Artificial Intelligence I

Lab 4 - Winter Semester 2023 / 2024
https://moodle.haw-landshut.de/course/view.php?id=10282

1. Knowledge Representation.

Three friends work in a company: a C++ programmer, a Java programmer and a Python programmer. Their names are Emil, Paul and Felix. The C++ programmer has no siblings (A); he is the youngest of the friends (B). Felix, who is married to Emil's sister (C), is older than the Java programmer (D).

Knowledge often exists in verbal form and must be transferred into a formal representation so that a knowledge link can take place (think about how this can be achieved here). For the solution, general knowledge has to be considered and modeled in addition to the existing specialized knowledge. The general knowledge is not fomulated, but assumed by the reader. *What belongs to general knowledge in the example given above?*

Who programs in which language? Use as short notation the equal sign = as "programs in the language". Thus the relation $X = Y$ says that the person X programs in the language Y. We understand the $\neq$ sign analogously. Now try to deduce who programs in which language with the help of the statements from the text (A)-(D) and the signs = and $\neq$.

2. Consider a vocabulary with only four propositions, A, B, C and D. How many models are there for the following sentences?

(a) $B \vee C$.

(b) $\neg A \vee \neg B \vee \neg C \vee \neg D$.

(c) $(A \Rightarrow B) \wedge A \wedge \neg B \wedge C \wedge D$.

3. Use resolution to prove the sentence $\neg A \wedge \neg B$ from the clauses

S1: $(\neg A \vee B \vee E) \wedge (\neg B \vee A) \wedge (\neg E \vee A)$.

S2: $E \Rightarrow D$.

S3: $B \wedge F \Rightarrow \neg C$.

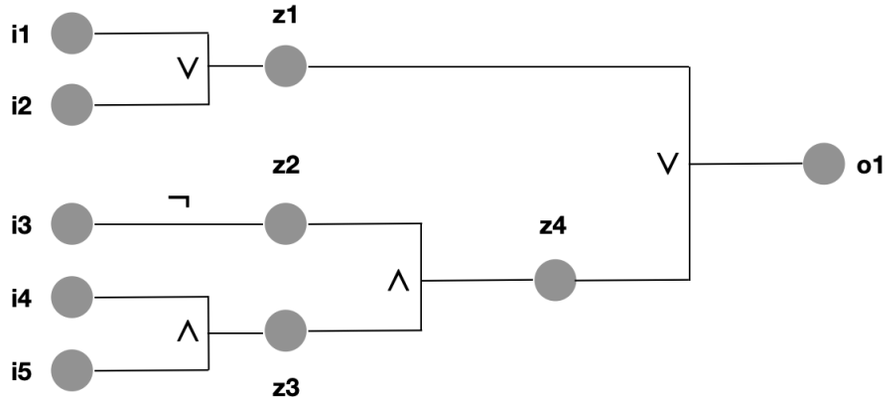
S4: $E \Rightarrow B$.

S5: $B \Rightarrow F$.

S6: $B \Rightarrow C$.

Convert the clauses to causal form first.

4. Let the following electronic circuit be given



Signals (0 for false and 1 for true) can be applied to the five input points. These are converted into an output signal by the links And, Or and Negation. The circuit can be described by propositional logic formulas.

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5 · i + 4 · z + 1 · o = 10 propositions = 2^{10} = 1024

(1) $i1 \vee i2 \Leftrightarrow z1$

(2) $i3 \Leftrightarrow \neg z2$

(3) $i4 \wedge i5 \Leftrightarrow z3$

(4) $z2 \wedge z3 \Leftrightarrow z4$

(5) $z1 \vee z4 \Leftrightarrow o1$

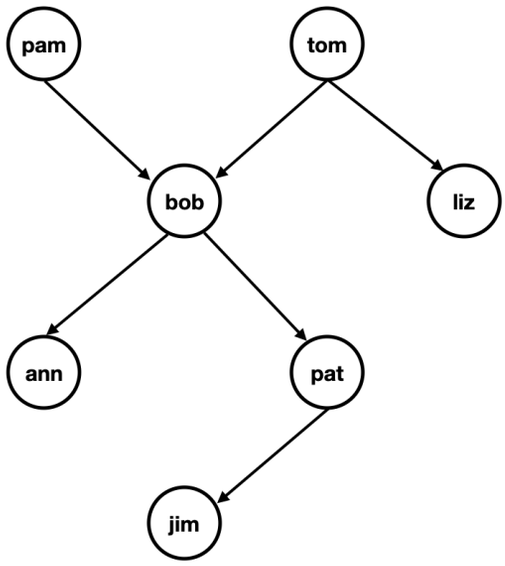
We claim that the output signal is always 1 when the input signal $i1 = 1$:

$$i1 \Rightarrow o1.$$

1. How many possible combinations would have to be considered if you wanted to prove this statement using a truth table?
2. How many combinations are necessary considering that the values of the intermediate nodes can be calculated from the input signals? all i nodes = 5 ⇒ 2^5 = 32
3. Transform statements (1)-(5) into CNF.
4. Use resolution to prove $i1 \Rightarrow o1$.

5. Knowledge Representation in Prolog.

Read A.2.1 - A.2.7 in P.M. Nugues, Language Processing with Perl and Prolog, then visit https://swish.swi-prolog.org/ and create a notebook based on an empty profile. Try to represent the following family relation



in Prolog by choosing `parent` as the name of the relation and e.g. stating the fact that Tom is a parent of Bob as: `parent(tom, bob)`.

(a) Execute the following queries

- `?- parent(bob, pat)`
- `?- parent(liz, pat)`
- `?- parent(tom, ben)`
- `?- parent(X, liz)`
- `?- parent(bob, X)`
- `?- parent(X, Y)`
- `?- parent(Y, jim), parent(X, Y)`

(b) Now, ask the following questions in Prolog:

- Who is Pat's parent?
- Does Liz have a child?
- Who is Pat's grandparent?

  - Do Ann and Pat have a common parent?

(c) Extend your current program (KB) by adding the information on the sex of the people that occur in the `parent` relation, e.g. `female(pam)`, `male(tom)`. In addition, introduce the `offspring` relation as the inverse of the `parent` relation. Now, translate the following logical statements into Prolog rules

  - `mother`: For all X and Y, X is the mother of Y if X is a parent of Y and X is a female.
  - `sister`: For any X and Y, X is a sister of Y if both X and Y have the same parent, and X is a female.

(d) Define the relation `grandchild` using the `parent` relation.

(e) Define the relation `aunt` in terms of the relations `parent` and `sister`.

1,

(A) C++ has no siblings
(B) C++ is the youngest
(C) Emil has a sister
(D) Felix is older than Java

(B,D) => (W1) Python is the oldest
(A,C) => (W2) Emil != C++
(B,W1) => (W3) Javas age is between Python and C++
(D,W1,W3) => (W4) Felix = Python
(W2,W4) => (W5) Emil = Java
(W4,W5) => (W6) Paul = C++

In this case general knowledge could be meanings of words like 'siblings', 'youngest', 'married' or 'sister' and what they bring with them in terms of information.

2 a) $B \vee C \rightarrow 12$
b) $\neg A \vee \neg B \vee \neg C \vee \neg D \rightarrow 15$
c) $(A \Rightarrow B) \wedge A \wedge \neg B \wedge C \wedge D \rightarrow 0$

a) $B \vee C \rightarrow$ 3x True

B	C	B ∨ C
0	0	F
0	1	T
1	0	T
1	1	T

2

3

$3 \cdot 2^2 = 12$

$4 - 2 = 2$

A B C D

4

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3) S1 – $\neg A \vee B \vee E$
S2 – $\neg B \vee A$
S3 – $\neg E \vee A$
S4 – $\neg E \vee D$
S5 – $\neg B \vee \neg F \vee \neg C$
S6 – $\neg E \vee B$
S7 – $\neg B \vee F$
S8 – $\neg B \vee C$
S9 – $A \vee B$

} KB

$KB \wedge \neg \alpha$

$\alpha = \neg A \wedge \neg B$

$\neg \alpha = \neg (\neg A \wedge \neg B)$

$= A \vee B$

$KB \wedge A \vee B$

gesucht: $\neg A \vee \neg B \wedge A \vee B$

$\neg B \vee A \wedge A \vee B \Rightarrow A \vee A \Rightarrow A$

$\neg A \vee B \vee E \wedge \neg E \vee B \Rightarrow \neg A \vee B$

$\neg B \vee \neg F \vee \neg C \wedge \neg B \vee C \Rightarrow \neg B \vee \neg F \vee \neg B \Rightarrow \neg B \vee \neg F$

$\neg B \vee \neg F \wedge \neg B \vee F \Rightarrow \neg B$

$\neg A \vee B \wedge \neg B \Rightarrow \neg A$

$\neg A \vee A \Rightarrow \lightning \Rightarrow$ Widerspruch!

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4)

$i1 \Rightarrow o1$ is to be proven.

$i1 \vee i2 \Leftrightarrow z1$

$((i1 \vee i2) \Rightarrow z1) \wedge (z1 \Rightarrow (i1 \vee i2))$

$(\neg(i1 \vee i2) \vee z1) \wedge (\neg z1 \vee (i1 \vee i2))$ (1)

$((i1 \wedge \neg i2) \vee z1) \wedge (\neg z1 \vee i1 \vee i2)$

$(z1 \vee \neg i1) \wedge (z1 \vee \neg i2) \wedge (\neg z1 \vee i1 \vee i2)$

a b c

$i3 \Leftrightarrow \neg z2$

$(i3 \Rightarrow \neg z2) \wedge (\neg z2 \Rightarrow i3)$ (2)

$(\neg i3 \vee \neg z2) \wedge (z2 \vee i3)$

d e

$i4 \wedge i5 \Leftrightarrow z3$

$((i4 \wedge i5) \Rightarrow z3) \wedge (z3 \Rightarrow (i4 \wedge i5))$

$(\neg(i4 \wedge i5) \vee z3) \wedge (\neg z3 \vee (i4 \wedge i5))$ (3)

$(\neg i4 \vee \neg i5 \vee z3) \wedge (\neg z3 \vee i4) \wedge (\neg z3 \vee i5)$

f g h

$z2 \wedge z3 \Leftrightarrow z4$

$(\neg z2 \vee \neg z3 \vee z4) \wedge (\neg z4 \vee z2) \wedge (\neg z4 \vee z3)$ (4)

i i k

$z1 \vee z4 \Leftrightarrow o1$

$(\neg z1 \vee o1) \wedge (\neg z4 \vee o1) \wedge (\neg o1 \vee z1 \vee z4)$ (5)

l m n

$i1 \Rightarrow o1$

$\neg i1 \vee o1$

$\neg \alpha = \neg(\neg i1 \vee o1)$

$= i1 \wedge \neg o1$

x z

$q + x = (z1 \vee \neg i1) \wedge (i1) = z1$ o

$l + z = (\neg z1 \vee o1) \wedge (\neg o1) = \neg z1$ p

$o + p = z1 \vee \neg z1 = 1$ = widerspruch

5,

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parent(pam,bob).
parent(tom,bob).
parent(tom,liz).
parent(bob,ann).
parent(bob,pat).
parent(pat,jim).

% a)
?- parent(bob,pat).
?- parent(liz,pat).
?- parent(tom, ben).
?- parent(X,liz).
?- parent(bob, X).
?- parent(X,Y).
?- parent(Y, jim), parent(X,Y).


% b)
?- parent(X,pat).
?- parent(liz, X).
?- parent(Y, _X), parent(_X, pat).
?- parent(X, ann), parent(X, pat).

% c)
male(bob).
male(tom).
male(jim).

female(pam).
female(liz).
female(ann).
female(pat).

offspring(X,Y) :- parent(Y,X).

mother(X,Y) :- parent(X,Y), female(X).
sister(X,Y) :- parent(Z,X), parent(Z,Y), female(Y), not(X = Y).

grandchild(X,Y) :- parent(Y, A), parent(A, X).

aunt(X,Y) :- sister(X,A), parent(A, Y).
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