

Chapter 10: Question 13

*Lecturer: Vijay Garg**Scribe: Beth Richardson***10.1 Question**

Suppose you want to design an auction for the following type of situation: you have two identical copies of a valuable object, and there are four potential buyers for the object. Each potential buyer i wants at most one copy, and has a value v_i for either copy.

You decide to design the auction by analogy with the way in which we derived the single-item ascending-bid (English) auction from the general procedure for matching markets. In the present case, as there, you want to create a bipartite graph that encodes the situation, and then see what prices the bipartite graph auction procedure comes up with.

- (a) Describe how this construction would work using an example with four potential buyers. In creating your example, first choose specific valuations for the potential buyers, and then show how the auction proceeds and what the market-clearing prices are.
- (b) In the case of the single-item auction, the bipartite graph procedure yielded the simple rule from the ascending-bid (English) auction: sell to the highest bidder at the second-highest price. Describe in comparably simple terms what the rule is for the current case of two identical items (i.e. your description should not involve the terms bipartite, graph, or matching)

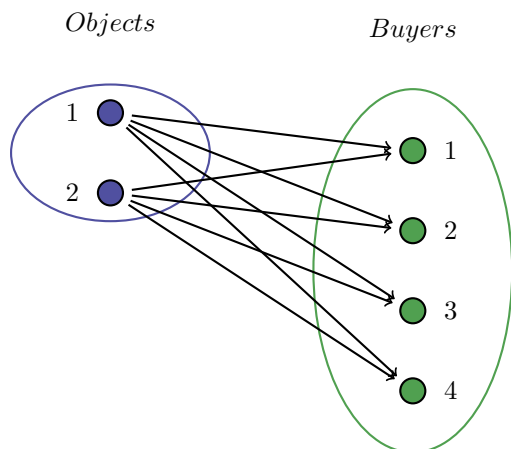
10.2 Answer - a

We have 2 copies of a valuable object and 4 potential buyers. The following buyers value the object at these valuations:

Buyer1	Buyer2	Buyer3	Buyer4
1	2	3	4

10.2.1 Auction Begins

At the beginning of the auction we would have the following bipartite graph:

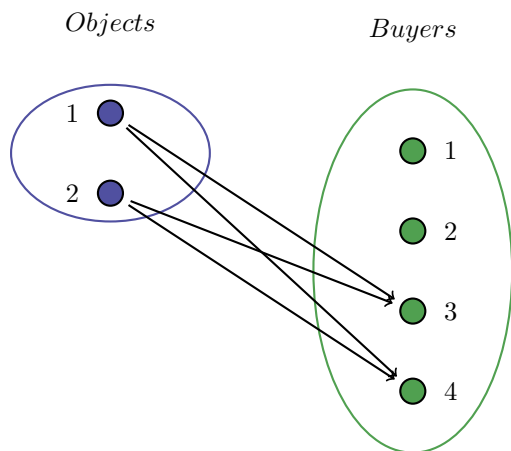


10.2.2 Auction Steps

In the first step of the auction, both sellers have a price of 0 and all buyers are willing to purchase the items. In the second step of the auction, both sellers have a price of 1 and buyers 2, 3 and 4 are willing to purchase the items. In the third step of the auction, both sellers have a price of 2 and buyers 3 and 4 are both willing to purchase both items. There are no further steps of the auction as both buyers then have purchased items and both items have been purchases.

10.2.3 Auction Ends

At the end of the auction we would have the following bipartite graph, in which Buyer 4 would buy the first copy of the object for the valuation of Buyer 2 (2). Buyer 3 would get the second copy of the object for the valuation of Buyer 2 (2).



10.3 Answer - b

In this auction, the top and second highest bidder both buy for the valuation of the third highest bidder.