```
smoothmatrix_SS<-function(x,lam){
sx<-sort(x)
n<-length(x)
D<-cbind(rep(1,n), sx)

for (i in 1:(n-2)){

temp<-(sx-sx[i])^3*((sx-sx[i])>0)-(sx-sx[n])^3*((sx-sx[n])>0)
temp<-temp/(sx[n]-sx[i]) #d_i
temp1<-(sx-sx[n-1])^3*((sx-sx[n-1])>0)-(sx-sx[n])^3*((sx-sx[n])>0)
temp1<-temp1/(sx[n]-sx[n-1]) #d(n-1)

D<-cbind(D,temp-temp1)
}

z<-seq(0,1,0.01)
J<-length(z)

B<-cbind(rep(1,J), z)

for (i in 1:(n-2)){

temp<-(z-sx[i])^3*((z-sx[i])>0)-(z-sx[n])^3*((z-sx[n])>0)
temp<-temp/(sx[n]-sx[i]) #d_i (z)

temp1<-(z-sx[n-1])^3*((z-sx[n-1])>0)-(z-sx[n])^3*((z-sx[n])>0)
temp1<-temp1/(sx[n]-sx[n-1]) #d(n-1)(z)

B<-cbind(B,temp-temp1)
}

#as a result, we get B:J x no. of basis functions containing Bk(zi) as (i,k) entry

#diff
Bd1<-matrix(NA, J-2, n)
for (j in 2:(J-1)){
Bd1[j-1,]<-(B[j+1,]-B[j-1,])/(z[j+1]-z[j-1]) #the first derivative of basis functions
}

#diff
Bd2<-matrix(NA, J-4, n)
for (j in 2:(J-3)){
Bd2[j-1,]<-(Bd1[j+1,]-Bd1[j-1,])/(z[j+1]-z[j-1]) #the second derivative of basis functions
}

W<-(t(Bd2)%*%Bd2)/(J-4 )



S<-D%*% solve(t(D)%*% D+lam*W) %*% t(D)
return(S)
}
```

```
n<-5
x<-sort(runif(n))
lam=1

S<-smoothmatrix_SS(x,lam)

xminus1<-x[-1]
Sminus1<-smoothmatrix_SS(xminus1,lam)

for (k in 1:(n-1)){
Sminus1[k,]
S[k+1,-1]/sum(S[k+1,-1])
print(max(abs(Sminus1[k,]-S[k+1,-1]/sum(S[k+1,-1]))))
}
```