中山大学软件学院 2013 级软件工程专业(2013 学年秋季学期)

《SE-103 线性代数》期末试题(A 卷)

(考试形式：开/闭 卷　考试时间：2 小时)

警示　《中山大学授予学士学位工作细则》第六条

考试作弊不授予学士学位

方向：______________　姓名：__________　学号：____________

1. Fill in the blanks（6×4=24 Pts）

(1) Let $A=\begin{bmatrix}1&0&1\\0&2&0\\-2&0&1\end{bmatrix}$ and $A^2B-A-B=I$, where A and B are 3×3 matrices, then $|B|=$________.

(2) Given a subspace $H=\left\{\begin{bmatrix}a-3b+6c\\5a\\b-2c-d\\0\end{bmatrix}: a,b,c,d \text{ in } R\right\}$, a basis is ________________, and the dimension of H is ______________.

(3) Let $A=\begin{bmatrix}0&1\\-8&4\end{bmatrix}$ act on C^2. Then an eigenvalues of A is $\lambda=$______________. And a basis for the eigenspace corresponding to λ is ________________________.

(4) Let $y=\begin{bmatrix}5\\-9\\5\end{bmatrix}$ and $W=\text{span}\{u_1,u_2\}$, where $u_1=\begin{bmatrix}-3\\-5\\1\end{bmatrix}$ and $u_2=\begin{bmatrix}-3\\2\\1\end{bmatrix}$, then

$\text{proj}_W y$ = ______________, and the distance from y to W is ____________.

(5) Let $A=\begin{bmatrix}1 & 5\\ 3 & 1\\ -2 & 4\end{bmatrix}$ and $b=\begin{bmatrix}4\\ -2\\ -3\end{bmatrix}$, then a least-squares solution of $Ax=b$ is ____________, and the associated least-squares error is ____________________.

(6) Let A be the matrix of the quadratic form $(x_1+x_2)^2+(x_2-x_3)^2+(x_3+x_1)^2$, then $A=$ __________________________, and rank A is ________________.

2. Make each statement True or False, and descript your reasons. (6×3 = 18 Pts)

(1) If A is a positive definite symmetric $n\times n$ matrix, then A^{-1} is also positive definite.

(2) Suppose a 3×5 matrix A has dim Row A =3. Then the equation $Ax=b$ always has a unique solution.

(3) If V is a vector space having dimension n, and if S is a subset of V with n vectors, then S is linearly independent if and only if S spans V.

(4) Two distinct eigenvectors corresponding to the same eigenvalue are always linearly dependent.

(5) If A is produced by multiplying row 2 of B by 3, then det A =3det B .

(6) If a matrix U has orthonormal columns, then $UU^T=I$, where I is the $n\times n$ identity matrix.

3. Calculation (5 ×8 = 40 Pts)

(1) Let $A=\begin{bmatrix}1 & 0\\ -1 & 2\\ 0 & 1\end{bmatrix}$, $B=A^TA$

a. Find the eigenvalues and the corresponding eigenvectors of B.

b. Computer B^k, where k represents an arbitrary positive integer.

(2) Find a QR factorization of $A=\begin{bmatrix}5 & 9\\ 1 & 7\\ -3 & -5\\ 1 & 5\end{bmatrix}$.

(3) If $B=\begin{bmatrix}2 & 3 & 0\\ 3 & 1 & 1\\ 0 & 1 & 0\end{bmatrix}$, and if A and B are similar.

a. Find $|A-2I|$.

b. Let $\vec{x}=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$. Computer x^TBx for the matrix B.

(4) Suppose $A=\begin{bmatrix}1 & 0 & -2 & 1\\ 0 & 1 & a & 0\\ 2 & 1 & 0 & a\\ 0 & 0 & 0 & a-1\end{bmatrix}$. If the columns of the matrix A are linearly dependent and $a\neq 1$.

a. Find a.

b. Find bases for the null space , the column space, and the row space of the matrix A.

(5) If T is the linear transformation from P_2 to P_2.

a. Suppose the set $B=\{1+t, 1+t^2, t+t^2\}$ is a basis for P_2. Find the coordinate vector of $p(t)=1+3t-2t^2$ relative to B.

b. IF $T(a_0+a_1t+a_2t^2)=2a_1+4a_2t^2$, find the C-matrix for T, when C is the basis $\{1, t, t^2\}$.

4. Prove issue（$2\times9=18$ Pts）

(1) Suppose $A=I-3uu^T$, where u is a unit vector in R^n and I is the $n\times n$ identity matrix.

a. Let u be an eigenvector of A, find the corresponding eigenvalue.

b. If v is any vector orthogonal to u, show that v is an eigenvector of A and find the eigenvalue.

(2) Suppose A and B are both $n\times n$ symmetric matrices. Show that AB is an symmetric matrix if and only if $AB=BA$.