

6.8 Problem Overview

The problem is a coordination game with the payoff matrix

1,1	0,0
0,0	4,4

6.8.1 Part a

Q: Find all pure-strategy Nash equilibria for this game.

A: (U, L) and (D, R) are both NE.

6.8.2 Part b

Q: Find the mix-strategy NE, and give an explanation.

A:

A plays U with prob p

B plays L with prob q

Expected payoff for A to play U: $q \cdot 1 + (1-q) \cdot 0 = q$

Expected payoff for A to play D: $q \cdot 0 + (1-q) \cdot 4 = 4 - 4q$

Indifference for B:

$$q = 4 - 4q, 5q = 4, q = 4/5.$$

Expected payoff for B to play L:

$$p \cdot 1 + (1-p) \cdot 0 = p$$

Expected payoff for B to play R:

$$p \cdot 0 + (1-p) \cdot 4 = 4 - 4p$$

Indifference for A:

$$p = 4 - 4p, 5p = 4, p = 4/5.$$

Mixed strategy NE: $((4/5, 1/5), (4/5, 1/5))$ When player A expect player B to play L with the probability of $4/5$, then A will try to match the probability as well.

6.8.3 Part c

Q: With Schelling's focal point idea, what equilibrium do you think is the best prediction of how the game will be played?

A: When a Schelling focal point can be expected, player A would expect player B to play R, since it has the better payoff of the two NE. Likewise for player B to expect A to play D. Thus (D, R) would be the converged focal point.

6.8 Problem Overview

Find all Nash equilibria given the payoff matrices:

6.8.1 Part a

8,4	5,5
3,3	4,8

Pure strategy NE:

(U,R)

Mixed strategy NE:

A plays U with prob p

B plays L with prob q

Expected payoff for A to play U:

$$q \cdot 8 + (1-q) \cdot 5 = 5 + 3q$$

Expected payoff for A to play D:

$$q \cdot 3 + (1-q) \cdot 4 = 4 - q$$

Indifference for B:

$$5 + 3q = 4 - q, 4q = -1, q = -1/4. \text{ i.e. B always plays R}$$

Expected payoff for B to play L:

$$p \cdot 4 + (1-p) \cdot 3 = 1 + p$$

Expected payoff for B to play R:

$$p \cdot 5 + (1-p) \cdot 8 = 8 - 3p$$

Indifference for A:

$$1 + p = 8 - 3p, 4p = 7, p = 7/4. \text{ i.e. A always plays U}$$

This just shows that if a pure strategy NE exists, the mixed strategy NE would be the same (i.e. U,R in this case)

6.8.2 Part b

0,0	-1,1
-1,1	2,-2

Pure strategy NE:

Does not exist

Mixed strategy NE:

A plays U with prob p

B plays L with prob q

Expected payoff for A to play U:

$$q \cdot 0 + (1-q) \cdot (-1) = -1 + q$$

Expected payoff for A to play D:

$$q \cdot (-1) + (1-q) \cdot 2 = -q + 2 - 2q = 2 - 3q$$

Indifference for B:

$$-1 + q = 2 - 3q, 4q = 3, q = 3/4.$$

Expected payoff for B to play L:

$$p \cdot 0 + (1-p) \cdot 1 = 1 - p$$

Expected payoff for B to play R:

$$p \cdot 1 + (1-p) \cdot (-2) = -2 + 3p$$

Indifference for A:

$$1 - p = -2 + 3p, 3 = 4p, p = 3/4.$$

$$((3/4, 1/4), (3/4, 1/4))$$