# **Models in Machine Learning**

**An Overview** 



# LINEAR MODEL – FUNCTIONALITY

SUPERVISED PARAMETRIC WHITE-BOX

#### General idea

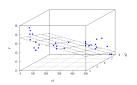
A linear model (LM) represents the target as a linear combination of the input variables.

### Hypothesis space

 $\mathcal{H} = \{ f : \mathcal{X} \to \mathbb{R} \mid f(\mathbf{x}) = \phi(\boldsymbol{\theta}^{\top}\mathbf{x}) \}, \text{ where } \phi(\cdot) \text{ is a transformation function.}$ 

For different  $\phi(\cdot)$  it results in different models, e.g.:

- identity function  $\phi(\theta^{\top}\mathbf{x}) = \theta^{\top}\mathbf{x}$ : linear regression models a continous output with the linear combination of the features  $\theta^T \mathbf{x}$ .
- logistic sigmoid function  $\phi(\theta^{\top}\mathbf{x}) = \frac{1}{1+\exp(-\theta^{\top}\mathbf{x})} = \pi(\mathbf{x})$ : logistic regression models a probability  $\pi(\mathbf{x}) = \mathbb{P}(y = 1 \mid \mathbf{x})$  belonging to one of two classes. Applying a decision rule (e.g.,  $\pi(\mathbf{x}) > 0.5$ ) results in a separating hyperplane.



Linear regression: surface



Logisitc regression: surface



Logistic regression: classification

### LINEAR MODEL – FUNCTIONALITY

### **Empirical risk**

- Typically, in **linear regression** the **ordinary least squares (OLS)** with a squared loss function is used for regression:  $\mathcal{R}_{emp}(\theta) = \sum_{i=1}^{n} \left( y^{(i)} \theta^T \mathbf{x}^{(i)} \right)^2$
- Alternatively, the absolute loss or the **Huber loss** can be used.
- For logistic regression the risk function is based on the Bernoulli loss

$$\mathcal{R}_{\text{emp}}(\boldsymbol{\theta}) = \sum_{i=1}^{n} -y^{(i)}\log\left(\pi\left(\mathbf{x}^{(i)}\right)\right) - (1-y^{(i)})\log\left(1-\pi\left(\mathbf{x}^{(i)}\right)\right).$$

### Optimization

- for **OLS**: analytically with  $\hat{\theta} = (\mathbf{X}^{\top}\mathbf{X})^{-1}\mathbf{X}^{\top}\mathbf{y}$  (with  $\mathbf{X} \in \mathbb{R}^{n \times p}$ : matrix of feature vectors)
- for other loss functions: numerical optimization

Hyperparameters None

# **LINEAR MODEL - PRO'S & CON'S**

#### Advantages

- simple and fast implementation; cheap computational costs
- intuitive interpretability: mean influence of features on the output and feature importance
- fits linearly separable data sets very well
- + works well independent of data size
- basis for many machine learning algorithms

### Disadvantages

- not suitable for data based on a non-linear data generating process → strong simplification of real-world problems
- strong assumptions: data is independent (multi-collinearity must be removed)
- tend to **overfit** (can be reduced by regularization)
- sensitive to outliers and noisy data

Simple method with good interpretability for linear problems, but strong assumptions and simplification of real-world problems

# **LINEAR MODEL – PRACTICAL HINTS**

### Check assumptions??

This model is very effective if the following assumptions are fulfilled:

- **linearity**: The expected response is a linear combination of the features.
- homoscedasticity: The variance of residuals is equal for all features.
- independence: All observations are independent of each other.
- normality: Y is normally distributed for any fixed value of the features

### Implementation

- R: mlr3 learner LearnerRegrLM, calling stats::lm()
- Python: LinearRegression from package sklearn.linear\_model, package for advanced statistical parameters statsmodels.api

### Regularization

In practice, we often use regularized models in order to **prevent overfitting** or perform feature selection. More details will follow in the subsequent chapter.

### REGULARIZED LM – FUNCTIONALITY

SUPERVISED PARAMETRIC WHITE-BOX

#### General idea

- Linear model (LM) can overfit if we operate in high-dimensional space with only a few observations.
- We can find a compromise between generalizing the model (simple model, underfitted) and corresponding closely to the data (complex model, overfitted).

### Hypothesis space

 $\mathcal{H} = \{ f : \mathcal{X} \to \mathbb{R} \mid f(\mathbf{x}) = \phi(\boldsymbol{\theta}^{\top}\mathbf{x}) \}, \text{ where } \phi(\cdot) \text{ is a transformation function.}$ 

### **Empirical risk**

- We minimize the empirical risk function  $\mathcal{R}_{emp}(\theta)$  plus a complexity penalty  $J(\theta)$ , controlled by shrinkage parameter  $\lambda$ :  $\mathcal{R}_{req}(\theta) = \mathcal{R}_{emp}(\theta) + \lambda \cdot J(\theta)$ .
- Ridge regression uses the L2-penalty  $J(\theta) = \|\theta\|_2^2$ .
- Alternativly, **LASSO** uses the L1-penalty  $J(\theta) = \|\theta\|_1$ .

### Optimization

- for **Ridge** regression: analytically with  $\hat{\theta}_{Ridge} = (\mathbf{X}^T \mathbf{X} + \lambda \mathbf{I})^{-1} \mathbf{X}^T \mathbf{y}$
- for LASSO regression: e. g. (sub)-gradient descent

Hyperparameters Shrinkage parameter  $\lambda$ 

### **REGULARIZED LM – PRO'S & CON'S**

### ALMOST SAME LIKE LINEAR MODEL?-> does this make sense?

**Advantages** 

Disadvantages

- simple and fast implementation;
  cheap computational costs
- + intuitive interpretability: mean influence of features on the output and feature importance
- fits linearly separable data sets very well
- + works well independent of data size
- basis for many machine learning algorithms
- + less tendency to overfit

- not suitable for data based on a non-linear data generating process
   → strong simplification of real-world problems
- strong assumptions: data is independent (multi-collinearity must be removed and normal distributed residuals ??
- sensitive to outliers and noisy data

Simple method with good interpretability for linear problems, but strong assumptions and simplification of real-world problems.

### REGULARIZED LM – PRACTICAL HINTS

### Choice of regularization parameter $\lambda$

Choose  $\lambda$  with e. g. the smallest sum of squared residuals through cross-validation. In the R package glmnet lambda.min is the value of  $\lambda$  that gives minimum mean cross-validated error.

### Ridge vs. LASSO

- ullet neither is overall better o elastic net as a compromise
- Ridge works better, if there are many influential and high correlated features.
- In contrast, LASSO is more suitable if the underlying structure is sparse (only a few features influence the output y).
- LASSO can set coefficients to zero, thus performing variable selection.

### Implementation

- R: mlr3 learners LearnerClassifGlmnet / LearnerRegrGlmnet, calling glmnet::glmnet()
- Python: LinearRegression from package sklearn.linear\_model, package for advanced statistical parameters statsmodels.api

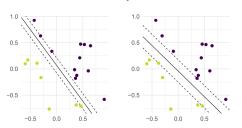
# LINEAR SVM - FUNCTIONALITY

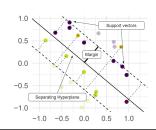
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#### General idea

- The support vector machine (SVM) algorithm finds a decision boundary (separating hyperplane) that maximizes the distance (margin  $\gamma$ ) to the closest members (support vectors, SV) of the separate classes. (hard margin)
- $\bullet$  In a **soft-margin SVM** also "violations" of the margin are allowed  $\rightarrow$  not only the margin should be maximized, but also the margin violations minimized.
- There are three different types of training points: **Non-SVs** which do not influence the hyperplane, SVs which are exactly on the hyperplane, and margin violators.

Hypothesis space 
$$\mathcal{H} = \{ f : \mathcal{X} \to \mathbb{R} \mid f(\mathbf{x}) = \boldsymbol{\theta}^{\top} \mathbf{x} + \theta_0 \}$$





# **LINEAR SVM – FUNCTIONALITY**

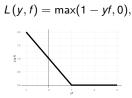
**Empirical risk** 

Soft-margin SVMs can also be interpreted as **L2-regularized ERM**:

$$\frac{1}{2}\|\boldsymbol{\theta}\|^2 + C\sum_{i=1}^n L\left(y^{(i)}, f\left(\mathbf{x}^{(i)}\right)\right)$$

with  $\|\boldsymbol{\theta}\| = 1/\gamma$ ,

C as a cost parameter for margin violation, and L(y, f) as the hinge loss.



**Dual problem** 

$$\begin{aligned} \max_{\in \mathbb{R}^n} \quad & \sum_{i=1}^n \alpha_i - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y^{(i)} y^{(j)} \left\langle \mathbf{x}^{(i)}, \mathbf{x}^{(j)} \right\rangle \\ \text{s.t.} \quad & 0 \leq \alpha_i \leq C, \\ & \sum_{i=1}^n \alpha_i y^{(i)} = 0, \end{aligned}$$

Optimization not differentiable problem → mostly subgradient methods

Hyperparameters C: penalization for missclassified data points

# LINEAR SVM - PRO'S & CON'S

#### **Advantages**

- + high accuaracy
- + often **sparse** solution
- robust against overfitting (regularized); especially in high-dimensional space
- stable solutions, as the non-SV do not influence the separating hyperplane

### Disadvantages

- costly implementation; long training times
- does not scale well to larger data sets ??
- only linear separation → possible with non-linear SVMs which are explained in the following slides.
- poor interpretability

Very accurate solution for high-dimensional data that is linearly separable

# **LINEAR SVM – PRACTICAL HINTS**

#### Preprocessing

Features must be rescaled before applying SVMs.

### **Tuning**

The cost parameter C must be tuned, as it has a strong influence on the resulting separating hyperplane.

### Implementation

- R: mlr3 learners LearnerClassifSVM / LearnerRegrSVM, calling svm() from libsvm
- Python: sklearn.svm.SVC from package scikit-learn / package libSVM

# NON-LINEAR SVM – FUNCTIONALITY

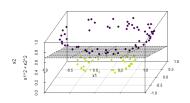
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#### General idea

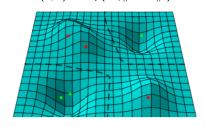
- Non-linear SVMs construct a separating hyperplane in a higher dimensional dimension
- Kernels = feature map + inner product  $k(\mathbf{x}, \tilde{\mathbf{x}})$  transform the input space into a higher dimensional space and calculates the inner product.

Hypothesis space 
$$\mathcal{H} = \{f(\mathbf{x}) = \sum_{i=1}^{n} \alpha_i y^{(i)} k(\mathbf{x}^{(i)}, \mathbf{x}) + \theta_0 | \theta_0, \alpha_i \in \mathbb{R} \}$$

$$\phi(x_1, x_2) = (x_1, x_2, x_1^2 + x_2^2)$$



$$k(\mathbf{x}, \tilde{\mathbf{x}}) = \exp(-\gamma ||\mathbf{x} - \tilde{\mathbf{x}}||^2)$$



### NON-LINEAR SVM – FUNCTIONALITY

#### **Dual Problem**

We kernelize the dual (soft-margin) SVM problem by replacing all inner products  $\left\langle \mathbf{x}^{(i)}, \mathbf{x}^{(j)} \right\rangle$  by kernels  $k(\mathbf{x}^{(i)}, \mathbf{x}^{(j)})$ 

$$\begin{aligned} \max_{\alpha} & & \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j \mathbf{y}^{(i)} \mathbf{y}^{(j)} k(\mathbf{x}^{(i)}, \mathbf{x}^{(j)}) \\ \text{s.t.} & & 0 \leq \alpha_i \leq C, \\ & & \sum_{i=1}^{n} \alpha_i \mathbf{y}^{(i)} = 0. \end{aligned}$$

### **Hyperparameters**

- C: penalization for missclassified data points
- Kernel parameters: depending on which kernel is used (e. g. degree of the polynomial kernel or width of RBF kernel)

### NON-LINEAR SVM - PRO'S & CON'S

### **Advantages**

- + high accuaracy
- can learn non-linear decision boundaries
- + often sparse solution
- robust against overfitting (regularized); especially in high-dimensional space
- stable solutions, as the non-SV do not influence the separating hyperplane

### Disadvantages

- costly implementation; long training times
- does not scale well to larger data sets ??
- only linear separation → possible with non-linear SVMs which are explained in the following slides.
- poor interpretability
- not easy tunable as it is highly important to choose the right kernel

Non-linear SVMs perform very well for non-linear separable data, but are hard to interpret and need a lot of tuning.

# **NON-LINEAR SVM – PRACTICAL HINTS**

#### Kernels

Mainly, these three types of kernels are used:

- Linear kernel: the dot product of the given observations
- Polynomial kernel: curved lines in the input space
- Radial basis function (RBF) kernel: complex regions in the input space (e. g. spirals)

#### **Tuning**

- SVMs are sensitive to its hyperparameters and should always be tuned.
- For the RBF kernel the RBF sigma heuristic is used.

### Implementation

- R: mlr3 learners LearnerClassifSVM / LearnerRegrSVM, calling e1071::svm() (interface to libSVM)
- Python: sklearn.svm.SVC from package scikit-learn / package libSVM

# K-NEAREST NEIGHBORS – FUNCTIONALITY

SUPERVISED NON-PARAMETRIC WHITE-BOX

#### General idea

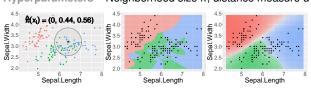
- The k-nearest neighbors (k-NN) model is based on inter-observational distances, thus heavily depending on the chosen distance measure.
- It builds upon the rationale that closeness in feature space infers closeness in target space.
- **Prediction** for  $\mathbf{x}^{(i)}$ : construct k-neighborhood  $N_k(\mathbf{x}^{(i)})$  (i.e., the k points closest to  $\mathbf{x}^{(i)}$  in feature space), then predict
  - the (weighted) mean target for **regression**:

$$\hat{y} = 1/(\sum_{i:\mathbf{x}^{(i)} \in N_k(\mathbf{x})} w_i) \sum_{i:\mathbf{x}^{(i)} \in N_k(\mathbf{x})} w_i y^{(i)}$$

• the most frequent class for classification:

$$\hat{h}(\mathbf{x}) = \mathop{\mathrm{arg\,max}}_{\ell \in \{1,\dots,g\}} \sum_{i: \mathbf{x}^{(i)} \in \mathcal{N}_k(\mathbf{x})} \mathbb{I}(y^{(i)} = \ell)$$

#### Hyperparameters Neighborhood size k, distance measure $d(\cdot)$



Left: Neighborhood for exemplary observation in iris. k = 50Right: Prediction surfaces for k = 1 and k = 50

# K-NEAREST NEIGHBORS – PRO'S & CON'S

### **Advantages**

- + **Easy** to explain and implement
- Few assumptions therefore (in theory) able to model data situations of arbitrary complexity
- + No training period
- + Constant evolvement with new data
- Ability to learn non-linear decision boundaries
- + No optimization required
- + Only one hyperparameter to tune

### Disadvantages

- Sensitivity toward noisy or irrelevant features
- Bad performance when feature scales are not consistent with importance
- Sensitivity toward outliers
- No handling of missing data
- Bad with data imbalances
- Large memory consumption for distance computation

Easy and intuitive for small, well-behaved data sets, but not suitable for large-scale data with problematic features

# **K-NEAREST NEIGHBORS – PRACTICAL HINTS**

### Non-parametric behavior

- k-NN is a lazy classifier its parameters are the training data; there is no real compression of information.
- The number of parameters is thus equal to the number of observations (which have to be stored for prediction!).

#### Distance measures

- For numerical features, typically Manhattan or Euclidean distance; in presence of categorical features, Gower distance might be considered.
- Can be weighted to account for different scales or importance of features.

### Neighborhoods in higher dimensions

*k*-NN is vulnerable to the **curse of dimensionality**: in very high-dimensional feature spaces, finding meaningful neighborhoods may become difficult.

### Implementation

- R: mlr3 learners LearnerClassifKKNN / LearnerRegrKKNN, calling kknn::kknn()
- Python: KNeighborsClassifier / KNeighborsRegressor from package scikit-learn

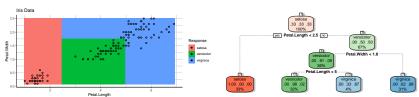
### **CART – FUNCTIONALITY**

SUPERVISED NON-PARAMETRIC WHITE-BOX FEATURE SELECTION

#### General idea

- Starting from a root node, classification & regression trees (CART) perform repeated binary splits of the data according to feature values, thereby subsequently dividing the input space  $\mathcal{X}$  into T rectangular partitions  $Q_t$ .
- Observations are passed along until each ends up in exactly one leaf node (unless stopped early or pruned).
- In each step, CART find the optimal feature-threshold combination to split by.
- Leaf node t is assigned response c<sub>t</sub>.

Hypothesis space 
$$\mathcal{H} = \{ f(\mathbf{x}) : f(\mathbf{x}) = \sum_{t=1}^{T} c_t \mathbb{I}(\mathbf{x} \in Q_t) \}$$



Prediction surface for iris data, 3 splits

Corresponding classification tree

### CART – FUNCTIONALITY

#### **Empirical risk**

- ullet Empirical risk is calculated for each potential terminal node  $\mathcal{N}_t$  of a split.
- In general, trees can handle any type of loss function. Typical choices are:
  - Classification (for g classes):

• Using Brier score 
$$\mathcal{R}(\mathcal{N}_t) = \sum_{(\mathbf{x}, y) \in \mathcal{N}_t} \sum_{k=1}^g (\mathbb{I}(y=k) - \pi_k(\mathbf{x}))^2$$

• Using Bernoulli loss 
$$\mathcal{R}(\mathcal{N}_t) = \sum_{(\mathbf{x},y) \in \mathcal{N}_t} \sum_{k=1}^g \mathbb{I}(y=k) \cdot \log(\pi_k(\mathbf{x}))$$

$$ullet$$
 Regression: Using **quadratic loss**  $\mathcal{R}(\mathcal{N}_t) = \sum\limits_{(\mathbf{x},y) \in \mathcal{N}_t} (y-c_t)^2$ 

#### Optimization

**Exhaustive** search over all (randomly selected) split candidates in each node to minimize empirical risk in the child nodes (greedy optimization)

Hyperparameters Complexity, i.e., number of leaves T

### CART – PRO'S & CON'S

### Advantages

- + **Easy** to understand, interpret & visualize
- + Automatic handling of non-numerical features
- Built-in feature selection
- Automatic handling of missings
- **Interaction** effects between features easily possible, even of higher orders
- + Fast computation and good scalability
- + High **flexibility** (custom split criteria or leaf-node prediction rules)

#### **Disadvantages**

- Rather **low accuracy** (at least, without bagging or boosting)
- High variance/instability: strong dependence on training data
- Therefore, poor generalization & risk of overfitting
- Several steps required for modeling linear relationships
- In presence of categorical features, bias towards features with many categories

Simple and good with feature selection, but not the best predictor

### CART – PRACTICAL HINTS

#### Pruning / early stopping

Unless interrupted, splitting will go on until each leaf node contains a single observation (expensive + overfitting!)

→ Use pruning and stopping criteria to limit complexity

### Implementation

- R: mlr3 learners LearnerClassifRpart / LearnerRegrRpart, calling rpart::rpart()
- Python: DecisionTreeClassifier / DecisionTreeRegressor from package scikit-learn
- Complexity controlled via tree depth, minimum number of observations per node, maximum number of leaves, minimum risk reduction per split, ...

### Bagging / boosting

Since CART are instable predictors on their own, they are typically ensembled to form a random forest (bagging) or used in combination with boosting.

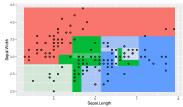
### **RANDOM FOREST – FUNCTIONALITY**

SUPERVISED NON-PARAMETRIC BLACK-BOX FEATURE SELECTION

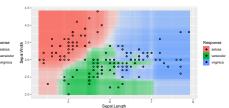
#### General idea

- Random forests (RF) perform bagging: they combine M trees (base learners) to form a strong ensemble learner.
- They use complex trees with low bias and compensate for the resulting variance by aggregating them in a **decorrelated** manner.
- Each tree is trained on a bootstrap sample of the data and only on a random subset of features to incur variability.
- Aggregation via averaging (regression) or majority voting (classification).

Hypothesis space 
$$\mathcal{H} = \left\{ f(\mathbf{x}) : f(\mathbf{x}) = \frac{1}{M} \sum_{m=1}^{M} \sum_{t=1}^{T^{[m]}} c_t^{[m]} \mathbb{I}(\mathbf{x} \in Q_t^{[m]}) \right\}$$



Prediction surface for iris data with a single tree



Prediction surface for iris data with 500-tree ensemble

# RANDOM FOREST – FUNCTIONALITY

#### **Empirical risk**

- Applicable with any kind of loss function (just like tree base learners)
- Computation of empirical risk for all potential child nodes in all trees

#### Optimization

**Exhaustive** search over all (randomly selected) split candidates in each node of each tree to minimize empirical risk in the child nodes (greedy optimization)

### **Hyperparameters**

- Ensemble size, i.e., number of trees
- Complexity, i.e., number of leaves T of each base learner
- Number of split candidates, i.e., number of features to be considered as splitting variables at each split
  - $\rightarrow$  Frequently used heuristics:  $|\sqrt{p}|$  for classification and |p/3| for regression

# **RANDOM FOREST – PRO'S & CON'S**

#### **Advantages**

- Translation of most of base learners' advantages (e.g., inherent variable selection, handling of missing data)
- + Fairly good at **prediction**: improved accuracy through bagging
- Inherent computation of feature importance
- + Quite stable wrt changes in the data
- Good with high-dimensional data, even in presence of noisy covariates
- + Applicable to unbalanced data
- Easy to parallelize
- + Rather easy to tune

#### Disadvantages

- Translation of some of base learners' disadvantages (e.g., trouble to model linear relations, bias towards features with many categories)
- Loss of single trees' interpretability
  black-box method
  - Hard to visualize
- Often suboptimal for regression
- Often still inferior in performance to other methods (e.g., boosting)

### Fairly good predictor, but black-box method

# **RANDOM FOREST – PRACTICAL HINTS**

#### **Pre-processing**

Inherent feature selection of random forests, but high **computational costs** for large number of features

→ Upstream feature selection (e.g., via PCA) might be advisable

### Implementation

- R: mlr3 learners LearnerClassifRanger / LearnerRanger, calling ranger::ranger()
- Python: RandomForestClassifier / RandomForestRegressor from package scikit-learn

### Tuning

- Overall limited tunability
- Number of split candidates often more impactful than number of trees

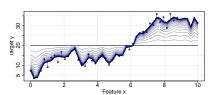
# GRADIENT BOOSTING – FUNCTIONALITY

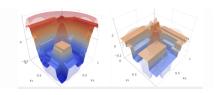
SUPERVISED NON-PARAMETRIC BLACK-BOX FEATURE SELECTION

#### General idea

- Gradient boosting (GB) is an ensemble method that constructs a strong learner from weak base learners (frequently, CART).
- As opposed to bagging, however, base learners are assembled in a sequential, stage-wise manner: in each iteration, GB improves the current model by adding a new component that minimizes empirical risk.
- Each base learner is fitted to the current **point-wise residuals**  $\rightarrow$  One boosting iteration  $\hat{=}$  one approximate gradient step in function space
- The final model is a weighted sum of base learners  $b(\mathbf{x}, \theta^{[m]})$  with weights  $\beta^{[m]}$ .

Hypothesis space 
$$\mathcal{H} = \left\{ f(\mathbf{x}) : f(\mathbf{x}) = \sum_{m=1}^{M} \beta^{[m]} b(\mathbf{x}, \theta^{[m]}) \right\}$$





Left: univariate GB, right: bivariate GB (1: target function (transparent) & current ensemble, 2: residual (transparent) & next base learner)

### GRADIENT BOOSTING – FUNCTIONALITY

#### **Empirical risk**

- Outer loss: Loss used to compute pseudo-residuals how large is the error of the current model fit?
  - → Arbitrary **differentiable** loss function
- Inner loss: Loss used to fit next base learner component to current pseudo-residuals
  - → Typically, **quadratic loss** (desirable optimization properties)

Optimization Functional gradient descent for outer optimization loop, procedure for inner one depending on inner loss

### **Hyperparameters**

- Ensemble size, i.e., number of base learners
- Learning rate, i.e., impact of single base learner
- Complexity of base learners (depending on type used)

# **GRADIENT BOOSTING – PRO'S & CON'S**

### **Advantages**

- Powerful off-the-shelf method for supercharging weak learners' performance
- Translation of most of base learners' advantages (e.g., for tree boosting: inherent variable selection, handling of missing data)
- High predictive accuracy that is hard to outperform
- High flexibility (custom loss functions, many tuning options)
- + Applicable to unbalanced data

#### **Disadvantages**

- Hardly interpretable black-box method
- Hard to visualize
- Prone to overfitting
- Sensitive to outliers
- Hard to tune (high sensitivity to variations in hyperparameter values)
- Rather slow in training
- Hard to parallelize

High-performing predictor, but rather delicate to handle

### **GRADIENT BOOSTING – PRACTICAL HINTS**

XGBoost (extreme gradient boosting)

Fast, efficient implementation of gradient-boosted decision trees that has become **state of the art** for many machine learning problems

→ Clever modeling techniques + computational speed

Stochastic gradient boosting (SGB)

Faster, approximate version of GB that performs each iteration only on **random subset** of the data

### Implementation

- R: mlr3 learners LearnerClassifXgboost / LearnerXgboost, calling xgboost::xgb.train()
- Python: GradientBoostingClassifier / GradientBoostingRegressor from package scikit-learn, XGBClassifier / XGBRegressor from package xgboost

#### Tuning

- Overall limited tunability
- Number of split candidates often more impactful than number of trees

### NEURAL NETWORK – FUNCTIONALITY

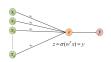
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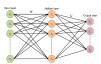
#### General idea

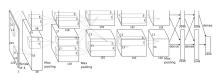
- A neural network (NN) is a model architecture loosely inspired by the human brain. It consists of various **neurons**, organized in **layers** assembled through weighted functional connections.
- Batches of data enter in the input layer and sequentially pass through h hidden **layers**, each of which performs a linear transformation  $\phi^{(j)}$  and a non-linear **activation**  $\sigma^{(j)}$ , thus creating intermediary representations of the data.
- The **output layer** yields predictions after a final transformation  $\phi$  and scaling  $\tau$ .
- The resulting loss is used to update the weights for the next **epoch**.

### Hypothesis space

$$\mathcal{H} = \left\{ f(\mathbf{x}) : f(\mathbf{x}) = \tau \circ \phi \circ \sigma^{(h)} \circ \phi^{(h)} \circ \sigma^{(h-1)} \circ \phi^{(h-1)} \circ \dots \circ \sigma^{(1)} \circ \phi^{(1)}(\mathbf{x}) \right\}$$







Left: a single neuron, middle: feedforward network with one hidden layer, right: convolutional network

### **NEURAL NETWORK – FUNCTIONALITY**

Empirical risk Any differentiable loss function

#### Optimization

NNs are optimized by **backpropagation** which consists of two steps:

- Forward pass: Predict result with current weights and compute empirical risk according to chosen loss function.
- Backward pass: Calculate error contribution of each weight by means of gradient descent – which essentially means applying the chain rule to the composition of functions applied in each layer – and update weights accordingly.

### Hyperparameters

- Number of hidden layers (depth), number of neurons per layer
- Activation function(s)
- Learning rate for backpropagation
- Number of iterations (epochs), batch size
- Initial weights
- •

# **NEURAL NETWORK – PRO'S & CON'S**

### **Advantages**

- Able to solve complex, non-linear regression or classification problems
- Therefore, typically very good performance
- Built-in feature extraction obtained by intermediary representations
- Suitable for unstructured data (e. g. image, audio, text data)
- Easy handling of high-dimensional or missing data
- Parallelizable structure

### Disadvantages

- Computationally expensive
  → slow to train and forecast
- Large amounts of data required
- Faster-than-linear scaling of weight matrices with increased network size
- Network architecture requiring much expertise in tuning
- Black-box model hard to interpret or explain
- Tendency towards overfitting

Able to learn extremely complex functions, but computationally expensive and hard to get right

### **NEURAL NETWORK – PRACTICAL HINTS**

Types of neural networks (RNNs, CNNs)

- Recurrent neural networks (RNNs: Deep NN that make use of sequential information with temporal dependencies
  - → Frequently applied to natural language processing
- Convolutional neural networks (CNNs): Regularized version of the fully connected feed-forward NN (where each neuron is connected to all neurons of the subsequent layer) that abstracts inputs to feature maps via convolution
  - ightarrow Frequently applied to image recognition

Problem of neural architecture search (NAS)

NN are **not off-the-shelf** methods – the network architecture needs to be tailored to each problem anew

ightarrow Automated machine learning attempts to learn architectures

### Implementation

- R: package neuralnet
- Python: libraries PyTorch, keras