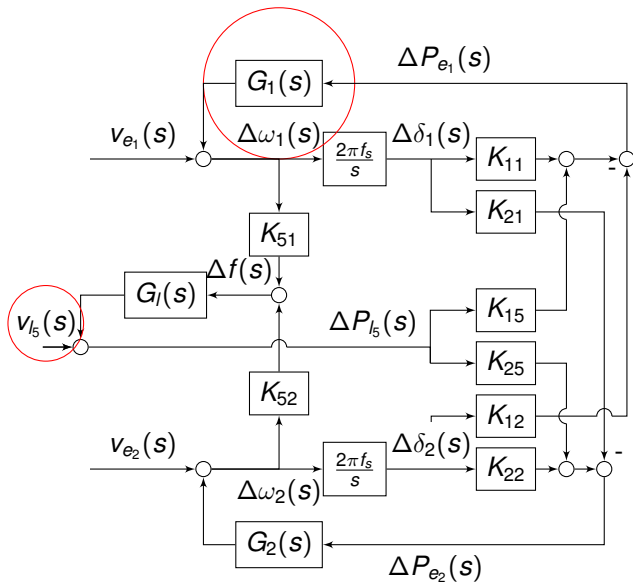
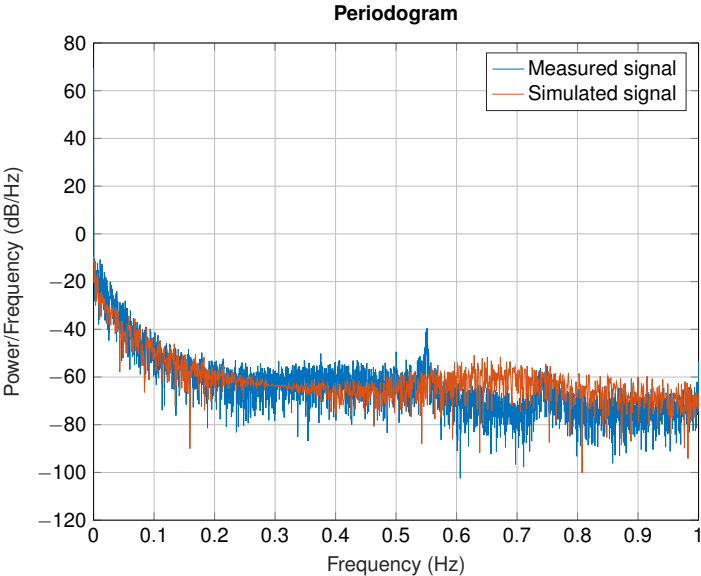


Figure: Single line diagram

Test system



Simulation Result



Main contributions



- Developed simple test system for analysing power plant identifiability using PMUs.
- Developed simple test system used in the proceeding papers for simulations.

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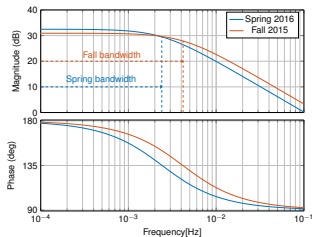
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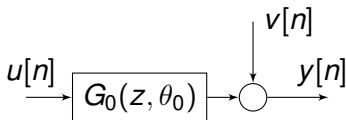
Background

- Why do we get different results?



Background

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- The signals we use are corrupted by noise.



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- The signals we use are corrupted by noise.
- From system identification we have that the error will be asymptotic normally distributed

$$\sqrt{N}(\hat{\theta}_n - \theta^*) \in AsN(0, P_\theta)$$

Background

- Why do we get different results?
- The signals we use are corrupted by noise.
- From system identification we have that the error will be asymptotic normally distributed
- However, first we need to prove the identifiability of the system

True system: \mathcal{S}

x: unbiased

x: biased



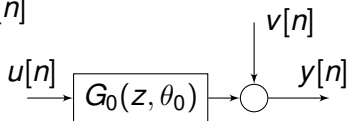
System identification basic

- Assume that a data set $Z^N = \{u[n], y[n] | n = 1 \dots N\}$ has been collected.
- The dataset Z^N is assumed generated by

$$\mathcal{S} : y[n] = G_0(z, \theta_0)u[n] + H_0(z, \theta_0)e[n] \quad (11)$$

- Using the data set Z^N we want to find the parameter vector θ^N minimizing

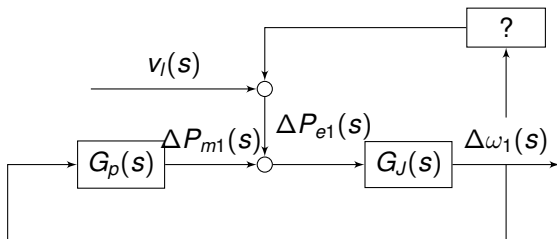
$$\hat{\theta}_N = \arg \min_{\theta} \frac{1}{N} \sum_{n=1}^N \epsilon^2(n, \theta) \quad (12)$$



Modeling used for the validation

- The system we are identifying

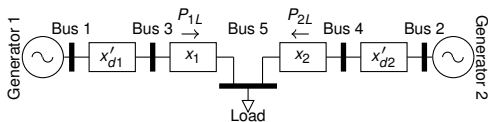
$$G_1(s) = \frac{G_p(s)}{1 + G_p(s)G_J(s)} \quad (13)$$



Modeling used for the validation

- The system we are identifying
- We use a small power system

(13)

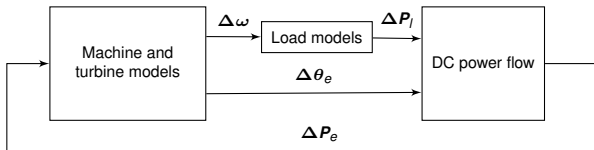


Modeling used for the validation



- The system we are identifying
- We use a small power system
- We use a dc power flow

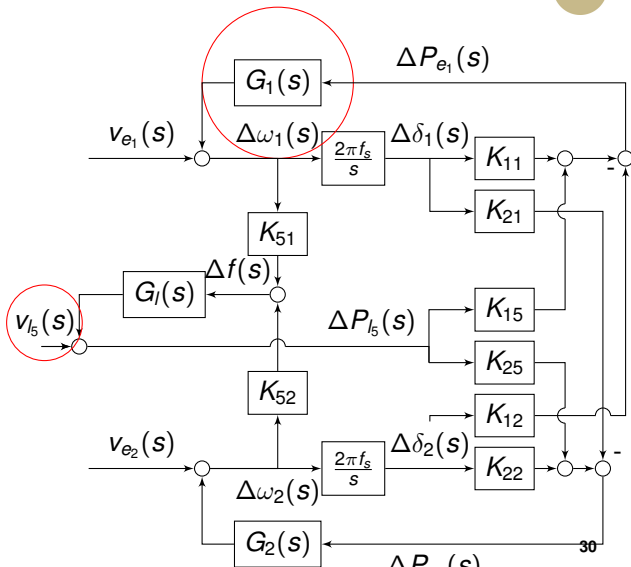
(13)



Modeling used for the validation

(13)

- The system we are identifying
- We use a small power system
- We use a dc power flow
- This results in the following block diagram



Results from the theoretical validation



- A consistent estimate of the closed loop transfer function of the turbine and electromechanical dynamics can be obtained by using:

Results from the theoretical validation



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 - Measured PMU frequency as the output $u[n]$

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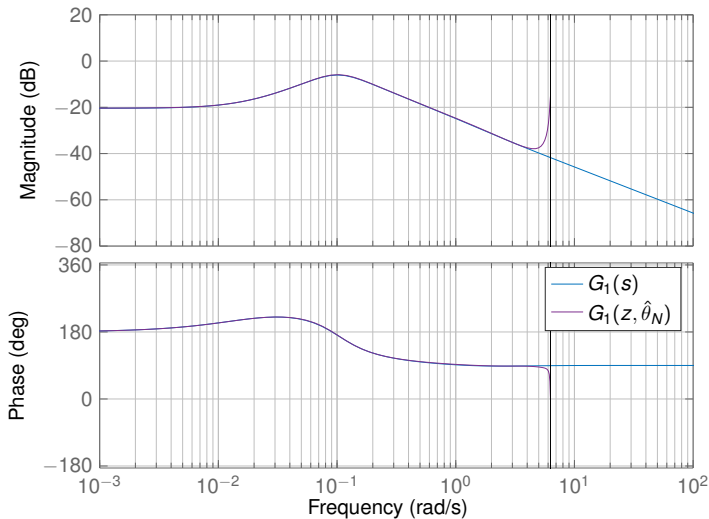
Results from the theoretical validation



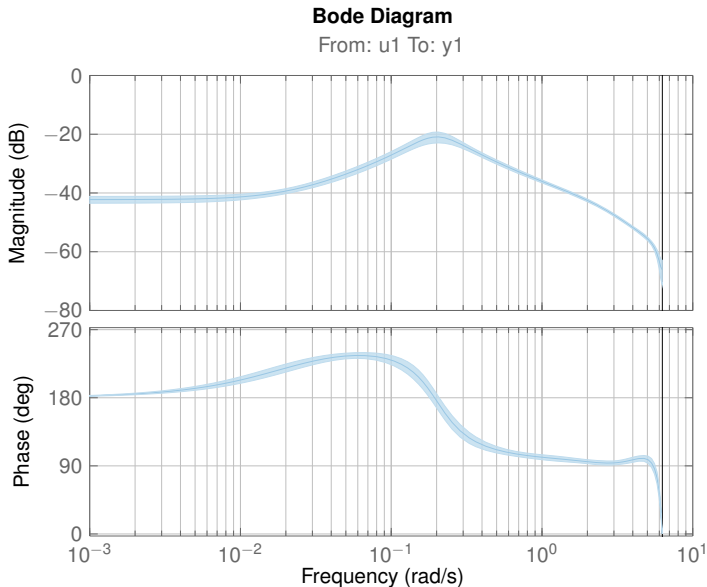
- A consistent estimate of the closed loop transfer function of the turbine and electromechanical dynamics can be obtained by using:
 - Measured PMU frequency as the output $u[n]$
 - Measured PMU power as the input $y[n]$
- The proof was done with the following assumptions.
 - The system is excited by a load acting as a filtered white noise process
 - The measurement error of the electrical power is negligible.
 - The measured frequency is a good estimate of the generator speed.

Comparison of bode plots from simulation

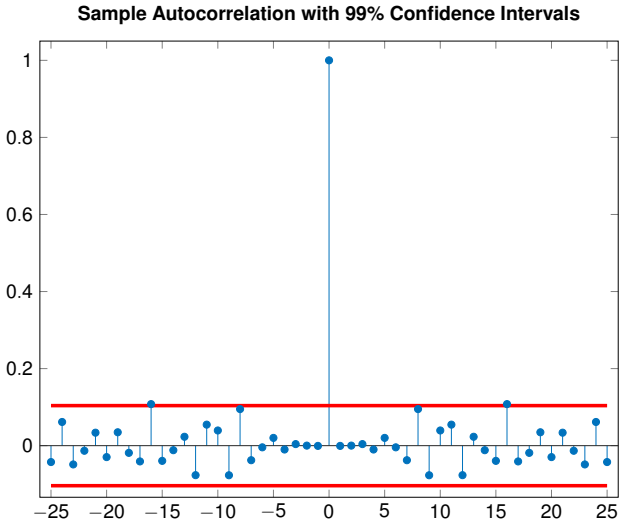
Bode Diagram



Model obtained using PMU data



Whiteness test on model identified using PMU data



Main contributions



- To show that the transfer function one is identifying using PMUs is $G_1(s)$.
- To prove under which conditions a consistent estimate of $G_1(s)$ is possible.
- To demonstrate the theory for identification of $G_1(s)$ on real datasets.

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Motivation



- Relate the results from Paper III and the new requirements.

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- Test the methods on more real datasets.
- Demonstrate that industry proposed tests can be done easier.

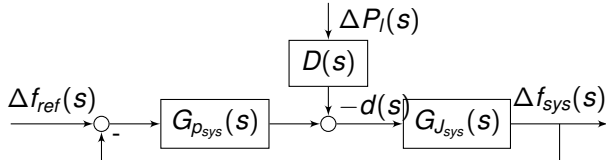
Motivation



- Relate the results from Paper III and the new requirements.
- Test the methods on more real datasets.
- Demonstrate that industry proposed tests can be done easier.
- Less theoretical presentation in a more industry focused conference.

The new requirements

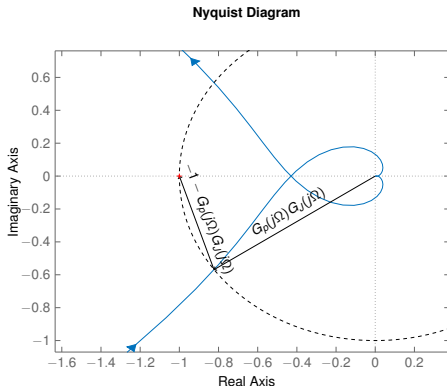
- Puts requirements on an aggregated system model.



The new requirements

- Puts requirements on an aggregated system model.
- Stability requirement

$$\begin{aligned} M_S &= \max \left| \frac{1}{1 + G_p(j\Omega)G_J(j\Omega)} \right| \\ &= \max |S(j\Omega)| \end{aligned} \quad (14)$$



The new requirements

- Puts requirements on an aggregated system model.
- Stability requirement

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- Performance requirement

$$|G_1(j\Omega)| < \frac{\sigma_{\omega_{req}}}{\sqrt{\phi_{P_e}(j\Omega)}} \quad (15)$$



The new requirements

- Puts requirements on an aggregated system model.
- Stability requirement

$$\begin{aligned} M_S &= \max \left| \frac{1}{1 + G_p(j\Omega)G_J(j\Omega)} \right| \\ &= \max |S(j\Omega)| \end{aligned} \quad (14)$$

- Performance requirement

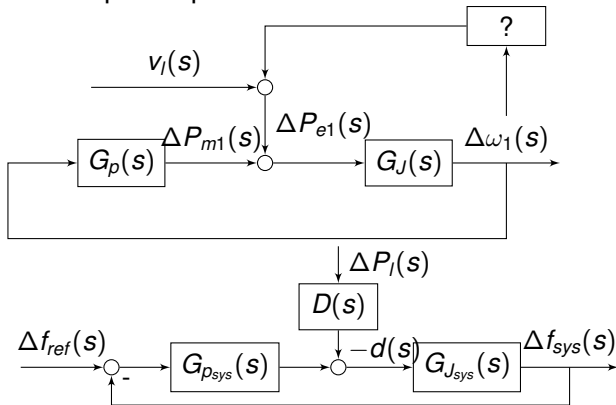
$$|G_1(j\Omega)| < \frac{\sigma_{\omega_{req}}}{\sqrt{\phi_{P_e}(j\Omega)}} \quad (15)$$

- Requirement per plant stated using a per unit conversion



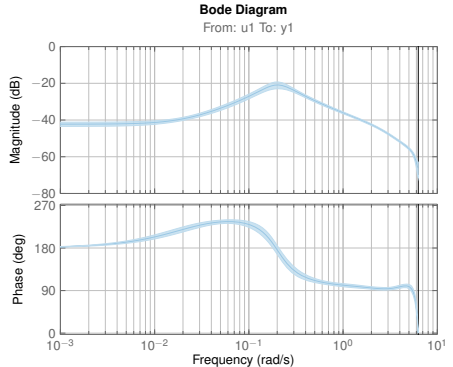
Alternative requirements

- Place requirements directly on one power plant.



Alternative requirements

- Place requirements directly on one power plant.
- We already have an estimate of $G_1(s)$.



Alternative requirements



- Place requirements directly on one power plant.
- We already have an estimate of $G_1(s)$.
- We need to find $S(s)$

Estimating $S(s)$



$$G_1(s) = G_J(s)S(s)$$

(16)

Estimating $S(s)$



$$G_1(s) = G_J(s)S(s) \quad (16)$$

$$G_J(s) = \frac{1}{2Hs + K_d} \quad (17)$$

Estimating $S(s)$



$$G_1(s) = G_J(s)S(s) \quad (16)$$

$$G_J(s) = \frac{1}{2Hs + K_d} \quad (17)$$

$$2H \gg K_d \quad (18)$$

Estimating $S(s)$



$$G_1(s) = G_J(s)S(s) \quad (16)$$

$$G_J(s) = \frac{1}{2Hs + K_d} \quad (17)$$

$$2H \gg K_d \quad (18)$$

$$S(s) \approx 2HsG_1(s) \quad (19)$$

Estimating $S(s)$



—

$$G_1(s) = G_J(s)S(s) \quad (16)$$

—

$$G_J(s) = \frac{1}{2Hs + K_d} \quad (17)$$

—

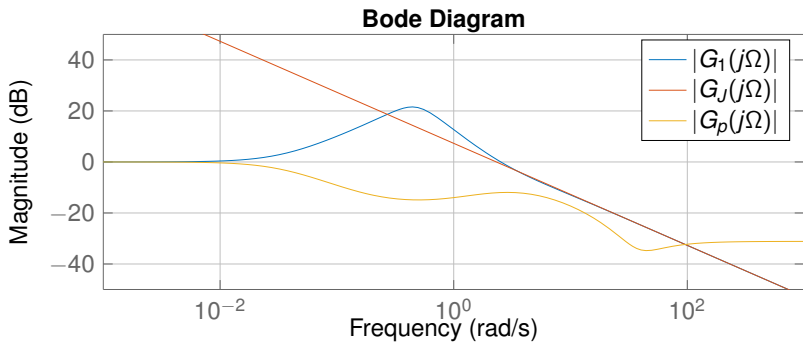
$$2H \gg K_d \quad (18)$$

—

$$S(s) \approx 2HsG_1(s) \quad (19)$$

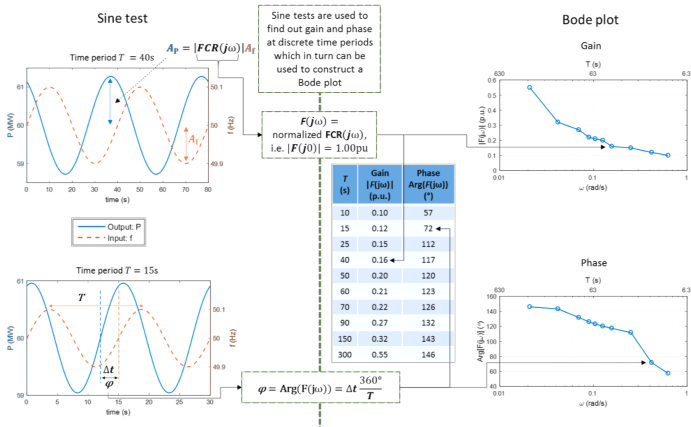
— Need to estimate H

Estimating H



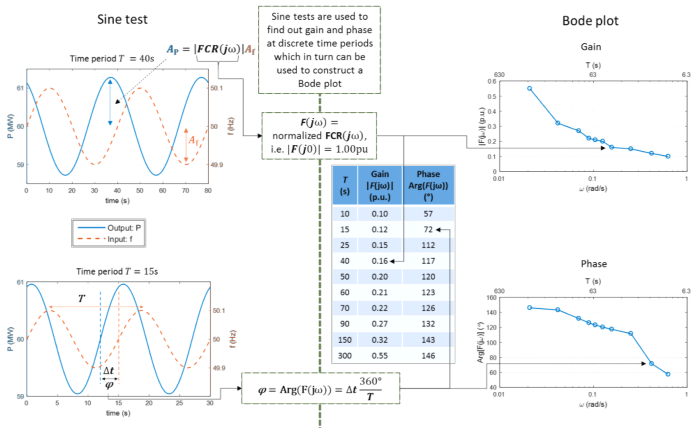
Dataset from Statkraft

— One of Norway's biggest power producers.



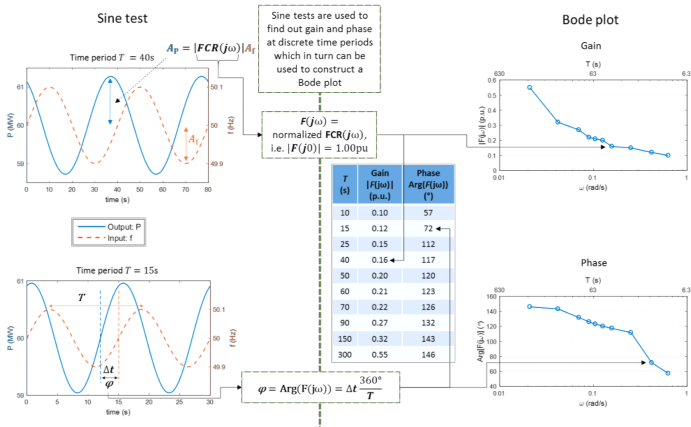
Dataset from Statkraft

- One of Norway's biggest power producers.
- They performed the tests from the draft requirements

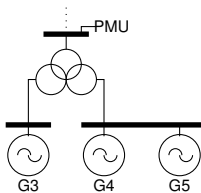


Dataset from Statkraft

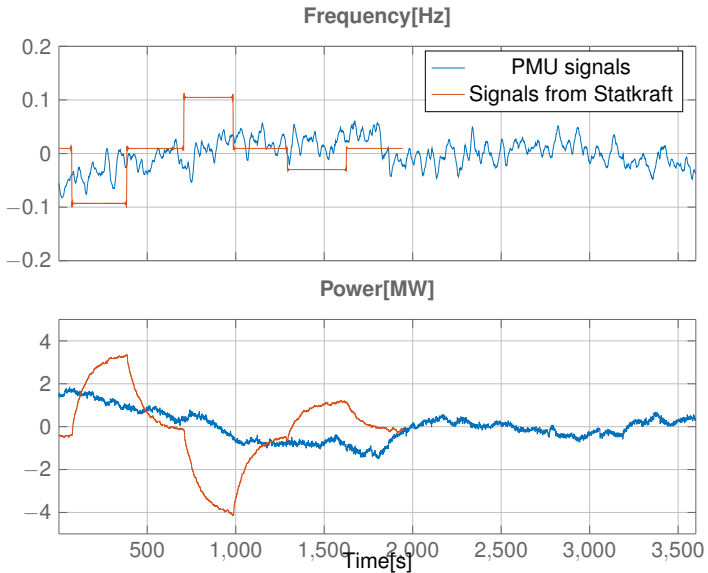
- One of Norway's biggest power producers.
- They performed the tests from the draft requirements
- By chance I had PMU measurements from the same plant.



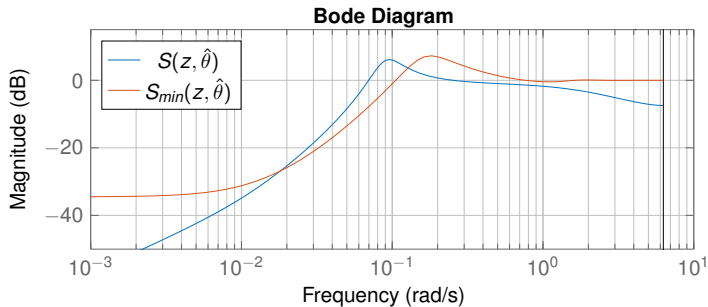
Single line diagram of the plant



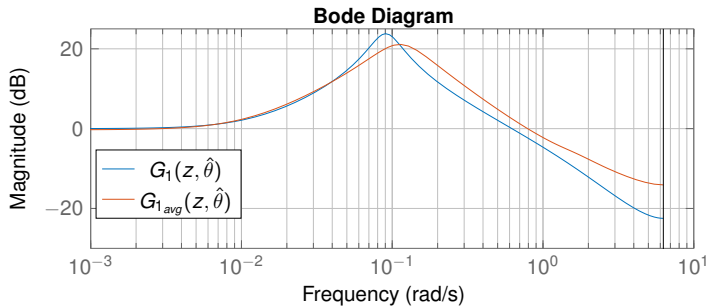
Datasets used



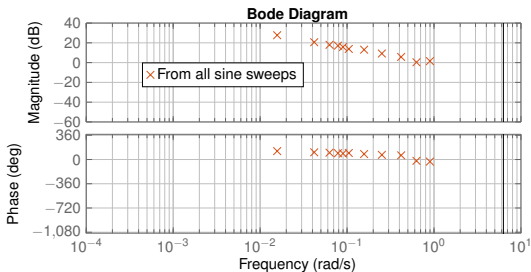
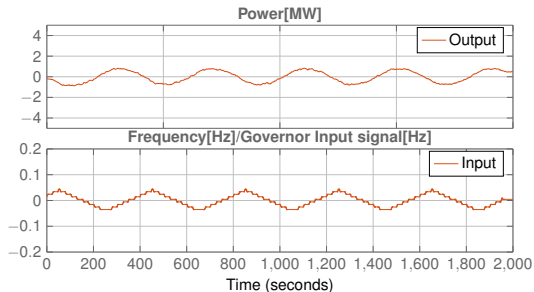
Estimated sensitivity functions



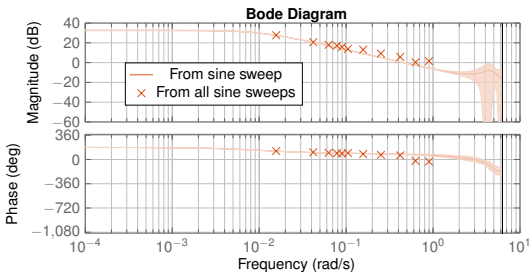
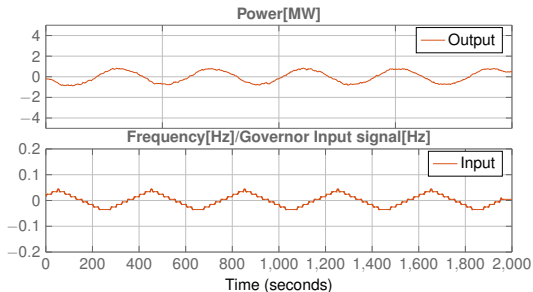
Estimated $G_1(s)$



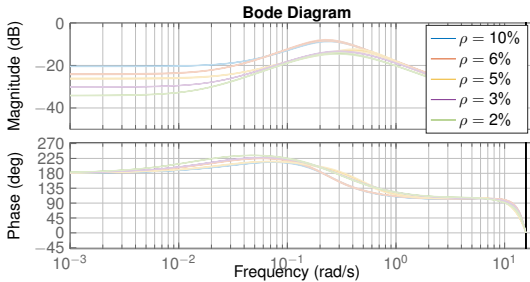
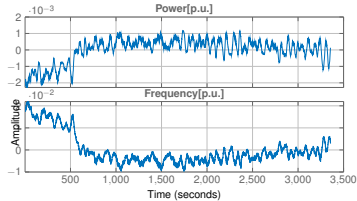
Can the industry proposed tests be done easier?



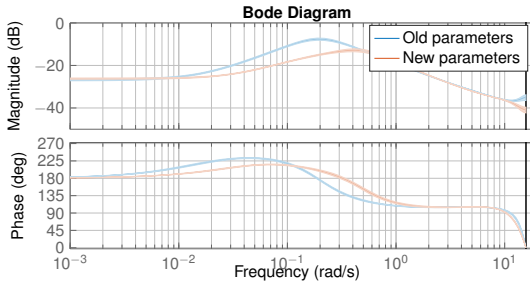
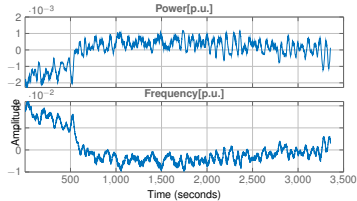
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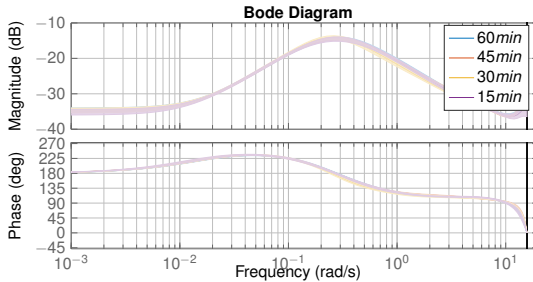
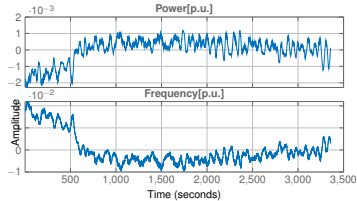
Control system data in closed loop



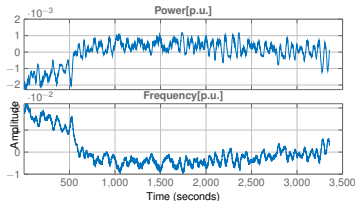
Control system data in closed loop



Control system data in closed loop



Control system data in closed loop



Droop	60min	45min	30min	15min
10%	9.5%	9.5%	9.5%	9.5%
6%	6.2%	6.0%	5.9%	6.1%
5%	4.9%	4.9%	5.0%	5.1%
3%	3.1%	3.1%	3.1%	2.9%
2%	2.0%	1.8%	1.8%	1.7%

Main Contributions



- Proposal for alternative tests.
- Demonstrating that the proposed methods can detect parameter changes.
- Demonstrated that the industry proposed tests can be done easier.

Outline



Problem

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An extension of Paper IV, with more discussion, simulation comparisons and more simulation validations Paper V

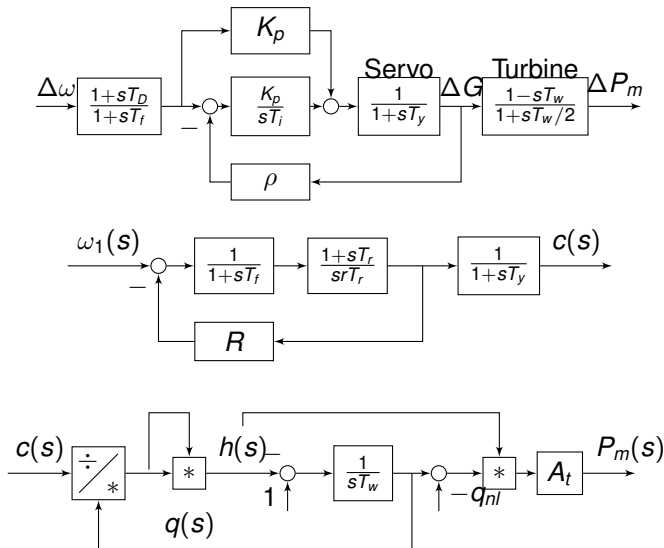
Checking the requirements using measurements from the control system of a hydro power plant Paper VI

Motivation



- Test with a more detailed power plant model.
- Test with a more detailed power system model.
- Investigate some of the assumptions from previous papers.

More detailed power plant model



More detailed power system model



- Added the frequency divider formula to the simple test system.

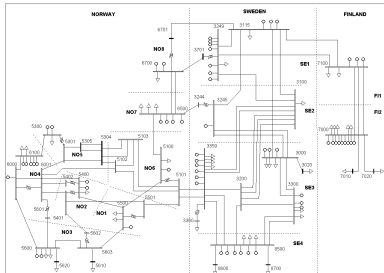
$$\omega_I = \mathbf{1} + \mathbf{D}(\omega_e - \mathbf{1}) \quad (20)$$

where

$$\mathbf{D} = -\mathbf{B}_{22}^{-1} \mathbf{B}_{21} \quad (21)$$

More detailed power system model

- Added the frequency divider formula to the simple test system.
- Used the Nordic 44 test system in PSS/E.

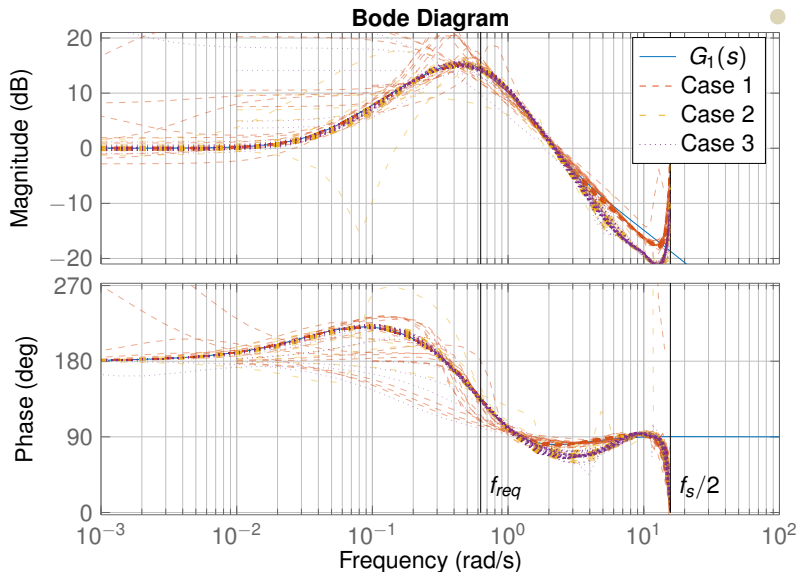


Identification cases

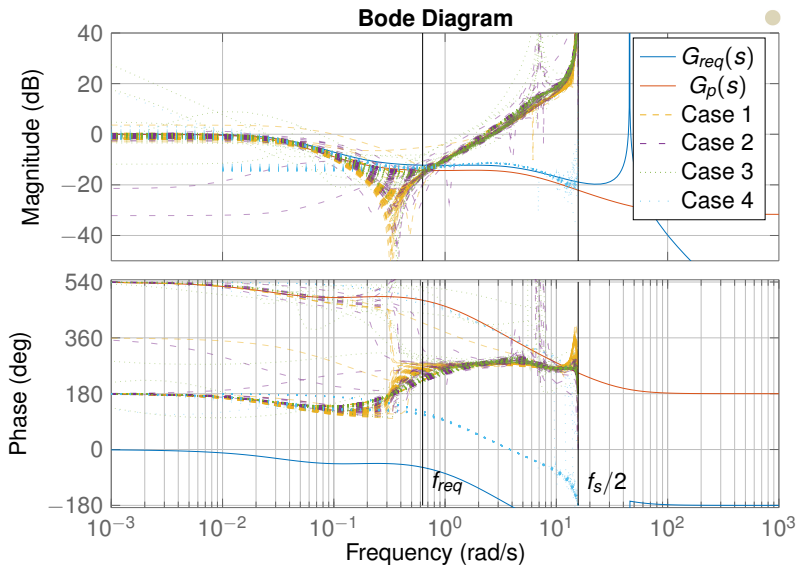


1. The plant is operating under normal conditions with speed feedback and we have access to the control system signals.
2. The plant is still operating under speed feedback, but we don't have access to the control system signals for the identification and therefore use frequency as an estimate for the speed.
3. The plant is operating with frequency feedback and we have access to the frequency measurements.
4. In this case we have disconnected the input to the governor and replaced it with our own signal.

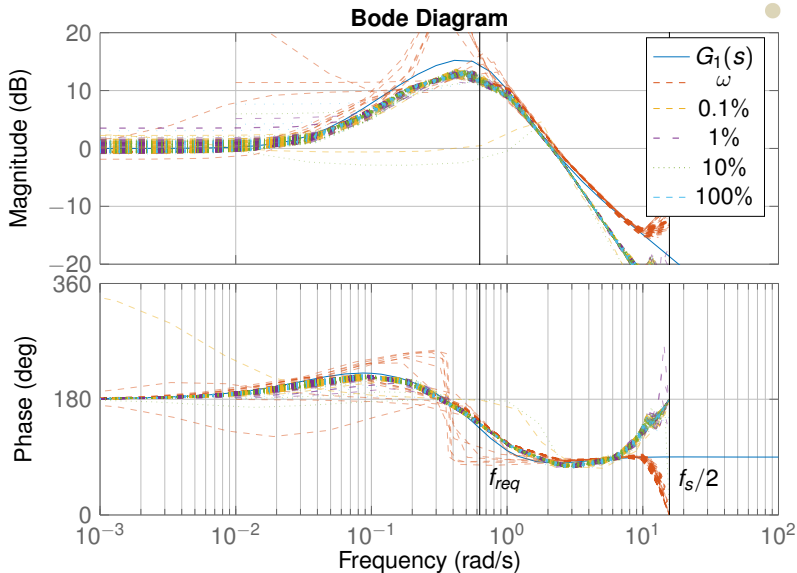
Test the different cases



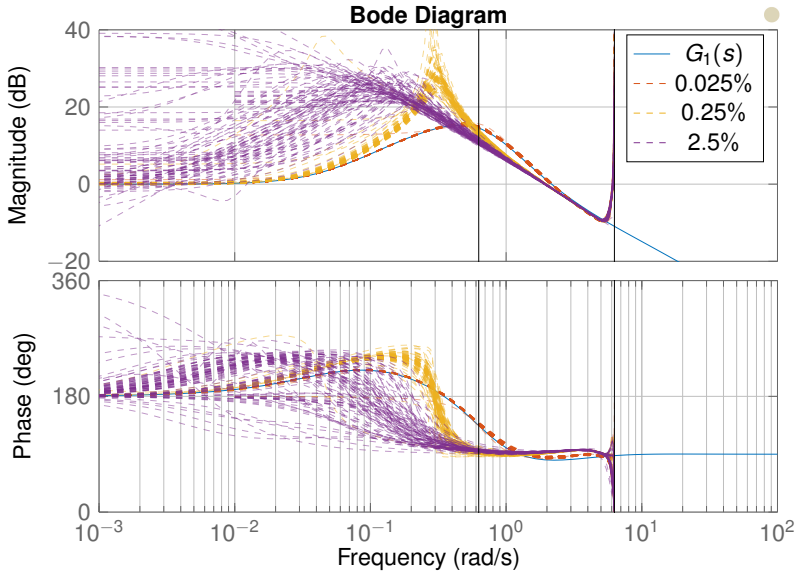
Test the different cases



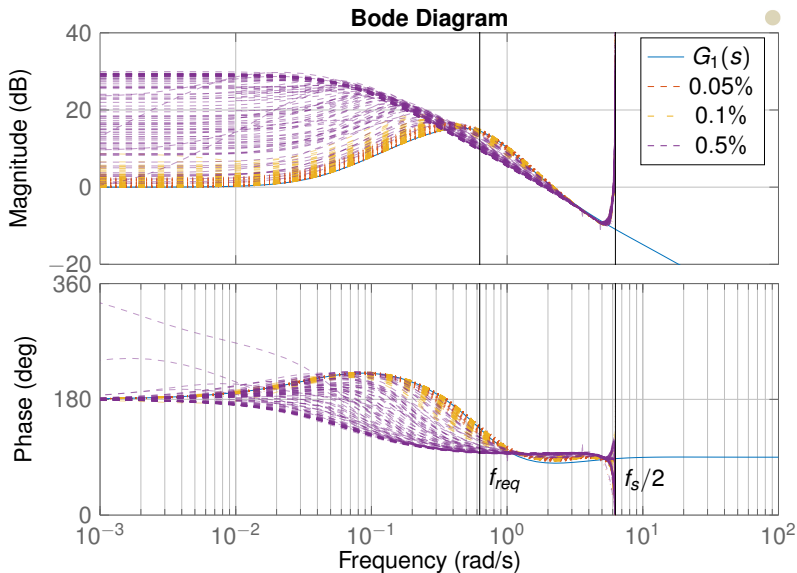
Test frequency assumption



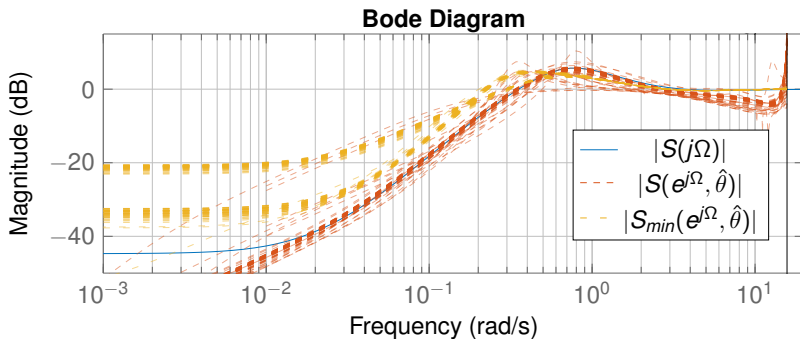
Test backlash



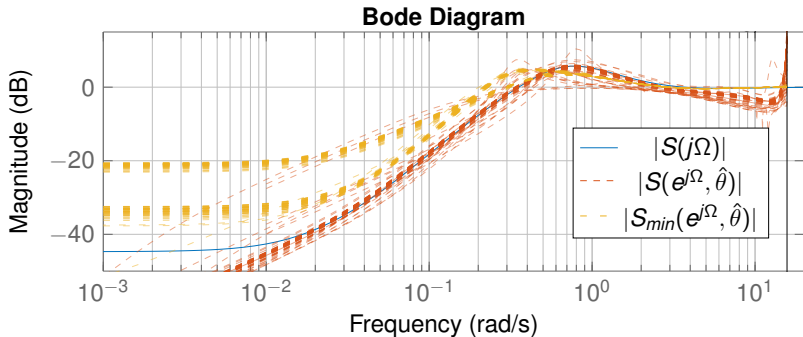
Test deadband



Sensitivity function $S(s)$



Disturbance rejection function $G_1(s)$



Comparison of stability margins



Method	Median	Root mean square error (RMSE)
$\max S(j\Omega) $	1.84	0
$\max S(e^{j\Omega}, \hat{\theta}) $, Case 1	1.84	0.25
$\max S(e^{j\Omega}, \hat{\theta}) $, Case 2	1.75	0.34
$\max S(e^{j\Omega}, \hat{\theta}) $, Case 3	1.74	0.39
$\max S_{\min}(e^{j\Omega}, \hat{\theta}) $	1.66	0.25

Comparison of estimated inertias



Case	Median	RMSE
Actual	3.5	0
Case 1	3.40	0.46
Case 2	3.33	0.40
Case 3	3.27	0.43

Major contributions



- Tested the methods with a more detailed power plant model.
- Tested the methods with a more detailed power system model.
- Investigate some of the assumptions from previous papers.

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Motivation



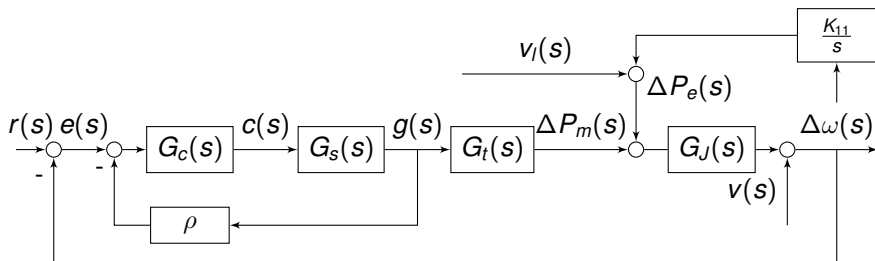
- How to best identify hydro power plant dynamics given access to control system data.

Systems to be identified

$$e(s) = G_1(s)\Delta P_{e1}(s) + S(s)r(s) \quad (22)$$

$$\Delta\omega_1(s) = G_s(s)G_t(s)G_J(s)c(s) - G_J(s)\Delta P_{e1}(s) \quad (23)$$

$$G_p(s) = \frac{G_c(s)G_s(s)G_t(s)G_J(s)}{G_J(s)(1 + \rho G_c(s)G_s(s))} \quad (24)$$



Systems to be identified



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— Two approaches.

Systems to be identified



$$e(s) = G_1(s)\Delta P_{e1}(s) + S(s)r(s) \quad (22)$$

$$\Delta\omega_1(s) = G_s(s)G_t(s)G_J(s)c(s) - G_J(s)\Delta P_{e1}(s) \quad (23)$$

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- Two approaches.
- Extra excitation is needed.

Systems to be identified



$$e(s) = G_1(s)\Delta P_{e1}(s) + S(s)r(s) \quad (22)$$

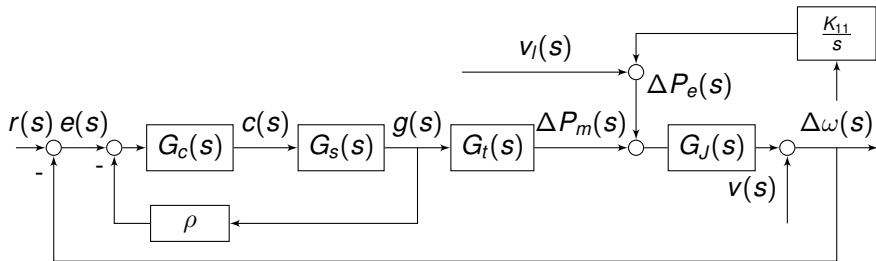
$$\Delta\omega_1(s) = G_s(s)G_t(s)G_J(s)c(s) - G_J(s)\Delta P_{e1}(s) \quad (23)$$

$$G_p(s) = \frac{G_c(s)G_s(s)G_t(s)G_J(s)}{G_J(s)(1 + \rho G_c(s)G_s(s))} \quad (24)$$

- Two approaches.
- Extra excitation is needed.
- PMU approach is a special case without extra excitation.

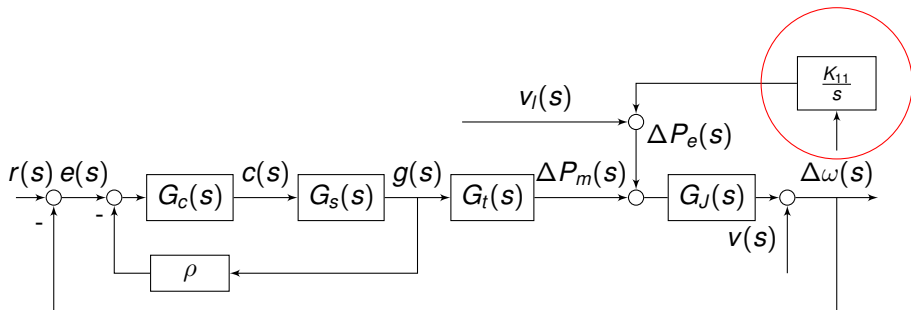
Identifiability

- The systems can be identified



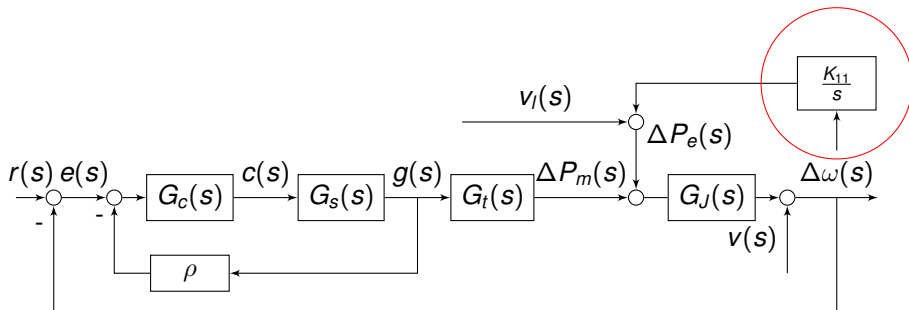
Identifiability

- The systems can be identified
- However, there is a lack of delay



Identifiability

- The systems can be identified
- However, there is a lack of delay
- This is no problem if $v(s) \ll v_I(s)$.



Identifying $G_p(s)$ and $G_J(s)$

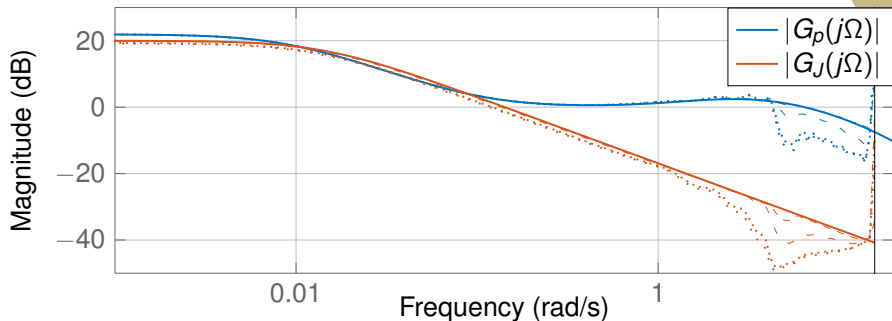


Figure: The mean value of $|G_p(e^{j\Omega}, \hat{\theta}_N)|$ and $|G_J(e^{j\Omega}, \hat{\theta}_N)|$ calculated from the MCS. The solid lines are the analytical calculated versions and the dashed loosely dashed dotted and loosely dotted lines represent an SNR of 50dB, 26dB, 6dB, and 3dB respectively

Identifying $S(s)$

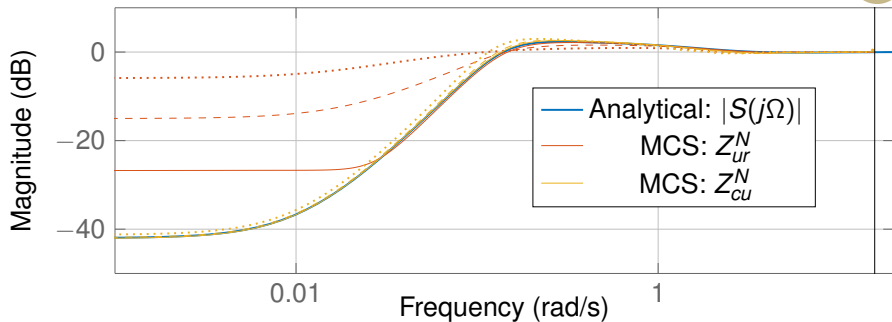


Figure: The mean value of $|S(e^{j\Omega}, \hat{\theta}_N)|$ calculated analytical and from the MCS. The solid, dashed and dotted lines represent an SNR of 50dB, 26dB, and 6dB respectively

Identifying $G_1(s)$

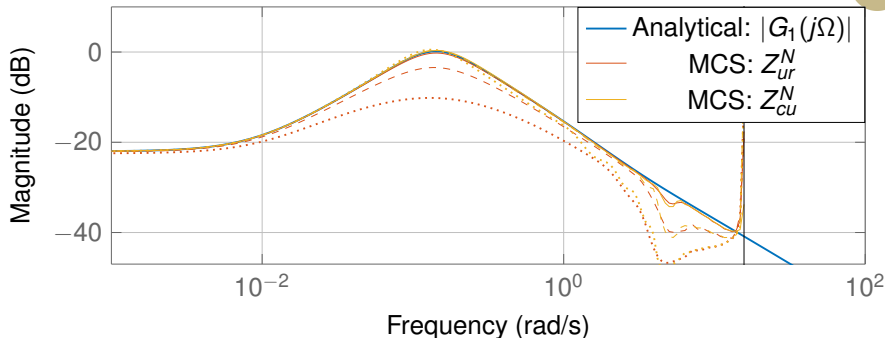


Figure: The mean value of $|G_1(e^{j\Omega}, \hat{\theta}_N)|$ calculated analytical and from the MCS. The solid, dashed and dotted lines represent an SNR of 50dB, 26dB, and 6dB respectively

Major contributions



- Demonstrated two methods for finding transfer functions for checking the requirements in closed loop.
- Analytical validation of the demonstrated methods.
- Addressed the delay condition.