EE 382C/361C: Multicore Computing **Fall 2016**

Lecture 1: November 16

Lecturer: Vijay Garg *Scribe: Guillermo León*

1.1 Chapter 6 Exercises

10. Given payoff matrix.

		Player B	
		L	R
Player A	U	3,3	1,2
	D	2,1	3,0

(a) All pure strategy Nash equilibria of this game.

Let's try first finding dominant strategies. In this game we can see that Player B has L as a strictly dominant strategy since $P_B(U,L) > P_B(U,R)$ and $P_B(D,L) > P_B(D,R)$. In other words, strategy L is always the strict best response to whichever strategy Player A chooses. Because this is the case, it is expected that Player B will go with L, therefore the best response from Player A to L is U because it maximizes her payoff. The only pure strategy Nash equilibrium is (U,L).

		Player B	
		L	R
Player A	U	3,3	1,2
	D	2,1	3,0

(b) Changing player A's payoff from (U,L) to not have pure-strategy Nash equilibrium.

No, as it was stated in the answer above, Player B is indifferent of the payoff of Player A. In the original case Player A does not have a dominant strategy, let's say that we modify $P_A(U,L) = 1$. In this case Player A now has a strictly dominant strategy D, however the best response from Player B is still to go with L. This only moves the Nash equilibrium from (U,L) to (D,L).

		Player B	
		L	R
Player A	U	**1**,3	1,2
	D	2,1	3,0

Finally let's suppose we change the payoff $P_A(D,L) = 3$, in this case we now have 2 Nash equilibria because U and D are both best responses to L from player B.

		Player B	
		L	R
Player A	U	3,3	1,2
	D	**3**,1	3,0

(c) Changing player B's payoff from (U, L) to not have pure-strategy Nash equilibrium.

Yes, since Player B has a strictly dominant strategy it is just a matter of removing it. If we set the payoff for player B for the pair of strategies U and L to 1 or 0. In this case neither of the players have a dominant strategy. Let's analyze then the best responses for each player and each strategy. Player A chooses U, then best response from player B is R, but if player B chooses R then best response from player A is D. Finally if player A chooses D, then the best response from B is L and when player B chooses L, the best response from player A is U, which is the initial strategy. Therefore there is no pure strategy Nash equilibrium.

		Player B	
		L	R
Player A	U	3,**1**	1,2
	D	2,1	3,0

11. Explain why the strategies used in an equilibrium of this game will not be dominated strategies.

Let's recall that strategies in a pure strategy Nash equilibrium are the best responses to each other from both players. More precisely, let's suppose that Player 1 chooses strategy S and Player 2 chooses T, then we say that (S, T) are in a Nash equilibrium if S is a best response to T, and T is a best response to S. Because they are considered best strategies, that means that they produce a payoff that is at least as good as any other strategy. In other words, let's suppose that strategy S is indeed a dominated strategy, this means that there is a strategy $S\prime$ with a greater payoff, therefore $S\prime$ is a best response to a given strategy T and because of this S cannot be part of the Nash equilibrium.