EE 382V: Social Computing **Fall 2018**

Lecture 18: October 27

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18.1 Pareto Dominant Strategies

In a multi-participant game, one strategy profile is *Pareto dominant* compared to another if it leaves at least one participant better off and no-one worse off.

18.1.1 Definition

Strategy profile $(s_1, s_2, ...s_n)$ *Pareto dominates* strategy profile $(s_1^{'}, s_2^{'}, ...s_n^{'})$ iff:

$\forall i \in N : U_i(s_1, s_2, ...s_n) \geq U_i(s_1^{'}, s_2^{'}, ...s_n^{'})$

and

$\exists j \in N : U_j(s_1, s_2, ...s_n) > U_j(s_1^{'}, s_2^{'}, ...s_n^{'})$

18.1.2 Example

In the game represented below, strategy pair (A_1, A_1) Pareto dominates (A_1, A_2), because both players receive a higher payoff. (A_2, A_1) Pareto dominates (A_2, A_2) because Player 2 receives the same payoff as in (A_2, A_2), but Player 1 receives a higher payoff. (A_1, A_2) Pareto dominates (A_2, A_2) because Player 1 receives the same payoff, but Player 2 receives a higher payoff. (A_2, A_2) is Pareto dominated by all other strategies.

x	A_1	A_2
A_1	3, 5	2, 4
A_2	7, 2	2, 2

18.2 Pareto Optimal Strategies

Strategy profile $(s_1, s_2, ...s_n)$ is *Pareto optimal* if there does not exist any strategy profile that *Pareto dominates* $(s_1, s_2, ...s_n)$.

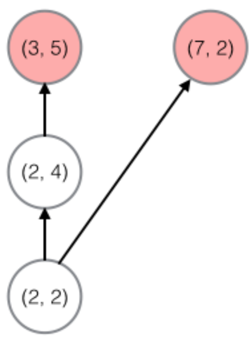
18.2.1 Example

In our same game, strategy pairs (A_1, A_1) and (A_2, A_1) are Pareto optimal because no other pair Pareto dominates them.

x	A_1	A_2
A_1	3, 5	2, 4
A_2	7, 2	2, 2

18.2.2 Poset example

This same game can be represented as a poset, where it is immediately evident that these strategy pairs represent the maximal elements in the poset. Note that representative payoffs (3, 5) and (7, 2) are incomparable, which allows this game to have multiple Pareto optimal strategy profiles.



18.3 Pareto Optimality, Nash Equilibrium and Strictly Dominant Strategies

Some interesting relationships to note for Pareto optimality are that strategy profiles that are Pareto optimal, may not be Nash Equilibrium or Strictly Dominant Strategies. In our example game of the Prisoner's Dilemma, you can see that the Nash Equilibrium and Strictly Dominant Strategy are for both suspects to confess, however, all strategy profiles *except* a double confession are considered Pareto optimal.

x	don't confess	confess
don't confess	-1, -1	-10, 0
confess	0, -10	-4, -4

18.4 Social Optimality

Strategy profile $(s_1, s_2, ...s_n)$ is *Socially optimal* if the sum of the payoffs in the strategy are maximized for the choice.

Lemma 18.1 *Social optimality implies Pareto optimality.*

x	A_1	A_2
A_1	$3 + 5 = 8$	$2 + 4 = 6$
A_2	$7 + 2 = 9$	$2 + 2 = 4$