Dynamics and Control of Robot-Environment Interaction
(HW 06 Fall 2021)

Due to December 9th, 2021

Problem 1 (Robot Experiment) Closed-loop inverse kinematics(CLIK) (30)

✓ Using Franka Panda that have 7dof, implement the following CLIK algorithm.

✓ Choose proper gain.

✓ Desired position and orientation, x_d, is expressed in base frame.

✓ Using Direction Cosines to represent the orientation.

✓ **Specify position and orientation trajectory on your results.**

$$x_d = [x_p^T \quad x_r^T]^T, \quad x_r = [s_1^T \, s_2^T \, s_3^T]^T$$

1. Design and Implement a controller using CLIK.

$$\dot{q}_d(t) = J^{\dagger}(\dot{x}_d + k_p(x_d - x(q_d(t)))), \quad J^{\dagger} = J(q_d(t))^T(J(q_d(t))J(q_d(t))^T)^{-1} \tag{1}$$

$$q_d(t + \delta t) = q_d(t) + \delta t \dot{q}_d(t) \tag{2}$$

where

$$k_p = diag(k_{p1}, k_{p2}, k_{p3}, k_{p4}, k_{p5}, k_{p6}) \tag{3}$$

- Move the manipulator from initial position to $q_d = [0° \, 0° \, 0° \, -90° \, 0° \, 90° \, 0°]^T$ in joint space. And then, Plot a result of trajectory tracking of the end effector to $x_p = [0.25, 0.28, 0.65]^T$, $s_1 = [0 \, -1 \, 0]^T$, $s_2 = [-1 \, 0 \, 0]^T$, $s_3 = [0 \, 0 \, -1]^T$. **Do not use step command! Use cubic spline trajectory. Here, every J, q, and x are calculated using desired values, not current values. Use gain lower than 1.5*Hz.**

Problem 2 (Robot Experiment) Torque control using a real robot in Joint Space and Task Space (30)

✓ Implement the following controllers in both Joint Space and Task space.

✓ For each problem choose proper gain.

1. Design and Implement a Joint Space PD controller with dynamic compensation. Design the controller to have $w_n = 20$ rad/sec and critically damped. **Design trajectories to start from initial position,** $q_{init} = [0° \, 0° \, 0° \, -30° \, 0° \, 90° \, 0°]^T$**.**

$$\tau = M\{k_p(q_d - q) + k_v(\dot{q}_d - \dot{q})\} + G \tag{4}$$

- Plot a result of trajectory tracking of the Joint 4, starting from $-30°$ to $-60°$ using cubic spline.

2. Design and Implement a Task Space PD controller with dynamic compensation. Design the controller to have $w_n = 20$ rad/sec and critically damped.

$$\tau = J^T \Lambda F_0^\star + [I - J^T \overline{J}^T]\tau_0 + G, \quad F_0^\star = [F^{\star T}\, M^{\star T}]^T \tag{5}$$

$$F^\star = k_p(x_d - x) + k_v(\dot{x}_d - \dot{x}) \tag{6}$$

$$M^\star = -k_p \delta\Phi - k_v \omega \tag{7}$$

$$\tau_0 = A\{k_p(q_{init} - q) - k_v \dot{q}\} \tag{8}$$

- Move the robot to the configuration of $q_{init} = [0\,0\,0\ -90°\,0\,90°\,0]^T$ using the Joint Space Controller (the PD controller with dynamic compensation). Command the end-effector to move $10cm$ in the y_0 direction using a cubic spline trajectory. Simultaneously, maintain the initial orientaion of end-effector. Plot the response of the end-effector.

3. Design and Implement Velocity saturation controller in Task Space.

$$\tau = J^T \Lambda F_0^\star + [I - J^T \overline{J}^T]\tau_0 + G, \quad F_0^\star = [F^{\star T}\, M^{\star T}]^T \tag{9}$$

$$F^\star = k_v(\dot{x}_d - \dot{x}). \quad \dot{x}_d = \begin{cases} \frac{k_p}{k_v}(x_d - x) & |\frac{k_p}{k_v}(x_d - x)| < |\dot{x}_{max}| \\ \frac{|\dot{x}_{max}|}{|x_d - x|}(x_d - x) & |\frac{k_p}{k_v}(x_d - x)| \geq |\dot{x}_{max}| \end{cases} \tag{10}$$

$$M^\star = -k_p \delta\Phi - k_v \omega \tag{11}$$

$$\tau_0 = A\{k_p(q_{init} - q) - k_v \dot{q}\} \tag{12}$$

- Move the robot to the configuration of $q_{init} = [0\ -60°\,0\ -90°\,0\,30°\,0]^T$ using the Joint Space Controller (the PD controller with dynamic compensation). Command the end-effector to move to $x_d = [0.3\ -0.012\,0.52]^T$ maintaining the initial orientaion of end-effector. Use $\dot{x}_{max} = 0.3$, and choose proper gains. Plot the response of the end-effector.