Models in Machine Learning

An Overview



LINEAR MODEL – FUNCTIONALITY

SUPERVISED PARAMETRIC WHITE-BOX

General idea

A linear model (LM) represents the target as a linear combination of the input variables.

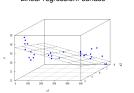
Hypothesis space

 $\mathcal{H} = \{ f : \mathcal{X} \to \mathbb{R} \mid f(\mathbf{x}) = \phi(\boldsymbol{\theta}^{\top}\mathbf{x}) \}, \text{ where } \phi(\cdot) \text{ is a transformation function.}$

For different $\phi(\cdot)$ it results in different models, e.g.:

- identity function $\phi(\theta^{\top}\mathbf{x}) = \theta^{\top}\mathbf{x}$: linear regression models a continous output with the linear combination of the features $\theta^T \mathbf{x}$.
- logistic sigmoid function $\phi(\theta^{\top}\mathbf{x}) = \frac{1}{1+\exp(-\theta^{\top}\mathbf{x})} = \pi(\mathbf{x})$: logistic regression models a probability $\pi(\mathbf{x}) = \mathbb{P}(y = 1 \mid \mathbf{x})$ belonging to one of two classes. Applying a decision rule (e.g., $\pi(\mathbf{x}) > 0.5$) results in a separating hyperplane.

Linear regression: surface



Logisitc regression: surface



Logistic regression: classification



LINEAR MODEL – FUNCTIONALITY

Empirical risk

- Typically, in **linear regression** the **ordinary least squares (OLS)** with a squared loss function is used for regression: $\mathcal{R}_{emp}(\theta) = \sum_{i=1}^{n} \left(y^{(i)} \theta^T \mathbf{x}^{(i)} \right)^2$
- Alternatively, the absolute loss or the **Huber loss** can be used.
- For logistic regression the risk function is based on the Bernoulli loss

$$\mathcal{R}_{\text{emp}}(\boldsymbol{\theta}) = \sum_{i=1}^{n} -y^{(i)}\log\left(\pi\left(\mathbf{x}^{(i)}\right)\right) - (1-y^{(i)})\log\left(1-\pi\left(\mathbf{x}^{(i)}\right)\right).$$

Optimization

- for **OLS**: analytically with $\hat{\theta} = (\mathbf{X}^{\top}\mathbf{X})^{-1}\mathbf{X}^{\top}\mathbf{y}$ (with $\mathbf{X} \in \mathbb{R}^{n \times p}$: matrix of feature vectors)
- for other loss functions: numerical optimization

Hyperparameters None

LINEAR MODEL - PRO'S & CON'S

Advantages

- simple and fast implementation; cheap computational costs
- intuitive interpretability: mean influence of features on the output and feature importance
- fits linearly separable data sets very well
- + works well independent of data size
- basis for many machine learning algorithms

Disadvantages

- not suitable for data based on a non-linear data generating process → strong simplification of real-world problems
- strong assumptions: data is independent (multi-collinearity must be removed)
- tend to **overfit** (can be reduced by regularization)
- sensitive to outliers and noisy data

Simple method with good interpretability for linear problems, but strong assumptions and simplification of real-world problems

LINEAR MODEL – PRACTICAL HINTS

Check assumptions??

This model is very effective if the following assumptions are fulfilled:

- **linearity**: The expected response is a linear combination of the features.
- homoscedasticity: The variance of residuals is equal for all features.
- independence: All observations are independent of each other.
- normality: Y is normally distributed for any fixed value of the features

Implementation

- R: mlr3 learner LearnerRegrLM, calling stats::lm()
- Python: LinearRegression from package sklearn.linear_model, package for advanced statistical parameters statsmodels.api

Regularization

In practice, we often use regularized models in order to **prevent overfitting** or perform feature selection. More details will follow in the subsequent chapter.

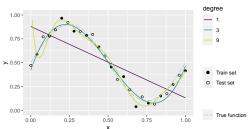
REGULARIZED LM – FUNCTIONALITY

SUPERVISED PARAMETRIC WHITE-BOX

General idea

- Linear model (LM) can overfit if we operate in high-dimensional space with not that many observations.
- When features are highly correlated, the OLS estimate becomes highly sensitive to random errors in the observed response, producing a large variance in the fit.
- We can find a compromise between generalizing the model (simple model.) underfitted) and corresponding closely to the data (complex model, overfitted).

Hypothesis space $\mathcal{H} = \{f : \mathcal{X} \to \mathbb{R} \mid f(\mathbf{x}) = \phi(\boldsymbol{\theta}^{\top}\mathbf{x})\}, \text{ where } \phi(\cdot) : \mathbb{R} \to \Phi \subseteq \mathbb{R}$ is some transformation function.



REGULARIZED LM – FUNCTIONALITY

Empirical risk

- Therefore, we minimize the empirical risk function $\mathcal{R}_{emp}(\theta)$ plus a complexity penalty $J(\theta)$, controlled by shrinkage parameter λ : $\mathcal{R}_{reg}(\theta) = \mathcal{R}_{emp}(\theta) + \lambda \cdot J(\theta).$
- Ridge regression uses the L2-penalty as a complexity measure with $J(\theta) = \|\theta\|_2^2$.
- Alternativly, LASSO (least absolute shrinkage and selection operator) uses the L1-penalty ($J(\theta) = \|\theta\|_1$).
- While both regularization methods shrink the coefficients of the model; LASSO also performs feature selection.
- Elastic net uses a convex combination of Ridge and LASSO with $J(\theta) = \lambda_1 \|\theta\|_1 + \lambda_2 \|\theta\|_2^2$.

Optimization

- for **Ridge** regression: analytically with $\hat{\theta}_{\text{Ridge}} = (\mathbf{X}^T \mathbf{X} + \lambda \mathbf{I})^{-1} \mathbf{X}^T \mathbf{y}$
- for LASSO regression: e. g. (sub)-gradient descent

Hyperparameters Shrinkage parameter λ [or λ_1 and λ_2 for elastic net]

REGULARIZED LM – PRO'S & CON'S

ALMOST SAME LIKE LINEAR MODEL? -> does this make sense?

Advantages

- Disadvantages + simple and fast implementation;
- + intuitive interpretability: mean influence of features on the output and feature importance

cheap computational costs

- + fits linearly separable data sets very well
- + works well independent of data size
- + basis for many machine learning algorithms
- + less tendency to overfit

- not suitable for data based on a non-linear data generating process → strong simplification of real-world problems
- strong assumptions: data is independent (multi-collinearity must be removed and normal distributed residuals??
- sensitive to outliers and noisy data

Simple method with good interpretability for linear problems, but strong assumptions and simplification of real-world problems.

REGULARIZED LM – PRACTICAL HINTS

Choice of regularization parameter λ

Choose λ with e. g. the smallest sum of squared residuals through cross-validation. In the R package glmnet lambda.min is the value of λ that gives minimum mean cross-validated error.

Ridge vs. LASSO

- ullet neither is overall better o elastic net as a compromise
- Ridge works better, if there are many influential and high correlated features.
- In contrast, LASSO is more suitable if the underlying structure is sparse (only a few features influence the output y).
- LASSO can set coefficients to zero, thus performing variable selection.

Implementation

- R: mlr3 learners LearnerClassifGlmnet / LearnerRegrGlmnet, calling glmnet::glmnet()
- Python: LinearRegression from package sklearn.linear_model, package for advanced statistical parameters statsmodels.api

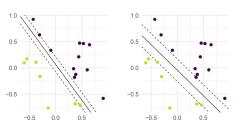
LINEAR SVM - FUNCTIONALITY

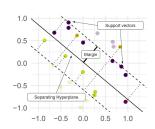
SUPERVISED PARAMETRIC BLACK-BOX

General idea

- The support vector machine (SVM) algorithm finds a decision boundary (separating hyperplane) that maximizes the distance (margin γ) to the closest members (support vectors, SV) of the separate classes. (hard margin)
- \bullet In a **soft-margin SVM** also "violations" of the margin are allowed \rightarrow not only the margin should be maximized, but also the margin violations minimized.
- There are three different types of training points: **Non-SVs** which do not influence the hyperplane, SVs which are exactly on the hyperplane, and margin violators.

Hypothesis space
$$\mathcal{H} = \{f : \mathcal{X} \to \mathbb{R} \mid f(\mathbf{x}) = \boldsymbol{\theta}^{\top} \mathbf{x} + \theta_0 \}$$





LINEAR SVM - FUNCTIONALITY

Empirical risk

Soft-margin SVMs can also be interpreted as **L2-regularized ERM**:

$$\frac{1}{2}\|\boldsymbol{\theta}\|^{2} + C\sum_{i=1}^{n} L\left(y^{(i)}, f\left(\mathbf{x}^{(i)}\right)\right); \ L(y, f) = \max(1 - yf, 0),$$

with L(y, f) as the hinge loss, $\|\theta\| = 1/\gamma$ and C as cost parameter for margin violation.

Optimization As the problem is **not differentiable**, there are the following solutions:

- Use a smoothed loss (squared hinge, huber), then do gradient descent. NB: Will not create a sparse SVM if we do not add extra tricks.
- 2 Use subgradient methods.
- O Do stochastic subgradient descent. Pegasos: Primal Estimated sub-GrAdient SOlver for SVM.

Hyperparameters C: penalization for missclassified data points

LINEAR SVM - PRO'S & CON'S

Advantages

- + high accuaracy
- + often **sparse** solution
- robust against overfitting (regularized); especially in high-dimensional space
- stable solutions, as the non-SV do not influence the separating hyperplane

Disadvantages

- costly implementation; long training times
- does not scale well to larger data sets ??
- only linear separation → possible with non-linear SVMs which are explained in the following slides.
- poor interpretability

Very accurate solution for high-dimensional data that is linearly separable

LINEAR SVM - PRACTICAL HINTS

Preprocessing

Features must be rescaled before applying SVMs.

Tuning

The cost parameter C must be tuned, as it has a strong influence on the resulting separating hyperplane.

Implementation

- R: mlr3 learners LearnerClassifSVM / LearnerRegrSVM, calling svm() from libsvm
- Python: sklearn.svm.SVC from package scikit-learn / package libSVM

NON-LINEAR SVM – FUNCTIONALITY

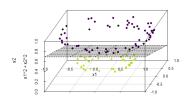
SUPERVISED NON PARAMETRIC BLACK-BOX

General idea

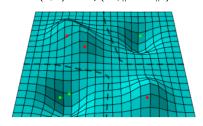
- Non-linear SVMs construct a separating hyperplane in a higher dimensional dimension
- Kernels = feature map + inner product $k(\mathbf{x}, \tilde{\mathbf{x}})$ transform the input space into a higher dimensional space and calculates the inner product.

Hypothesis space
$$\mathcal{H} = \{f(\mathbf{x}) = \sum_{i=1}^{n} \alpha_i \mathbf{y}^{(i)} \mathbf{k}(\mathbf{x}^{(i)}, \mathbf{x}) + \theta_0 | \theta_0, \alpha_i \in \mathbb{R} \}$$

$$\phi(x_1,x_2)=(x_1,x_2,x_1^2+x_2^2)$$



$$k(\mathbf{x}, \tilde{\mathbf{x}}) = \exp(-\gamma ||\mathbf{x} - \tilde{\mathbf{x}}||^2)$$



NON-LINEAR SVM - FUNCTIONALITY

Empirical risk

SVMs can also be interpreted as **L2-regularized ERM**:

$$\frac{1}{2}\|\boldsymbol{\theta}\|^{2} + C\sum_{i=1}^{n} L\left(y^{(i)}, f\left(\mathbf{x}^{(i)}\right)\right); L(y, f) = \max(1 - yf, 0),$$

with L(y, f) as the hinge loss, $\|\theta\| = 1/\gamma$ and C as cost parameter for margin violation.

Optimization As the problem is **not differentiable**, there are the following solutions:

- Use a smoothed loss (squared hinge, huber), then do gradient descent. NB: Will not create a sparse SVM if we do not add extra tricks.
- Use subgradient methods.
- Do stochastic subgradient descent. Pegasos: Primal Estimated sub-GrAdient SOlver for SVM.

Hyperparameters

- C: penalization for missclassified data points
- Kernel parameters: depending on which kernel is used (e. g. degree of the polynomial kernel or width of RBF kernel)

NON-LINEAR SVM – PRO'S & CON'S

Advantages

- + high accuaracy
- can learn non-linear decision boundaries
- + often sparse solution
- robust against overfitting (regularized); especially in high-dimensional space
- stable solutions, as the non-SV do not influence the separating hyperplane

Disadvantages

- costly implementation; long training times
- does not scale well to larger data sets ??
- only linear separation → possible with non-linear SVMs which are explained in the following slides.
- poor interpretability
- not easy tunable as it is highly important to choose the right kernel

Non-linear SVMs perform very well for non-linear separable data, but are hard to interpret and need a lot of tuning.

NON-LINEAR SVM – PRACTICAL HINTS

Kernels

Mainly, these three types of kernels are used:

- Linear kernel: the dot product of the given observations
- Polynomial kernel: curved lines in the input space
- Radial basis function (RBF) kernel: complex regions in the input space (e. g. spirals)

Tuning

- SVMs are sensitive to its hyperparameters and should always be tuned.
- For the RBF kernel the RBF sigma heuristic is used.

Implementation

- R: mlr3 learners LearnerClassifSVM / LearnerRegrSVM, calling e1071::svm()
- Python: sklearn.svm.SVC from package scikit-learn / package libSVM

K-NEAREST NEIGHBORS – FUNCTIONALITY

SUPERVISED NON-PARAMETRIC WHITE-BOX

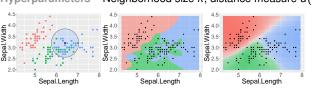
General idea

- The *k*-nearest neighbors (*k*-NN) model is based on inter-observational distances, thus heavily depending on the chosen distance measure.
- It builds upon the rationale that closeness in feature space infers closeness in target space.
- The prediction for $\mathbf{x}^{(i)}$ is the (weighted) **mean target** (regression) or **most frequent class** (classification) within its k-neighborhood $N_k(\mathbf{x}^{(i)})$ (i.e., the kpoints closest to $\mathbf{x}^{(i)}$ in feature space). only true for specific loss functions???

Hypothesis space $\mathcal{H} = \{f(\mathbf{x}) : f(\mathbf{x}) \text{ is a step function over a tesselation of } \mathcal{X}\}$

Empirical risk Any loss function applicable to regression/classification Optimization Not necessary

Hyperparameters Neighborhood size k, distance measure $d(\cdot)$



Left: Neighborhood for exemplary observation in iris. k = 50Right: Prediction surfaces for k = 1 and k = 50

K-NEAREST NEIGHBORS – PRO'S & CON'S

Advantages

- + **Easy** to explain and implement
- Few assumptions therefore (in theory) able to model data situations of arbitrary complexity
- + No training period
- Constant evolvement with new data
- Ability to learn non-linear decision boundaries
- + No optimization required
- + Only one hyperparameter to tune

Disadvantages

- Sensitivity toward noisy or irrelevant features
- Bad performance when feature scales are not consistent with importance
- Sensitivity toward outliers
- No handling of missing data
- Bad with data imbalances
- Large memory consumption for distance computation

Easy and intuitive for small, well-behaved data sets, but not suitable for large-scale data with problematic features

K-NEAREST NEIGHBORS – PRACTICAL HINTS

Non-parametric behavior

- k-NN is a lazy classifier its parameters are the training data; there is no real compression of information.
- The number of parameters is thus equal to the number of observations (which have to be stored for prediction!).

Distance measures

- For numerical features, typically Manhattan or Euclidean distance; in presence of categorical features, Gower distance might be considered.
- Can be weighted to account for different scales or importance of features.

Neighborhoods in higher dimensions

k-NN is vulnerable to the **curse of dimensionality**: in very high-dimensional feature spaces, finding meaningful neighborhoods may become difficult.

Implementation

- R: mlr3 learners LearnerClassifKKNN / LearnerRegrKKNN, calling kknn::kknn()
- Python: KNeighborsClassifier / KNeighborsRegressor from package scikit-learn

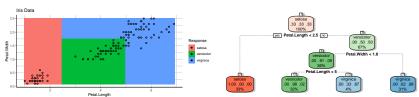
CART – FUNCTIONALITY

SUPERVISED NON-PARAMETRIC WHITE-BOX FEATURE SELECTION

General idea

- Starting from a root node, classification & regression trees (CART) perform repeated binary splits of the data according to feature values, thereby subsequently dividing the input space \mathcal{X} into T rectangular partitions Q_t .
- Observations are passed along until each ends up in exactly one leaf node (unless stopped early or pruned).
- In each step, CART find the optimal feature-threshold combination to split by.
- Leaf node t is assigned response c_t.

Hypothesis space
$$\mathcal{H} = \{ f(\mathbf{x}) : f(\mathbf{x}) = \sum_{t=1}^{T} c_t \mathbb{I}(\mathbf{x} \in Q_t) \}$$



Prediction surface for iris data, 3 splits

Corresponding classification tree

CART – FUNCTIONALITY

Empirical risk

- Empirical risk is calculated for each potential terminal node \mathcal{N}_t of a split.
- In general, trees can handle any type of loss function. Typical choices are:
 - Classification (for g classes):

• Using Brier score
$$\mathcal{R}(\mathcal{N}_t) = \sum_{(\mathbf{x}, y) \in \mathcal{N}_t} \sum_{k=1}^g (\mathbb{I}(y = k) - \pi_k(\mathbf{x}))^2$$

• Using Bernoulli loss
$$\mathcal{R}(\mathcal{N}_t) = \sum_{(\mathbf{x},y) \in \mathcal{N}_t} \sum_{k=1}^g \mathbb{I}(y=k) \cdot \log(\pi_k(\mathbf{x}))$$

$$ullet$$
 Regression: Using **quadratic loss** $\mathcal{R}(\mathcal{N}_t) = \sum\limits_{(\mathbf{x},y) \in \mathcal{N}_t} (y-c_t)^2$

Optimization

Exhaustive search over all (randomly selected) split candidates in each node to minimize empirical risk in the child nodes (greedy optimization)

Hyperparameters Complexity, i.e., number of leaves T

CART - PRO'S & CON'S

Advantages

- Easy to understand, interpret & visualize
- Automatic handling of non-numerical features
- + Built-in feature selection
- + Automatic handling of missings
- Interaction effects between features easily possible, even of higher orders
- Fast computation and good scalability
- High flexibility (custom split criteria or leaf-node prediction rules)

Disadvantages

- Rather low accuracy (at least, without bagging or boosting)
- High variance/instability: strong dependence on training data
- Therefore, poor generalization & risk of overfitting
- Several steps required for modeling linear relationships
- In presence of categorical features, bias towards features with many categories

Simple and good with feature selection, but not the best predictor

CART – PRACTICAL HINTS

Pruning / early stopping

Unless interrupted, splitting will go on until each leaf node contains a single observation (expensive + overfitting!)

→ Use pruning and stopping criteria to limit complexity

Implementation

- R: mlr3 learners LearnerClassifRpart / LearnerRegrRpart, calling rpart::rpart()
- Python: DecisionTreeClassifier / DecisionTreeRegressor from package scikit-learn
- Complexity controlled via tree depth, minimum number of observations per node, maximum number of leaves, minimum risk reduction per split, ...

Bagging / boosting

Since CART are instable predictors on their own, they are typically ensembled to form a random forest (bagging) or used in combination with boosting.

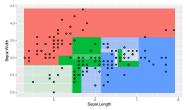
RANDOM FOREST – FUNCTIONALITY

SUPERVISED NON-PARAMETRIC BLACK-BOX FEATURE SELECTION

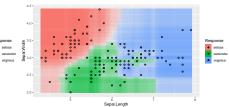
General idea

- Random forests (RF) perform bagging: they combine M trees (base learners) to form a strong ensemble learner.
- They use complex trees with low bias and compensate for the resulting variance by aggregating them in a **decorrelated** manner.
- Each tree is trained on a bootstrap sample of the data and only on a random subset of features to incur variability.
- Aggregation via averaging (regression) or majority voting (classification).

Hypothesis space
$$\mathcal{H} = \left\{ f(\mathbf{x}) : f(\mathbf{x}) = \frac{1}{M} \sum_{m=1}^{M} \sum_{t=1}^{T[m]} c_t^{[m]} \mathbb{I}(\mathbf{x} \in Q_t^{[m]}) \right\}$$



Prediction surface for iris data with a single tree



Prediction surface for iris data with 500-tree ensemble

RANDOM FOREST – FUNCTIONALITY

Empirical risk

- Applicable with any kind of loss function (just like tree base learners)
- Computation of empirical risk for all potential child nodes in all trees

Optimization

Exhaustive search over all (randomly selected) split candidates in each node of each tree to minimize empirical risk in the child nodes (greedy optimization)

Hyperparameters

- Ensemble size, i.e., number of trees
- Complexity, i.e., number of leaves T of each base learner
- Number of split candidates, i.e., number of features to be considered as splitting variables at each split
 - ightarrow Frequently used heuristics: $\lfloor \sqrt{p}
 floor$ for classification and $\lfloor p/3
 floor$ for regression

RANDOM FOREST – PRO'S & CON'S

Advantages

- Translation of most of base learners' advantages (e.g., inherent variable selection, handling of missing data)
- + Fairly good at **prediction**: improved accuracy through bagging
- Inherent computation of feature importance
- + Quite stable wrt changes in the data
- Good with high-dimensional data, even in presence of noisy covariates
- + Applicable to unbalanced data
- Easy to parallelize
- Rather easy to tune

Disadvantages

- Translation of some of base learners' disadvantages (e.g., trouble to model linear relations, bias towards features with many categories)
- Loss of single trees' interpretability
 black-box method
- Hard to visualize
- Often suboptimal for regression
- Often still inferior in performance to other methods (e.g., boosting)

Fairly good predictor, but black-box method

RANDOM FOREST – PRACTICAL HINTS

Pre-processing

Inherent feature selection of random forests, but high **computational costs** for large number of features

→ Upstream feature selection (e.g., via PCA) might be advisable

Implementation

- R: mlr3 learners LearnerClassifRanger / LearnerRanger, calling ranger::ranger()
- Python: RandomForestClassifier / RandomForestRegressor from package scikit-learn

Tuning

- Overall limited tunability
- Number of split candidates often more impactful than number of trees

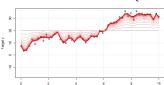
GRADIENT BOOSTING – FUNCTIONALITY

SUPERVISED NON-PARAMETRIC BLACK-BOX FEATURE SELECTION

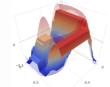
General idea

- Gradient boosting (GB) is an ensemble method that constructs a strong learner from weak base learners (frequently, CART).
- As opposed to bagging, however, base learners are assembled in a sequential, stage-wise manner: in each iteration, GB improves the current model by adding a new component that minimizes empirical risk.
- Each base learner is fitted to the current point-wise residuals \rightarrow One boosting iteration $\widehat{=}$ one approximate gradient step in function space
- The final model is a weighted sum of base learners $b(\mathbf{x}, \theta^{[m]})$ with weights $\beta^{[m]}$.

Hypothesis space
$$\mathcal{H} = \left\{ f(\mathbf{x}) : f(\mathbf{x}) = \sum_{m=1}^{M} \beta^{[m]} b(\mathbf{x}, \theta^{[m]}) \right\}$$



Gradient boosting for univariate function WHICH FIGURE



Gradient boosting for bivariate function WHICH FIGURE

GRADIENT BOOSTING – FUNCTIONALITY

Empirical risk

- Outer loss: Loss used to compute pseudo-residuals how large is the error of the current model fit?
 - → Arbitrary differentiable loss function
- Inner loss: Loss used to fit next base learner component to current pseudo-residuals
 - → Typically, **quadratic loss** (desirable optimization properties)

Optimization Functional gradient descent for outer optimization loop, procedure for inner one depending on inner loss

Hyperparameters

- Ensemble size, i.e., number of base learners
- Learning rate, i.e., impact of single base learner
- Complexity of base learners (depending on type used)

GRADIENT BOOSTING – PRO'S & CON'S

Advantages

- Powerful off-the-shelf method for supercharging weak learners' performance
- Translation of most of base learners' advantages (e.g., for tree boosting: inherent variable selection, handling of missing data)
- High predictive accuracy that is hard to outperform
- High flexibility (custom loss functions, many tuning options)
- + Applicable to unbalanced data

Disadvantages

- Hardly interpretable black-box method
- Hard to visualize
- Prone to overfitting
- Sensitive to outliers
- Hard to tune (high sensitivity to variations in hyperparameter values)
- Rather slow in training
- Hard to parallelize

High-performing predictor, but rather delicate to handle

GRADIENT BOOSTING – PRACTICAL HINTS

XGBoost (extreme gradient boosting)

Fast, efficient implementation of gradient-boosted decision trees that has become **state of the art** for many machine learning problems

ightarrow Clever modeling techniques + computational speed

Stochastic gradient boosting (SGB)

Faster, approximate version of GB that performs each iteration only on **random subset** of the data

Implementation

- R: mlr3 learners LearnerClassifXgboost / LearnerXgboost, calling xgboost::xgb.train()
- Python: GradientBoostingClassifier / GradientBoostingRegressor from package scikit-learn, XGBClassifier / XGBRegressor from package xgboost

Tuning

- Overall limited tunability
- Number of split candidates often more impactful than number of trees

NEURAL NETWORK – FUNCTIONALITY

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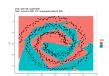
General idea

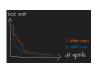
- A **neural network (NN)** is a model architecture loosely inspired by the human brain. It consists of various neurons, organized in layers assembled through weighted functional connections.
- Batches of data enter in the input layer and sequentially pass through h hidden **layers**, each of which performs a linear transformation $\phi^{(j)}$ and a non-linear **activation** $\sigma^{(j)}$, thus creating intermediary representations of the data.
- The **output layer** yields predictions after a final transformation ϕ and scaling τ .
- The resulting loss is used to update the weights for the next **epoch**.

Hypothesis space

$$\mathcal{H} = \left\{ f(\mathbf{x}) : f(\mathbf{x}) = \tau \circ \phi \circ \sigma^{(h)} \circ \phi^{(h)} \circ \sigma^{(h-1)} \circ \phi^{(h-1)} \circ \dots \circ \sigma^{(1)} \circ \phi^{(1)}(\mathbf{x}) \right\}$$







Classification of spirals data with X & Y layers/epochs

Classification error vs layers/epochs

NEURAL NETWORK – FUNCTIONALITY

Empirical risk Any differentiable loss function

Optimization

NNs are optimized by **backpropagation** which consists of two steps:

- Forward pass: Predict result with current weights and compute empirical risk according to chosen loss function.
- Backward pass: Calculate error contribution of each weight by means of gradient descent – which essentially means applying the chain rule to the composition of functions applied in each layer – and update weights accordingly.

Hyperparameters

- Number of hidden layers (depth), number of neurons per layer
- Activation function(s)
- Learning rate for backpropagation
- Number of iterations (epochs), batch size
- Initial weights
- •

NEURAL NETWORK – PRO'S & CON'S

Advantages

- Able to solve complex, non-linear regression or classification problems
- Therefore, typically very good performance
- Built-in feature extraction obtained by intermediary representations
- Suitable for unstructured data (e. g. image, audio, text data)
- Easy handling of high-dimensional or missing data
- Parallelizable structure

Disadvantages

- Computationally expensive
 → slow to train and forecast
- Large amounts of data required
- Faster-than-linear scaling of weight matrices with increased network size
- Network architecture requiring much expertise in tuning
- Black-box model hard to interpret or explain
- Tendency towards overfitting

Able to learn extremely complex functions, but computationally expensive and hard to get right

NEURAL NETWORK – PRACTICAL HINTS

Types of neural networks (RNNs, CNNs)

- Recurrent neural networks (RNNs: Deep NN that make use of sequential information with temporal dependencies
 - → Frequently applied to natural language processing
- Convolutional neural networks (CNNs): Regularized version of the fully connected feed-forward NN (where each neuron is connected to all neurons of the subsequent layer) that abstracts inputs to feature maps via convolution
 - \rightarrow Frequently applied to image recognition

Problem of neural architecture search (NAS)

NN are **not off-the-shelf** methods – the network architecture needs to be tailored to each problem anew

ightarrow Automated machine learning attempts to learn architectures

Implementation

• R: package neuralnet

• Python: libraries PyTorch, keras