EE 382V: Social Computing	Fall 2018

Lecture 17: October 26

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17.1 Games with Multiple Nash Equilibria

There are certain games in which more than one Nash Equilibrium exists. The following figure depicts a game in which you and a partner are preparing slides for a presentation. Each of you can choose to prepare the slides with either Microsoft PowerPoint or Apple Keynote.

	ppt	keynote
ppt	1,1	0,0
keynote	0,0	1,1

Table 17.1: Combining Presentation Slides

If you choose a different presentation tool than your partner, you will both get a payoff of 0, as you will be unable to combine your slides. Your best response is to choose the same tool as your partner, regardless of the tool chosen. For this reason, (ppt, ppt) and (keynote, keynote) are both Nash Equilibrium.

17.1.1 Battle Of The Sexes

Another game with multiple Nash Equilibria is the Battle of the Sexes, where you and your significant other go to see a movie together and must choose between Action and Romance.

	romance	action
romance	2,1	0,0
action	0,0	1,2

Table 17.2: Battle of the Sexes

In this game, both (romance, romance) and (action, action) are Nash Equilibrium, but one player will receive a higher payoff.

17.2 Schelling's Focal Point

For games in which there are multiple Nash Equilibria, there may be one NE that is more likely to be chosen because of information available outside of the context of the game. For example, in the game combining presentation slides, (ppt, ppt) may be more likely due to PowerPoint being more popular than Keynote. We call this point the **Schelling Focal Point** and define it as the point most likely to be chosen in the absence of communication.

17.3 Games with No Nash Equilibrium

Consider the following game, in which you and a friend flip two pennies. If the pennies match, you receive a payoff of 1 and your friend receives a payoff of -1. Otherwise, you receive a payoff of -1 and your friend receives a payoff of 1.

	heads	tails
heads	1,-1	-1,1
tails	-1,1	1,-1

Table 17.3: Matching Pennies

This game is also called a **Zero-Sum Game** as the overall payoff is always zero. It should be clear that there is no Nash Equilibrium, as there is no pair of strategies that are best responses to each other.

17.3.1 Mixed Strategy for Matching Pennies

As we learned in the previous lecture, there exists at least one Nash Equilibrium in a finite game where each player is using a mixed strategy. To derive the NE for Matching Pennies, we define the following:

- Player2 chooses heads with probability q and tails with probability $1 - q$.
- Player1 chooses heads with probability p and tails with probability $1 - p$.

Payoffs for player1:

- Payoff for choosing heads:

$$(q)(1) + (1 - q)(-1) \to 2q - 1 \tag{17.1}$$

- Payoff for choosing tails:

$$(q)(-1) + (1 - q)(1) \to 1 - 2q \tag{17.2}$$

Player1 must be "indifferent" about his responses, so we solve:

$$1 - 2q = 2q - 1 \to q = \frac{1}{2} \tag{17.3}$$

In solving for player2's expected payoff w/ respect to p, we see that $p = 1/2$ from the symmetric principle. Therefore, the Nash Equilibrium for Matching Pennies is:

$$[(\frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2})] \tag{17.4}$$

In other words, player1 chooses heads with probability 0.5 and tails with probability 0.5, as does player2.

17.3.2 Football Plays

Consider the following game of football. An offense can choose to pass or run the ball, while a defense can choose to defend against the pass or the run. The following matrix defines the payoff for each strategy.

defense

offense

	defend pass	defend run
pass	0,0	10,-10
run	5,-5	0,0

Table 17.4: Football Plays

Our goal is to compute a Nash Equilibrium vector for each team's mixed strategy, and so we define the following:

- Offense passes with probability p
- Defense defends the pass with probability q

Expected payoff for offense when choosing to pass:

$$(q)(0) + (1-q)(10) \rightarrow 10 - 10q \tag{17.5}$$

Expected payoff for offense when choosing to run:

$$(q)(5) + (1-q)(-5) \rightarrow 5q \tag{17.6}$$

By the indifference principle:

$$10 - 10q = 5q \rightarrow q = 2/3 \tag{17.7}$$

By the symmetric principle, we could solve for the defense's expected payoff and get $p = \frac{1}{3}$. Therefore, the Nash Equilibrium vector for this game is:

$$[(pass = \frac{1}{3}, run = \frac{2}{3}), (defendPass = \frac{2}{3}, defendRun = \frac{1}{3})] \tag{17.8}$$