EE 382V: Social Computing **Fall 2018**

Lecture 7, Session 1: November 9

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7.1 Voting

Voting is the aggregation of beliefs / rankings. Examples include gathering Concensus, or Search Engines.

If we consider a group of people on an island, and they all vote given two choices, it's easy to determine what is preferred by the majority. But, when you add a third choice, it makes things complicated. In particular if Alice gives the following preference for choices A, B, and C.

$$\text{We have a Problem!} \begin{cases} A > B \\ B > C \\ C > A \end{cases}$$

Definition 7.1 (Plurality) *When chosing order, we want the following properties to produce a ranking.*

1. *Transitivity* - $\forall x, y, z : (x > y) \wedge (y > z) \implies (x > z)$
2. *Completeness* - $\forall x, y : (x > y) \vee (y > x)$

7.1.1 Sample Voting Ballot with Three Choices

Each voter is represented by V_i, and the columns represent their preferences with their first choice on top, second below their first etc.

V_1	V_2	V_3	Final Ranking
A	A	A	A
B	B	C	B
C	C	B	C

Table 7.1: Simple Case

V_1	V_2	V_3	
A	B	C	(A > B) : 2
B	C	A	(B > C) : 2
C	A	B	(C > A) : 2

Table 7.2: Problematic Case

This lead to the discovery of **Condorcet's Paradox (1700's)**.

College	National Rank	Class size	Money Offered
X	4	40	3K
Y	8	18	1K
Z	12	24	8K

Table 7.3: Best College Example

Voting in context of plurality is related to comparing 2 choices. In Table 7.3, how do you decide which college is the best?

7.2 Voting Systems

7.2.1 Axioms

1. **Unanimity** (U)
   If all voters prefer A over B, then in the final ranking, A must be placed over B.

2. **Independence of Irrelevant Alternative** (IIA)
   For each pair of alternatives x, y the group ranking should depend only on how each individual voter ranks x and y reltative to each other.

3. **Non-Dictatorial** (ND)
   Voting system is not a Dictatorial system.

Critic	Unanimous Vote	IIA Vote
C_1	CK > GF	CK > GF > PF
C_2	CK > GF	CK > GF > PF
C_3	CK > GF	CK > GF > PF
C_4	GF > CK	GF > PF > CK
C_5	GF > CK	GF > PF > CK

Table 7.4: Movie Preferences for Unanimous and IAA Votes

In the first vote, (unanimous vote), CK is the obvious winner, 3 to 2. In the second case, it is not so obvious. If first choice gets 2 points, second 1 point, and third 0 points, the totals are as follows:

Movie	Vote Totals
CK	6
GF	7
PF	2

Table 7.5: Vote totals for IAA Example

Once we add a third option, now the GF becomes the winner. Furthermore, PF is also now an Irrelevant Alternative.

7.2.2 Systems

1. **Majority Rule** [U ✓] [IIA ✗] [ND ✓]
   A system where the option with the most votes or highest ranking is adopted. This system can lead to the Condorcet Paradox

2. **Positional Voting** [U ✓] [IIA ✗] [ND ✓]
   This system uses Bordat Counting, to assign a number to each choice and the choice with the largest sum wins. This does not solve the Condorcet Paradox, if selections are symmetric.

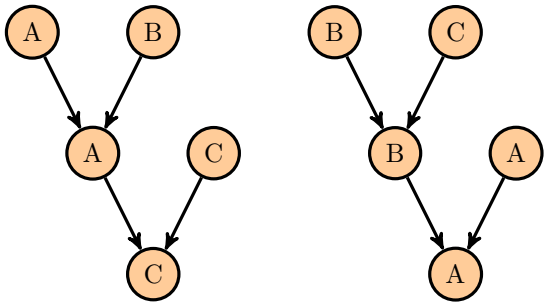
V_1	V_2	V_3	Option Totals
A (2)	A (2)	C (2)	A : 5 (winner)
B (1)	C (1)	A (1)	B : 1
C (0)	B (0)	B (0)	C : 3

Table 7.6: Sample Positional Voting with Assigned Values for each choice

V_1	V_2	V_3
A	B	C
B	C	A
C	A	B

Table 7.7: Symmetric Vote Choices

As you reconsider Table 7.7, if we instead of summing point values (Bordat Counting), only compare two choices at a time, we create a new problem. A problem of order matters, see below:



In the depiction above, which is comparing two elements at a time, and then comparing the winner to the next element, demonstrates the problem. Outcomes will vary simply based on what order you perform your comparison.

Definition 7.2 (Social Choice) *The top most choice.*

Definition 7.3 (Social Welfare) *The complete ranking of all choices.*

Note: Each definition above can be deduced from the other...

3. **Dictatorial** [U ✓] [IIA ✓] [ND ✗]
   All voters agree with the dictator.

7.3 Arrow's Theorem

Theorem 7.4 (Arrow's Theorem) *There is no voting system that satisfies Unanimity, Independence of Irrelevant Alternatives, and Non-Dictatorial. - Kenneth Arrow (1950s)*

Definition 7.5 (Quorum) *S is a quorum if all the members unanimously prefer A over B and all members not in S prefer B over A, then the final ranking places A over B.*

Lemma 7.6 *If S is a quorum and T is a quorum, then $S \cap T$ is a a quorum.*

Given options $A, B,$ and C

S: All voters prefer A to B and All voters in S' prefer B to A.

T: All voters prefer B to C and all voters in T' prefer C to B.

$$A > B$$

$$B > C$$

By transitivity, $A > C$ in final ranking.

Now focus on $S \cap T$:

$$(S \cap T)^c = S^c \cap T^c$$

$$v = w, x, y, z$$

$$s_w = x, y, z$$

$$s_x = w, y, z$$

$$s_y = w, x, z$$

$$s_z = w, x, y$$

The intersection of all of these sets is the null set.

$\therefore$ All the axioms can not be met by any voting system.