Let $\{\hat{r}_h\}$ be pred. profit at moment K_h, w_h – weight of coin at K_h, z_h - trades at K_h, then

total profit $\underbrace{\hat{R} = \hat{r}_H w_H - \sum_{h=1}^{H-1} \hat{r}_h z_h}$, but that is a random variable, not a number.

We want to maximize profit, but include risk in our model.

Instead of risk penalty for mean profit here was used max profit from $\varepsilon\%$ worst cases.

So that problem may be formulated as:

$$
\begin{cases}
\bar{R} - \sum_{h=0}^{H-1} \Psi(z_h) - \sum_{h=1}^{H} \varphi(w_h) \to \max \\
\mathbb{P}[R \geq \hat{R}] \leq \varepsilon, \ R \in \mathbb{R} \\
z_h = w_{h+1} - w_h, \ \forall h \in 0..H-1 \\
w_h \in \mathcal{W}, \ z_h \in \mathcal{Z}, \ \forall h \in 0...H
\end{cases}
\implies
\begin{cases}
R - \sum_{h=0}^{H-1} \Psi(z_h) - \sum_{h=1}^{H} \varphi(w_h) \to \max \\
R + \beta_\varepsilon(w, z) \leq \bar{R}, \ R \in \mathbb{R} \\
z_h = w_{h+1} - w_h, \ \forall h \in 0..H-1 \\
w_h \in \mathcal{W}, \ z_h \in \mathcal{Z}, \ \forall h \in 0...H
\end{cases}
$$

Case 1: $\hat{r} = N(\bar{r}, \Sigma)$ we get $\beta(z, w) = \Phi^{-1}(\varepsilon) \cdot \sqrt{\mathbb{E}(\hat{R} - \bar{R})^2}$

It can be simplified into $\Phi^{-1}(\varepsilon) \cdot \sqrt{\sum x_i^2}$, but it still non-linear.

Case 2: $\hat{r}_h = \bar{r}_h + \sigma_h \hat{\eta}_h$, $\hat{\eta}_h \in [-1, 1]$ and $\varepsilon = 0$. In such case $\beta = \sigma_H |w_H| + \sum_{h=1}^{H-1} \sigma_h |z_h|$

Obviosly it has simple linearisation, but such strategy are too risk-avoiding.

Case 3: $\hat{r}_h = \bar{r}_h + \sigma_h \hat{\eta}_h$, $\hat{\eta}_h \in [-1, 1]$ and independent, $\varepsilon \in [0, 1]$. Then

$$
\begin{cases}
\sigma_H |w_H| \cdot y_H + \sum_{h=1}^{H-1} \sigma_h |z_h| y_h \to \max_y = \beta(z, w) \\
\sum_{h=1}^{H} y_h \leq \Gamma \\
y_h \in [0, 1], \ \forall h \in 1...H
\end{cases}
$$

Complicated, but it can be linearized.

Γ also complicated, but may be used as hyp. param, instead of ε.

$$\begin{cases} R - \sum_{h=0}^{H-1} \Psi(z_h) - \sum_{h=1}^{H} \varphi(w_h) \to \max \\ R + y\Gamma + \sum_{h=1}^{H} p_h \leqslant \bar{R}, \; R \in \mathbb{R} \\ y \geq 0, \; p_h \geq 0, \; \forall h \in 1 \dots H \\ y + p_H \geqslant \sigma_H |w_H|, \; y + p_h \geq \sigma_h |z_h| \; \forall h \in 1 \dots H-1 \\ z_h = w_{h+1} - w_h, \; \forall h \in 0 .. H-1 \\ w_h \in \mathcal{W}, \; z_h \in \mathcal{Z}, \; \forall h \in 0 \dots H \end{cases}$$

Geom. Walks approach. Let $\hat{S}(t)$ be a price of coin, and let $d\hat{S}(t) = \hat{S}(\mu dt + \sigma d\hat{w})$.

Then $\hat{S}(t) = S(0)\exp\left[(\mu - \frac{1}{2}\sigma^2)t + \sigma\hat{w}\right]$, and $\hat{r}_h = \frac{\hat{S}(k_h)}{S(0)} - 1$, and $\bar{r}_h$ is one of realisation for process.

We will estimate μ and σ based on such assumption.

Let $\hat{a}(t) = \frac{1}{t}\ln(1 + \hat{r}_t) = (\mu - \frac{1}{2}\sigma^2) + \sigma\frac{\hat{w}}{2} \Rightarrow (\mu - \frac{1}{2}\sigma^2) \approx \frac{1}{H}\sum_{h=1}^{H}\frac{1}{K_h}\ln(1 + \bar{r}_h)$

In same way we able to estimate σ: let $\hat{b}(t) = \ln(1 + \hat{r}_t) - (\mu - \frac{1}{2}\sigma^2)t = \sigma\hat{W}$.

Then $\sigma^2 = \frac{1}{H}\sum_{h=0}^{H-1}\frac{(\bar{b}(k_{h+1}) - \bar{b}(k_h))^2}{k_{h+1} - k_h}$. When we got μ and σ we able to make

classical 1 step portfolio optimisation:

$$\mu w - \gamma\sigma^2 w^2 - s|w - w_0| \to \max_{w \in [0,1]} \Rightarrow w = \operatorname{argmax}\left\{\mu w - \gamma\sigma^2 w^2 - s|w - w_0| \;\middle|\; \left\{0, w_0, 1, \frac{\mu \pm s}{2\gamma\sigma^2}\right\} \cap [0,1]\right\}$$